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Multi-objective optimization of series-parallel system with mixed subsystems failure dependencies using NSGA-II and MOHH

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Abstract

In complex systems, failure dependencies play a crucial role in determining their overall performance. This paper explores the multi-objective optimization of series-parallel systems with mixed failure dependencies. By optimizing system cost and availability, the study aims to identify the most efficient redundancy and repair strategies. Two optimization algorithms, the non-dominated sorting genetic algorithm II (NSGA-II) and a novel multi-objective algorithm named the multi-objective hoopoe heuristic (MOHH), are utilized alongside constraint handling techniques to produce Pareto fronts. These fronts illustrate the trade-offs between cost and availability. Additionally, a fuzzy decision method is utilized to determine the best compromise solutions from each optimization technique. Comparing the results, NSGA-II consistently outperforms MOHH in providing better compromise solutions across five independent runs. However, MOHH demonstrates a better standard deviation in its performance.

Keywords

Multi-objective system availability and cost, mixed failure dependencies, non-dominated sorting genetic algorithm II (NSGA-II), multi-objective hoopoe heuristic (MOHH), fuzzy decision method, best compromise solution

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Introduction

Failures are an unavoidable aspect of industrial systems, where the interdependence of subsystems can significantly impact overall system performance and availability.[1–5] In various industrial sectors, from manufacturing to transportation, energy production to healthcare, the failure of even a single component can lead to cascading effects, causing widespread disruption and costly downtime.[6,7] Understanding the intricate relationships and dependencies among subsystems within these complex systems is paramount for mitigating risks and ensuring robust operational resilience.[8,9]

Failure dependency of subsystems in systems refers to the phenomenon wherein the failure of one subsystem can propagate and trigger failures in interconnected subsystems, ultimately compromising the functionality of the entire system. This concept underscores the importance of not only identifying and addressing individual component failures but also comprehensively assessing the potential ripple effects across the system as a whole.

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In industrial contexts, failure dependency manifests in multifaceted ways. For instance, in manufacturing processes, the breakdown of a critical machine may halt production lines, affecting downstream operations and supply chains. Similarly, in transportation systems, a malfunction in signaling equipment can lead to delays, impacting schedules and passenger services. Moreover, in critical infrastructure such as power grids or healthcare facilities, failures in essential subsystems can have far-reaching consequences, jeopardizing public safety and service delivery.

Addressing failure dependency requires a holistic approach that considers the complex interactions and dependencies among subsystems. Advanced analytical techniques, such as fault tree analysis, reliability and availability modeling, and system simulation, play a crucial role in assessing the vulnerability of interconnected systems and identifying potential failure scenarios. By systematically evaluating the interdependencies and vulnerabilities, stakeholders can implement targeted mitigation strategies, redundancy measures, repairs, and contingency plans to enhance system resilience and minimize the impact of failures.

In Das et al.,[10] failures in multi-layer complex networks (MLCNs), focusing on a specific type resembling computer networks, were explored. Through simulations on a three-layer MLCN, the impact of edge and node failures was investigated. In Xiang et al.,[11] a reliability analysis framework for failure-dependent manufacturing systems was presented, integrating copula functions, fuzzy inference, and Bayesian networks. It characterized failure correlations, deriving subsystem and system reliability. In Yu et al.,[12] the impact of failure dependencies in multi-component systems was addressed, emphasizing the role of redundancy in system reliability. The authors highlighted two aspects of system reliability reduction due to component failures: the loss of reliability contribution and system reconfiguration, including the redistribution of loading. The authors of Hu et al.[13] conducted an analysis of steady-state availability within a repairable series-parallel system featuring redundant dependencies. This analysis considers various component types and repairmen, taking into account the dynamic nature of component failure rates influenced by the presence of other failed components. By introducing a modified failure dependence function to quantify redundant dependency, the paper utilized Markov theory and matrix analysis to compute the steady-state probability vectors of subsystems and overall system availability. In Mohamed and Zulkernine,[14] an approach for assessing the reliability of fault-tolerant component-based software systems, taking into account component failure dependencies, was investigated. A machine learning-based approach[15] for enhancing failure dependency resilience in edge computing services was used in Aral and Brandic.[16] In Hu et al.,[17] the authors explored the design of a repairable series-parallel system with failure dependencies, emphasizing their impact on subsystem state transitions and system availability. They introduced a dependence function to quantify failure rates and utilized a Markov model to determine subsystem state distributions. An optimal allocation problem was proposed, aiming to minimize system cost while ensuring availability constraints, and genetic algorithms were employed for this purpose. An adaptive cuckoo optimization algorithm was developed in Mellal and Zio[18] to effectively deal with this complex problem. In Mellal et al.[19] and Mellal and Zio,[20] the flower pollination algorithm, plant propagation algorithm, particle swarm optimization, and cuckoo optimization algorithm were implemented to consider the system cost and availability using the weighted sum method. Linear, weak, and strong dependencies were considered, but a similar type of dependencies was assumed for the subsystems in the system. Later, in Mellal et al.,[21] the model was improved by considering mixed failure dependencies. However, the objective was solely to minimize the system cost under a system availability constraint. Differential evolution, manta ray foraging optimization, and shuffled frog leaping algorithm with constraint handling were implemented to minimize the system cost under three scenarios of system availability. A comprehensive literature review on dependent failures in systems was conducted in Zeng et al.[22]

The present work addresses both objectives: minimizing system cost and maximizing the system availability of the series-parallel system with mixed failure dependencies through Pareto fronts. The non-dominated sorting genetic algorithm II and the single-objective hoopoe heuristic, transformed into a multi-objective hoopoe heuristic, are implemented to solve the problem. Penalty functions are used to handle the nonlinear system availability constraint. The best compromise solutions are identified and compared using the fuzzy decision method. The remainder of the paper is organized as follows: "Multi-objective optimization of series-parallel system with mixed subsystems failure dependencies" section describes the multi-objective optimization of the series-parallel system with mixed failure dependencies. "Non-dominated sorting genetic algorithm II (NSGA-II)" and "Multi-objective hoopoe heuristic (MOHH)" sections present the principles of the implemented non-dominated sorting genetic algorithm II and the multi-objective hoopoe heuristic. "Best compromise solution based on fuzzy method" section illustrates the method for identifying the best compromise solutions. The results and discussion are provided in "Results and discussion" section. Finaly, the conclusions are presented in the last section.

Multi-objective optimization of series-parallel system with mixed subsystems failure dependencies

Based on the single-objective mixed subsystem failure dependencies model presented in Mellal et al.,[21] the multi-objective problem can written as follows:

$$\text{Minimize } C_s(n,r) = \sum_{i=1}^{10} \left(n_i C_i^c + r_i C_i^r\right) \tag{1}$$

$$\text{Maximize } A_s(n,r) = \prod_{i=1}^{10} A_{i,D\in\{L,W,S\}} \tag{2}$$

where C_s and A_s denote the system cost and system availability, respectively, depending on the redundancy vector n of n_i levels and on the repair teams vector r of r_i levels. The cost of a component at subsystem I is denoted by C_i^c and the cost of the repair teams at subsystem i is denoted by C_i^r. The expression of the subsystem availability A_i depends on the type of failure dependency D, where L stands for linear dependency, W for weak dependency, and S for strong dependency, as follows[17,18,21]:

$$A_{i,D=L} = 1 - \left[1 + \sum_{j=1}^{n_i-r_i} r_i^j \left(\frac{\mu_i}{\lambda_i}\right)^j + \sum_{j=n_i-r_i+1}^{n_i} \frac{r_i^{n_i-r_i} r_i!}{(n_i-j)!}\left(\frac{\mu_i}{\lambda_i}\right)^j\right]^{-1} \tag{3}$$

$$A_{i,D=W} = 1 - \left[1 + \sum_{j=1}^{n_i-r_i} r_i^j (j!)^{-0.5} \left(\frac{\mu_i}{\lambda_i}\right)^j + \sum_{j=n_i-r_i+1}^{n_i} \frac{r_i^{n_i-r_i} r_i!(j!)^{-0.5}}{(n_i-j)!}\left(\frac{\mu_i}{\lambda_i}\right)^j\right]^{-1} \tag{4}$$

$$A_{i,D=S} = 1 - \left[1 + \sum_{j=1}^{n_i-r_i} r_i^j (j!)^{0.5} \left(\frac{\mu_i}{\lambda_i}\right)^j + \sum_{j=n_i-r_i+1}^{n_i} \frac{r_i^{n_i-r_i} r_i!(j!)^{0.5}}{(n_i-j)!}\left(\frac{\mu_i}{\lambda_i}\right)^j\right]^{-1} \tag{5}$$

The failure rate of a component within subsystem i is denoted by λ_i, while its repair rate is represented by μ_i. The data of the system are represented in Table 1.

The problem is subject to,

$$0.90 \leqslant A_s(n,r) \leqslant 0.9999 \tag{6}$$

$$n_i,\ r_i \in \mathbb{Z}^+ \quad (i = 1,2,\ldots,10) \tag{7}$$

$$r_i \leqslant n_i \quad (i = 1,2,\ldots,10) \tag{8}$$

$$n_i \leqslant 10 \quad (i = 1,2,\ldots,10) \tag{9}$$

$$r_i \leqslant 10 \quad (i = 1,2,\ldots,10) \tag{10}$$

Table 1. Data of the system.

Subsystem i	D	λ_i	μ_i	C_i^c	C_i^r
1	L	0.07	0.25	30	20
2	L	0.04	0.12	55	35
3	W	0.02	0.27	40	25
4	L	0.03	0.10	75	45
5	W	0.08	0.15	60	30
6	S	0.05	0.26	80	60
7	L	0.01	0.18	70	30
8	S	0.06	0.30	45	25
9	L	0.09	0.14	50	30
10	W	0.05	0.28	25	20

Non-dominated sorting genetic algorithm II (NSGA-II)

The Non-Dominated Sorting Genetic Algorithm II (NSGA-II)[23] has gained significant recognition as a leading approach for addressing multi-objective optimization problems. It represents a notable advancement over its precursor, NSGA, by incorporating innovative strategies to explore and exploit solution spaces while maintaining diversity among generated solutions. NSGA-II has demonstrated superior performance compared to other techniques, such as the Pareto envelope based selection algorithm II (PESA-II) and the multi-objective particle swarm optimization (MOPSO).[24,25] Algorithm 1 illustrates the pseudo-code of the implemented NSGA-II.

Algorithm 1. Pseudo-code of the implemented NSGA-II.

```
Initialize population P with random solutions
Evaluate each solution in P for system cost and availability
Handle the constraints
Assign each solution in P to a front based on non-dominance
Sort individuals in each front based on crowding distance
while maximum number of generations is not reached do
  Create an empty offspring population Q
  while size of Q < size of P do
    Select parents from P using binary tournament selection
    Perform crossover and mutation to create offspring
    Evaluate each offspring for system cost and availability
    Handle the constraints
    Add offspring to Q
  end while
  Combine P and Q to create R
  Assign each individual in R to a front based on non-
  dominance
  Sort individuals in each front based on crowding distance
  Select individuals from R to form the next generation P
end while
Display the optimal results.
```

Multi-objective hoopoe heuristic (MOHH)

The single-objective hoopoe heuristic (HH) was initially introduced in El-Dosuky et al.[26] and later modified in Mellal and Williams.[27] It is has proven to be effective in solving various engineering problems. Inspired by the behavior of the hoopoe bird, a distinctive species found across Europe, Asia, and Africa, the heuristic mimics the foraging behavior of the bird in open woodlands, savannas, and grasslands, where it hunts for insects and small invertebrates. The single-objective HH is extended to address the presented multi-objective problem, as depicted in Algorithm 2.

Algorithm 2. Pseudo-code of the implemented MOHH.

```
Initialize a random population of solutions
while maximum number of generations is not reached do
    Perform ground-probing with a fixed value of digging radius
    Perform a Lévy flight
    Evaluate each solution for system cost and availability
    Handle the constraints
    Classify the solutions into Pareto fronts based on dominance
    Identify the best solutions in each Pareto front
    Select solutions for the next generation of hoopoes
end while
Display the optimal results.
```

Best compromise solution based on fuzzy method

The NSGA-II and MOHH are run over different runs and the best compromise solution of each run is identified through the following fuzzy method[24,28–31]:

$$\mu^k = \frac{\sum_{x=1}^{2} \mu_x^k}{\sum_{j=1}^{NS} \sum_{x=1}^{2} \mu_x^j} \tag{11}$$

with,

$$\mu_1 = \begin{cases} 1, & C_s \leqslant C_s^{\min} \\ \dfrac{C_s^{\max} - C_s}{C_s^{\max} - C_s^{\min}}, & C_s^{\min} < C_s < C_s^{\max} \\ 0, & C_s \geqslant C_s^{\max} \end{cases}$$

$$\mu_2 = \begin{cases} 0, & A_s \leqslant A_s^{\min} \\ \dfrac{A_s - A_s^{\min}}{A_s^{\max} - A_s^{\min}}, & A_s^{\min} < A_s < A_s^{\max} \\ 1, & A_s \geqslant A_s^{\max} \end{cases}$$

The parameters $C_s^{\max}$, $C_s^{\min}$, $A_s^{\max}$, and $A_s^{\min}$ represent the maximum and minimum values of system cost and system availability from the Pareto front, respectively. The best compromise solution is taken from the Pareto front of the technique with the largest normalized membership value denoted by μ^k. NS is the number of solutions in the Pareto front.

Results and discussion

The problem-solving algorithms NSGA-II and MOHH were implemented using MATLAB 2023a and executed on a PC with the following specifications: 12th generation Intel Core i7 processor running at 2.30 GHz, paired with 16 GB of RAM. Each algorithm utilized a population size and maximum number of generations set to 100 and run over five independent runs. The parameters were determined through trial-and-error experimentation and guided by prior experience. The best results are highlighted in bold type.

Tables 2 and 3 report the five Pareto fronts obtained by NSGA-II and MOHH, respectively. The normalized membership values of these Pareto fronts are reported in Tables 4 and 5, respectively. From Table 4, it can be observed that the best normalized membership values of the five runs of NSGA-II are 0.0114232903444178, 0.0119524684646373, 0.0122520665697145, 0.01152330 16383350, and 0.0122428003975620, whereas, as reported in Table 5, those of the MOHH are 0.0117459168952589, 0.0114796420578444, 0.0114968441366447, 0.011770377 0641603, and 0.0114684699765886. Therefore, as compared in Table 6 and shown in Figure 1, the best normalized membership value of NSGA-II was obtained in run #3 and is equal to 0.0122520665697145 with a standard deviation (SD) of 0.000349605318, whereas that of MOHH was obtained in run #4 and is equal to 0.0117703770641603 with an SD of 0.000135975797. NSGA-II provided better normalized membership value than the MOHH, but the SD of latter is smaller. The best compromise solution provided by NSGA-II is [C_s = 3675, A_s = 0.99976] with a redundancy vector of (5, 5, 5, 5, 7, 4, 3, 4, 7, 6) and (4, 4, 3, 3, 5, 2, 3, 3, 4, 3) of repair teams vector. The best compromise solution provided by MOHH is [2645, 0.98967] with a redundancy vector of (4, 4, 3, 4, 5, 3, 3, 3, 5, 3) and (2, 3, 2, 2, 4, 1, 1, 2, 3, 2) of repair teams vector. Figure 2 shows the box diagram of the normalized membership values of NSGA-II and the MOHH. It can be seen that the MOHH has a lower median value than the NSGA-II but a narrower spread of data points.

Conclusions

The aim of this paper was to address the multi-objective optimization of the series-parallel system with mixed failure dependencies. The system cost and system availability were optimized to find the optimal redundancy

Table 2. Pareto fronts obtained by NSGA-II.

No.	Run #1		Run #2		Run #3		Run #4		Run #5	
	C_s	A_s	C_s	A_s	C_s	A_s	C_s	A_s	C_s	A_s
1	2465	0.9535453790	2625	0.9663869072	3380	0.9981515096	3105	0.9934731051	3005	0.95931378072
2	2515	0.9784125507	2635	0.9833751619	3385	0.9983191453	3105	0.9934731051	3025	0.96302028695
3	2540	0.9826377541	2650	0.9833849821	3385	0.9983191453	3125	0.9936138148	3050	0.96302032854
4	2540	0.9826377541	2675	0.9836739760	3405	0.9988055641	3125	0.9936138148	3055	0.99198118806
5	2560	0.9837790522	2680	0.9858011072	3435	0.9988190648	3135	0.9971902240	3060	0.99239668619
6	2560	0.9837790522	2705	0.9860908112	3435	0.9988190648	3170	0.9978003148	3090	0.99251717618
7	2565	0.9846471144	2710	0.9887410745	3460	0.9988192325	3195	0.9982438722	3120	0.99574857964
8	2580	0.9849182424	2710	0.9887410745	3465	0.9990287399	3210	0.9982488333	3175	0.99666449683
9	2590	0.9854185487	2720	0.9890463279	3470	0.9992017686	3230	0.9983727942	3200	0.99666450079
10	2595	0.9855252883	2720	0.9890463279	3495	0.9992508871	3230	0.9983727942	3205	0.99687926062
11	2605	0.9882719289	2745	0.9893369855	3540	0.9993297725	3255	0.9984041893	3215	0.99727957913
12	2625	0.9894163217	2745	0.9893369855	3540	0.9993297725	3260	0.9985255555	3215	0.99727957913
13	2635	0.9897146616	2765	0.9924972418	3550	0.9994287265	3280	0.9985364370	3250	0.99730188773
14	2660	0.9900216564	2790	0.9927889136	3580	0.9996021500	3280	0.9985364370	3250	0.99730188773
15	2660	0.9900216564	2820	0.9951749409	3595	0.9996675509	3295	0.9988558137	3260	0.99779621784
16	2670	0.9903201789	2880	0.9958356213	3625	0.9996810632	3295	0.9988558137	3260	0.99779621784
17	2675	0.9916161815	2880	0.9958356213	3630	0.9996846233	3300	0.9988663204	3285	0.99828605981
18	2695	0.9919337779	2885	0.9963978527	3655	0.9997042959	3320	0.9988683389	3300	0.99872398094
19	2705	0.9921082583	2910	0.9964638199	3675	0.9997626969	3325	0.9990698117	3320	0.99878092753
20	2715	0.9923556621	2930	0.9967475015	3675	0.9997626969	3345	0.9990806991	3320	0.99878092753
21	2715	0.9923556621	2935	0.9972260359	3695	0.9997647171	3345	0.9990806991	3335	0.99885296501
22	2730	0.9927556604	2960	0.9972920579	3715	0.9997652480	3360	0.9991988405	3335	0.99885296501
23	2745	0.9928481058	2960	0.9972920579	3745	0.9997892243	3360	0.9991988405	3345	0.99895927388
24	2755	0.9939224571	2980	0.9975759753	3755	0.9997929848	3380	0.9992808642	3345	0.99895927388
25	2755	0.9939224571	2985	0.9975851387	3755	0.9997929848	3390	0.9993498748	3350	0.99898086355
26	2775	0.9944561680	3005	0.9976420204	3775	0.9998124595	3390	0.9993498748	3380	0.99908828834
27	2795	0.9946636575	3015	0.9977177312	3775	0.9998124595	3405	0.9994800596	3380	0.99908828834
28	2805	0.9949568133	3025	0.9977648852	3800	0.9998173750	3405	0.9994800596	3405	0.99909005575
29	2830	0.9951286509	3035	0.9980562932	3805	0.9998200771	3430	0.9994888978	3410	0.99913815419
30	2840	0.9957050164	3050	0.9980581051	3825	0.9998212860	3435	0.9994935694	3410	0.99913815419
31	2860	0.9959432959	3060	0.9982231184	3830	0.9998406109	3450	0.9994997898	3420	0.99924942893
32	2870	0.9961991221	3075	0.9983600237	3850	0.9998504081	3455	0.9995343013	3420	0.99924942893
33	2885	0.9962359803	3100	0.9984261207	3870	0.9998595188	3455	0.9995343013	3440	0.99933327726
34	2895	0.9965526238	3100	0.9984261207	3890	0.9998655580	3475	0.9995678020	3440	0.99933327726
35	2915	0.9966146050	3120	0.9984830503	3890	0.9998655580	3480	0.9995724739	3460	0.99934416753
36	2920	0.9968454874	3140	0.9985987822	3905	0.9998706102	3490	0.9995847335	3470	0.99934540766
37	2940	0.9971523459	3160	0.9987259370	3930	0.9998722494	3500	0.9996195065	3470	0.99934540766
38	2940	0.9971523459	3175	0.9987945776	3955	0.9998831561	3500	0.9996195065	3490	0.99943583009
39	2965	0.9972977895	3185	0.9988569986	3955	0.9998831561	3505	0.9996400472	3515	0.99943597724
40	2970	0.9973446688	3205	0.9989516948	3980	0.9998849249	3525	0.9996509408	3530	0.99951312545
41	2990	0.9976516810	3230	0.9990618126	3990	0.9998864663	3530	0.9996854575	3530	0.99951312545
42	3010	0.9976867735	3230	0.9990618126	3995	0.9998898475	3555	0.9996906172	3545	0.99955732188
43	3020	0.9978194004	3270	0.9991776397	4015	0.9998941932	3560	0.9996989700	3545	0.99955732188
44	3035	0.9980677697	3300	0.9992286823	4040	0.9998970484	3575	0.9997051917	3555	0.99959010344
45	3065	0.9981027096	3310	0.9992915728	4060	0.9998990689	3580	0.9997181661	3555	0.99959010344
46	3070	0.9981966690	3330	0.9993009029	4100	0.9998995171	3595	0.9997291636	3575	0.99965729367
47	3085	0.9983248181	3335	0.9993577315	4165	0.9998998913	3595	0.9997291636	3575	0.99965729367
48	3115	0.9984926507	3355	0.9993656122	4165	0.9998998913	3615	0.9997411305	3580	0.99966029179
49	3125	0.9985278412	3365	0.9994299615	4290	0.9998998924	3625	0.9997509788	3610	0.99967491687
50	3155	0.9986957078	3385	0.9994869483	4290	0.9998998924	3640	0.9997664167	3610	0.99967491687
51	3155	0.9986957078	3400	0.9994936126	4315	0.9998998959	3640	0.9997664167	3620	0.99967760531
52	3175	0.9987545441	3405	0.9995349389	4315	0.9998998959	3655	0.9997841411	3630	0.99968903653
53	3185	0.9988169626	3430	0.9995658524	4360	0.9998998960	3655	0.9997841411	3645	0.99969670523
54	3210	0.9988835321	3440	0.9995785558	4360	0.9998998960	3670	0.9998122689	3645	0.99969670523
55	3240	0.9989733212	3460	0.9996265508	4440	0.9998999029	3700	0.9998216716	3650	0.99969941939
56	3275	0.9991196578	3460	0.9996265508	4440	0.9998999029	3715	0.9998393970	3650	0.99969941939
57	3280	0.9991764761	3480	0.9996474767	4465	0.9998999151	3715	0.9998393970	3665	0.99975652442
58	3310	0.9992486929	3490	0.9996601811	4465	0.9998999151	3740	0.9998516865	3665	0.99975652442
59	3310	0.9992486929	3510	0.9996953443	4485	0.9998999284	3740	0.9998516865	3680	0.99976675965

(continued)

Table 2. (continued)

No.	Run #1		Run #2		Run #3		Run #4		Run #5	
	C_s	A_s	C_s	A_s	C_s	A_s	C_s	A_s	C_s	A_s
60	3355	0.9993275782	3510	0.9996953443	4485	0.9998999284	3765	0.9998581621	3695	0.99977975889
61	3355	0.9993275782	3530	0.9997193159	4505	0.9998999365	3785	0.9998672728	3700	0.99981652854
62	3390	0.9994985800	3530	0.9997193159	4525	0.9998999522	3790	0.9998702004	3725	0.99981829724
63	3415	0.9995199857	3555	0.9997352187	4545	0.9998999525	3810	0.9998709579	3755	0.99982614985
64	3450	0.9995466179	3565	0.9997438629	4550	0.9998999619	3820	0.9998789648	3760	0.99984284665
65	3470	0.9996197671	3585	0.9997529726	4570	0.9998999646	3830	0.9998818540	3780	0.99984414598
66	3495	0.9996209512	3600	0.9997611810	4570	0.9998999646	3845	0.9998836552	3790	0.99985039396
67	3500	0.9996643121	3610	0.9997738869	4590	0.9998999703	3845	0.9998836552	3810	0.99985391943
68	3535	0.9996648301	3625	0.9997991166	4590	0.9998999703	3860	0.9998894721	3815	0.99985508426
69	3555	0.9997293805	3645	0.9998132380	4610	0.9998999730	3880	0.9998900761	3835	0.99986063833
70	3595	0.9997526106	3645	0.9998132380	4620	0.9998999746	3890	0.9998982367	3855	0.99987414540
71	3635	0.9997987546	3665	0.9998223484	4650	0.9998999753	3910	0.9998988406	3855	0.99987414540
72	3660	0.9998174177	3670	0.9998291423	4675	0.9998999804	3925	0.9998998031	3860	0.99987753645
73	3690	0.9998472782	3685	0.9998392783	4700	0.9998999817	3950	0.9998999504	3880	0.99987883582
74	3715	0.9998511083	3710	0.9998606913	4725	0.9998999826	3950	0.9998999504	3880	0.99987883582
75	3735	0.9998652304	3710	0.9998606913	4745	0.9998999828	4310	0.9998999845	3900	0.99987979880
76	3790	0.9998866796	3730	0.9998698021	4750	0.9998999839	4310	0.9998999845	3900	0.99987979880
77	3815	0.9998955214	3730	0.9998698021	4790	0.9998999853	4335	0.9998999851	3910	0.99989063824
78	3880	0.9998999751	3750	0.9998753129	4810	0.9998999855	4335	0.9998999851	3970	0.99989078461
79	3880	0.9998999751	3755	0.9998786437	4840	0.9998999856	4355	0.9998999852	3980	0.99989710339
80	4190	0.9998999815	3780	0.9998831824	4855	0.9998999859	4355	0.9998999852	3980	0.99989710339
81	4190	0.9998999815	3780	0.9998831824	4885	0.9998999860	4380	0.9998999852	4040	0.99989716794
82	4235	0.9998999859	3800	0.9998922933	4910	0.9998999862	4380	0.9998999852	4040	0.99989716794
83	4235	0.9998999859	3800	0.9998922933	4930	0.9998999862	4400	0.9998999852	4085	0.99989716829
84	4270	0.9998999939	3835	0.9998972982	4955	0.9998999863	4405	0.9998999852	4100	0.99989869299
85	4295	0.9998999988	3840	0.9998995378	4955	0.9998999863	4450	0.9998999852	4100	0.99989869299
86	4320	0.9998999991	3865	0.9998998451	4985	0.9998999864	4450	0.9998999852	4160	0.99989966026
87	4365	0.9998999991	3865	0.9998998451	5025	0.9998999865	4845	0.9998999872	4255	0.99989985319
88	4390	0.9998999999	3920	0.9998999119	5055	0.9998999865	4865	0.9998999884	4255	0.99989985319
89	4390	0.9998999999	3920	0.9998999119	5075	0.9998999865	4875	0.9998999905	4265	0.99989993795
90	4435	0.9998999999	4060	0.9998999475	5095	0.9998999865	4900	0.9998999915	4265	0.99989993795
91	4465	0.9999000000	4060	0.9998999475	5130	0.9998999865	4925	0.9998999917	4280	0.99989994598
92	4525	0.9999000000	4120	0.9998999735	5130	0.9998999865	4935	0.9998999926	4280	0.99989994598
93	4550	0.9999000000	4120	0.9998999735	5155	0.9998999865	4950	0.9998999966	4290	0.99989998113
94	4580	0.9999000000	4145	0.9998999867	5155	0.9998999865	4970	0.9998999972	4325	0.99989998564
95	4610	0.9999000000	4160	0.9998999949	5215	0.9998999865	4975	0.9998999976	4330	0.99989999416
96	4665	0.9999000000	4185	0.9998999959	5225	0.9998999865	5000	0.9998999979	4350	0.99989999885
97	4680	0.9999000000	4185	0.9998999959	5255	0.9998999865	5030	0.9998999980	4355	0.99989999915
98	4725	0.9999000000	4210	0.9998999961	5255	0.9998999865	5040	0.9998999980	4375	0.99989999978
99	4765	0.9999000000	4225	0.9998999963	5300	0.9998999866	5070	0.9998999980	4400	0.99989999987
100	4765	0.9999000000	4225	0.9998999963	5300	0.9998999866	5105	0.9998999984	4400	0.99989999987

Table 3. Pareto fronts obtained by MOHH.

No.	Run #1		Run #2		Run #3		Run #4		Run #5	
	C_s	A_s	C_s	A_s	C_s	A_s	C_s	A_s	C_s	A_s
1	1995	0.90043555	1970	0.90007018	2015	0.92122122	1965	0.90142398	2005	0.90386969
2	1995	0.90043555	1970	0.90007018	2060	0.92627403	1980	0.90224925	2020	0.90556418
3	2030	0.90497817	1985	0.90115125	2090	0.93177471	1985	0.90444199	2030	0.90811724
4	2030	0.90497817	1995	0.90839523	2090	0.93177471	2045	0.90961523	2045	0.91737188
5	2030	0.90497817	1995	0.90839523	2115	0.93377474	2045	0.90961523	2045	0.91737188
6	2035	0.91060970	2025	0.91019841	2150	0.94374397	2045	0.90961523	2045	0.91737188
7	2055	0.91139334	2035	0.92392910	2150	0.94374397	2050	0.91677443	2045	0.91737188
8	2055	0.91139334	2100	0.92642635	2170	0.95077289	2050	0.91677443	2080	0.91940146
9	2085	0.91718717	2100	0.92642635	2170	0.95077289	2055	0.92426769	2080	0.91940146

(continued)

Table 3. (continued)

No.	Run #1		Run #2		Run #3		Run #4		Run #5	
	C_s	A_s	C_s	A_s	C_s	A_s	C_s	A_s	C_s	A_s
10	2105	0.92362085	2105	0.93154629	2210	0.95390280	2055	0.92426769	2080	0.91940146
11	2110	0.93272663	2130	0.93335462	2235	0.95397252	2075	0.92646332	2080	0.91940146
12	2145	0.93479018	2135	0.93707828	2235	0.95397252	2075	0.92646332	2115	0.93350720
13	2165	0.93758015	2150	0.93975390	2235	0.95397252	2080	0.92750233	2140	0.93677418
14	2170	0.94005242	2150	0.93975390	2235	0.95397252	2080	0.92750233	2160	0.94438705
15	2185	0.94747377	2180	0.94112581	2240	0.95529536	2105	0.92849500	2185	0.94769210
16	2220	0.95250683	2180	0.94112581	2265	0.95920343	2110	0.93301029	2185	0.94769210
17	2240	0.95616940	2205	0.94541988	2275	0.96161173	2135	0.94325059	2205	0.94898324
18	2270	0.95730128	2205	0.94541988	2290	0.96356810	2140	0.94364456	2205	0.94898324
19	2295	0.95950720	2235	0.95464888	2290	0.96356810	2140	0.94364456	2230	0.94951986
20	2300	0.96066539	2235	0.95464888	2295	0.96469126	2165	0.94603954	2230	0.94951986
21	2335	0.96543657	2265	0.95800826	2295	0.96469126	2190	0.94844077	2240	0.95143630
22	2345	0.96671931	2265	0.95800826	2310	0.96602622	2190	0.94844077	2260	0.95273254
23	2350	0.96713545	2265	0.95800826	2315	0.96698048	2215	0.94929796	2260	0.95273254
24	2350	0.96713545	2265	0.95800826	2315	0.96698048	2220	0.95177830	2280	0.95647287
25	2375	0.96869801	2285	0.95831509	2345	0.96839213	2220	0.95177830	2280	0.95647287
26	2390	0.97086399	2335	0.96037002	2345	0.96839213	2255	0.95279512	2310	0.95970907
27	2400	0.97446455	2340	0.96212328	2380	0.97116632	2260	0.95491152	2335	0.96348262
28	2435	0.97456713	2365	0.96871672	2395	0.97481530	2280	0.96072096	2335	0.96348262
29	2445	0.97609956	2365	0.96871672	2405	0.97590411	2280	0.96072096	2430	0.96952453
30	2480	0.97620231	2415	0.96978303	2435	0.97933828	2305	0.96315928	2430	0.96952453
31	2485	0.97931284	2415	0.96978303	2455	0.97967373	2345	0.96632996	2430	0.96952453
32	2515	0.98187194	2415	0.96978303	2480	0.98009600	2365	0.96768502	2435	0.97045161
33	2535	0.98306980	2415	0.96978303	2480	0.98009600	2370	0.97162313	2460	0.97192372
34	2600	0.98624035	2415	0.96978303	2510	0.98152680	2410	0.97482167	2505	0.97355446
35	2620	0.98628849	2470	0.97223986	2530	0.98385596	2470	0.97502599	2535	0.97367266
36	2665	0.98669984	2475	0.97328714	2530	0.98385596	2475	0.97959064	2540	0.97495579
37	2675	0.98768864	2475	0.97328714	2560	0.98731811	2510	0.97971102	2545	0.97717398
38	2690	0.98772604	2475	0.97328714	2560	0.98731811	2540	0.98177919	2550	0.97813023
39	2705	0.98817877	2500	0.97449172	2605	0.98764300	2540	0.98177919	2615	0.98149615
40	2745	0.99172357	2510	0.97544043	2605	0.98764300	2560	0.98178474	2630	0.98213614
41	2755	0.99216726	2535	0.97700946	2605	0.98764300	2560	0.98178474	2680	0.98426313
42	2755	0.99216726	2550	0.97857160	2630	0.98765871	2570	0.98299418	2710	0.98707837
43	2805	0.99299193	2585	0.97963034	2630	0.98765871	2585	0.98520728	2765	0.98808707
44	2805	0.99299193	2590	0.98234119	2650	0.98965514	2635	0.98622556	2775	0.98865556
45	2865	0.99409217	2590	0.98234119	2695	0.99081293	2645	0.98967953	2785	0.98931560
46	2895	0.99428340	2610	0.98259040	2725	0.99160159	2645	0.98967953	2835	0.98933031
47	2910	0.99433043	2615	0.98427445	2730	0.99195685	2710	0.99025961	2840	0.99068032
48	2930	0.99478323	2650	0.98444769	2790	0.99205005	2765	0.99067820	2845	0.99121214
49	2940	0.99543221	2685	0.98681189	2795	0.99249390	2785	0.99228641	2860	0.99150517
50	2995	0.99581685	2710	0.98873192	2800	0.99283945	2810	0.99320435	2870	0.99272686
51	3030	0.99606372	2725	0.98910707	2845	0.99354734	2810	0.99320435	2900	0.99338972
52	3050	0.99645571	2755	0.99039086	2875	0.99426408	2840	0.99426688	2945	0.99412531
53	3065	0.99653709	2785	0.99198072	2915	0.99440240	2855	0.99453510	2945	0.99412531
54	3100	0.99683821	2815	0.99199413	2925	0.99500198	2895	0.99573924	2955	0.99496265
55	3100	0.99683821	2820	0.99358643	2950	0.99541869	2935	0.99583440	2965	0.99501218
56	3130	0.99686536	2845	0.99367027	2980	0.99584016	2945	0.99589253	2990	0.99508490
57	3145	0.99694913	2865	0.99403574	3020	0.99614340	2955	0.99593710	3010	0.99547787
58	3150	0.99713908	2890	0.99408470	3050	0.99650691	2960	0.99635577	3010	0.99547787
59	3155	0.99727742	2900	0.99564290	3065	0.99653614	2980	0.99659824	3030	0.99554182
60	3155	0.99727742	2930	0.99565635	3075	0.99686053	3020	0.99664478	3055	0.99577216
61	3155	0.99727742	2955	0.99594074	3120	0.99727628	3040	0.99697075	3060	0.99630529
62	3180	0.99753627	2975	0.99619054	3170	0.99777568	3045	0.99745759	3065	0.99643652
63	3205	0.99792217	3015	0.99649246	3190	0.99802953	3110	0.99784618	3070	0.99661288
64	3220	0.99801828	3030	0.99657767	3205	0.99809033	3150	0.99822815	3135	0.99670607
65	3235	0.99806344	3035	0.99683184	3245	0.99846569	3170	0.99851104	3160	0.99677552
66	3250	0.99840233	3060	0.99684410	3290	0.99862687	3215	0.99853505	3165	0.99751861
67	3300	0.99843819	3100	0.99724413	3305	0.99867484	3215	0.99853505	3165	0.99751861
68	3300	0.99843819	3110	0.99736595	3320	0.99870000	3220	0.99860071	3250	0.99813426

(continued)

Table 3. (continued)

No.	Run #1		Run #2		Run #3		Run #4		Run #5	
	C_s	A_s	C_s	A_s	C_s	A_s	C_s	A_s	C_s	A_s
69	3320	0.99859959	3110	0.99736595	3330	0.99894034	3235	0.99861985	3270	0.99819307
70	3365	0.99883442	3150	0.99741464	3375	0.99894208	3240	0.99868876	3325	0.99843384
71	3415	0.99909457	3170	0.99790794	3400	0.99908077	3245	0.99870291	3380	0.99863348
72	3415	0.99909457	3200	0.99792972	3420	0.99916223	3270	0.99876903	3420	0.99865021
73	3480	0.99921239	3250	0.99797416	3455	0.99922083	3280	0.99885307	3430	0.99881637
74	3490	0.99932660	3310	0.99855357	3460	0.99929713	3295	0.99892914	3500	0.99891357
75	3495	0.99943825	3315	0.99868132	3470	0.99953458	3330	0.99896075	3520	0.99891628
76	3550	0.99946055	3420	0.99870754	3515	0.99953987	3365	0.99926923	3525	0.99896638
77	3570	0.99946108	3425	0.99879837	3560	0.99962129	3440	0.99928841	3535	0.99906027
78	3605	0.99961748	3500	0.99909008	3565	0.99962149	3440	0.99928841	3565	0.99907916
79	3645	0.99965313	3575	0.99927571	3565	0.99962149	3480	0.99931595	3595	0.99914925
80	3655	0.99965527	3600	0.99935895	3565	0.99962149	3500	0.99941212	3635	0.99916129
81	3695	0.99978764	3665	0.99949920	3590	0.99962164	3505	0.99945024	3635	0.99916129
82	3695	0.99978764	3710	0.99967950	3635	0.99963622	3550	0.99946683	3645	0.99919109
83	3780	0.99981757	3800	0.99973407	3635	0.99963622	3590	0.99953690	3660	0.99930899
84	3795	0.99982216	3815	0.99974490	3670	0.99965192	3590	0.99953690	3700	0.99934885
85	3820	0.99983100	3830	0.99981002	3685	0.99965687	3615	0.99963879	3710	0.99956626
86	3820	0.99983100	3910	0.99982037	3685	0.99965687	3640	0.99964835	3760	0.99971455
87	3830	0.99983809	3960	0.99982809	3685	0.99965687	3665	0.99968931	3805	0.99971756
88	3850	0.99984602	3980	0.99985490	3735	0.99966868	3670	0.99971156	3920	0.99973739
89	3855	0.99984625	4000	0.99987718	3745	0.99972168	3670	0.99971156	3950	0.99975023
90	3895	0.99984666	4030	0.99989261	3795	0.99976761	3695	0.99971783	3965	0.99983094
91	3910	0.99984798	4130	0.99989470	3835	0.99981794	3715	0.99973404	3995	0.99985279
92	3915	0.99985214	4130	0.99989470	3860	0.99981809	3725	0.99979787	4020	0.99985456
93	3930	0.99985449	4205	0.99989770	3865	0.99983146	3750	0.99980551	4020	0.99985456
94	3960	0.99987779	4280	0.99989831	3910	0.99984121	3780	0.99982029	4065	0.99986998
95	3990	0.99988756	4280	0.99989831	3930	0.99986206	3785	0.99985313	4100	0.99988044
96	3990	0.99988756	4330	0.99989920	4035	0.99987185	3840	0.99986838	4120	0.99988061
97	4035	0.99989375	4400	0.99989957	4060	0.99987637	3900	0.99987500	4130	0.99988571
98	4055	0.99989648	4400	0.99989957	4075	0.99987864	3920	0.99989867	4150	0.99989354
99	4085	0.99989883	4400	0.99989957	4100	0.99989789	4015	0.99989890	4310	0.99989882
100	4190	0.99989963	4535	0.99989976	4375	0.99989872	4055	0.99989916	4375	0.99989897

Table 4. Normalized memberships of NSGA-II.

No.	μ^k				
	Run #1	Run #2	Run #3	Run #4	Run #5
1	0.00653589106373510	0.00688056926892292	0.00693055037087011	0.00645686419608747	0.00674431796702845
2	0.00990001840827446	0.0103254227352333	0.00757697043297500	0.00645686419608747	0.00726354505061552
3	0.0104247197217261	0.0102629335827042	0.00757697043297500	0.00653366143365230	0.00714268604834677
4	0.0104247197217261	0.0102147579991490	0.00943282657934108	0.00653366143365230	0.0119310144805511
5	0.0105288061979338	0.0106299773330006	0.00937805031984769	0.0100944643845639	0.0119758857065944
6	0.0105288061979338	0.0105819475400116	0.00937805031984769	0.0105944051321669	0.0118508687466489
7	0.0106369924400669	0.0111045710776389	0.00928847350180070	0.0109593201960998	**0.0122428003975620**
8	0.0106325954652662	0.0111045710776389	0.0101008631053770	0.0109158779500082	0.0121290957381344
9	0.0106747205432987	0.0111242390674430	0.0107686597472998	0.0109758482819439	0.0120082304827903
10	0.0106755621136253	0.0111242390674430	0.0108731123531997	0.0109758482819439	0.0120197445015915
11	0.0110344149732857	0.0110764050579832	0.0110233606958044	0.0109266789860435	0.0120379201078828
12	0.0111389377948164	0.0110764050579832	0.0110233606958044	0.0110324689994382	0.0120379201078828
13	0.0111525860918839	0.0116392298750663	0.0113794943925274	0.0109788326017936	0.0118724149074280
14	0.0111248293327177	0.0115916040909160	0.0119586143872311	0.0109788326017936	0.0118724149074280
15	0.0111248293327177	**0.0119524684646373**	0.0121637032023081	0.0112512721878044	0.0119062126666635
16	0.0111385033759499	0.0118300913497660	0.0121089729224781	0.0112512721878044	0.0119062126666635
17	0.0113070281943853	0.0118300913497660	0.0121050360108102	0.0112456857238043	0.0118667450719285
18	0.0112949746943819	0.0119240212482959	0.0120927720152533	0.0111831449948290	0.0118669960210513

(continued)

Table 4. (continued)

No.	μ^k				
	Run #1	Run #2	Run #3	Run #4	Run #5
19	0.0112911590971979	0.0118300560732891	**0.0122520665697145**	0.0113694151738754	0.0117797662538333
20	0.0112976255255837	0.0118022916008704	**0.0122520665697145**	0.0113157847037445	0.0117797662538333
21	0.0112976255255837	0.0118790376759580	0.0121878809329296	0.0113157847037445	0.0117192173618716
22	0.0113113989558900	0.0117850837519342	0.0121177920622628	0.0113860505577659	0.0117192173618716
23	0.0112818081604218	0.0117850837519342	0.0121045386100265	0.0113860505577659	0.0116885366189128
24	0.0114048722150815	0.0117573676915920	0.0120833477284283	0.0114038881215780	0.0116885366189128
25	0.0114048722150815	0.0117377472494455	0.0120833477284283	0.0114409362216432	0.0116679510480487
26	**0.0114232903444178**	0.0116634185102271	0.0120883476122123	0.0114409362216432	0.0115407630097188
27	0.0113957120339804	0.0116359591297445	0.0120883476122123	**0.0115233016383350**	0.0115407630097188
28	0.0114086293853054	0.0116026367552563	0.0120175899533827	**0.0115233016383350**	0.0114201907914638
29	0.0113618157831186	0.0116194621472483	0.0120102521321131	0.0114514702515634	0.0114040102520990
30	0.0114146650270459	0.0115553288113760	0.0119428506935096	0.0114400214764104	0.0114040102520990
31	0.0113914280332132	0.0115462041190298	0.0120014017302032	0.0113978444028839	0.0113741547007192
32	0.0113990819999222	0.0115098067967999	0.0119680422868622	0.0114163746818293	0.0113741547007192
33	0.0113616535417593	0.0114158682710192	0.0119319617194629	0.0114163746818293	0.0112913952658511
34	0.0113778826027958	0.0114158682710192	0.0118837064506153	0.0113854629655830	0.0112913952658511
35	0.0113297879689120	0.0113415493661454	0.0118837064506153	0.0113740144918291	0.0111965121997307
36	0.0113481333781147	0.0112793031561257	0.0118495872548538	0.0113540469417367	0.0111483719100146
37	0.0113345658598076	0.0112194021806640	0.0117658431156870	0.0113566977796597	0.0111483719100146
38	0.0113345658598076	0.0111689894390983	0.0117188331603629	0.0113566977796597	0.0110667049105020
39	0.0112840307665372	0.0111388015309889	0.0117188331603629	0.0113611921077809	0.0109458634494139
40	0.0112764321772550	0.0110722364833192	0.0116356027250244	0.0113075678665629	0.0108861638207955
41	0.0112628863302816	0.0109873358639748	0.0116056158545006	0.0113261033697577	0.0108861638207955
42	0.0112110004537935	0.0109873358639748	0.0116009698254346	0.0112505763288283	0.0108209885088896
43	0.0112012836128773	0.0108391020835969	0.0115460019220753	0.0112428259204241	0.0108209885088896
44	0.0111936777156274	0.0107205709613588	0.0114670777200607	0.0112006501529601	0.0107780895409203
45	0.0111133534052346	0.0106904794462516	0.0114028932724047	0.0111975428967252	0.0107780895409203
46	0.0111123929978258	0.0106063878923707	0.0112602833654167	0.0111601652005202	0.0106925619860973
47	0.0110878363389493	0.0105965535880923	0.0110271385985976	0.0111601652005202	0.0106925619860973
48	0.0110262495824751	0.0105121644580296	0.0110271385985976	0.0111076192644829	0.0106688870088453
49	0.0110027944414218	0.0104823724490174	0.0105759352523011	0.0110852291696385	0.0105262782004955
50	0.0109412124788666	0.0104080652878704	0.0105759352523011	0.0110523125806026	0.0105262782004955
51	0.0109412124788666	0.0103449281977999	0.0104857075841880	0.0110523125806026	0.0104783785802322
52	0.0108926744215794	0.0103319111177212	0.0104857075841880	0.0110216931543319	0.0104319317705402
53	0.0108730583636217	0.0102307490707620	0.0103232732062471	0.0110216931543319	0.0103606865504021
54	0.0108114022102664	0.0101903536467275	0.0103232732062471	0.0110015256428265	0.0103606865504021
55	0.0107388115063275	0.0101142003799420	0.0100345276240999	0.0109141192280295	0.0103369643867758
56	0.0106599854062599	0.0101142003799420	0.0100345276240999	0.0108835008064221	0.0103369643867758
57	0.0106537881922858	0.0100324895587024	0.00994433444073469	0.0108835008064221	0.0102739341306237
58	0.0105787198359346	0.00999209433997781	0.00994433444073469	0.0108151368142935	0.0102739341306237
59	0.0105787198359346	0.00991330657750369	0.00987219392576734	0.0108151368142935	0.0102031153974744
60	0.0104619663456849	0.00991330657750369	0.00987219392576734	0.0107409318098752	0.0101327559670809
61	0.0104619663456849	0.00983222106865351	0.00980003279922672	0.0106855163543584	0.0101146928931134
62	0.0103866179800220	0.00983222106865351	0.00972790179729357	0.0106723154463113	0.00999412088922117
63	0.0103185938385254	0.00972797717624531	0.00965570975339257	0.0106085078368432	0.00985038668185520
64	0.0102228897034018	0.00968674835822874	0.00963769870451768	0.0105842677548937	0.00982898805001269
65	0.0101763697308964	0.00960261155404400	0.00956551617365087	0.0105548861072835	0.00973251123235998
66	0.0101054943920028	0.00953979148300379	0.00956551617365087	0.0105082692254510	0.00968520311014889
67	0.0100973997167076	0.00949939657224408	0.00949334553407648	0.0105082692254510	0.00958909621599026
68	0.00999801354163334	0.00944007114275562	0.00949334553407648	0.0104656867702601	0.00956511659615407
69	0.00995028115882529	0.00935696329024114	0.00942116300320967	0.0104017249449667	0.00946934679975213
70	0.00983988887687716	0.00935696329024114	0.00938507272871738	0.0103776392797751	0.00937489857411766
71	0.00973272740020831	0.00927282662977332	0.00927678565380720	0.0103136773540153	0.00937489857411766
72	0.00966431655864933	0.00925271970579779	0.00918656432771711	0.0102662178610256	0.00935128889106935
73	0.00958327606496449	0.00919029539013789	0.00909632793932306	0.0101856550454869	0.00925481208006356
74	0.00951277380576351	0.00908718279638952	0.00900608996542324	0.0101856550454869	0.00925481208006356
75	0.00945793115286329	0.00908718279638952	0.00893389752514623	0.00902345374921155	0.00915827937025346
76	0.00930466239175775	0.00900304621804567	0.00891585357702919	0.00902345374921155	0.00915827937025346
77	0.00923486677031019	0.00900304621804567	0.00877147266023916	0.00894274354955852	0.00911173422293439

(continued)

Table 4. (continued)

No.	μ^k				
	Run #1	Run #2	Run #3	Run #4	Run #5
78	0.00905078476654903	0.00891817052402991	0.00869928021996215	0.00894274354955852	0.00882168035348906
79	0.00905078476654903	0.00889735259130898	0.00859099076679375	0.00887817500806398	0.00877438399629877
80	0.00816986122121297	0.00879077553656542	0.00853684703115022	0.00887817500806398	0.00877438399629877
81	0.00816986122121297	0.00879077553656542	0.00842855757798182	0.00879746420561289	0.00848431653061049
82	0.00804198571209461	0.00870663897875256	0.00833831682944733	0.00879746420561289	0.00848431653061049
83	0.00804198571209461	0.00870663897875256	0.00826612359641743	0.00873289556365202	0.00826675794467320
84	0.00794252762823610	0.00855715408155775	0.00817588245150651	0.00871675340316180	0.00819449176001914
85	0.00787148602495339	0.00853611211466427	0.00817588245150651	0.00857147395874983	0.00819449176001914
86	0.00780044377308165	0.00842866631157234	0.00806759299833811	0.00857147395874983	0.00790457430166938
87	0.00767256764357378	0.00842866631157234	0.00792320692865475	0.00729624528934921	0.00744531589047812
88	0.00760152546220084	0.00819216045765504	0.00781491707910991	0.00723167785298446	0.00744531589047812
89	0.00760152546220084	0.00819216045765504	0.00774272384608001	0.00719939564179703	0.00739698360991350
90	0.00747364933269298	0.00759011795565704	0.00767053061305011	0.00711868584400926	0.00739698360991350
91	0.00738839859378750	0.00759011795565704	0.00754419245524779	0.00703797524249086	0.00732446539624700
92	0.00721789708777702	0.00733210194613007	0.00754419245524779	0.00700569182570740	0.00732446539624700
93	0.00714685479360599	0.00733210194613007	0.00745395091396042	0.00695726936289017	0.00727612487185920
94	0.00706160404060075	0.00722459576139394	0.00745395091396042	0.00689270132372736	0.00710691334255503
95	0.00697635328759550	0.00716009210803904	0.00723737121487073	0.00687655956510252	0.00708274157566814
96	0.00682006024041923	0.00705258341852202	0.00720127459835578	0.00679584906405035	0.00698604962430760
97	0.00677743486391661	0.00705258341852202	0.00709298474881094	0.00669899620157538	0.00696187649148187
98	0.00664955873440875	0.00694507456475709	0.00709298474881094	0.00666671188059494	0.00686518386546052
99	0.00653589106373510	0.00688056926892292	0.00693055037087011	0.00656985891765363	0.00674431796702845
100	0.00653589106373510	0.00688056926892292	0.00693055037087011	0.00645686419608747	0.00674431796702845

Bold type represents the best value.

Table 5. Normalized memberships of MOHH.

No.	μ^k				
	Run #1	Run #2	Run #3	Run #4	Run #5
1	0.00740112494997144	0.00715075462668567	0.00714459438054445	0.00749293954329965	0.00730929972878118
2	0.00740112494997144	0.00715075462668567	0.00746720135862615	0.00750195695027340	0.00739201499289447
3	0.00762112804820390	0.00718637397871553	0.00787588934618570	0.00765087602247176	0.00755550105401200
4	0.00762112804820390	0.00767737930933110	0.00787588934618570	0.00782939760318760	0.00821365958235204
5	0.00762112804820390	0.00767737930933110	0.00798182502710620	0.00782939760318760	0.00821365958235204
6	0.00802331129705570	0.00772290584590704	0.00878115897452114	0.00782939760318760	0.00821365958235204
7	0.00801418575295545	0.00867855173240262	0.00878115897452114	0.00835621275563904	0.00821365958235204
8	0.00801418575295545	0.00867622058769288	0.00935889801659362	0.00835621275563904	0.00826019847602182
9	0.00834415047652754	0.00867622058769288	0.00935889801659362	0.00890844640830376	0.00826019847602182
10	0.00875544456544793	0.00902902083847627	0.00952202597180743	0.00890844640830376	0.00826019847602182
11	0.00941614685310689	0.00908885536042169	0.00945267288556186	0.00900380830145716	0.00826019847602182
12	0.00945168228391885	0.00934164060893672	0.00945267288556186	0.00900380830145716	0.00922591819675517
13	0.00959184781073460	0.00949147696748892	0.00945267288556186	0.00906494049030795	0.00939748311905710
14	0.00975895043541086	0.00949147696748892	0.00945267288556186	0.00906494049030795	0.00991525732889630
15	0.0102605961309655	0.00950611179894248	0.00955766129416279	0.00905084391517505	0.0100897199615703
16	0.0105170928831931	0.00950611179894248	0.00983686343451392	0.00937648494651368	0.0100897199615703
17	0.0107221886001975	0.00974399892619524	0.0100252841039727	0.0100660370725947	0.0101263136463193
18	0.0107052574950391	0.00974399892619524	0.0101575287860011	0.0100780884095698	0.0101263136463193
19	0.0107851047906091	0.0103214341087049	0.0101575287860011	0.0100780884095698	0.0100900563334918
20	0.0108544266860587	0.0103214341087049	0.0102443845314403	0.0101706930768308	0.0100900563334918
21	0.0110914369402269	0.0104784306559736	0.0102443845314403	0.0102637733042497	0.0102050858572559
22	0.0111531675500733	0.0104784306559736	0.0103201998353871	0.0102637733042497	0.0102420677301557
23	0.0111672734844531	0.0104784306559736	0.0103917180066676	0.0102393681066870	0.0102420677301557
24	0.0111672734844531	0.0104784306559736	0.0103917180066676	0.0104101705551092	0.0104650823044786
25	0.0111992483445889	0.0104446524001410	0.0104290868653799	0.0104101705551092	0.0104650823044786
26	0.0113098418127633	0.0104524553215545	0.0104290868653799	0.0103620601590517	0.0106188839335552
27	0.0115440414730603	0.0105641015718245	0.0105750492344936	0.0105051705492579	0.0108290065953675

(continued)

Table 5. (continued)

No.	μ^k				
	Run #1	Run #2	Run #3	Run #4	Run #5
28	0.0114336610718136	0.0109666916925784	0.0108609974771900	0.0108755058910607	0.0108290065953675
29	0.0115139711214815	0.0109666916925784	0.0109295970943888	0.0108755058910607	0.0109958997228951
30	0.0114036033699396	0.0109036801415778	0.0111506281821569	0.0109714082826804	0.0109958997228951
31	0.0116181989389086	0.0109036801415778	0.0111205425202373	0.0110692585941912	0.0109958997228951
32	0.0117074673135131	0.0109036801415778	0.0110832040093005	0.0111006618310424	0.0110510442535982
33	0.0117291638910951	0.0109036801415778	0.0110832040093005	0.0113823854524753	0.0110859920649086
34	**0.0117459168952589**	0.0109036801415778	0.0111223118528864	0.0114823556209457	0.0110713321957611
35	0.0116820627757704	0.0109263319134093	0.0112732722239804	0.0112827939409304	0.0109878062456416
36	0.0115609398496715	0.0109874090863443	0.0112732722239804	0.0116121907561717	0.0110700516373366
37	0.0116007983759698	0.0109874090863443	**0.0114968441366447**	0.0114958705764489	0.0112234694235804
38	0.0115530041473393	0.0109874090863443	**0.0114968441366447**	0.0115456827005924	0.0112808342382748
39	0.0115361146355218	0.0110039972267477	0.0113901152733021	0.0115456827005924	0.0113365663021978
40	0.0116650108653387	0.0110440747754499	0.0113901152733021	0.0114744022272633	0.0113390179575034
41	0.0116643077414264	0.0110867683299369	0.0113901152733021	0.0114744022272633	0.0113467098541632
42	0.0116643077414264	0.0111568465420387	0.0113158576140529	0.0115305766782593	**0.0114684699765886**
43	0.0115570809134291	0.0111351100333525	0.0113158576140529	0.0116451935498532	0.0113756224043812
44	0.0115570809134291	0.0113153480890070	0.0114366032364800	0.0115434171665865	0.0113880522776074
45	0.0114366411476591	0.0113153480890070	0.0114055088640599	**0.0117703770641603**	0.0114074505087154
46	0.0113497162499256	0.0112774425346162	0.0113863048670875	**0.0117703770641603**	0.0112543655294982
47	0.0113026385898621	0.0113841312990618	0.0114034286812271	0.0115814811290441	0.0113417015140401
48	0.0112688952321908	0.0112989667641139	0.0112302498136265	0.0114161488657198	0.0113667607019103
49	0.0112834677428692	0.0113707398558382	0.0112554183624892	0.0114668141912049	0.0113428033845161
50	0.0111266392158184	0.0114385754060540	0.0112716604250298	0.0114470314383765	0.0114049517871621
51	0.0110269954374715	0.0114236299785259	0.0111997112591578	0.0114470314383765	0.0113628828092316
52	0.0109887272060146	0.0114319528080888	0.0111739763069841	0.0114203247917768	0.0112800883081029
53	0.0109442055304032	0.0114621993146680	0.0110654421338526	0.0113869564730012	0.0112800883081029
54	0.0108485984960601	0.0113795253108009	0.0110896154549846	0.0113351734928594	0.0113129817860972
55	0.0108485984960601	**0.0114796420578444**	0.0110517720481786	0.0111990086401003	0.0112859108509528
56	0.0107494644009783	0.0114159520207746	0.0109992240378531	0.0111675803446418	0.0112143436394757
57	0.0107051205653332	0.0113863741257098	0.0109056660206141	0.0111351202738649	0.0111825728239661
58	0.0107023956955429	0.0113201856475830	0.0108478547432770	0.0111490510245838	0.0111825728239661
59	0.0106958305242220	0.0114039207318316	0.0108050985214317	0.0110957977052706	0.0111257585446634
60	0.0106958305242220	0.0113212495931491	0.0108042822212924	0.0109559333749315	0.0110661886305029
61	0.0106958305242220	0.0112719248758611	0.0107058042282165	0.0109090335393272	0.0110913475294470
62	0.0106307962871187	0.0112340615829447	0.0105997855228312	0.0109281513197999	0.0110859156798091
63	0.0105752158440372	0.0111441752543322	0.0105622898783695	0.0107246849823433	0.0110839189147283
64	0.0105317902281342	0.0111084615355174	0.0105224004843288	0.0106103433950041	0.0108905460754241
65	0.0104845734213188	0.0111127285425479	0.0104353915840130	0.0105601656183437	0.0108187299668387
66	0.0104592130719457	0.0110439112574602	0.0103137964431954	0.0104006612960603	0.0108598700391401
67	0.0102932908699823	0.0109610525106168	0.0102727419746936	0.0104006612960603	0.0108598700391401
68	0.0102932908699823	0.0109419002449368	0.0102296161618073	0.0103877316481673	0.0106445825575881
69	0.0102378644302334	0.0109419002449368	0.0102211673980324	0.0103354109255328	0.0105873770455217
70	0.0101036066460412	0.0108338751482946	0.0100850937325027	0.0103227285689219	0.0104360782328100
71	0.0099543738691952I	0.0108134536672299	0.0100220037160601	0.0103058795444319	0.0102816487968359
72	0.0099543738691952I	0.0107313792032610	0.00996885357539225	0.0102212821270173	0.0101595584976983
73	0.00974397314943654	0.0105951714950655	0.00986821699240579	0.0101918253137493	0.0101413648927532
74	0.00971875340959099	0.0104694052976440	0.00986000883681590	0.0101438363734806	0.00993287681233414
75	0.00971020223463021	0.0104646168884351	0.00985129763615930	0.0100207617177237	0.00987140123060996
76	0.00952641197813520	0.0101737740673745	0.00971554634119363	0.00991875399651173	0.00985979414476289
77	0.00945901519722692	0.0101663410926890	0.00958670830567557	0.00965132800521436	0.00983609968537639
78	0.00935263954367216	0.00997814977580535	0.00957158961489634	0.00965132800521436	0.00974501472246350
79	0.00922041982471444	0.00978235998910147	0.00957158961489634	0.00951001797199537	0.00965782686412239
80	0.00918686095306227	0.00971862697426120	0.00957158961489634	0.00944563274050705	0.00953537958411672
81	0.00906183817198565	0.00954746482119604	0.00949591897368201	0.00943060758433452	0.00953537958411672
82	0.00906183817198565	0.00943492780566836	0.00936101129069936	0.00927053867703165	0.00950680689363635
83	0.00877746133666769	0.00918793296278142	0.00936101129069936	0.00913246473557483	0.00946951949960829
84	0.00872722571502769	0.00914689143301990	0.00925647901836176	0.00913246473557483	0.00934918974790705
85	0.00864358822689402	0.00910973867519032	0.00921151796347574	0.00905058904405566	0.00933489705330001
86	0.00864358822689402	0.00888745455629835	0.00921151796347574	0.00896168799743651	0.00919197955901840

(continued)

Table 5. (continued)

No.	μ^k				
	Run #1	Run #2	Run #3	Run #4	Run #5
87	0.00861039768487907	0.00874861660854971	0.00921151796347574	0.00887517616505044	0.00905342449407717
88	0.00854355153817592	0.00869478062723567	0.00906122188814339	0.00885894346652508	0.00870026319900042
89	0.00852670959792655	0.00864062016375453	0.00903576103944903	0.00885894346652508	0.00860871773837934
90	0.00839186766959197	0.00855809085171469	0.00888856335387569	0.00876979208503885	0.00856859961546408
91	0.00834138872716132	0.00827945870082432	0.00877203893118683	0.00869932272711498	0.00847773995390925
92	0.00832483921832090	0.00827945870082432	0.00869636828997250	0.00866832814252048	0.00840077236021799
93	0.00827443691821881	0.00807058719708355	0.00868244554858148	0.00857928100382084	0.00840077236021799
94	0.00817501634453023	0.00786154449855705	0.00854709926001454	0.00847285145235822	0.00826316188644348
95	0.00807458900316660	0.00786154449855705	0.00848844520951975	0.00845742454296434	0.00815601480778923
96	0.00807458900316660	0.00772221732052116	0.00817146032328336	0.00826140229018899	0.00809434589314294
97	0.00792331811023649	0.00752709652372602	0.00809618651569291	0.00804679769126035	0.00806389315416799
98	0.00785608503138919	0.00752709652372602	0.00805098209373321	0.00797689596194090	0.00800280728290099
99	0.00775510556761988	0.00752709652372602	0.00797704589696193	0.00763632530149564	0.00750975433779941
100	0.00740112494997144	0.00715075462668567	0.00714459438054445	0.00749293954329965	0.00730929972878118

Bold type represents the best value.

Table 6. Comparison of normalized memberships.

Technique	Best μ^k	SD	$[C_s, A_s]$	n	r
NSGA-II	Run #3 **0.0122520665697145**	0.000349605318	[3675, 0.99976]	(5, 5, 5, 5, 7, 4, 3, 4, 7, 6)	(4, 4, 3, 3, 5, 2, 3, 3, 4, 3)
MOHH	Run #4 0.0117703770641603	**0.000135975797**	[2645, 0.98967]	(4, 4, 3, 4, 5, 3, 3, 3, 5, 3)	(2, 3, 2, 2, 4, 1, 1, 2, 3, 2)

Bold type represents the best value.

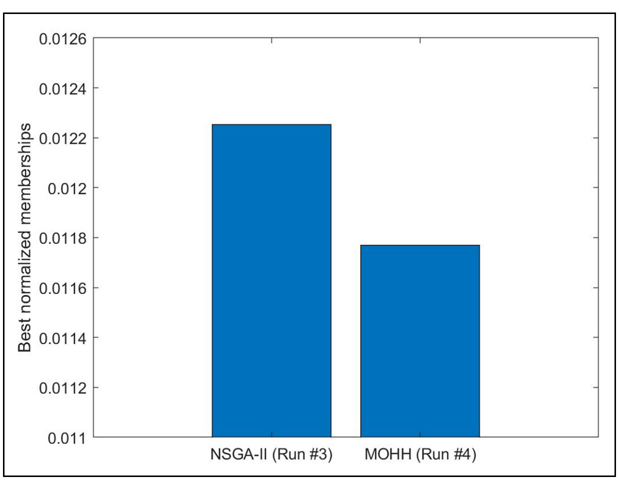

Figure 1. Best normalized membership values.

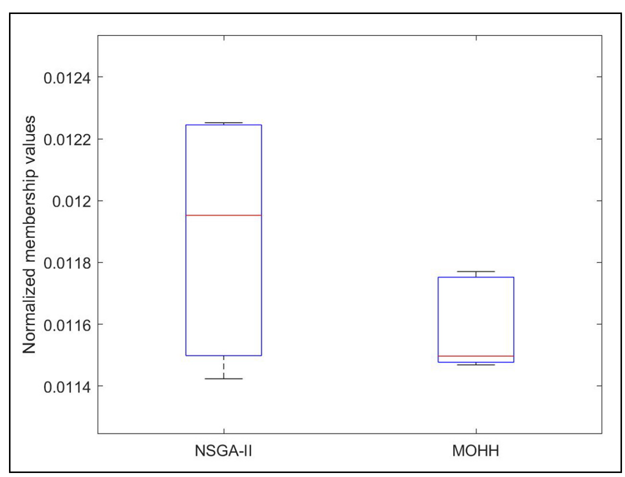

Figure 2. Box diagram of the normalized membership values.

and repair teams. The non-dominated sorting genetic algorithm II (NSGA-II) and the multi-objective hoopoe heuristic (MOHH) were implemented to generate the Pareto fronts. A fuzzy decision method was used to identify the best compromise solution of each optimization technique. Comparison of the results revealed that NSGA-II consistently provided a better best compromise solution over five independent runs compared to the MOHH. However, the latter exhibited a better standard deviation. The decision between MOHH and NSGA-II depends on computational resources and the preference for consistency. Future works will consider heterogeneity in redundancy.

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