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ADAPTIVE BACKSTEPPING CONTROLLER DESIGN USING TUNING FUNCTIONS APPROACH: APPLICATION TO ELECTRO-HYDRAULIC SERVO SYSTEM

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ABSTRACT

Although classical control is still the workhorse in the majority of control engineering applications, it is well recognized that this linear control method is not always the optimum way to deal with the typical highly nonlinear complex plants. For such systems, the control problem is very complicated and becomes even more difficult to deal with when parameters are unknown or uncertain.

Backstepping can be used to relax the matching condition, which blocked the traditional Lyapunov-based design. A major advantage of backstepping is that it has the flexibility to avoid cancellations of useful nonlinearities and achieve regulation and tracking properties. The tuning functions avoid the overparametrization problem and reduce the dynamic order of the controller to its minimum.

This report presents new extension for the design and analysis of nonlinear system via backstepping with tuning functions presented in (Krstic et al., 1995).In the new extension the system presents an unknown virtual control which not constant. The control law is derived and the stability and boundedness of the system is proved .The proposed scheme is applied in an area of engineering systems that is one of the most challenged in nonlinear control systems, i.e. electro-hydraulic actuators , it is used to track the load position and ensure a good transient performance .A single rod electro-hydraulic servo actuator is used to demonstrate the effectiveness of the proposed controller.

Keywords: *Adaptive backstepping control, tuning functions, electro-hydraulic servo system.*

RESUME

Bien que, le contrôle classique est le plus utilisé dans la majorité de contrôle des applications d'ingénierie, il est bien reconnu que cette méthode de contrôle linéaire n'est pas toujours la meilleure façon de commander les systèmes complexes non linéaire. Pour ces systèmes, le problème de contrôle est très complexe et devient encore plus difficile à traiter lorsque les paramètres sont inconnus.

L'approche Backstepping peut être utilisée pour détendre l'état correspondant, qui étais bloqué par la conception traditionnelle à base de Lyapunov. Un avantage majeur de backstepping, c'est qu'il a la possibilité d'éviter les annulations de non-linéarités utile et améliorer les propriétés de régulation et de poursuite.

Cette thèse présente une nouvelle extension pour le design et l'analyse de système non-linéaire via backstepping avec des fonctions de réglage présentées dans (Krstic et al., 1995). Dans la nouvelle extension le système présente un contrôle virtuel inconnu et non constant. La loi de commande est développée et la stabilité est prouvée. Le plan proposé est appliqué dans un secteur des systèmes d'ingénierie qui est un du plus défié dans des systèmes de commande non-linéaires, les actionneurs électro-hydrauliques, il est utilisé pour suivre la trajectoire de la position de la masse et assurer une bonne performance transitoire. Un actionneur en simple tige électro-hydraulique est utilisé pour démontrer l'efficacité du contrôleur proposé.

Mots-clés: *Commande adaptative par backstepping, les fonctions de réglage, Systèmes électro-hydraulique.*

خلاصة الرسالة

على الرغم من أنالتحكم الكلاسيكي هو المحرك لأغلبية تطبيقات هندسة التحكم. إلا أنهمن المعروف أن نظرية التحكم الخطي ليست دائما الطريقة المثلى للتعامل مع الأنظمة غير الخطية المعقدة للغاية . بالنسبة لهذه النظم ،مشكلة التحكم معقدة للغاية ويصبح الأمر أكثر تعقيدا عند إذا كانت المعلمات غير معروفة.يمكن استخدام (backstepping) للتخلص من الشروط المتطابقة والتي أوقفت التصميم التقليدي القائم على أساس(Lyapunov) . الميزة الرئيسية ل(backstepping) هو أن لديه المرونة اللازمة لتجنب إلغاء قياسات لا خطية مفيدة وتحقيق التنظيم و خصائص التتبع.وظائف الضبط تجنب مشكلة الإفراط في المعلمات (overparametrization) وتخفض النظام الديناميكي للتحكم إلى حده الأدنى.

هذا البحث يعرض امتدادا جديدا لتصميم و تحليل الأنظمة الغير خطية عن طريق مبدأ(*backstepping with tuning functions*) الموصوف في(*Krstic et al., 1995*). الامتداد الجديد يحوي تحكم ظاهري غير معروف وغير ثابت. قانون التحكم يتم إيجاده مرحليا و الاستقرار يثبت نظريا.

المخطط المقترح تم تطبيقه في احد النظم الهندسية التي هي واحدة من أكثر الأنظمة تحديا في السيطرة اللاخطية. ألا و هو المحركات الهيدروكهريائية أحادية الساق، حيث يضمن التتبع الشامل لخط مقترح قبلا، و الاستقرار المثالي .

كلمات البحث: *تحكم متكيف على أساس نهج (backstepping) ، وظائف الضبط ، المحركات الهيدروكهريائية أحادية الساق*

Nomenclature

ABC : Adaptive Backstepping Controller

CLF : Control Lyapunov function

EHSS : Electro-Hydraulic Servo System

FL : Feedback Linearization

GA : Genetic Algorithm

IO : Input-Output

LF : Lyapunov Function

MIMO : Multi Input Multi Output

NN Neural Networks

PID : Proportional Integral derivative

PSO : Particle Swarm Optimization

Introduction

Engineering is concerned with understanding and controlling the materials and forces of natures for the benefits of humankind. Control system engineering is concerned by understanding and controlling segments of their environments (system) to provide useful products to society. The two objectives of understanding and control are complementary because effective system requires that the system be understood and modelled . Perhaps, the most characteristics quality of control engineering is opportunity to control machines and industrial and economic processes for the benefit of the society. Control engineering is based on the foundation of feedback theory and linear system analysis. Therefore its applications are not restricted to any engineering area but it can be equally used aeronautical, chemical, environmental, civil and electrical engineering, etc. Due to the increasing of complexity of the system under control and the interest of achieving optimum performance, the importance of the control system engineering has grown in the past decades. Furthermore as the system becomes more and more complex, the interrelationship between variables must be considered in control scheme. One of the reasons of the emergency of the adaptive control is its capability to build systems capable to control unknown plants or adapting unpredictable changes in environment.

It is widely known that the cost of computers has dramatically dropped. This fact has given arise to their integration as a part of the control systems. Therefore, the research in adaptive control algorithms has increased and the application of the modern control theory are not strictly related to the engineering, even with application in different sciences such a biology, biomedicine and economy.

While in this report we will be preoccupied with nonlinear systems, we must not forget that the control of linear plants with unknown parameters was a formidable problem which took almost twenty years to solve. By early 1980's, several types of adaptive schemes were proven to provide stable operation and asymptotic tracking. We refer to the results from that period to *adaptive linear control* or *traditional adaptive control.* Traditional adaptive schemes are classified as "direct" and "indirect" and as "Lyapunov-based" and "estimation-based". They involve parameter identification with "parameter estimators" or "identifiers". The vital part of identifier is parameter adaptation algorithm, commonly referred us to "the parameter update law". The direct-indirect classification reflects the fact that update parameters are either those of the control (direct) or those of the plant (indirect). According to this classification all the schemes in the thesis are indirect.

The distinction between Lyapunov-based and estimation-based schemes is more substantial and is dictated in part by the type of parameter update law and the corresponding proof of stability and convergence. Lyapunov-based design is one of the oldest results of adaptive control. Until recently, however, its applicability was restricted to linear plants with relative degree one or two. This limitation has been removed by the recursive design procedures presented in [1], commonly referred to as *backstepping*.

An important feature of traditional adaptive control is its reliance on "certainty equivalence" controllers. This means that a controller is first designed as if all the plant parameters were known. The controller parameters are determined as functions of the plant parameters. Given the true values of the plant parameters, the controller parameters are calculated by solving design equations for model-matching, pole-zero placement, or optimality. When the true plant parameters are unknown, the controller parameters are either estimated directly (direct schemes) or computed by solving the same design equations with plant parameter estimates (indirect schemes). The resulting controller, which is either estimated (direct) or designed for the estimated plant (indirect), is called a *certainty equivalence* controller. Such an approach has been studied extensively and a number of results have been established [2-8]. Certain schemes have also been proposed to study the robustness issues in the context of both single loop control [9-16] and decentralized control of multi-loop systems[17-25].However, transient performance is difficult to be ensured with this approach.

It is not at all obvious that a certainty equivalence controller will work inside an adaptive feedback loop and achieve stabilization and tracking. Even when the plant is stable, bad parameter estimates may yield a destabilizing controller. The situation is more critical when the plant is unstable, because then the controller must achieve stabilization in addition to its tracking task. It is therefore significant that certainty equivalence controllers have been proven to be satisfactory for adaptive control of linear systems.

In spite of major advances in the development of adaptive control schemes for linear systems, they have not yet become tools for systematic engineering design. Each adaptive scheme leaves up to the designer the choice of various filters, design coefficients, initialization rules, and so on. It is still unclear how the adaptive system's performance, especially its transient performance, depends on these design choices. Certain research activity is aimed at providing the designer with clearer choices and trade-offs between transient performance and robustness.

In the beginning of 1990s, a new approach called “backstepping” was proposed for the design of adaptive controllers. Backstepping is a recursive Lyapunov-based scheme for the class of “strict feedback" systems. In fact, when the controlled plant belongs to the class of systems transformable into the parametric-strict feedback form, this approach guarantees global or regional regulation and tracking properties. An important advantage of the backstepping design method is that it provides a systematic procedure to design stabilizing controllers, following a step by-step algorithm. With this method the construction of feedback control laws and Lyapunov functions is systematic. Another advantage of backstepping is that it has the flexibility to avoid cancellations of useful nonlinearities and achieve stabilization and tracking. A number of results using this approach has been obtained[26-31].Research on decentralized adaptive control using backstepping approach has also received great attentions, certain decentralized control results using such technique has been addressed [32-36]. Due to a number of its advantages such as improving transient performance [1].Several applications have been investigated in different industry fields [37- 41].

Electro-hydraulic actuator system has become one of the most important actuators in the recent decades. It offers many advantages such as good capability in positioning, fast and smooth response characteristics and high power density. Due to its capability in positioning, it has given a significant impact in modern equipments for position control applications. Its applications in position control can be found in production assembly lines, robotics, aircrafts equipments and submarine operations. However, excellent positioning in these applications requires an accurate electro-hydraulic actuator. Position tracking performance of an electro-hydraulic actuator can be assured when its robustness and tracking accuracy are guaranteed. Therefore, the development of a suitable controller which could reflect robustness and tracking accuracy is very significant. There are number of problems appear in the position tracking performance of the system such as the highly nonlinear dynamics of hydraulic systems [42]. The system may be subjected to non-smooth and discontinuous nonlinearities due to control input saturation, directional change of valve opening, friction, and valve overlap. Valves also contain non-measurable states (position and velocity). Aside from the nonlinear nature of hydraulic dynamics, EHSS also have large extent of model uncertainties, such as the external disturbances and leakage that cannot be modelled exactly; and the nonlinear functions that describe them may not be known.

In the past, much of the work in the control of hydraulic systems uses linear control theory [43, 44, 45] and feedback linearization techniques (FL) [46, 47]. In [48], nonlinear adaptive control is applied to the force control of an active suspension driven by a double-rod cylinder where only the parametric uncertainties of the cylinder are considered. Adaptive sliding method has been also used, in [49] an adaptive sliding mode controller combined with novel-type Lyapunov function has been developed to compensate nonlinear uncertain parameters caused by the various original control volumes.

In [50] novel approach has decomposed the system into subsystems using graph theoretic decomposition then back integrating to construct the Lyapunov function .

During the last decade, backstepping based design have emerged as powerful tools for stabilizing nonlinear systems for tracking and regulation purposes [1]. An integrator backstepping is used to construct a controller which includes the friction compensation. The exponential stability of the resulting closed-loop system for the trajectory tracking is proved in the absence of both parametric uncertainties and uncertain nonlinearities. Simulation and experimental results show that the proposed nonlinear controller outperforms a PID controller [51, 52]. Zeng and Sepehri in [53, 54] presented an adaptive backstepping control of hydraulic manipulators with friction compensation. A third-order nonlinear dynamic model is used for the controller design while LuGre dynamic friction model characterizes the friction forces. Choux in [55] has addressed an adaptive backstepping controller with considering valve dynamics, the results show that this controller achieves significantly better tracking performance than the PI controller, while handling uncertain parameters related to internal leakage, friction, the orifice equation and oil characteristics.

In our work, we develop an adaptive backstepping controller for a complete fifth order dynamic model of electro-hydraulic servo system, which includes the valve dynamics to handle internal leakage and unknown friction in a cylinder, unknown volumes in the orifice equation and temperature dependent oil characteristics in nonlinear hydraulic mechanical system. The friction force is assumed nonlinear and the same as the practical assumption presented in [48, 52]. The developed controller handles internal leakage and unknown nonlinear friction in the cylinder, unknown volumes in the orifice equation and temperature dependent oil characteristics.

This report is organized as follows:

the first chapter presents a brief review of Lyapunov stability and its requirements,the idea of backstepping is emphasized, then; the adaptive backstepping with tunning function design is discussed .

In the second chapter, the backstepping with tuning functions design for tracking is presented in detail. The advantage of tuning functions design over traditional certainty equivalence adaptive design is emphasized, the issue of tracking performance is also discussed.

A new extension of the tuning function design is presented in the third chapter. The system considered is frequent in various ranges of applications from electric motors and manipulator robots to flight dynamics. The tracking objective is achieved as well as the stability and boundedness of states.

The fourth chapter presents an application of the new extension presented in previous chapter. The model of electro-hydraulic servo system (EHSS) is described at first, then the adaptive backstepping controller (ABC) is designed to achieve tracking performance. Simulation results demonstrate the effectiveness of the proposed controller.

In the last chapter, the results are summarized and special topics are discussed. The foreseen future improvements that can be done to the proposed controller and its applications are also discussed.

Chapter 1

Preliminaries

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Control systems have one main goal to achieve, and that is the stability of the controlled system. There are different kinds of stability problems which occur when studying dynamical systems. Here we are concerned with stability of equilibrium points. Let us first briefly review Lyapunov stability and formalize this requirement.

1.1 Stability

The concept of stability is concerned with the investigation and characterization of the behavior of dynamic systems. Stability plays a crucial role in system theory and control engineering, and has been investigated extensively in the past century. Some of the most fundamental concepts of stability were introduced by the Russian mathematician and engineer Alexander Lyapunov in [57]. The work of Lyapunov was extended and brought to the attention of the larger control engineering and applied mathematics community by Krasovskii [58], Kalman and Bertram [59], and many others.

In control systems, we are concerned with changing the properties of dynamic systems so that they can exhibit acceptable behavior when they are perturbed from their operating point by external forces. Stability is the primary requirement for adaptive control Systems. Stability concepts that are widely used in control theory are Lyapunov stability and input-output stability. This chapter deals with Lyapunov stability, which now briefly reviews [1, 56].

1.1.1 Lyapunov Stability [1]

Consider the time-varying System

$$\dot{x} = f\big(x(t)\big) \tag{1.1}$$

where $x \in \mathbb{R}^n$, and $f: \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ is piecewisecontinuous in t and locally Lipschitz in x. The solution of (1.1) which starts from the point x_0 at time $t_0 \geq 0$ is denoted as $x(t; x_0, t_0)$ with $x(t_0; x_0, t_0) = x_0$. Lyapunov stability concepts describe continuity properties of $x(t; x_0, t_0)$ with respect to x_0. If the initial condition x_0 is perturbed to $\tilde{x}_0$, then, for stability, the resulting perturbed solution $x(t; \tilde{x}_0, t_0)$ is required to stay close to $x(t; x_0, t_0)$ for all $t > t_0$. In addition, for asymptotic stability, the error $x(t; \tilde{x}_0, t_0) - x(t; x_0, t_0)$ is required to vanish as$t \longrightarrow \infty$.So, the solution $x(t; x_0, t_0)$ of (1.1) is

- *bounded*, if there exists a constant $B(x_0, t_0) > 0$ such that

$$|x(t; x_0, t_0)| < B(x_0, t_0)\,, \qquad \forall t \geq t_0 \tag{1.2}$$

- *stable*, if for each $\varepsilon > 0$ there exists a $\delta(\varepsilon, t_0) > 0$ such that

$$|\tilde{x}_0 - x_0| < \delta \Longrightarrow |x(t; \tilde{x}_0, t_0) - x(t; x_0, t_0)| < \varepsilon\,, \forall t \geq t_0 \tag{1.3}$$

- *attractive*, if there exist an $r(t_0) > 0$ and, for each $\varepsilon > 0$, a $T(\varepsilon, t_0) > 0$ such that

$$|\tilde{x}_0 - x_0| < r \Longrightarrow |x(t; \tilde{x}_0, t_0) - x(t; x_0, t_0)| < \varepsilon\,, \forall t \geq t_0 + T \tag{1.4}$$

- *asymptotically stable*, if it is stable and attractive;
- *unstable*, if it is not stable.

The stability properties of $x(t; x_0, t_0)$ in generaldepend on the initial time t_0. For differentt_0, different values of $B(x_0, t_0)$,$\delta(x_0, t_0)$,$r(t_0)$, and $T(\varepsilon, t_0)$ may be needed to satisfy (1.2), (1.3) and (1.4). When these constants are independent of t_0, the corresponding properties are uniform. For adaptive systems, *uniform stability* is more desirable than just stability

1.1.2 Uniform Stability [1]

Let $x = 0$ be an equilibrium point of (1.1) and $D = \{x \in \mathbb{R}^n \ |x| < r$. Let $V: D \times \mathbb{R}^n \to \mathbb{R}_+$ be a continuously differentiable function such that $\forall t \geq 0, \forall x \in D$, such that

$$\gamma_1(|x|) \leq V(t,x) \leq \gamma_2(|x|)$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x,t) \leq -\gamma_3(|x|)$$

Then the equilibrium $x = 0$ is

- *uniformly stable*, if γ_1 and γ_2 are class κ functions on $[0,r)$ and $\gamma_3(.) \geq 0$ on $[0,r)$.
- *uniformly asymptotically stable*, if γ_1, γ_2 and γ_3 are class κ functions on $[0,r)$;
- *exponentially stable*, if $\gamma_{\mathrm{i}}(\rho) = k_{\mathrm{i}}\rho^{\alpha}$ on $[0,r)$, $k_{\mathrm{i}} > 0$, $\alpha > 0$, $i = 1,2,3$;
- *globally uniformly stable*, if $D = \mathbb{R}^n$, γ_1 and γ_2 are class κ_∞ functions, and $\gamma_3(.) \geq 0$ on $\mathbb{R}_+$;
- *globally uniformly asymptotically stable*, if $D = \mathbb{R}^n$, γ_1 and γ_2 are class κ_∞ functions, and γ_3 is a class of κ function on $\mathbb{R}_+$; and
- *globally exponentially stable* ,if $D = \mathbb{R}^n$ and $\gamma_{\mathrm{i}}(\rho) = k_{\mathrm{i}}\rho^{\alpha}$ on $\mathbb{R}_+ k_{\mathrm{i}} \geq 0$ $\alpha > 0, i = 1,2,3$.

1.1.3 LaSalle-Yoshizawa Theorem [1]

Let $x = 0$ be an equilibrium point of a time varying system (1.1) and suppose f is locally Lipschitz in $x$uniformly in t. Let $V: R^n \to R_+$be a continuously differentiable function such that

$$\gamma_1(|x|) \leq V(x,t) \leq \gamma_2(|x|) \tag{1.5}$$

$$V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x,t) \leq -W(x) \leq 0 \tag{1.6}$$

$\forall t \geq 0, \forall x \in R^n$ where γ_1 and γ_2 are class k_∞ functions and W is a continuous function. Then, all solutions of (1.1) are globally uniformly bounded and satisfy

$$\lim_{t \to \infty} W(x(t)) = 0 \tag{1.7}$$

In addition, if $W(x)$ is positive definite, then the equilibrium $x = 0$ is globally uniformly asymptotically stable.

Proof. The proof of the above theorem is found in [1].

Now that we laid the foundation of Lyapunov stability the main question appearing is how to find these functions. The theorems above do not offer any systematic method of finding these functions. In the case of electrical or mechanical systems there are natural Lyapunov function candidates like total energy functions. In other cases, it is basically a matter of trial and error.

The backstepping approach is so far the only systematic and recursive method for constructing a Lyapunov function, along the design of the stabilizing control law. Yet, the system must have a lower triangular structure in order to apply the method, as we will see later. Before we can explore this state-of-the-art technique in adaptive control of nonlinear systems, we have to extend the systems handled so far to those including a control input.

1.2 Control Lyapunov functions (clf)

Let us now add a control input and consider the system

$$\dot{x} = f(x, u) \tag{1.8}$$

Our main objective of this thesis is the design of a closed-loop system with desirable stability properties, rather than to analyze the properties of the system itself. Therefore we are interested in an extension of the Lyapunov function concept. This concept is called *control lyapunov function* and labelled (*clf)* for convenience. Given the stability results from the previous section, we want to find a control law

$$u = \alpha(x) \tag{1.9}$$

such that the desired state of the closed-loop system

$$\dot{x} = f\big(x, \alpha(x)\big) \tag{1.10}$$

is a globally asymptotically stable equilibrium point. Once again we consider the origin to be the goal state for simplicity. We can choose a function $V(x)$ as a Lyapunov candidate, and require that its derivative along the solutions of (1.10) satisfy $\dot{V}(x) \leq -W(x)$, where $W(x)$ is positive definite function. Then closed loop stability follows from LaSalle's theorem. We therefore need to find $\alpha(x)$ to guarantee that for all $x \in \mathbb{R}^n$

$$\dot{V}(x) = \frac{\partial V}{\partial x} f\big(x, \alpha(x)\big) \leq -W(x) \tag{1.11}$$

The pair $V$and Wmust be chosen carefully otherwise (1.11) will not be solvable. This motivates the following definition, which can be found in [1].

Control Lyapunov function (clf): A smooth positive definite and radially unbounded function$V : \mathbb{R}^n \to \mathbb{R}_+$ is called a control Lyapunov function (clf) for (1.8) if for every $x \neq 0$

$$\dot{V}(x) = V_x(x) f(x, u) \leq 0 \qquad for\ some\ u \tag{1.12}$$

The significance of this definition is in establishing the fact that, the existence of a globally stabilizing control law is equivalent to the existence of a clf. If we have a clf for the system then we can certainly find a globally stabilizing control law. The reverse is also true. This is known as Artestin's theorem and can be found in [6]. Now that we defined the concept clf, we can move on and explore the backstepping theory, which is the main tool have been utilized in this thesis.

1.3 Adaptive backstepping and tuning functions

The main deficiency of the clf concept as a design tool is that for most nonlinear systems a clf is not known. The task of finding an appropriate clf may be as complex as that of designing a stabilizing feedback law. The *backstepping* procedure solves these two problems for us simultaneously. Backstepping is a recursive Lyapunov-based scheme proposed in the beginning of 1990s. The technique was comprehensively addressed by Krstic, Kanellakopoulos and Kokotovic in [1]. The idea of backstepping is to design a controller recursively by considering some of the state variables as "virtual controls" and designing for them intermediate control laws. Backstepping achieves the goals of stabilization and tracking. The proof of these properties is a direct consequence of the recursive procedure, because a Lyapunov function is constructed for the entire system including the parameter estimates.

1.3.1 First Lyapunov based example

Let us start this section applying the Lyapunov-based approach to the adaptive control problem for nonlinear plant

$$\dot{x} = u + \theta x^2 \tag{1.13}$$

where$u$ is control and θ is unknown constant. In this procedure we seek a parameter update law for the estimate $\hat{\theta}$

$$\dot{\hat{\theta}} = \tau(x, \hat{\theta}) \tag{1.14}$$

which, along the control law$u = \alpha(x, \hat{\theta})$, will make the Lyapunov function

$$V(x, \hat{\theta}) = \frac{1}{2}x^2 + \frac{1}{2}(\hat{\theta} - \theta)^2 \tag{1.15}$$

nonincreasing function of time

To this end, we express $\dot{V}$ as function of $u$and seek $\alpha(x, \hat{\theta})$ and $\tau(x, \hat{\theta})$ to guarantee that $\dot{V} \leq cx^2$ with $c > 0$, namely

$$\begin{aligned}\dot{V} &= x\dot{x} + (\hat{\theta} - \theta)\dot{\hat{\theta}} \\ &= x(u + \theta x^2) + (\hat{\theta} - \theta)\dot{\hat{\theta}} \\ &= xu + \dot{\hat{\theta}}\hat{\theta} + \theta\left(x^3 - \dot{\hat{\theta}}\right)\end{aligned} \tag{1.16}$$

The requirement $\dot{V} \leq -cx^2$ impose the following condition of the choice of an update law $\dot{\hat{\theta}}$ and a control law for:

$$xu + \dot{\hat{\theta}}\hat{\theta} + \theta\left(x^3 - \dot{\hat{\theta}}\right) \leq -cx^2 \tag{1.17}$$

To eliminate the unknown parameter θ, a possible choice of the update law is $\tau = x^3$, that is

$$\dot{\hat{\theta}} = x^3 \tag{1.18}$$

So that (1.17) reduces to

$$xu + x^3\hat{\theta} \leq -cx^2 \tag{1.19}$$

The condition allows us to select $\alpha(x, \hat{\theta})$ in various way. One of them are, for example,

$$u = -cx - \hat{\theta}x^2 \tag{1.20}$$

1.3.2 Backstepping preview with a generic third order system

Consider now, for example, the class of pure feedback system

$$\begin{aligned}
\dot{x}_1 &= x_2 + \phi_1^T(x_1)\theta \\
\dot{x}_2 &= x_3 + \phi_2^T(x_1, x_2)\theta \\
\dot{x}_3 &= u + \phi_3^T(x_1, x_2, x_3)\theta
\end{aligned} \tag{1.21}$$

Where θ is constant and unknown.

The idea of backstepping is to design a controller for (1.21) recursively by considering some of the states variables as virtual controls and designing for them intermediate control laws. In (1.21) the first virtual control is x_2. It is used to stabilize the first equation as a separate system. Since θ is unknown, this task is solved with an adaptive controller consisting of the control law $\alpha_1(x_1)$ and the update law $\dot{\hat{\theta}} = \tau(x_1)$, as in the previous example.

In the next step the state x_3 is the virtual control which is used to stabilize the subsystem consisting of the first two equations of (1.21).

This is again an adaptive control task, and a new update law is to be designed.

However an update law $\dot{\hat{\theta}} = \tau(x_1)$ has already been designed in the first step and this does not seem to allow any freedom to proceed further. We can treat this in two ways:

- ✓ **Adaptive backstepping with overparametrization.** In this case the parameter θ in the second equation of (1.21) is treated as a new parameter and assigns to it a new estimate with a new update law. As result, there are several estimates for the same parameter (*overparametrization*).

- ✓ **Adaptive backstepping with tuning function.** The overparametrization is avoided by considering that in the first step $\dot{\hat{\theta}} = \tau(x_1)$ is not an update law but only a function $\tau(x_1)$. This *tuning function* is used in subsequent recursive steps and the discrepancy $\dot{\hat{\theta}} - \tau(x_1)$ is compensated with additional terms in the controller. Whenever the second derivative $\ddot{\hat{\theta}}$ would appear, it is replaced by the analytic expression for the first derivative of $\tau(x_1)$.

1.4 Structural constraints

The main topic of this work is the design of feedback controllers for nonlinear systems with unknown constant parameters. The most important design specification is to achieve asymptotic tracking of a known reference trajectory with the strongest possible form of stability. Another key requirement is that the designed controller should provide effective means for shaping the transient performance and thus allow different performance robustness trade-offs. The stated design problem of the largest classes of nonlinear systems is solvable with either state feedback or output feedback controllers.

1.4.1 Full State Feedback Form

Backstepping tools will now be employed to form systematic design procedures for general classes of nonlinear Systems. In increasing order of complexity, the classes considered are strict-feedback systems, pure-feedback systems, and block-strict-feedback systems.

State feedback solutions are given for the so-called class of “parametric pure-feedback Systems.” They are first presented for the subclass of *"parametric strict-feedback systems"* for which the achieved stability and tracking properties are global.

$$
\begin{aligned}
\dot{x}_1 &= x_2 + \varphi_1^T(x_1)\theta \\
\dot{x}_2 &= x_3 + \varphi_2^T(x_1, x_2)\theta \\
\vdots \quad & \quad \vdots \\
\dot{x}_{n-1} &= x_n + \varphi_{n-1}^T(x_1, \cdots, x_{n-1})\theta \\
\dot{x}_n &= bu + \varphi_n^T(x)\theta
\end{aligned}
\tag{1.22}
$$

where $x = [x_1, \cdots, x_n]^T \in R^n$, the vector $\theta \in R^r$ is constant and unknown, $\phi_i \in R^r, i = 1, \dots, n$ are known nonlinear functions, and the high frequency gain b is an unknown constant.

By analogy with linear systems, strict feedback systems are also called “triangular”.

1.4.2 Output Feedback Form

Output feedback solutions are restricted to a narrower class of minimum phase systems in which the non-uncertainties depend only on the output variable.

$$\dot{x}_1 = x_2 + \varphi_1^T(y)\theta$$
$$\vdots$$
$$\dot{x}_{\rho-1} = x_\rho + \varphi_{\rho-1}^T(y)\theta$$
$$\dot{x}_\rho = x_{\rho+1} + \varphi_\rho^T(y)\theta + b_m u \quad (1.23)$$
$$\vdots$$
$$\dot{x}_{n-1} = x_n + \varphi_{n-1}^T(y)\theta + b_1 u$$
$$\dot{x}_n = \varphi_n^T(y)\theta + b_0 u$$
$$y = x_1$$

where $x_1, \cdots, x_n, y$ and u , are system states, output and input, the vector $\theta \in R^r$ is constant and unknown, $\varphi_i(y) \in R^r, i = 1, \ldots, n$ are known nonlinear functions ,and $b_m, \cdots, b_0$ are unknown constants.

Both designs of backstepping achieve the goals of the stabilization and tracking. The proof of these properties is a direct consequence of the recursive procedure during which a Lyapunov function is constructed for the entire system, including the parameter estimates.

The tuning functions approach is an advanced form of adaptive backstepping. It has the advantage that the dynamic order of the adaptive controller is minimal. The dimension of the set to which the states and parameter estimates converge is also minimal.

Chapter 2

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Tuning Functions Design For Nonlinear Systems

We now present an approach of adaptive control of nonlinear system via backstepping tuning functions control design. This design removes several obstacles from adaptive nonlinear control. Since the design is based on single Lyapunov function incorporating both the state of the error and the update law, the proof of global uniform stability is direct and simple. Moreover, all the error states except for the parameter error converge to zero.

However, the main advantage of tuning functions design over traditional certainty equivalence adaptive design is in the transient performance. The nonlinear control law which incorporates the parameter update law keeps the parameter estimation transient from causing bad tracking transients. The performance bounds obtained for the tuning functions scheme are computable and can be used for systematic improvement of transient performance.

2.1 Tuning Functions Design

The adaptive backstepping solution to the problem of nonlinear stabilization and tracking in the presence of unknown parameters is a starting point for more elaborate adaptive designs which lead to new properties of the designed controller and the resulting feedback system.

One of the improvements to be achieved with the tuning functions design in this chapter is the reduction of the dynamic order of the adaptive controller to its minimum: The number of parameter estimates is equal to the number of unknown parameters. This minimum-order design is advantageous not only for implementation, but also because it guarantees the strongest achievable stability and convergence properties

In the tuning functions procedure the parameter update law is designed recursively. At each consecutive step we design a tuning function as a potential update law. In contrast to adaptive traditional backstepping with overparametrization, these intermediate update laws are not implemented. Instead, the controller uses them to compensate for the effect of parameter estimation transients. Only the final tuning function is used as the parameter update law.

In this section, we will consider unknown parameters which appear linearly in system equations. An adaptive controller is designed by combining a parameter estimator, which provides estimates of unknown parameters, with a control law. The parameters of the controller are adjusted during the operation of the plant. In the presence of such parametric uncertainties, the adaptive controller is able to ensure the boundedness of the closed-loop states and asymptotic tracking. The following are standard results and can be found in [1] and [61]. To illustrate the idea of adaptive backstepping, let us first consider a class of nonlinear system as in the following parametric strict-feedback form ([1], sec. 4.3).

$$
\begin{aligned}
\dot{x}_1 &= x_2 + \varphi_1^T(x_1)\theta \\
\dot{x}_2 &= x_3 + \varphi_2^T(x_1, x_2)\theta \\
&\vdots \\
\dot{x}_{n-1} &= x_n + \varphi_{n-1}^T(x_1, \cdots, x_{n-1})\theta \\
\dot{x}_n &= \beta(x)u + \varphi_n^T(x)\theta
\end{aligned}
\tag{2.1}
$$

where $x = [x_1, \cdots, x_n]^T \in R^n$, the vector $\theta \in R^r$, is a vector of unknown constant parameters, $\Phi = [\varphi_1, \dots, \varphi_n]$ and $\beta(x)$ are known smooth nonlinear functions taking arguments in $\mathbb{R}^n$, and $\beta(x) \neq 0, \forall\, x \in \mathbb{R}^n$.

For the development of control laws, the following assumption is made

Assumption 1: The reference signal x_r and its first n order derivative areknown, piecewise continuous and bounded.

The control objective is to force x_1 to asymptotically track the reference output $x_r(t)$.

2.1.1 Design Procedure

We will start by adaptively stabilizing the first equation of (2.1) considering $x_2$to be its control. At each subsequent step we will augment the designed subsystem by one equation. At the *i*th step, an *i*th-order subsystem is stabilized with respect to a Lyapunov function V_i by the design of a *stabilizing function* α_i and a *tuning function* τ_n. The update law for the parameter estimate $\hat{\theta}(t)$ and the adaptive feedback control u are designed at the final step. The third step is crucial for understanding the general design procedure [1].

Step 1. Introducing the first two error variables

$$z_1 = x_1 - x_r \tag{2.2}$$

$$z_2 = x_2 - \alpha_1 - x_r^{(1)} \tag{2.3}$$

We rewrite $\dot{x}_1 = x_2 + \varphi_1^T(x_1)\theta$, the first equation of (2.1), as

$$\dot{z}_1 = z_2 + \alpha_1 + w_1^T(x_1)\theta \tag{2.4}$$

where, for uniformity with subsequent steps, we have defined the first regressor vector as

$$w_1(x_1) = \varphi_1(x_1) \tag{2.5}$$

Our task in this step is to stabilize (2.4) with respect to the Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{2.6}$$

where Γ is a positive definite matrix.

whose derivative along the solutions of (2.4) is

$$\begin{aligned}\dot{V}_1 &= z_1\dot{z}_1 - \theta_1^T\Gamma^{-1}\dot{\hat{\theta}} \\ &= z_1(z_2 + \alpha_1 + {w_1}^T\theta) - \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \Gamma w_1 z_1) \\ &= -c_1 z_1^2 - \hat{\theta}^T(\tau_1 - w_1 z_1) + z_1 z_2 \end{aligned} \tag{2.7}$$

We can eliminate $\tilde{\theta}$ from $\dot{V}_1$, with the update law $\dot{\hat{\theta}} = \Gamma\tau_1$,where

$$\tau_1(x_1) = w_1(x_1)z_1 \tag{2.8}$$

If x_2 were our actual control, we would let $z_2 \equiv 0$, that is, $x_2 \equiv \alpha_1$. Then to make $\dot{V}_1 = -c_1 z_1^2$, we would choose

$$\alpha_1(x_1, \hat{\theta}) = -c_1 z_1 - w_1^T(x_1)\hat{\theta} \tag{2.9}$$

Since x_2 is not our control we have $z_2 \not\equiv 0$, and we do not use $\dot{\hat{\theta}} = \Gamma\tau_1$, as an update law. Instead, we retain $\tau_1$as our first *tuning function* and tolerate the presence of $\tilde{\theta}$ in $\dot{V}_1$:

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2 - \tilde{\theta}^T(\Gamma^{-1}\dot{\hat{\theta}} - \tau_1) \tag{2.10}$$

The second term $z_1 z_2$ in $\dot{V}_1$ will be cancelled at the next step. With $\alpha_1(x_1, \hat{\theta})$as in (2.9), the z_1-system becomes

$$\dot{z}_1 = -c_1 z_1 + z_2 + w_1^T(x_1)\tilde{\theta} \tag{2.11}$$

Step 2. We now consider that x_3 is the control variable in the second equation of (2.1). Introducing

$$z_3 = x_3 - \alpha_2 - x_r^{(2)} \tag{2.12}$$

We rewrite $\dot{x}_2 = x_3 + \varphi_2^T(x_1, x_2)\theta$ as

$$\dot{z}_2 = z_3 + \alpha_2 - \frac{\partial\alpha_1}{\partial x_1}x_2 + w_2^T(x_1, x_2, \hat{\theta})\theta - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_1}{\partial x_r}\dot{x}_r \tag{2.13}$$

where the second regressor vector w_2 is defined as

$$w_2(x_1, x_2, \hat{\theta}) = \varphi_2 - \frac{\partial\alpha_1}{\partial x_1}\varphi_1 \tag{2.14}$$

Our task in this step is to stabilize the (z_1, z_2)-System (2.11), (2.13) with respect to

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{2.15}$$

whose derivative along the solutions of (2.11), (2.13) is

$$\dot{V}_2 = -c_1 z_1^2 + z_2\left(z_1 + z_3 + \alpha_2 - \frac{\partial \alpha_1}{\partial x_1} x_2 + w_2^T \hat{\theta} - \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} - \frac{\partial \alpha_1}{\partial x_r} \dot{x}_r\right)$$

$$+\tilde{\theta}^T \left(\tau_1 + w_2 z_2 - \Gamma^{-1} \dot{\hat{\theta}}\right) \tag{2.16}$$

We can eliminate $\hat{\theta}$ from $\dot{V}_2$with the update law $\dot{\hat{\theta}} = \Gamma \tau_2$, where

$$\tau_2(x_1, x_2, \hat{\theta}) = \tau_1 + w_2 z_2 \tag{2.17}$$

If x_3 were our actual control, we would let $z_3 \equiv 0$, we would achieve$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2$, we would choose by designing α_2 to make the bracketed term multiplying z_2 in (2.16) equal to $-c_2 z_2$, namely

$$\alpha_2(x_1, x_2, \hat{\theta}) = -z_1 - c_2 z_2 + \frac{\partial \alpha_1}{\partial x_1} x_2 - w_2^T \hat{\theta} + \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma \tau_2 + \frac{\partial \alpha_1}{\partial x_r} \dot{x}_r \tag{2.18}$$

We retain α_2 as our second timing function in the term $\Gamma\tau_2$, which replaces $\dot{\hat{\theta}}$ in (2.18). However, we do not use $\dot{\hat{\theta}} = \Gamma \tau_2$ as an update law, so that: the resulting $\dot{V}_2$ is

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 + z_2 z_3 + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \left(\Gamma \tau_2 - \dot{\hat{\theta}}\right) + \tilde{\theta}^T \left(\tau_2 - \Gamma^{-1} \dot{\hat{\theta}}\right) \tag{2.19}$$

The first two terms in $\dot{V}_2$, are negative definite, the third term will be cancelled at the next step, while the discrepancy between $\Gamma\tau_2$, and $\dot{\hat{\theta}}$ in the last two terms remains. By substituting (2.18) into (2.13), the (z_1, z_2)-subsystem becomes

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -c_1 & 1 \\ -1 & -c_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} w_1^T \\ w_2^T \end{bmatrix} \tilde{\theta} + \begin{bmatrix} 0 \\ z_3 + \frac{\partial \alpha_1}{\partial \hat{\theta}} \left(\Gamma \tau_2 - \dot{\hat{\theta}}\right) \end{bmatrix} \tag{2.20}$$

Step 3. Proceeding to the third equation in (2.1) we introduce

$$z_4 = x_4 - \alpha_3 - x_r^{(3)} \tag{2.21}$$

and rewrite $\dot{x}_3 = x_4 + \varphi_3^T(x_1, x_2, x_3)\theta$ as

$$\dot{z}_3 = z_4 + \alpha_3 - \left(\frac{\partial\alpha_2}{\partial x_1}x_2 + \frac{\partial\alpha_2}{\partial x_2}x_3\right) - \frac{\partial\alpha_2}{\partial x_r}\dot{x}_r - \frac{\partial\alpha_2}{\partial \dot{x}_r}\ddot{x}_r + w_3\left(x_1, x_2, x_3, \hat{\theta}\right)^T\theta - \frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}} \tag{2.22}$$

where the third regressor vector w_3 is defined as

$$w_3(x_1, x_1, x_3, \hat{\theta}) = \varphi_3 - \frac{\partial\alpha_2}{\partial x_1}\varphi_1 - \frac{\partial\alpha_2}{\partial x_2}\varphi_2 \tag{2.23}$$

Our task in this step is to stabilize the (z_1, z_2, z_3)-system with respect to

$$V_3 = V_2 + \frac{1}{2}z_3^2 \tag{2.24}$$

whose derivative along (2.22) and (2.23) is

$$\dot{V}_3 = -c_1z_1^2 - c_2z_2^2 + z_2z_3 + z_2\frac{\partial\alpha_1}{\partial\hat{\theta}}\left(\Gamma\tau_2 - \dot{\hat{\theta}}\right) + \tilde{\theta}^T\left(\tau_2 + z_3w_3 - \Gamma^{-1}\dot{\hat{\theta}}\right)$$

$$+z_3[z_4 + z_2 + \alpha_3 - \left(\frac{\partial\alpha_2}{\partial x_1}x_2 + \frac{\partial\alpha_2}{\partial x_2}x_3\right) - (\frac{\partial\alpha_2}{\partial x_r}\dot{x}_r + \frac{\partial\alpha_2}{\partial \dot{x}_r}\ddot{x}_r) + w_3{}^T\hat{\theta} - \frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}}] \tag{2.25}$$

We can eliminate $\hat{\theta}$ from $\dot{V}_3$ with the update law $\dot{\hat{\theta}} = \Gamma\tau_3$, where τ_3 is our tuning function

$$\tau_3\left(x_1, x_2, x_3, \hat{\theta}\right) = \tau_2 + w_3z_3 = [w_1, \quad w_2, \quad w_3]\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \tag{2.26}$$

If x_3 were our actual control, we would let $z_4 \equiv 0$, we would achieve $\dot{V}_3 = -c_1z_1^2 - c_2z_2^2 - c_3z_3^2$, we would choose by designing α_3 to make the bracketed term multiplying z_3 equal to $-c_3z_3$, namely

$$\alpha_3 = -z_2 - c_3z_3 + \left(\frac{\partial\alpha_2}{\partial x_1}x_2 + \frac{\partial\alpha_2}{\partial x_2}x_3\right) + (\frac{\partial\alpha_2}{\partial x_r}x_r^{(1)} + \frac{\partial\alpha_2}{\partial \dot{x}_r}x_r^{(2)})$$

$$-w_3{}^T\hat{\theta} + \frac{\partial\alpha_2}{\partial\hat{\theta}}\Gamma\tau_3 + v_3 \tag{2.27}$$

wherev_3 is a correction term yet to be chosen. Substituting (2.26) into (4.24), and noting that

$$\dot{\hat{\theta}} - \Gamma\tau_2 = \dot{\hat{\theta}} - \Gamma\tau_2 + \Gamma\tau_3 - \Gamma\tau_3$$

$$= \dot{\hat{\theta}} - \Gamma\tau_3 + \Gamma z_3w_3 \tag{2.28}$$

(2.24) is rewritten as

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 + \; + z_3\left(v_3 - \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma w_3 z_2\right) + \; z_3 z_4$$

$$+ \left(z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} + z_3 \frac{\partial \alpha_2}{\partial \hat{\theta}}\right)\left(\Gamma \tau_3 - \dot{\hat{\theta}}\right) + \tilde{\theta}^T \left(\tau_3 - \Gamma^{-1}\dot{\hat{\theta}}\right) \tag{2.29}$$

Step i, Repeating the procedure in a recursive manner, we derive the i-th tracking error for z_i

$$\dot{z}_i = z_{i+1} + \alpha_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} + w_i^T \hat{\theta} - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \; - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_r^{(j-1)}} x_r^{(j)} \tag{2.30}$$

Where the i-th regressor is

$$w_i\left(x_i, \hat{\theta}, x_r, x_r^{(i-2)}\right) = \varphi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_k \tag{2.31}$$

We select the stabilizing function α_i

$$\alpha_i = -c_i z_i - z_{i-1} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} - w_i^T \hat{\theta} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_r^{(j-1)}} x_r^{(j)} + v_i \tag{2.32}$$

and tuning function

$$\tau_i \; = \tau_{i-1} + w_i z_i \tag{2.33}$$

Our task in this step is to stabilize the $(z_1, \dots, z_i)$-system with respect to

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 \tag{2.34}$$

Its derivative is given as

$$\dot{V}_i = -\sum_{k=1}^{i-1} c_k z_k^2 + z_i \left[z_{i+1} + v_i - \sum_{k=1}^{i-2} z_{k+1} \frac{\partial \alpha_k}{\partial \hat{\theta}} \Gamma w_i \right]$$

$$+ \left(\sum_{k=1}^{i-2} z_{k+1} \frac{\partial \alpha_k}{\partial \hat{\theta}} \Gamma w_i \right) \left(\Gamma \tau_i - \dot{\hat{\theta}} \right) + \hat{\theta}^T \left(\tau_i - \Gamma^{-1} \dot{\hat{\theta}} \right) \tag{2.35}$$

and represent the $(z_1, \ldots, z_i)$-subsystem as

$$\begin{bmatrix} \dot{z}_1 \\ \vdots \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -c_1 & 1 & 0 & \ldots & 0 & 0 \\ -1 & -c_2 & 1 + \sigma_{23} & \ldots & \sigma_{2n} & 0 \\ 0 & -1 - \sigma_{23} & \ddots & \ddots & \vdots & 0 \\ \vdots & \vdots & \ddots & \ddots & 1 + \sigma_{n-1,n} & 0 \\ 0 & -\sigma_{2n} & \cdots & -1 - \sigma_{n-1,n} & -c_{i-1} & 1 \\ 0 & 0 & \cdots & 0 & -1 & -c_i \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_2 \end{bmatrix}$$

$$+ \begin{bmatrix} w_1^T \\ \vdots \\ w_2^T \end{bmatrix} \tilde{\theta} + \begin{bmatrix} 0 \\ \sigma_{2,i} z_i \\ \vdots \\ \sigma_{i-1,i} z_i \\ v_i \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ z_{i+1} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\partial \alpha_1}{\partial \hat{\theta}} \\ \vdots \\ \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \end{bmatrix} \left(\Gamma \tau_i - \dot{\hat{\theta}} \right) \tag{2.36}$$

where

$$\sigma_{jk} = -\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma w_k \tag{2.37}$$

Now the correction term is chosen as

$$v_i\left(x_1, \ldots, x_i, \hat{\theta}\right) = \sum_{k=1}^{i-2} z_{k+1} \frac{\partial \alpha_k}{\partial \hat{\theta}} \Gamma w_i = -\sum_{k=2}^{j-1} \sigma_{k,i} z_k \tag{2.38}$$

Because we do not use $\dot{\hat{\theta}} = \Gamma \tau_i$, as an update law, the resulting $\dot{V}_i$, is

$$\dot{V}_i = -\sum_{k=1}^{i} c_k z_k^2 + z_i z_{i+1} + \left(\sum_{k=1}^{i-1} z_{k+1} \frac{\partial \alpha_k}{\partial \hat{\theta}}\right) \left(\Gamma \tau_i - \dot{\hat{\theta}} \right) + \tilde{\theta}^T \left(\tau_i - \Gamma^{-1} \dot{\hat{\theta}} \right) \tag{2.39}$$

and the $(z_1, \ldots, z_i)$-subsystem becomes

$$\begin{bmatrix}\dot{z}_1\\ \vdots \\ \dot{z}_2\end{bmatrix} = \begin{bmatrix} -c_1 & 1 & 0 & \dots & 0 & 0 \\ -1 & -c_2 & 1+\sigma_{23} & \dots & \sigma_{2n} & 0 \\ 0 & -1-\sigma_{23} & \ddots & \ddots & \vdots & 0 \\ \vdots & \vdots & \ddots & \ddots & 1+\sigma_{n-1,n} & 0 \\ 0 & -\sigma_{2n} & \cdots & -1-\sigma_{n-1,n} & -c_{i-1} & 1 \\ 0 & 0 & \cdots & 0 & -1 & -c_i \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_2 \end{bmatrix}$$

$$+ \begin{bmatrix} w_1^T \\ \vdots \\ w_2^T \end{bmatrix} \tilde{\theta} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ z_{i+1} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\partial \alpha_1}{\partial \hat{\theta}} \\ \vdots \\ \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \end{bmatrix} \left(\Gamma \tau_i - \dot{\hat{\theta}}\right) \tag{2.40}$$

Step n. At the final step, we introduce

$$z_n = x_n - \alpha_{n-1} - x_r^{(n-1)} \tag{2.41}$$

and rewrite $\dot{x}_n = \beta(x)u + \varphi_n^T(x)\theta$ as

$$\dot{z}_n = \beta(x)u + \varphi_n^T\theta - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k}\left(x_{k+1} + \varphi_k^T\right) - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_r^{(k-1)}} x_r^{(k)} - x_r^{(n)} \tag{2.42}$$

where the last regressor vector is defined as

$$w_n\left(x, \hat{\theta}, x_r, x_r^{(n)}\right) = \phi_n - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k}\phi_k \tag{2.43}$$

In this equation, the actual control input is at our disposal. We are finally in the position to design our actual update law $\dot{\hat{\theta}} = \Gamma\tau_n$ and feedback control u to stabilize the full z-system with respect to

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 = \frac{1}{2}z^T z + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} \tag{2.44}$$

Our goal is to make $\dot{V}_n$ nonpositive:

$$\dot{V}_n = -\sum_{k=1}^{n} c_k z_k^2 + \left(\sum_{k=2}^{n} z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\theta}}\right)\left(\Gamma\tau_n - \dot{\hat{\theta}}\right) + \tilde{\theta}^T\left(\tau_{n-1} + w_n z_n - \Gamma^{-1}\dot{\hat{\theta}}\right)$$

$$+ z_n\left(z_{n-1} + \beta u - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}x_{k+1} + w_n^T\hat{\theta} - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_r^{(k-1)}}x_r^{(k)} - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\right) \qquad (2.45)$$

To eliminate $\hat{\theta}$ from $\dot{V}_n$ we choose the update law

$$\dot{\hat{\theta}} = \Gamma\tau_n(z,\hat{\theta}) = \Gamma\tau_{n-1} + \Gamma w_n z_n$$

$$= \Gamma W(z,\hat{\theta})z \qquad (2.46)$$

where the regressor matrix $W$is composed of the regressor vectors $w_1, \dots, w_n$:

$$W(z,\hat{\theta}) = [w_1, \dots, w_n] \qquad (2.47)$$

We choose the control u to make the bracketed term multiplying $z_n$equal to $-c_n z_n$:

$$u = \frac{1}{\beta}\left(-z_{n-1} - c_n z_n + \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}x_{k+1} - w_n^T z_n + \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\Gamma\tau_n + v_n\right) \qquad (2.48)$$

where v_n is a correction term yet to be chosen. With (2.48). $\dot{V}_n$ becomes

$$\dot{V}_n = -\sum_{k=1}^{n-1} c_k z_k^2 + \left(\sum_{k=1}^{n-2} z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\theta}}\right)\left(\Gamma\tau_{n-1} - \dot{\hat{\theta}}\right) + z_n v_n \qquad (2.49)$$

Then, noting that

$$\dot{\hat{\theta}} - \Gamma\tau_{n-1} = \Gamma\tau_n - \Gamma\tau_{n-1}$$

$$= \Gamma w_n z_n \qquad (2.50)$$

We rewrite $\dot{V}_n$ as

$$\dot{V}_n = -\sum_{k=1}^{n-1} c_k z_k^2 + z_n \left(v_n - \sum_{k=1}^{n-2} z_{k+1} \frac{\partial \alpha_k}{\partial \hat{\theta}} \Gamma w_n \right) \tag{2.51}$$

Now the correction term v_n is chosen as

$$v_n(x, \hat{\theta}) = \sum_{k=1}^{n-2} z_{k+1} \frac{\partial \alpha_k}{\partial \hat{\theta}} \Gamma w_n = \sum_{k=2}^{n-1} \sigma_{k,n} z_k \tag{2.52}$$

We have thus reached our goal :

$$\dot{V}_n = -\sum_{k=1}^{n-1} c_k z_k^2 \tag{2.53}$$

The overall closed-loop system is

$$\dot{z}_n = A_z(z, \hat{\theta}) z + W(z, \hat{\theta})^T \tilde{\theta} \tag{2.54}$$

$$\dot{\hat{\theta}} = \Gamma W(z, \hat{\theta}) z \tag{2.55}$$

where

$$A_z = \begin{bmatrix} -c_1 & 1 & 0 & & \cdots & 0 \\ -1 & -c_2 & & 1 + \sigma_{23} & \cdots & \sigma_{2n} \\ 0 & -1 - \sigma_{23} & \ddots & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & & 1 + \sigma_{n-1,n} \\ 0 & -\sigma_{2n} & \cdots & -1 - \sigma_{n-1,n} & & -c_n \end{bmatrix} \tag{2.56}$$

We can summarize the tracking tuning function design for nonlinear system as follows :

Coordinate transformation

$$z_i = x_i - \alpha_{i-1} - x_r^{(i-1)} \tag{2.57}$$

Regressor

$$w_i\left(x_i, \hat{\theta}, x_r, \bar{x}_r^{(i-2)}\right) = \varphi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_k \qquad i = 1, \dots, n \tag{2.58}$$

Tuning functions:

$$\tau_i\left(\bar{x}_i, \hat{\theta}, \bar{x}_r^{(i-1)}\right) = \tau_{i-1} + w_i z_i\,, \qquad\qquad i = 1, \dots, n \tag{2.59}$$

Stabilizing functions:

$$\alpha_i\left(\bar{x}_i, \hat{\theta}, \bar{x}_r^{(i)}\right) = -c_i z_i - z_{i-1} + w_i^T \hat{\theta} + \sum_{k=1}^{i-1}\left(\frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1}}{\partial x_r^{(k-1)}} x_r^{(k)}\right)$$

$$+ \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_i z_i \tag{2.60}$$

$\bar{x}_i = (x_1, \dots, x_i)$, $\quad \bar{x}_r^{(i)} = (x_r, \dot{x}_r, \dots, x_r^{(i)})$

Adaptive control law:

$$u = \frac{1}{\beta(x)}\left[\alpha_n\left(\bar{x}_i, \hat{\theta}, \bar{x}_r^{(n-1)}\right) + x_r^{(n)}\right] \tag{2.61}$$

Parameter update law:

$$\dot{\hat{\theta}} = \Gamma \tau_n\left(x, \hat{\theta}, \bar{x}_r^{(n-1)}\right) \tag{2.62}$$

Remark:

The design for set-point regulation is only a minor modification of the tracking procedure (see [1]; sec 4.2). As before, the first z-variable is the tracking error $z_1 = x_1 - x_{rs}$.However, because the reference signal x_{rs} is a constant, its derivative disappears in the definition of the i-th error state $z_i, i = 1, \dots, n$.

The only change this creates in the design is the elimination of the term $\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_r^{(k-1)}} x_r^{(k)}$ in the definition of α_i

2.2 Stability Analysis

For the adaptive scheme developed in the previous subsection, we establish the following results.

Theorem 2.1 [1]:

The closed-loop adaptive system consisting of the plant (2.1), the controller (2.61), and the update law (2.62) has a globally uniformly stable equilibrium $(z_i, \tilde{\theta}) = 0$ and $lim_{t\to\infty} z(t) = 0$ which means, in particular, that global asymptotic tracking is achieved :

$$\lim_{t\to\infty}[x(t) - x_r(t)] = 0 \tag{2.63}$$

Moreover, if $\lim_{t\to\infty} x_r^{(i)}(t) = 0, \quad i = 0, \dots, n-1$ and $\varphi(0) = 0$, then $\lim_{t\to\infty} x(t) = 0$.

Proof. Denote $c_0 = \min_{1\le i\le n} c_i$. The derivative of the Lyapunov function

$$V = \frac{1}{2}z^T z + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} \tag{2.64}$$

along the solutions of (2.61) and (2.62) is

$$\dot{V} = -\sum_{i=1}^{n} c_i z_i^2 \le c_0 |z|^2 \tag{2.65}$$

which proves that the equilibrium $(z_i, \tilde{\theta}) = 0$ is globally uniformly stable. From the LaSalle-Yoshizawa theorem (Theorem 2.1), it further follows that, as $t \to \infty$, all the solutions converge to the manifold $z = 0$. From the definitions in (2.57)-(2.58) we conclude that, if $\lim_{t\to\infty} x_r^{(i)}(t) = 0, i = 0, \dots, n-1$ and $\Phi(0) = 0$, then $\lim_{t\to\infty} x(t) = 0$.

The proof of Theorem 2.1 reveals the stabilization mechanism employed in the tuning functions design. The update law is chosen so as to make the derivative of the Lyapunov function nonpositive. The update law is fast because it does not use any form of normalization common in traditional certainty equivalence adaptive control. The speed of adaptation is dictated by the speed of the nonlinear behavior captured by the Lyapunov function. The tuning functions controller incorporates the knowledge of the update law and eliminates the disturbing effect of the parameter estimation transients on the error system. The controller and the update law designs are interlaced.

2.3 Illustrative Example

We consider the following second order nonlinear system.

$$\dot{x}_1 = x_2 + a_1 x_1^2$$

$$\dot{x}_2 = u + a_2 x_2 \tag{2.66}$$

where $\varphi_1 = [x_1^2, 0], \varphi_2 = [0, x_2],, \theta = [a_1, a_2]$ and $a_1 = 1, a_2 = 2$. The objective of the controller is to make the first state x_1 track a desired reference and stabilize system (2.66). The controller (2.61) is implemented

Our control objective is to asymptotically track a given reference $x_r\,(t)$ with the first state x_1. We use the error variables

$$z_1 = x_1 - x_r \tag{2.67}$$

$$z_2 = x_2 - \alpha_1 - x_r^{(1)} \tag{2.68}$$

And derive the stabilizing functions

$$\alpha_1 = -c_1 z_1 \tag{2.69}$$

$$\alpha_2 = -c_2 z_2 - z_1 + \varphi_1^T \hat{\theta} - c_1 (x_1 - x_r^{(1)}) \tag{2.70}$$

The design procedure from Section 4.5.1 results in an adaptive controller consisting of the control law

$$u = \alpha_2 + x_r^{(2)} \tag{2.71}$$

Tuning functions :

$$\tau_1 = \varphi_1 z_1 \tag{2.72}$$

$$\tau_2 = \tau_1 + (\varphi_2 + c_1 \varphi_1) z_2 \tag{2.73}$$

and the update law:

$$\dot{\hat{\theta}} = \Gamma\tau_2 \tag{2.74}$$

The design parameters are chosen $c_i = 1, i = 1,2$. The tracking of the reference signal $x_r(t) = \sin(2t)$ and its related control effort are presented in Fig 2.1 and Fig 2.2 respectively.

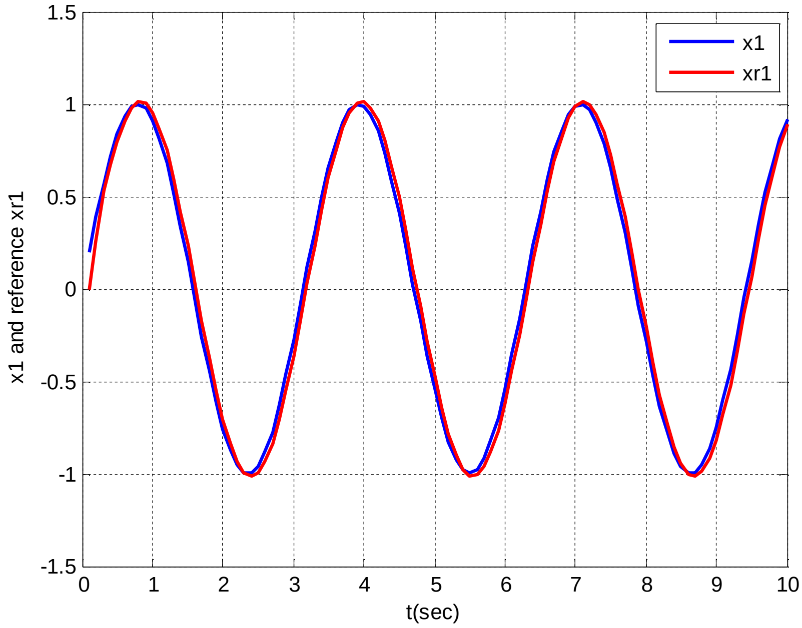

Fig 2.1 tracking response of the system using $sin(2t)$ as reference input

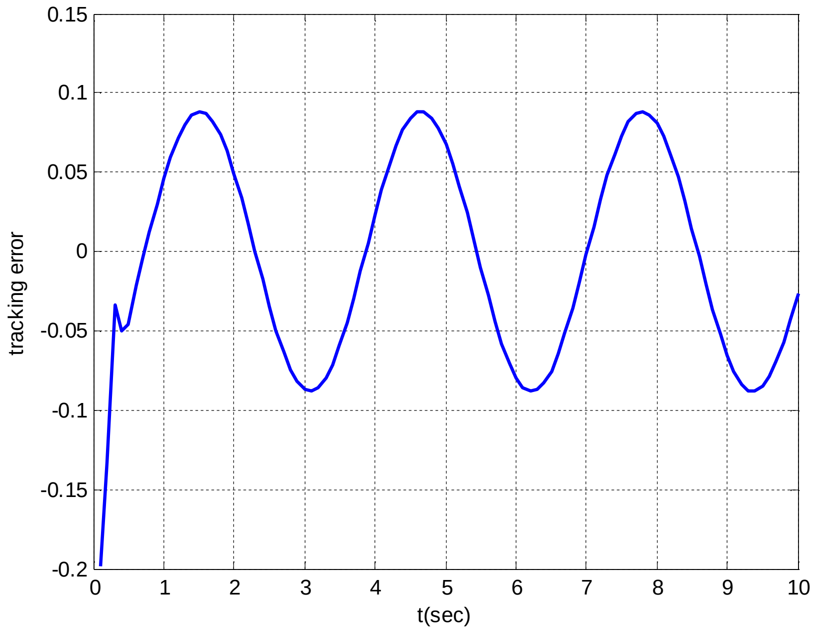

Fig 2.2 Tracking error of the system using $sin(2t)$ as reference input

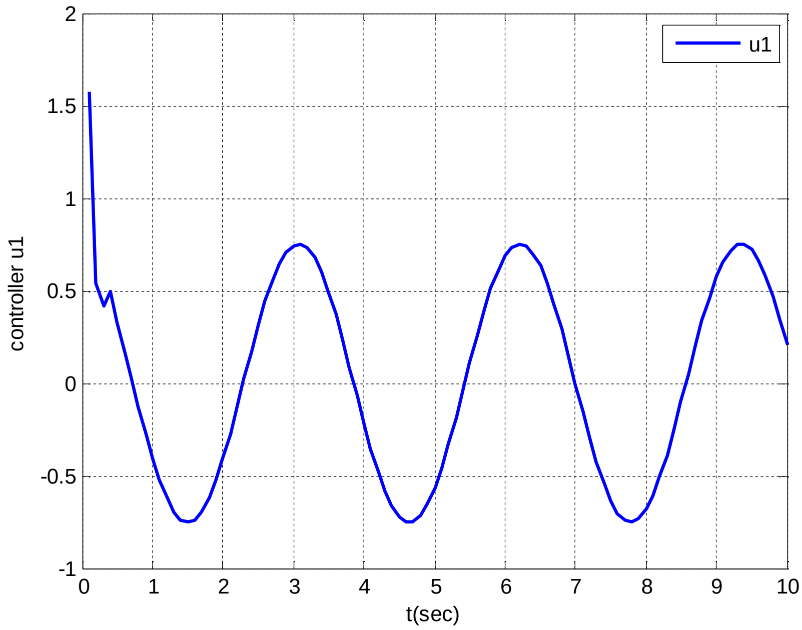

Fig 2.3 controller input of the system using $sin(2t)$ as reference input

The controller designed in this section achieves the goals of stabilization and tracking. The proof of these properties is a direct consequence of the recursive procedure, because a Lyapunov function is constructed for the entire system including the parameter estimates. The overparametrization problem is overcomed by using tuning functions. The number of parameter estimates are equal to the number of unknown parameters.

Chapter 3

New Extension Design

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In this chapter, we develop a new extension of the adaptive backstepping design presented in [1]. The specific model is presented at first, it is frequent in various range of applications from electric motors and manipulator robots to flight dynamics. The tracking objective is achieved as well as the stability and boundedness of states.

In [1] an adaptive backstepping design was presented for the class of parametric strict-feedback systems and it was extended to a three specific models strict-feedback systems with unknown virtual control coefficients, block-strict-feedback systems and parametric pure-feedback systems. We extend the design to a modified model of the first case. The description of the model considered for this extension is presented in the next part.

3.1 Problem Formulation

The system presented is for the class of parametric strict-feedback with unknown virtual control and has an unknown virtual function in the m-th order, the high frequency gain is considered and assumed with known sign.

The new extension is applied to the system of the form:

$$\dot{x}_i = x_{i+1} + \varphi_i^T(x_1, \cdots, x_i)\theta, \qquad i = 1, \dots, m-1, m+1, \dots, n-1$$

$$\dot{x}_m = b_m \beta(\bar{x}) x_{m+1} + \varphi_m^T(x_1, \cdots, x_m)\theta$$

$$\dot{x}_n = b_n u + \varphi_n^T(x)\theta \tag{3.1}$$

where $x = [x_1, \cdots, x_n]^T \in R^n$, the vector $\theta \in R^r$, is a vector of unknown constant parameters , $\Phi = [\varphi_1, \dots, \varphi_n]$ and $\beta(\bar{x})$; $\bar{x} = [x_1, \cdots, x_m]$ are known smooth nonlinear functions taking arguments in $\mathbb{R}^n$, and $\beta(\bar{x}) \neq 0$, $\forall\, x \in \mathbb{R}^n$.

In equation (3.1) b_n and $b_m, m < n.$ are the two unknown coefficients. From step m , the design procedure for this case differs considerably from the procedure in chapter 2.

For the development of control laws, the following assumptions are made

Assumption 1: The reference signal x_r and its first n order derivative are known, piecewise continuous and bounded.

Assumption 2: The sign of b_m and b_n are known.

Assumption 3: The nonlinear function $\beta(\bar{x})$ is known, piecewise continuous, derivative and bounded.

3.2 Design Procedure

We now need $\hat{b}_n, \hat{b}_m, \hat{\varrho}$ and $\hat{\lambda}$ the estimates of $b_n, b_m, \varrho = {}^1\!/_{b_m}$ and $\lambda = {}^1\!/_{b_n}$ respectively. The estimate ϱ and λ are introduced to avoid the division by $\hat{b}_m(\mathrm{t})$ or $\hat{b}_n(\mathrm{t})$which can occasionally take value zero. The new complete design procedure is given by the following expressions (which $z_0 = 0, \alpha_0 = 0, \tau_0 = 0$):

Coordinate transformation:

$$z_i = x_i - \alpha_{i-1} - x_r^{(i-1)}, \qquad i = 1, \dots, m \tag{3.2}$$

$$z_j = x_j - \alpha_{j-1} - \frac{\hat{\varrho}}{\beta(x)} x_r^{(j-1)}, \qquad j = m+1, \dots, n \tag{3.3}$$

Regressor:

$$w_i = \varphi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_j , \qquad i = 1, \dots, m \tag{3.4}$$

$$w_{m+1} = \varphi_{m+1} - \sum_{k=1}^{m} \frac{\partial}{\partial x_k} \left(\frac{\hat{\varrho}}{\beta(x)} x_r^{(m)} + \alpha_m\right) \varphi_k$$

$$= \varphi_{m+1} - \sum_{k=1}^{m} \frac{\partial \alpha_m}{\partial x_k} \varphi_k - \sum_{k=1}^{m} \frac{\partial}{\partial x_k} \left(\frac{1}{\beta(x)}\right) \varphi_k \, \hat{\varrho} x_r^{(m)}$$

$$w_j = \varphi_j - \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_k} \varphi_k - \sum_{k=1}^{j-1} \frac{\partial}{\partial x_k} \left(\frac{1}{\beta(x)}\right) \varphi_k \hat{\varrho} x_r^{(j-1)} , \quad j = m+1, \dots, n \tag{3.5}$$

Tuning functions for $\hat{\theta}$:

$$\tau_i = \tau_{i-1} + w_i z_i , \qquad i = 1, \dots, n \tag{3.6}$$

Tuning functions for $\hat{b}_m$:

$$\pi_m = z_{m+1} z_m \tag{3.7}$$

$$\pi_j = \pi_{j-1} - \frac{\partial}{\partial x_m} \left(\frac{\hat{\varrho}}{\beta(x)} x_r^{(j-1)} + \alpha_m\right) \beta(x) x_{m+1} z_j$$

$$= \pi_{j-1} - \frac{\partial \alpha_m}{\partial x_m} \beta(x) x_{m+1} z_j - \frac{\partial}{\partial x_m} \left(\frac{1}{\beta(x)}\right) x_r^{(j-1)} \beta(x) x_{m+1} z_j$$

$$j = m+1, \dots, n \tag{3.8}$$

Stabilizing functions:

$$\alpha_i \left(x_i, \hat{\theta}, x_r^{(i)}\right) = \bar{\alpha}_i, \qquad i = 1, \dots, m-1 \tag{3.9}$$

$$\alpha_m \left(x_m, \hat{\theta}, x_r^{(m-1)}, \hat{\varrho}\right) = \frac{\hat{\varrho}}{\beta(x)} \bar{\alpha}_m, \tag{3.10}$$

$$\alpha_j \left(x_j, \hat{\theta}, x_r^{(j-1)}, \hat{b}_m, \hat{\varrho}\right) = \bar{\alpha}_j, \qquad j = m+1, \dots, n \tag{3.11}$$

$$\bar{\alpha}_i = -c_i z_i - z_{i-1} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} - w_i^T \hat{\theta} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i$$

$$+ \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_i z_k + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_r^{(k-1)}} x_r^{(k)}, \qquad i = 1, \dots, m \tag{3.12}$$

$$\bar{\alpha}_{m+1} = -c_{m+1} z_{m+1} - \hat{b}_m \beta(x) z_m + \sum_{k=1}^{m-1} \frac{\partial \alpha_m}{\partial x_k} x_{k+1} + \hat{b}_m \frac{\partial \alpha_m}{\partial x_m} \beta(x) x_{m+1} - w_{m+1}^T \hat{\theta}$$

$$+ \frac{\partial \alpha_m}{\partial \hat{\theta}} \Gamma \tau_{m+1} + \left(\frac{x_r^{(m)}}{\beta(x)} + \frac{\partial \alpha_m}{\partial \hat{\varrho}} \right) \dot{\hat{\varrho}} + \sum_{k=2}^{m} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma w_{m+1} z_k + \sum_{k=1}^{m} \frac{\partial \alpha_m}{\partial x_r^{(k-1)}} x_r^{(k)}$$

$$+ \hat{\varrho} \left(\sum_{k=1}^{n} \frac{\partial}{\partial x_k} \left(\frac{1}{\beta(\bar{x})} \right) \left(\varphi_{k+1}^T \tilde{\theta} + \varphi_{k+1}^T \hat{\theta} \right) x_r^{(m)} + \sum_{\substack{k=1 \\ k \neq m}}^{n} \frac{\partial}{\partial x_k} \left(\frac{1}{\beta(\bar{x})} \right) x_{k+1} \right)$$

$$+ \frac{\partial}{\partial x_m} \left(\frac{1}{\beta(x)} \right) \hat{b}_m \hat{\varrho} x_r^{(m)} \beta(x) x_{m+1} \tag{3.13}$$

$$\bar{\alpha}_j = -c_j z_j - z_{j-1} - w_j^T \hat{\theta} + \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \tau_j + \sum_{\substack{k=1 \\ k \neq m}}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_k} x_{k+1} + \hat{b}_m \beta(x) \frac{\partial \alpha_{j-1}}{\partial x_m} x_{m+1}$$

$$+ \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_r^{(k-1)}} x_r^{(k)} + \sum_{k=2}^{j-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma w_i z_k + \frac{\partial \alpha_{j-1}}{\partial \hat{b}_m} \gamma \pi_j + \left(\frac{x_r^{(j-1)}}{\beta(x)} + \frac{\partial \alpha_{j-1}}{\partial \hat{\varrho}} \right) \dot{\hat{\varrho}}$$

$$+ \hat{\varrho} \left(\sum_{k=1}^{n} \frac{\partial}{\partial x_k} \left(\frac{1}{\beta(\bar{x})} \right) \left(\varphi_{k+1}^T \tilde{\theta} + \varphi_{k+1}^T \hat{\theta} \right) x_r^{(j-1)} + \sum_{\substack{k=1 \\ k \neq m}}^{n} \frac{\partial}{\partial x_k} \left(\frac{1}{\beta(\bar{x})} \right) x_{k+1} \right)$$

$$+ \frac{\partial}{\partial x_m} \left(\frac{1}{\beta(x)} \right) \hat{b}_m \hat{\varrho} x_r^{(m)} \beta(x) x_{m+1} + \sum_{k=m+1}^{j-1} \frac{\partial \alpha_{k-1}}{\partial \hat{b}_m} \gamma \frac{\partial \alpha_{j-1}}{\partial x_m} x_{m+1} z_k,$$

$$j = m + 2, \dots, n \tag{3.14}$$

Adaptive control law :

$$u = \hat{\lambda} \left(\alpha_n + \frac{\hat{\varrho}}{\beta(x)} x_r^{(n)} \right) \tag{3.15}$$

Parameter update law:

$$\dot{\hat{\theta}} = \Gamma\tau_n \tag{3.16}$$

$$\dot{\hat{b}}_m = \gamma\pi_n = \gamma\left[z_{m+1}z_m - \sum_{j=m+1}^{n}\frac{\partial\alpha_{j-1}}{\partial x_m}x_{m+1}z_j\right] \tag{3.17}$$

$$\dot{\hat{\varrho}} = -\gamma sign(b_m)(\bar{\alpha}_m + x_r^{(m)})z_m \tag{3.18}$$

$$\dot{\hat{\lambda}} = -\gamma sign(b_n)(\bar{\alpha}_n + x_r^{(n)})z_n \tag{3.19}$$

3.3 Error System

In this subsection, we illustrate the error system results during the design of each step of the adaptive backstepping controller (see Appendix A).

$$\dot{z}_i = -c_i z_i - z_{i-1} + z_{i+1} - \sum_{k=2}^{i-1}\sigma_{ki}z_k + \sum_{k=i+1}^{n}\sigma_{ik}z_k + \tilde{\theta}^T w_i$$

$$i = 1, \dots, m-1 \tag{3.20}$$

$$\dot{z}_m = -c_m z_m - z_{m-1} + \hat{b}_m\beta(\bar{x})z_{m+1} - \sum_{k=2}^{m-1}\sigma_{km}z_k + \sum_{k=m+1}^{n}\sigma_{mk}z_k + w_m^T\tilde{\theta}$$
$$-b_m(\bar{\alpha}_m + x_r^{(m)})\tilde{\varrho} + \tilde{b}_m\beta(\bar{x})z_{m+1} \tag{3.21}$$

$$\dot{z}_{m+1} = -c_{m+1}z_{m+1} - \sum_{k=2}^{m}\sigma_{k,m+1}z_k - z_{m+2} - \hat{b}_m z_m + \sum_{k=m+2}^{n}\sigma_{m+1,k}z_k$$
$$+w_{m+1}^T\tilde{\theta} + \tilde{b}_m(\frac{\partial}{\partial x_m}\left(\frac{1}{\beta(\bar{x})}\right)\hat{\varrho}x_r^{(m)} - \frac{\partial\alpha_m}{\partial x_m})\beta(\bar{x})x_{m+1} \tag{3.22}$$

$$\dot{z}_j = -c_j z_j - \sum_{k=2}^{j-1}\sigma_{kj}z_k - z_{j-1} + z_{j+1} + \sum_{k=j+1}^{n}\sigma_{jk}z_k + w_j^T\tilde{\theta}$$
$$-\tilde{b}_m\left(\frac{\partial}{\partial x_m}\left(\frac{1}{\beta(\bar{x})}\right)\hat{\varrho}x_r^{(j-1)} - \frac{\partial\alpha_{j-1}}{\partial x_m}\right)\beta(\bar{x})x_{m+1}, \qquad i = m+2, \dots, n-1 \tag{3.23}$$

$$\dot{z}_n \quad = -c_j z_j - \sum_{k=2}^{n-1} \sigma_{kj} z_k - z_{n-1} + \sum_{k=j+1}^{n} \sigma_{jk} z_k + w_j^T \tilde{\theta} - b_n(\alpha_n + \frac{\hat{\varrho}}{\beta(x)} x_r^{(n)})\hat{\lambda}$$

$$-\tilde{b}_m \left(\frac{\partial}{\partial x_m} \left(\frac{1}{\beta(\bar{x})} \right) \hat{\varrho} x_r^{(j-1)} - \frac{\partial \alpha_{j-1}}{\partial x_m} \right) \beta(\bar{x}) x_{m+1} , \tag{3.24}$$

where σ_{ik} is defined for $k = i + 1, \dots , n$ as

$$\sigma_{ik} = \begin{cases} 0, \quad i = 1 \\ -\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_k, \qquad i = 2, \dots , m + 1 \\ -\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_k + \frac{\partial \alpha_{i-1}}{\partial \hat{b}_m} \gamma \frac{\partial \alpha_{k-1}}{\partial x_m} x_{m+1} \qquad i = m + 2, \dots , n - 1 \end{cases} \tag{3.25}$$

3.4 Stability Analysis

A Lyapunov function for this system is

$$V = \frac{1}{2} z^T z + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{2\gamma} \tilde{b}_m^2 + \frac{|b_m|}{2\gamma} \tilde{\varrho}^2 + \frac{|b_n|}{2\gamma} \tilde{\lambda}^2 \tag{3.26}$$

The derivative is:

$$\dot{V} = z\dot{z} + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} + \frac{\tilde{b}_m}{\gamma} \dot{\tilde{b}}_m + \frac{\tilde{b}_n}{\gamma} \dot{\tilde{b}}_n + \frac{|b_m|}{\gamma} \tilde{\varrho} \dot{\tilde{\varrho}} + \frac{|b_n|}{\gamma} \tilde{\lambda} \dot{\tilde{\lambda}} \tag{3.27}$$

Its derivative along the solutions of (3.16)-(3.19) and (3.20)-(3.24),

$$\dot{V} = -\sum_{i=1}^{n} c_i z_i^2 \leq 0 \tag{3.28}$$

From the La Salle's Theorem (chapter 1), this Lyapunov function provides the proof of uniform stability, such that $z_1,\dots,z_n$, $\hat{\theta}$,$\hat{p}$ and $\hat{\lambda}$ are bounded and , $z_i \to 0$, $i = 1, \dots , n$. This further implies that $\lim_{t\to\infty}(x_1 - x_r) = 0$. Since $x_1 = z_1 + x_r$, x_1 is also bounded from the boundedness of z_1 and x_r. The boundedness of x_2 follows from boundedness of $\dot{x}_r$ and $\alpha_1$and the fact that $x_2 = z_2 + \alpha_1 + \dot{x}_r$. Similarly, the boundedness of x_i , $(i = 3, \dots , n)$ can be ensured from the boundedness of $x_r^{(i-1)}$, α_i , $\hat{\varrho}$ and $\beta(\bar{x})$ .Combining this with (3.15) we conclude that the control $u(t)$ is also bounded.Therefore boundedness of all signals and asymptotic tracking is achieved.

Chapter 4

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Application to Electro-Hydraulic Servo System

In this chapter the non-linear dynamic model of electro-hydraulic servo system is presented. A complete fifth order nonlinear dynamic with considering the valve dynamics which achieves the tracking performance is presented [55], the friction force is nonlinear. An adaptive backstepping controller is developed that ensure the tracking error signals asymptotically converge to zero despite the uncertainties in the system.

The proposed controller handles internal leakage and unknown nonlinear friction in the cylinder. Simulation results are presented verifying the effectiveness of the developed controller.

4.1 Introduction

When closed-loop hydraulic control systems first began to appear in industry, the applications were generally those in which very high performance was required. While hydraulic servo systems are still heavily used in high-performance applications such as the machine-tool industry, they are beginning to gain wide acceptance in a variety of industries. Examples are material handling, mobile equipment, plastics, steel plants, mining, oil exploration and automotive testing.

Closed loop servo drive technology is increasingly becoming the norm in machine automation, where the operators are demanding greater precision, faster operation and simpler adjustment. There is also an expectation that the price of increasing the level of automation should be contained within acceptable limits.

4.1.1 What is a servo?

In its simplest form a servo or a servomechanism is a control system which measures its own output and forces the output to quickly and accurately follow a command signal. In this way, the effect of anomalies in the control device itself and in the load can be minimized as well as the influence of external disturbances. A servomechanism can be designed to control almost any physical quantities, e.g. motion, force, pressure, temperature, electrical voltage or current.

4.1.2 Technology comparisons

The potential for alternative technologies should be assessed in the light of the well-known capabilities of electro-pneumatic and electro-mechanical servos. High performance actuation system is characterized by wide bandwidth frequency response, low resolution and high stiffness. Additional requirements may include demanding duty cycles and minimization of size and weight. The last mentioned requirements are of special interests in aerospace applications. The most important selection criteria can be summarized as follows:

- Customer performance
- Cost
- Size and weight
- Duty cycle
- Environment: vibration, shock, temperature, etc.

The performance available with electro-hydraulic servos encompasses every industrial and aerospace application. As indicated in Figure 4.1 electro-hydraulic servos will cover applications with higher performance then electro-mechanical and electro-pneumatic servos. This is easily explained because electro-hydraulic servo systems have been designed and developed to accomplish essentially every task that has appeared.

The figure indicates that applications in the lower range of power and dynamic response may also be satisfied with electro-pneumatic servos. However, the best choice is always determined by considerations, such as those selection criteria discussed above.

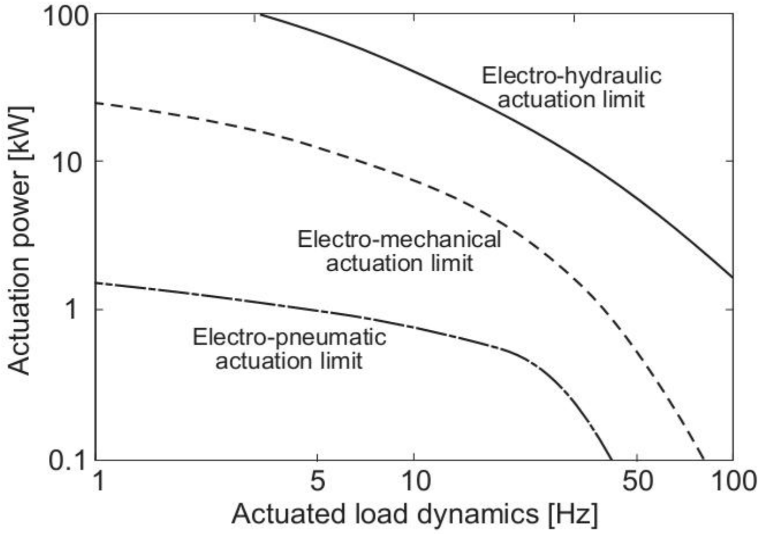

Fig 4.1 Typical performance characteristics for different types of servo actuators

In most applications the aspect of cost is generally dominant. Experience indicates that electro-mechanical or electro-pneumatic actuators tends to have lower cost than electro-hydraulic actuators in the low performance range. This cost difference rapidly dissipates for applications that require high power and/or high dynamic response.

In comparing costs, one must be careful to consider the total cost of entire servo-actuation system. The higher cost of an electro-hydraulic servo often results from the power conversion equipment needed to provide high pressure fluid with low contamination level. It is also clear that the relative cost of an alternative actuation system designed for a specific application will depend, primarily, on the actuation power level.

4.1.3 Capabilities of electro-hydraulic servos

When rapid and precise control of sizeable loads is required an electro-hydraulic servo is often the best approach to the problem. Generally speaking, the hydraulic servo actuator provides fast response, high force and short stroke characteristics. The main advantages of hydraulic components are.

- Easy and accurate control of work table position and velocity
- Good stiffness characteristics
- Zero backlash

- Rapid response to change in speed or direction
- Low rate of wear

There are several significant advantages of hydraulic servo drives over electric motor drives:

- Hydraulic drives have substantially higher power to weight ratios resulting in higher machine frame resonant frequencies for a given power level.
- Hydraulic actuators are stiffer than electric drives, resulting in higher loop gain capability, greater accuracy and better frequency response.
- Hydraulic servos give smoother performance at low speeds and have a wide speed range without special control circuits.
- Hydraulic systems are to a great extent self-cooling and can be operated in stall condition indefinitely without damage.
- Both hydraulic and electric drives are very reliable provided that maintenance is followed.
- Hydraulic servos are usually less expensive for system above several horsepower, especially if the hydraulic power supply is shared between several actuators.

4.1.4 Configuration of an electro-hydraulic servo

The basic elements of an electro-hydraulic servo are shown in Figure 4.2. The output of the servo is measured with a transducer device to convert it to an electric signal. This feedback signal is compared with the command signal. The resulting error signal is then amplified by the regulator and the electric power amplifier and then used as an input control signal to the servo valve. The servo valve controls the fluid flow to the actuator in proportion to the drive current from the amplifier. The actuator then forces the load to move. Thus, a change in the command signal generates an error signal, which causes the load to move in an attempt to zero the error signal. If the amplifier gain is high, the output will vary rapidly and accurately following the command signal.

External disturbances (forces or torque) can cause the load to move without any changes in the command signal. In order to offset the disturbance input an actuator output is needed in the opposite direction (see Figure 4.2).

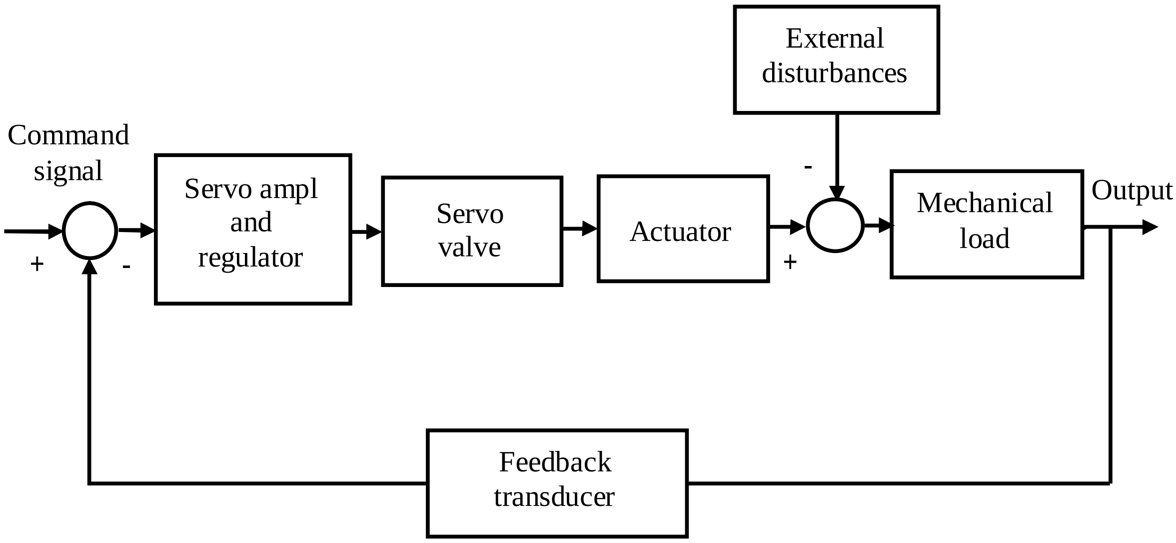

Fig 4.2 Components in an electro-hydraulic servomechanism

To provide this opposing output a finite error signal is required. The magnitude of the required error signal is minimized if the amplifier gain is high. Ideally, the amplifier gain would be set high enough that the accuracy of the servo becomes dependent only upon the accuracy of the transducer itself. However, since the control loop gain is proportional to the amplifier gain, this gain is limited by stability considerations. In some applications, stability may be critical enough that the desired performance is not possible to reach.

The three common types of electro-hydraulic servos are:

- Position servo (linear or angular)
- Velocity or speed servo (linear or angular)
- Force or torque servo

In this report the objective is to control the position of the load and force it to track a desired reference.

4.2 Dynamic Model of Electro-Hydraulic Servo System

In this part, the electro-hydraulic servo system shown in Fig 4.1 is considered [52]. The system parameters used in the model description are tabulated in Table 4.1. The goal of the controller is to make the mass position x_L track the reference.

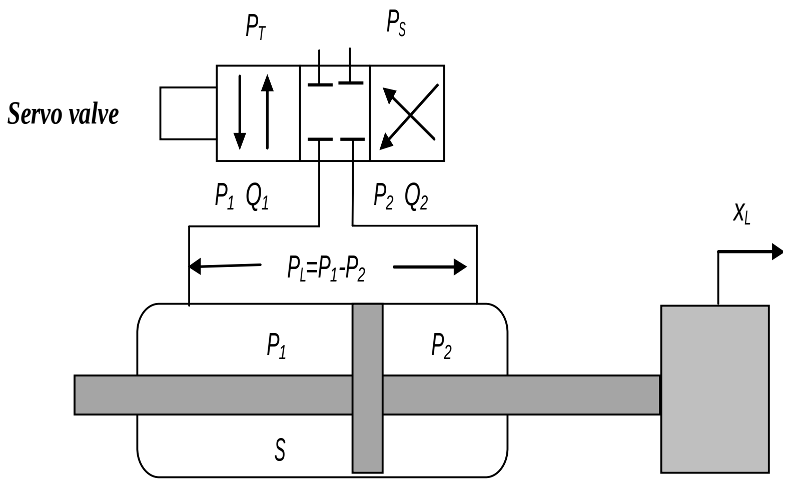

Fig 4.3 A hydraulic actuator with four-way valve configuration

The differential equations governing the dynamics of the hydraulic actuator in Fig 4.3 are given in [42]. The equation is divided in the following parts:

Dynamic pressure:

$$\frac{V_t}{4\beta_e}\dot{P}_L = -S\dot{x}_L - C_{tm}P_L + Q_L \tag{4.1}$$

The load pressure is $P_L = P_1 - P_2$.

Restriction flow:

The load flowQ_L is related to the spool valve displacement of the servo valvex_v, repressed by

$$Q_L = C_d w x_v\sqrt{\frac{P_s - sgn(x_v)P_L}{\rho}} = Cx_v\sqrt{P_s - sgn(x_v)P_L} \tag{4.2}$$

where $C = C_d w/\sqrt{\rho}$.

Valve dynamics:

In particular, the system that we have experimented with the valve dynamics can be considered to be a second order system [44]:

$$\ddot{x_v} = -w^2{}_e x_v - 2\xi_e w_e \dot{x_v} + w^2{}_e u \tag{4.3}$$

Mechanical coupling:

The dynamics of the inertia load can be described by

$$P_L S = m\ddot{x}_L + f_f \tag{4.4}$$

We consider f_f *is* an unknown nonlinear function, which is the combination of damping friction, viscous friction, and the external disturbance.

Parameter	Description
S	Ram area of cylinder.
V_t	The total volume of the cylinder and the hoses between the cylinder and servovalve.
P_L	The load pressure.
β_e	The effective bulk modulus.
C_{tm}	The coefficient of the total internal leakage of the cylinder due to pressure.
Q_L	The load flow.
x_v	Displacement of the servo valve.
C_d	The discharge coefficient.
w	The spool valve area gradient.
P_s	The supply pressure of the fluid.
ρ	Density of hydraulic oil.
m	The total mass of the actuator and the load.
x_L	The load displacement.

Table 4.1.Description of parameters in electro-hydraulic servo system (EHSS).

Equations.(4.1)−(4.4) completely described the fifth order nonlinear dynamics of the system under study. The corresponding state space representation of these dynamics follows. By defining

$$x = [x_1, x_2, x_3, x_4, x_5] = \left[x_L, \dot{x}_L, \frac{S}{m}P_L, x_v, \dot{x}_v\right]$$

Then the system can write

$\dot{x}_1 = x_2$

$$\dot{x}_2 = x_3 - \frac{1}{m} f_f(x_1, x_2)$$

$$\dot{x}_3 = \frac{-4\beta_e}{V_t} C \sqrt{P_s - sgn(x_4)\frac{m}{S} x_3} x_4 - \frac{4\beta_e}{V_t} S x_2 - \frac{4\beta_e}{V_t}\frac{m}{S} C_{tm} x_3 \tag{4.5}$$

$$\dot{x}_4 = x_5$$

$$\dot{x}_5 = w_e^2 u - w_e^2 x_4 - 2\xi_e w_e x_5$$

With respect to the assumptions made in the new extension described in the chapter 3, certain particular assumptions are also considered as follows:

Assumption 1: the f_f can be consider as $f_f \le \Delta(x_1^2, x_2^2)$, and $\Delta(x_1^2, x_2^2) = \xi_1 x_1^2 + \xi_2 x_2^2$ [48,52]. As maximum value we can consider $f_{f\,max} = \Delta(x_1^2, x_2^2)$.

Assumption 2: Assuming a symmetrical valve, where only positive spool displacement (x_v) can be studied the valve flow equation can now be simplified as $Q_L = C x_v \sqrt{P_s - P_L}$

Under the above assumption the system (4.5) can be written as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3 + \varphi_2^T(x_1, x_2)\theta$$

$$\dot{x}_3 = b_3 \beta(x) x_4 + \varphi_3^T(x_2, x_3)\theta \tag{4.6}$$

$$\dot{x}_4 = x_5$$

$$\dot{x}_5 = b_5 u + \varphi_5^T(x_4, x_5)\theta$$

with the nonlinear function $\beta(x) = \sqrt{P_s - \frac{m}{S} x_3}$, the vector of unknown parameters is :

$$\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6] = \left[-\frac{1}{m}\xi_1, -\frac{1}{m}\xi_2, -\frac{4\beta_e}{V_t} S, \frac{4\beta_e}{V_t}\frac{m}{S} C_{tm}, -w_e^2, -2\xi_e w_e\right];$$

and the nonlinear functions are $\varphi_2 = [x_1^2, x_2^2, 0,0,0,0]$, $\varphi_3 = [0,0, x_2, x_3, 0,0]$, $\varphi_5 = [0,0,0,0, x_4, x_5]$;

4.3 Adaptive Backstepping Controller Design

Now, we pass to the construction of controller. The nonlinear system described by (4.6) is in so-called strict feedback form and it has the same form as the system presented for the new extension in the previous chapter (3.1). Therefore, the new extension design can be used for controlling the single rod electro-hydraulic servo system; the results are presented as follows

4.3.1 Design Procedure

Coordinate transformation:

from Esq. (3.2) and (3.3) the coordinate transformation is

$$z_1 = x_1 - x_r \tag{4.7}$$

$$z_2 = x_2 - \alpha_1 - x_r^{(1)} \tag{4.8}$$

$$z_3 = x_3 - \alpha_2 - x_r^{(2)} \tag{4.9}$$

$$z_4 = x_4 - \alpha_3 - \frac{\hat{\varrho}}{\beta(x)} x_r^{(3)} \tag{4.10}$$

$$z_5 = x_5 - \alpha_4 - \frac{\hat{\varrho}}{\beta(x)} x_r^{(4)} \tag{4.11}$$

Regressor:

from Esq. (3.4) and (3.5) the controller's regressors are

$$w_1 = 0 \tag{4.12}$$

$$w_2 = \varphi_2 - \sum_{j=1}^{1} \frac{\partial \alpha_1}{\partial x_j} \varphi_j = \varphi_2 \tag{4.13}$$

$$w_3 = \varphi_3 - \sum_{j=1}^{2} \frac{\partial \alpha_2}{\partial x_j} \varphi_j = \varphi_3 - \frac{\partial \alpha_2}{\partial x_2} \varphi_2 \tag{4.14}$$

$$w_4 = \varphi_4 - \sum_{k=1}^{3} \frac{\partial \alpha_3}{\partial x_k}\varphi_k - \sum_{k=1}^{3} \frac{\partial}{\partial x_k}(\frac{1}{\beta(x)})\varphi_k \hat{\varrho} x_r^{(3)} = -\frac{\partial \alpha_3}{\partial x_2}\varphi_2 - \frac{\partial \alpha_3}{\partial x_3}\varphi_3 - \frac{\partial}{\partial x_3}(\frac{1}{\beta(x)})\varphi_3 \hat{\varrho} x_r^{(3)}$$

$$= -\frac{\partial \alpha_3}{\partial x_2}\varphi_2 - \frac{\partial \alpha_3}{\partial x_3}\varphi_3 + \frac{\hat{\varrho}}{2\beta(x)^3} x_r^{(3)} \varphi_3 \tag{4.15}$$

$$w_5 = \varphi_5 - \sum_{k=1}^{4} \frac{\partial \alpha_3}{\partial x_k}\varphi_k - \sum_{k=1}^{4} \frac{\partial}{\partial x_k}(\frac{\hat{\varrho}}{\beta(x)} x_r^{(4)})\varphi_k = \varphi_5 - \frac{\partial \alpha_4}{\partial x_2}\varphi_2 - \frac{\partial \alpha_4}{\partial x_3}\varphi_3 - \frac{\partial}{\partial x_3}(\frac{1}{\beta(x)})\varphi_3 \hat{\varrho} x_r^{(4)}$$

$$= \varphi_5 - \frac{\partial \alpha_4}{\partial x_2}\varphi_2 - \frac{\partial \alpha_4}{\partial x_3}\varphi_3 + \frac{\hat{\varrho}}{2\beta(x)^3} x_r^{(4)} \varphi_3 \tag{4.16}$$

Tuning functions for $\hat{\boldsymbol{\theta}}$:

We use Eq. (3.6) to derive the tuning functions of the parameters

$$\tau_1 = 0 \tag{4.17}$$

$$\tau_2 = \tau_1 + w_2 z_2 \tag{4.18}$$

$$\tau_3 = \tau_2 + w_3 z_3 \tag{4.19}$$

$$\tau_4 = \tau_3 + w_4 z_4 \tag{4.20}$$

$$\tau_5 = \tau_4 + w_5 z_5 \tag{4.21}$$

Tuning functions for $\hat{\boldsymbol{b}}_{\boldsymbol{3}}$:

The tuning functions for the virtual control coefficient is calculated by Eq. (3.7)

$$\pi_3 = z_4 z_3 \tag{4.22}$$

$$\pi_4 = \pi_3 - \frac{\partial \alpha_3}{\partial x_3}\beta(x) x_4 z_4 + \frac{\hat{\varrho}}{2\beta(x)^2} x_r^{(3)} x_4 z_4 \tag{4.23}$$

$$\pi_5 = \pi_4 - \frac{\partial \alpha_4}{\partial x_3}\beta(x) x_4 z_5 + \frac{\hat{\varrho}}{2\beta(x)^2} x_r^{(4)} x_4 z_5 \tag{4.24}$$

Stabilizing functions:

The stabilizing functions are calculated based on the Esq. (3.9)-(3.14)

$$\alpha_1 = \bar{\alpha}_1 \tag{4.25}$$

$$\alpha_2 = \bar{\alpha}_2 \tag{4.26}$$

$$\alpha_3 = \frac{\hat{\varrho}}{\beta(x)}\bar{\alpha}_3 \tag{4.27}$$

$$\alpha_4 = \bar{\alpha}_4 \tag{4.28}$$

$$\alpha_5 = \bar{\alpha}_5 \tag{4.29}$$

with

$$\bar{\alpha}_1 = -c_1 z_1 \tag{4.30}$$

$$\bar{\alpha}_2 = -c_2 z_2 - z_1 - \hat{\theta}^T w_2 + \frac{\partial \alpha_1}{\partial x_1} x_2 + \frac{\partial \alpha_1}{\partial x_r} x_r^{(1)} \tag{4.31}$$

$$\bar{\alpha}_3 = -c_3 z_3 - z_2 + \sum_{k=1}^{2}\left(\frac{\partial \alpha_2}{\partial x_k} x_{k+1} + \frac{\partial \alpha_2}{\partial x_r^{(k-1)}} x_r^{(k)}\right) - \hat{\theta}^T w_3 + \frac{\partial \alpha_2}{\partial \hat{\theta}} \Gamma \tau_3 \tag{4.32}$$

$$\begin{aligned}\bar{\alpha}_4 = {} & -c_4 z_4 - \hat{b}_3 \beta(x) z_3 - \hat{\theta}^T w_4 + \sum_{k=1}^{2} \frac{\partial \alpha_3}{\partial x_k} x_{k+1} + \hat{b}_3 \beta(x) \frac{\partial \alpha_3}{\partial x_3} x_4 + \frac{\partial \alpha_3}{\partial \hat{\theta}} \Gamma \tau_4 + \left(\frac{x_r^{(3)}}{\beta(x)} + \frac{\partial \alpha_3}{\partial \hat{\varrho}}\right)\dot{\hat{\varrho}} \\ & + \sum_{k=2}^{3} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma w_4 z_4 + \sum_{k=1}^{3} \frac{\partial \alpha_3}{\partial x_r^{(k-1)}} x_r^{(k)} - \frac{\hat{\varrho}\hat{b}_3}{2\beta(x)^2} x_r^{(3)} x_4\end{aligned} \tag{4.33}$$

$$\begin{aligned}\bar{\alpha}_5 = {} & -c_5 z_5 - z_4 + \sum_{\substack{k=1 \\ k\neq 3}}^{4} \frac{\partial \alpha_4}{\partial x_k} x_{k+1} - \hat{\theta}^T w_5 + \frac{\partial \alpha_4}{\partial \hat{\theta}} \Gamma \tau_5 + \hat{b}_3 \beta(x) \frac{\partial \alpha_4}{\partial x_3} x_4 + \sum_{k=2}^{4} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma w_5 z_k \\ & + \sum_{k=1}^{4} \frac{\partial \alpha_4}{\partial x_r^{(k-1)}} x_r^{(k)} + \frac{\partial \alpha_4}{\partial \hat{b}_3} \gamma \pi_5 + + \left(\frac{x_r^{(4)}}{\beta(x)} + \frac{\partial \alpha_4}{\partial \hat{\varrho}}\right)\dot{\hat{\varrho}} - \frac{\hat{\varrho}\hat{b}_3}{2\beta(x)^2} x_r^{(4)} x_4\end{aligned} \tag{4.34}$$

Adaptive control law:

The adaptive control law of the entire system is

$$u = \hat{\lambda}\left(\alpha_5 + \frac{\hat{\varrho}}{\beta(x)} x_r^{(5)}\right) \tag{4.35}$$

Parameter update law:

$$\dot{\hat{\theta}} = \Gamma\tau_5 \tag{4.36}$$

$$\dot{\hat{b}}_3 = \gamma\pi_5 \tag{4.37}$$

$$\dot{\hat{\varrho}} = -\gamma sign(b_3)\left(x_r^{(3)} + \bar{\alpha}_3\right)z_3 \tag{4.38}$$

$$\dot{\hat{\lambda}} = -\gamma sign(b_5)(\bar{\alpha}_5 + x_r^{(5)})z_5 \tag{4.39}$$

4.3.2 Calculation of the Error System

The error system results during the design of each step of the adaptive backstepping controller are illustrated as follows:

$$z_1 = x_1 - x_r$$

$$\dot{z}_1 = \dot{x}_1 - x_r^{(1)}$$

$$= x_2 - x_r^{(1)}$$

$$= z_2 + \alpha_1$$

$$= -c_1z_1 + z_2 \tag{4.40}$$

$$z_2 = x_2 - \alpha_1 - x_r^{(1)}$$

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 - x_r^{(2)}$$

$$= x_3 + \varphi_2^T\theta - \dot{\alpha}_1 - x_r^{(2)}$$

$$= z_3 + \alpha_2 + \varphi_2^T\hat{\theta} + \varphi_2^T\tilde{\theta} - \dot{\alpha}_1$$

$$= z_3 - c_2z_2 - z_1 - w_2^T\hat{\theta} + \frac{\partial\alpha_1}{\partial x_1}x_2 + \frac{\partial\alpha_1}{\partial x_r}x_r^{(1)} + \varphi_2^T\hat{\theta} + \varphi_2^T\tilde{\theta} - \dot{\alpha}_1$$

$$= z_3 - c_2z_2 - z_1 + (\varphi_2^T - w_2^T)\hat{\theta} + \frac{\partial\alpha_1}{\partial x_1}x_2 + \frac{\partial\alpha_1}{\partial x_r}x_r^{(1)} + \varphi_2^T\tilde{\theta} - \dot{\alpha}_1$$

$$\dot{z}_2 = z_3 - c_2 z_2 - z_1 + (\varphi_2^T - w_2^T)\hat{\theta} + \frac{\partial \alpha_1}{\partial x_1}\varphi_1^T\tilde{\theta} + \frac{\partial \alpha_1}{\partial x_1}x_2 + \frac{\partial \alpha_1}{\partial x_r}x_r^{(1)} + \varphi_2^T\tilde{\theta} - \frac{\partial \alpha_1}{\partial x_1}\varphi_1^T\tilde{\theta} - \dot{\alpha}_1$$

$$= z_3 - c_2 z_2 - z_1 + \frac{\partial \alpha_1}{\partial x_1}x_2 + \frac{\partial \alpha_1}{\partial x_r}x_r^{(1)} + \frac{\partial \alpha_1}{\partial x_1}\varphi_1^T\left(\tilde{\theta} + \hat{\theta}\right) + (\varphi_2^T - \frac{\partial \alpha_1}{\partial x_1}\varphi_1^T)\tilde{\theta} - \dot{\alpha}_1$$

$$= z_3 - c_2 z_2 - z_1 + w_2^T\tilde{\theta} \tag{4.41}$$

From Eq. (3.22)

$$\dot{z}_3 = -c_3 z_3 - z_2 + \hat{b}_3\beta(x)z_4 - \sum_{k=2}^{2}\sigma_{km}z_k + \sum_{k=4}^{5}\sigma_{mk}z_k + \tilde{\theta}^T w_3 - b_3(\bar{\alpha}_3 + x_r^{(3)})\tilde{\varrho} + \tilde{b}_3\beta(x)z_4$$

and

$$\sigma_{23} = -\frac{\partial \alpha_1}{\partial \hat{\theta}}\Gamma w_2 = 0$$

Then

$$\dot{z}_3 = -c_3 z_3 - z_2 + \hat{b}_3\beta(x)z_4 + \tilde{\theta}^T w_3 - b_3\left(\bar{\alpha}_3 + x_r^{(3)}\right)\tilde{\varrho} + \tilde{b}_3\beta(x)z_4 \tag{4.45}$$

From Eq. (3.23) :

$$\dot{z}_4 = -c_4 z_4 - \sum_{k=2}^{3}\sigma_{k,4}z_k - z_5 - \hat{b}_3 z_3 + \sum_{k=5}^{5}\sigma_{4,k}z_k + w_4^T\tilde{\theta} + \tilde{b}_3(\frac{\partial}{\partial x_3}\left(\frac{1}{\beta(x)}\right)\hat{\varrho}x_r^{(3)} - \frac{\partial \alpha_3}{\partial x_3})\beta(x)x_4$$

$$\dot{z}_4 = -c_4 z_4 - \sigma_{2,4}z_2 - \sigma_{3,4}z_3 - z_5 + \sigma_{4,5}z_5 - \hat{b}_3 z_3 + w_4^T\tilde{\theta} + \tilde{b}_3(\frac{\partial}{\partial x_3}\left(\frac{1}{\beta(x)}\right)\hat{\varrho}x_r^{(3)} - \frac{\partial \alpha_3}{\partial x_3})\beta(x)x_4$$

$$\sigma_{34} = -\frac{\partial \alpha_2}{\partial \hat{\theta}}\Gamma w_4$$

$$\sigma_{45} = -\frac{\partial \alpha_3}{\partial \hat{\theta}}\Gamma w_5$$

$$\dot{z}_4 = -c_4 z_4 - \hat{b}_3 z_3 - \frac{\partial \alpha_3}{\partial \hat{\theta}}\Gamma w_5 + w_4^T\tilde{\theta} - \frac{\partial \alpha_3}{\partial \hat{\theta}}\Gamma w_5 + \tilde{b}_3\left(\frac{\hat{\varrho}}{2\beta(x)^2}x_r^{(3)} - \beta(x)\frac{\partial \alpha_3}{\partial x_3}\right)x_4 \tag{4.46}$$

From Eq(3.25):

$$\dot{z}_n \quad = -c_j z_j - \sum_{k=2}^{n-1} \sigma_{kj} z_k - z_{n-1} + \sum_{k=j+1}^{n} \sigma_{jk} z_k + w_j^T \tilde{\theta} - b_n(\alpha_n + \frac{\hat{\varrho}}{\beta(x)} x_r^{(n)})\hat{\lambda}$$

$$-\tilde{b}_m \left(\frac{\partial}{\partial x_m} \left(\frac{1}{\beta(\bar{x})} \right) \hat{\varrho} x_r^{(j-1)} - \frac{\partial \alpha_{j-1}}{\partial x_m} \right) \beta(\bar{x}) x_{m+1} ,$$

$$\dot{z}_5 \quad = -c_5 z_5 - \sum_{k=2}^{4} \sigma_{k5} z_k - z_4 + \sum_{k=5}^{5} \sigma_{5k} z_k + w_5^T \tilde{\theta} - b_5(\alpha_5 + \frac{\hat{\varrho}}{\beta(x)} x_r^{(5)})\hat{\lambda}$$

$$- \tilde{b}_3 \left(\frac{\partial}{\partial x_3} \left(\frac{1}{\beta(x)} \right) \hat{\varrho} x_r^{(4)} - \frac{\partial \alpha_4}{\partial x_3} \right) \beta(x) x_4$$

$$\dot{z}_5 \quad = -c_5 z_5 - \sigma_{2,5} z_2 - \sigma_{3,5} z_3 - \sigma_{4,5} z_4 - z_4 + w_5^T \tilde{\theta} - b_5(\alpha_5 + \frac{\hat{\varrho}}{\beta(x)} x_r^{(5)})\hat{\lambda}$$

$$- \tilde{b}_3 \left(\frac{\partial}{\partial x_3} \left(\frac{1}{\beta(x)} \right) \hat{\varrho} x_r^{(4)} - \frac{\partial \alpha_{j4}}{\partial x_3} \right) \beta(x) x_4 \qquad (4.47)$$

with

$$\sigma_{25} = -\frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma w_5 = 0$$

$$\sigma_{35} = -\frac{\partial \alpha_2}{\partial \hat{\theta}} \Gamma w_5$$

$$\sigma_{45} = -\frac{\partial \alpha_3}{\partial \hat{\theta}} \Gamma w_5$$

4.4 Stability Analysis

A Lyapunov function for this system is

$$V = \frac{1}{2} z^T z + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{2\gamma} \tilde{b}_m^2 + \frac{1}{2\gamma} \tilde{b}_n^2 + \frac{|b_m|}{2\gamma} \tilde{\varrho}^2 + + \frac{|b_n|}{2\gamma} \tilde{\lambda}^2 \qquad (4.48)$$

Its derivative is:

$$\dot{V} = z\dot{z} + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} + \frac{\tilde{b}_m}{\gamma}\dot{\tilde{b}}_m + \frac{|b_m|}{\gamma}\tilde{\varrho}\dot{\tilde{\varrho}} + \frac{|b_n|}{\gamma}\tilde{\lambda}\dot{\tilde{\lambda}}$$

$$= z_1\dot{z}_1 + z_2\dot{z}_2 + z_3\dot{z}_3 + z_4\dot{z}_4 + z_5\dot{z}_5 + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} + \frac{\tilde{b}_m}{\gamma}\dot{\hat{b}}_m + \frac{\tilde{b}_n}{\gamma}\dot{\hat{b}}_n + \frac{|b_m|}{\gamma}\tilde{\varrho}\dot{\tilde{\varrho}} + \frac{|b_n|}{\gamma}\tilde{\lambda}\dot{\tilde{\lambda}}$$

$$\dot{V} = z_1(-c_1z_1 + z_2) + z_2(z_3 - c_2z_2 - z_1 + w_2^T\tilde{\theta}) + z_3(-c_3z_3 - z_2 + \hat{b}_3\beta(x)z_4 + w_3^T\tilde{\theta}$$

$$-b_3\left(\bar{\alpha}_3 + x_r^{(3)}\right)\tilde{\varrho} + \tilde{b}_3\beta(x)z_4) + z_4(-c_4z_4 - \sigma_{3,4}z_3 - z_5 - \hat{b}_3z_3 + w_4^T\tilde{\theta}$$

$$+\tilde{b}_3(\frac{\partial}{\partial x_3}\left(\frac{1}{\beta(x)}\right)\hat{\varrho}x_r^{(3)} - \frac{\partial\alpha_3}{\partial x_3})\beta(x)x_4) + z_5(-c_5z_5 - \sigma_{3,5}z_3 - \sigma_{4,5}z_4 - z_4$$

$$+w_5^T\tilde{\theta} - b_5\left(\alpha_5 + \frac{\hat{\varrho}}{\beta(x)}x_r^{(5)}\right)\hat{\lambda} - \tilde{b}_3\left(\frac{\partial}{\partial x_3}\left(\frac{1}{\beta(x)}\right)\hat{\varrho}x_r^{(4)} - \frac{\partial\alpha_{j4}}{\partial x_3}\right)\beta(x)x_4 - \frac{\tilde{b}_m}{\gamma}\dot{\hat{b}}_m$$

$$+\tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} + \frac{|b_m|}{\gamma}\tilde{\varrho}\dot{\tilde{\varrho}} + \frac{|b_n|}{\gamma}\tilde{\lambda}\dot{\tilde{\lambda}}$$

$$\dot{V} = -c_1z_1 - c_2z_2 - c_3z_3 - c_4z_4 - c_5z_5 + (z_2w_2 + z_3w_3 + z_4w_4 + z_5w_5 - \Gamma^{-1}\dot{\hat{\theta}})^T\tilde{\theta}$$

$$+\left(-b_3\left(\bar{\alpha}_3 + x_r^{(3)}\right)z_3 + \frac{|b_m|}{\gamma}\dot{\hat{\varrho}}\right)\tilde{\varrho} + \left(\frac{\partial}{\partial x_3}\left(\frac{1}{\beta(x)}\right)\hat{\varrho}x_r^{(4)} - \frac{\partial\alpha_{j4}}{\partial x_3}\right)\beta(x)x_4z_5$$

$$+(\frac{\partial}{\partial x_3}\left(\frac{1}{\beta(x)}\right)\hat{\varrho}x_r^{(3)} - \frac{\partial\alpha_3}{\partial x_3})\beta(x)x_4z_4) + \left(-b_5(\alpha_5 + \frac{\hat{\varrho}}{\beta(x)}x_r^{(5)})z_5 + \frac{|b_5|}{\gamma}\dot{\hat{\lambda}}\right)\tilde{\lambda}$$

Using Eqs. (4.37)-(4.39) the derivative of V becomes

$$\dot{V} = -\sum_{i=1}^{5} c_i\, z_i^2 \tag{4.49}$$

4.5 Simulations Results

Results of simulations are presented in this section, the model dynamics of the valve is represented by a second order transfer function, the friction in the cylinder is nonlinear and moreover the compressibility of the fluid is not neglected inside the load and thus can the cylinder accumulate fluid. The values of the system parameters used in the model are illustrated in Table 4.2.

Parameter	Value	Parameter	Value
S	0.002 m^2	w	255 rad/sec
V_t	0.001 m^3	P_s	21 Mpa
P_L	1.0457	ρ	900 kg/ m^3
β_e	700 Mpa	m	3.3 kg
C_{tm}	0.1	ξ_e	0.6
x_v	x_v mm	C	1.5×10^{-4}
ξ_1	10	ξ_2	0.3

Table 4.2.Values of the parameters of electro-hydraulic servo system (EHSS).

The model used to develop the backstepping controllers contains the following uncertainties (refereed by the *-superscript):$S^* = 0.9\,S, V_t^* = 0.9V_t$,$\beta_e^* = 1.1\beta_e, C_{tm}^* = 0.8C_{tm}, w^* = 0.9w$

$m^* = 0.8\,m,\ \xi_e^* = 1.1\,\xi_e$.

To verify the efficiency of the proposed controllers, we considered the following two cases:

Case 1:we seek to track a step response of 0.1m, the controller gains in this case are equal $[c_1, c_2, c_3, c_4, c_5,] = [100,100,50,50,50]$and $\gamma = 1$.

Case 2: In this case the same controller gains are used. The reference to track is sinusoidal

$x_r = 0.1\sin(t)$.

At the first case:

As shown in Fig 4.4 (a), which represents the entire response (blue) against the reference step (red), it can be seen that the mass position tracks the reference input very fast, Fig 4.4 (b) is zoomed at the transient response , we can see that the response track the reference in 0.23 sec.

Fig 4.5 represents the error between response system and the step reference, we can see that the error is small and tends to zero, which means a perfect tracking of the proposed controller.

Control effort delivered by the system is shown in Fig 4.6.

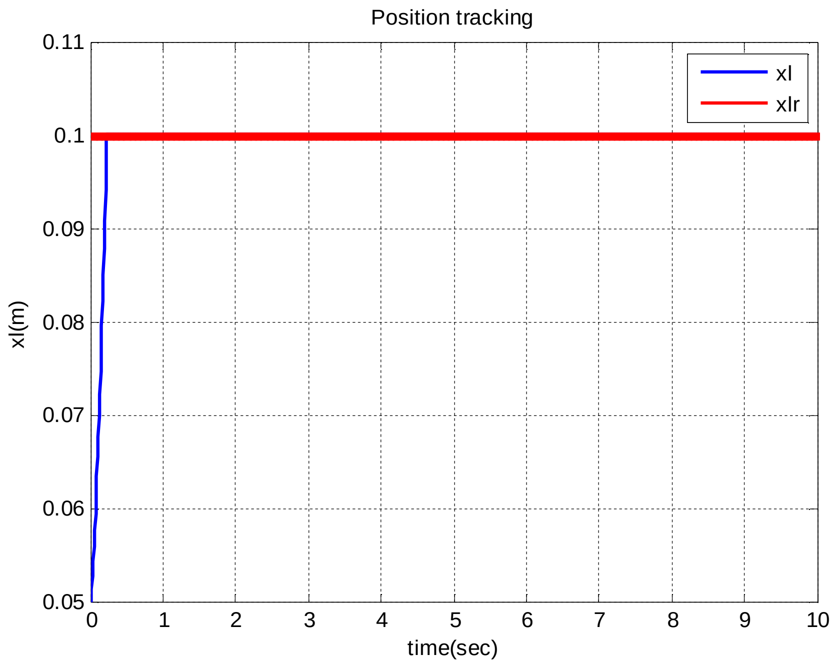

Fig 4.2 Position tracking of the system with a step

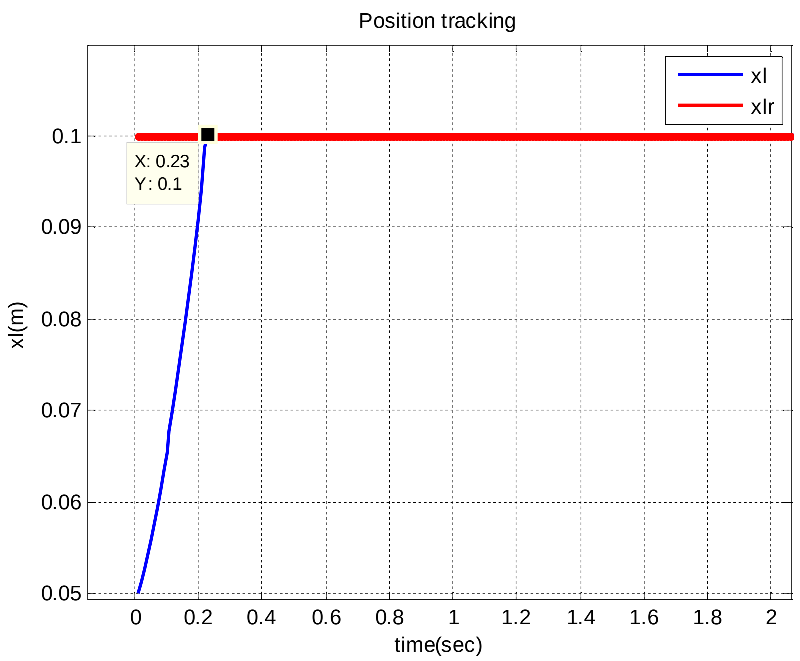

Fig 4.4 Tracking error of the system with a step (zoomed)

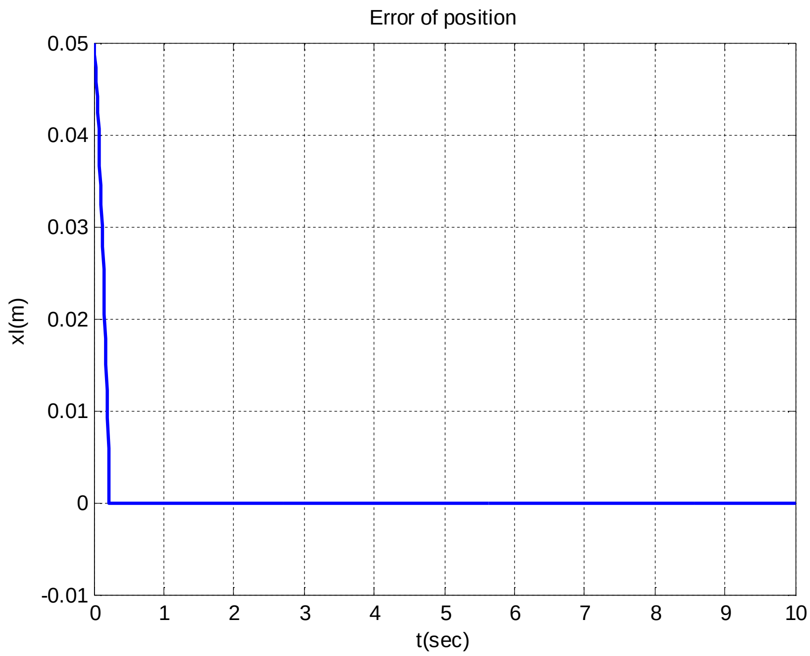

Fig 4.5 Tracking error of the system with a step

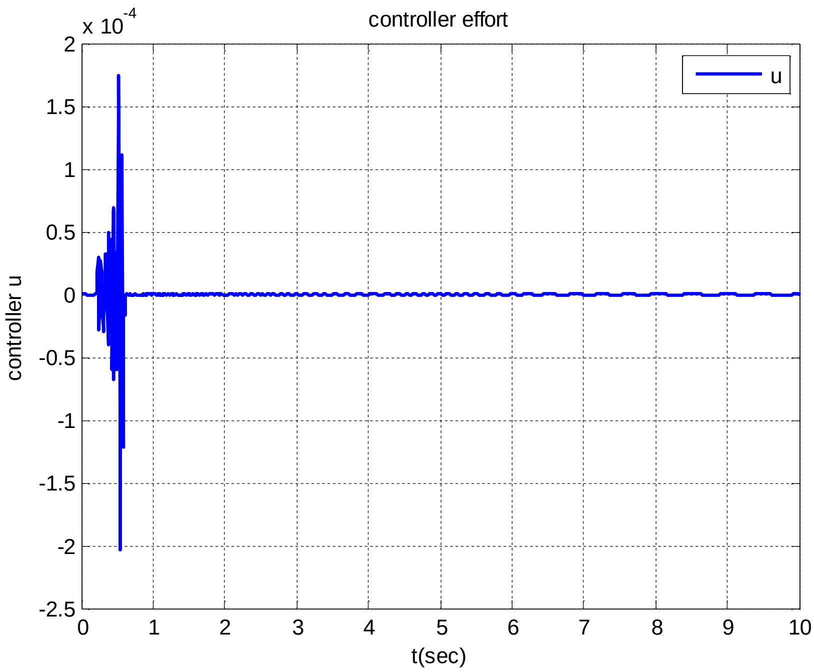

Fig 4.6 controller effort using step input

In second case:

The Figs 4.7-4.9 show the tracking of the system against a sinusoidal reference. In Fig 4.7 the response system (bleu) tracks the sinusoidal reference (red) quickly. The error is also very small in this case Fig 4.8 and change sinusoidally according to the reference changes. Control law is also changed in the same way as shown in Fig 4.9.

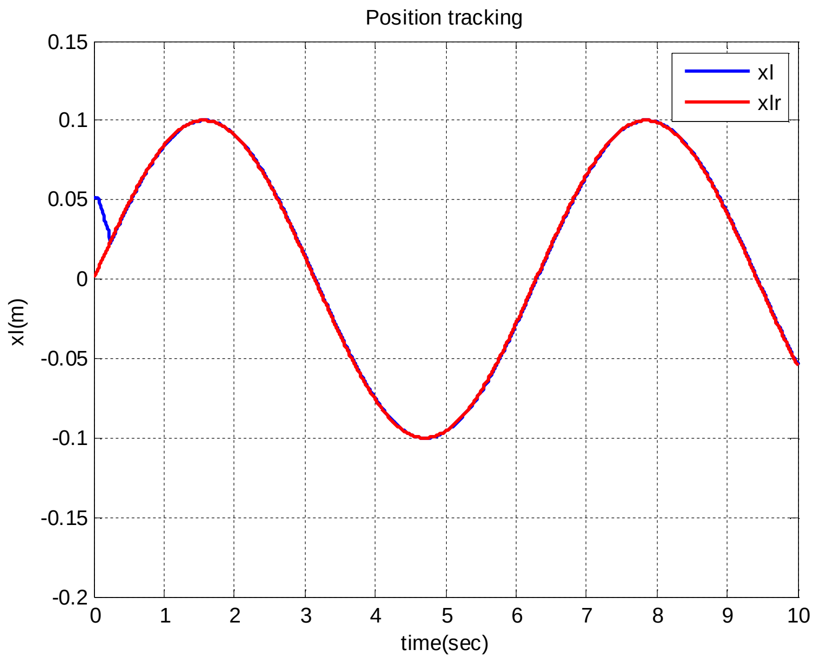

Fig 4.7 Position tracking of the system with a sinusoidal

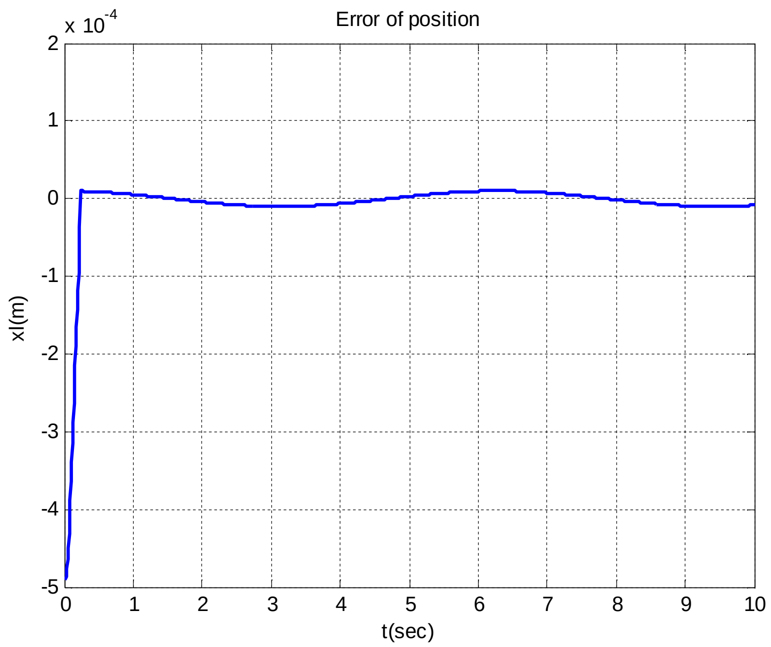

Fig 4.8 Tracking error of the system with a sinusoidal

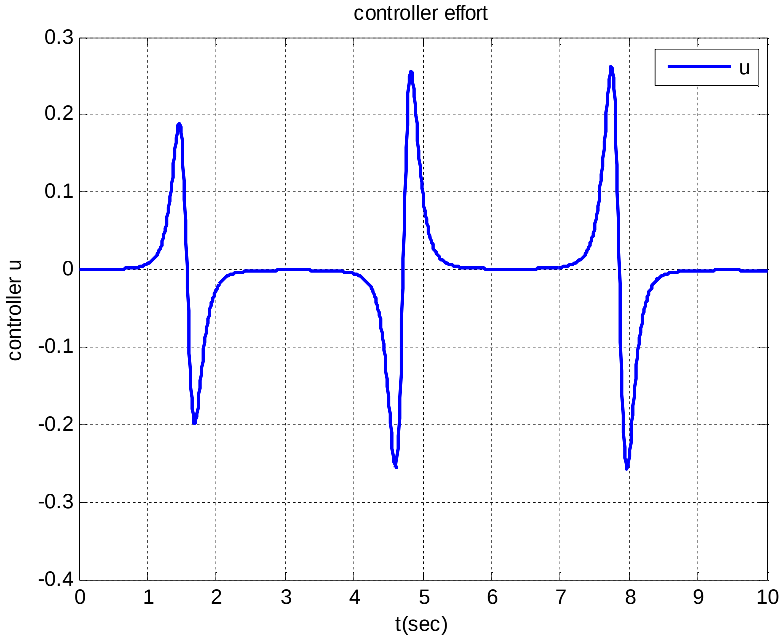

Fig 4.9 control effort using a sinusoidal input

4.5.1 Effect of Design Parameters

In this part, the ABC is designed with different values of c_i and γ_i in order to see the effect of those design parameters in the states.

- **Effect of c_i**

To see the effects of changing design parametersc_i, we fix $\gamma = 1$. The position tracking and the error for the two references (step and sinusoidal) are given in Figs 4.10 -4.114 with $c_i = 1,10,100$ for $i = 1,2,3,4,5$. As shown in Figures below the change of c_i values affect the response of the states. Figure 4.10 and 4.12 show the position tracking for three values of c_i , we can remark that increasing c_i make the tracking smoother, in other side it decrease the time response, the system follow the reference faster when the c_i is greater. This result can be checked by the tracking error represented in Fig 4.11 and 4.13.

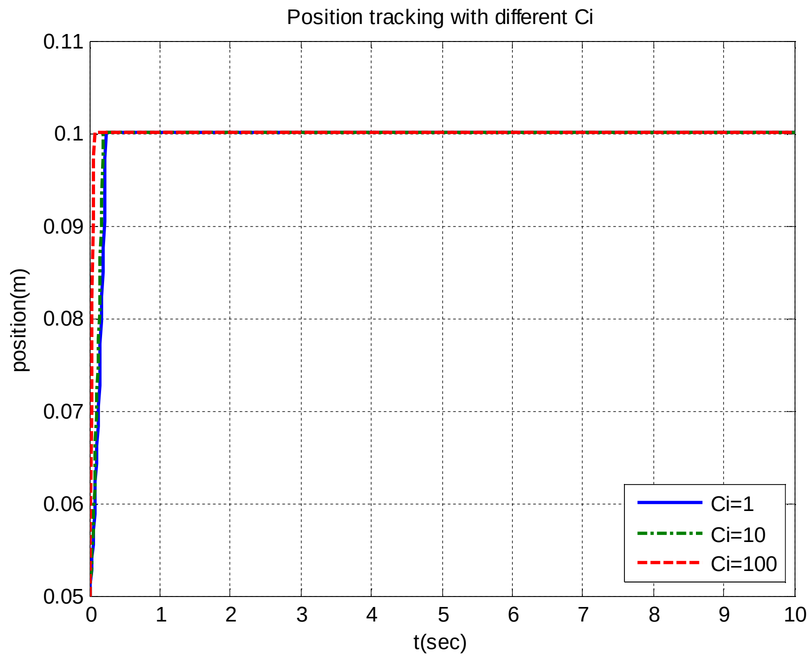

Fig 4.10 Position tracking with different c_i(Step reference)

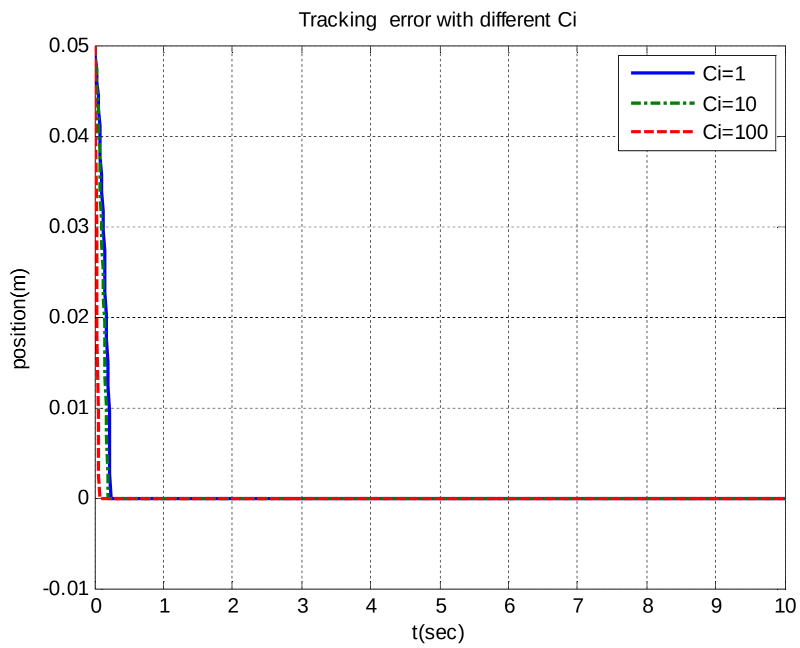

Fig 4.11 Tracking error with different c_i(Step reference)

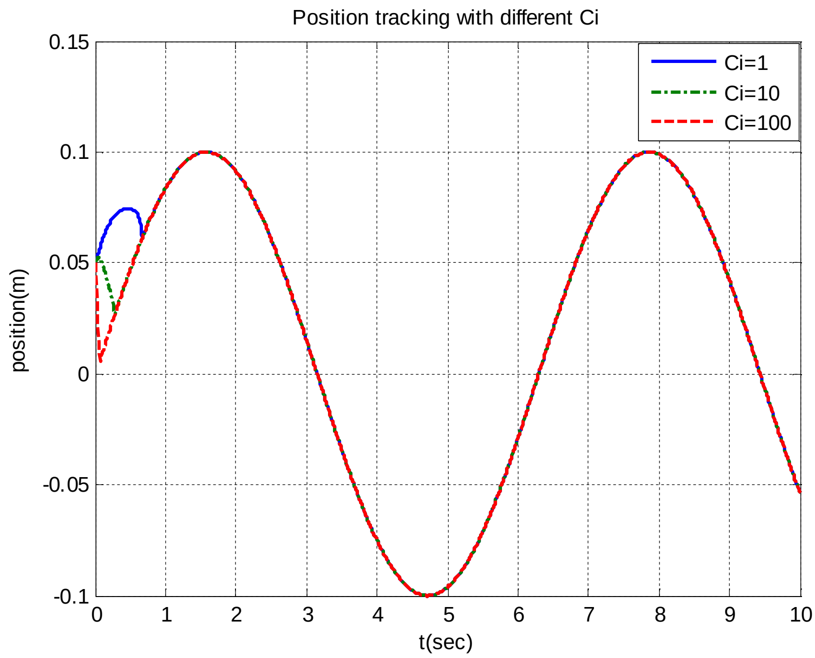

Fig 4.12 Position tracking with different c_i(sinusoidal reference)

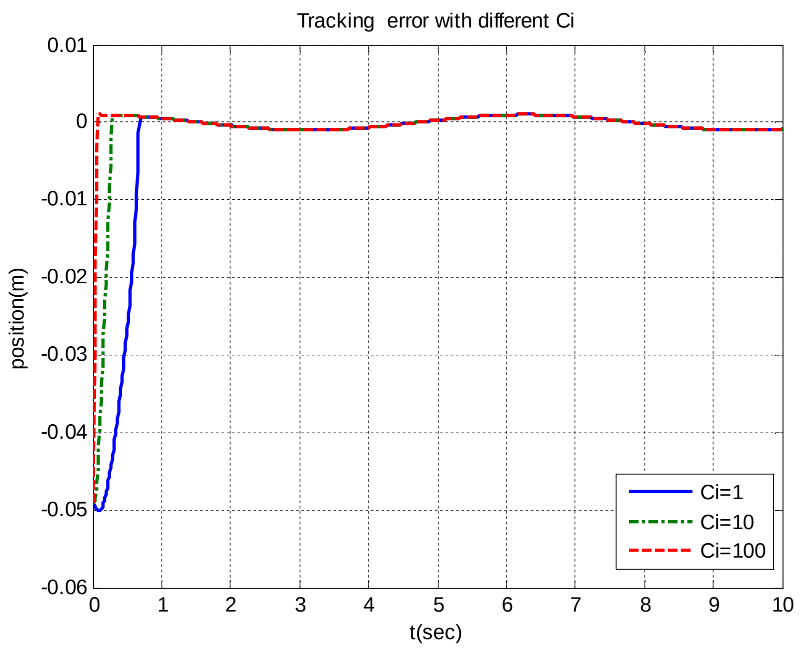

Fig 4.13 Tracking error with different c_i(sinusoidal reference)

- **Effect of γ_i**

To see the effects of changing design parameters γ,we fix $c_i = 10$ for $i = 1,2,3,4,5$.The position tracking and the error for the two references (step and sinusoidal) are given in Figs 4.9 -4.13with $\gamma = 0.1,0.5,1$

As shown in Figures below the change of c_i values affect the response of the states. Position tracking for three values of $\gamma$are given in Figs 4.14 and 4.16, for the two references tracking we see that increasing γ reduce the pick values and yield the system to achieve the tracking task faster. The response becomes smoother and the tracking error also decreases by this increase as shown in Fig 4.15 and 4.17.

As conclusion, the change of design parameters c_i and γ affect directly the response time and the tracking performance, the smoothness of response is also affected.

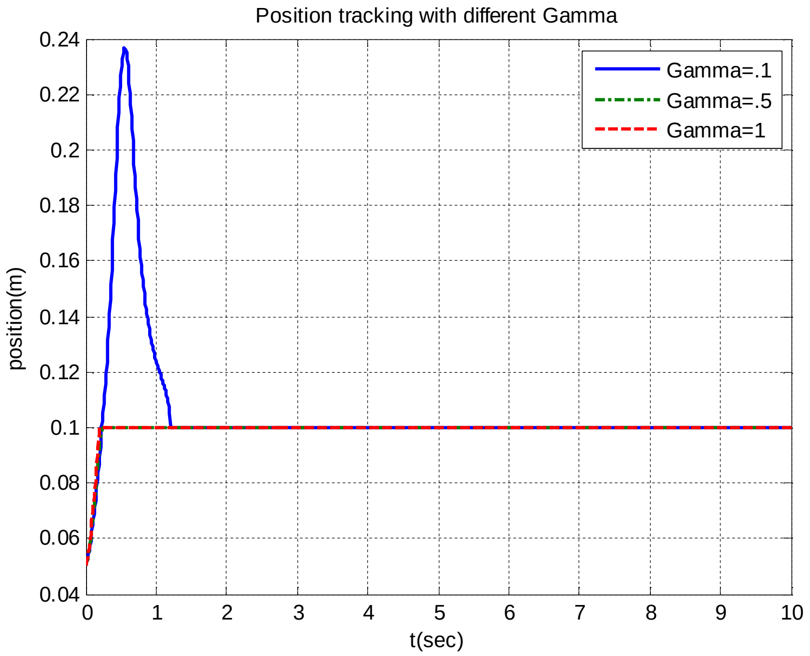

Fig 4.14 Position tracking with different γ(Step reference)

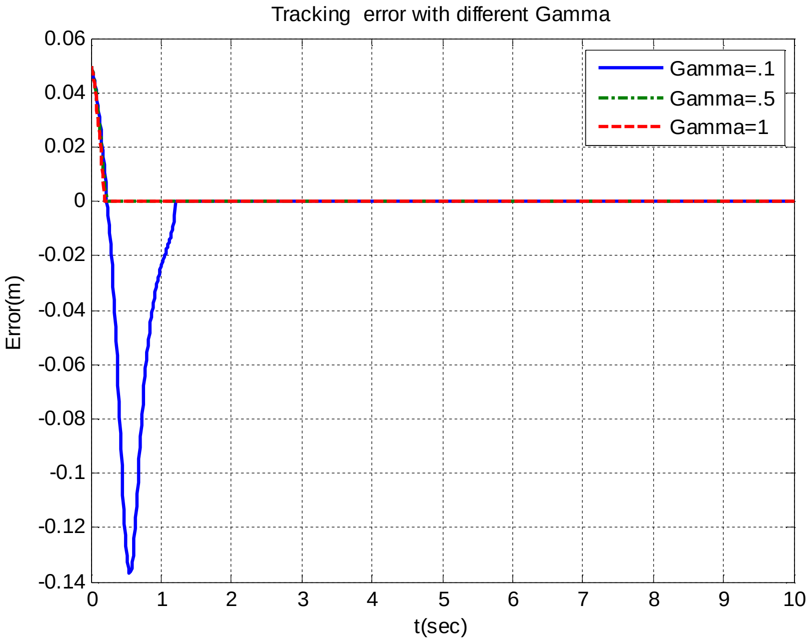

Fig 4.15 Tracking error with different γ(sinusoidal reference)

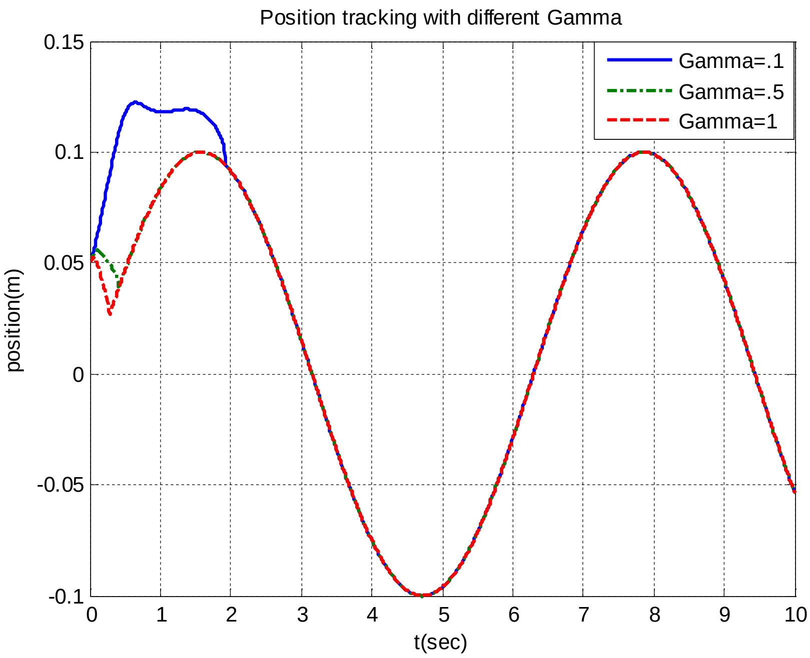

Fig 4.16 Position tracking with different γ(sinusoidal reference)

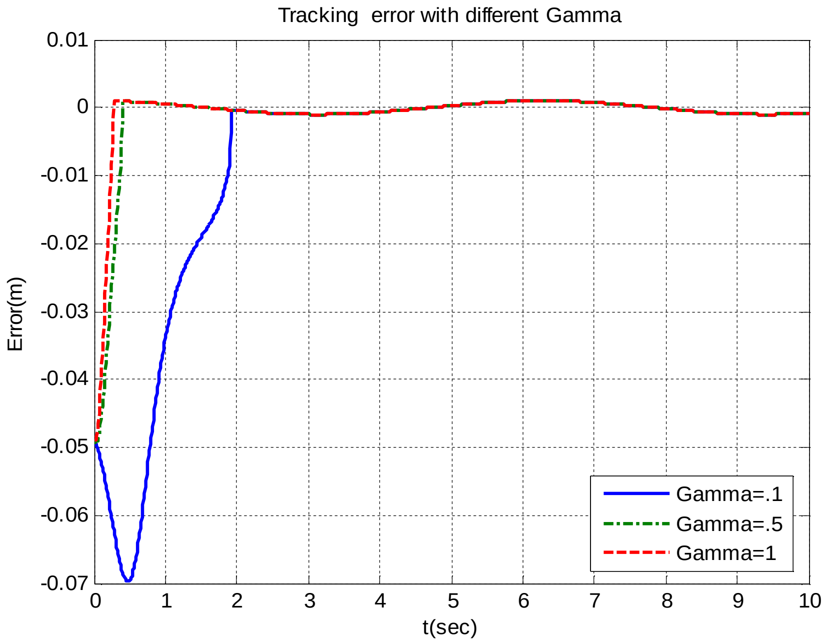

Fig 4.17 Tracking error with different γ(sinusoidal reference)

A systematic methodology for the design of a nonlinear adaptive backstepping controller for single rod electro-hydraulic servo actuator has been presented in this work. The model used for the controller design is a nonlinear fifth-order system model which takes into account the valve dynamic system. The friction force is considered nonlinear which has enhanced the modeling and as result the transient performance.

Finally, the simulations confirm that the new proposed control law is effective and robust against parametric uncertainties and achieve perfectly the tracking task in different inputs.

Conclusions & Future Work

In this part some conclusions are derived from this work. A brief summary of this thesis is also presented. Finally, some recommendation for future work and improvements is also given.

Conclusions

The thesis has dealt with the problem of controlling nonlinear uncertain systems.

The backstepping approach was employed to control the nonlinear systems. The advantage of backstepping design technique is that the controller and the adaptive update laws can be designed at the same time, and this can improve the system transient performance. Tuning functions approach was introduced completely remove the overparametrization problem presented by the classical backstepping approach. It has the advantage that the dynamic order of the adaptive controller is minimal. The dimension of the set to which the states and parameter estimates converge is also minimal.

Future work

Research is always considered as an iterative as well as exhaustive process. The work on the proposed controller and this topic, by a true measure, cannot be considered complete. There will always be points to improve upon and more problems to tackle.

This, when accompanied by the eagerness to excel in this field, gives rise to several recommendations. Rest assured, ample time and energy will be invested to further work on this vast topic and positively contribute to science. The many possible suggestions that come to mind in this regard are as follows:

- Developing an output feedback version of the proposed adaptive backstepping scheme used either the Kresseilmeier filter developed in [1] or the MT-filters which proposed in [63,64] to estimate the state of the system we can also compare the simplicity as well as the ability to compensate several type of disturbances in these two kinds of filters.

- The sensitivity analysis of adaptive backstepping design: the backstepping-based adaptive tuning functions design is a control of nonlinear uncertain systems that ensure reasonably good stability and performance properties of the closed loop system. Then, the issue of numerical sensitivity of the adaptive tuning functions can be discussed[62]. Also, various nonlinear optimization techniques such as the Particle Swarm Optimization (PSO) ,Neural Networks (NN) ,Genetic Algorithm (GA), etc. can be used to tune the optimal gains that can unsure the best transient performance.

- Extend the backstepping approach to systems with non-triangular forms : some encouraged researches are made like the combination of the backstepping and sliding mode control proposed in [65]

- The complexity in backstepping design: In order to reduce the explosion of complexity in traditional backstepping design several techniques are introduced once utilizes the differentiation of the first-order filter to replace the quantity of the differentiation of the virtual control in determining the next virtual control at each step of recursion [66].

- The presented scheme can be applied to other practical systems.

- Use another modeling of the friction force compensation as the LuGre dynamic friction model [53.54], it presents extra states but it can well modeling the friction.

- Implementation of the proposed controller on an experimental setup

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APPENDICES

Appendix A

Error System Calculation

- Calculation of error system for $(z_1, \dots, z_{m-1})$-subsystem

The error variable is

$$z_i = x_i - \alpha_{i-1} - x_r^{(i-1)} \qquad i = 1, \dots, m-1 \qquad (A.1)$$

Its derivative is written as

$$\dot{z}_i = \dot{x}_i - \dot{\alpha}_{i-1} - x_r^{(i)}$$

$$= x_{i+1} + \varphi_i^T \hat{\theta} + \varphi_i^T \tilde{\theta} - \dot{\alpha}_{i-1} - x_r^{(i)}$$

$$\dot{z}_i = z_{i+1} + \alpha_i + \varphi_i^T \hat{\theta} + \varphi_i^T \tilde{\theta} - \dot{\alpha}_{i-1}$$

with replacing $\alpha_i$by its equation

$$\bar{\alpha}_i = -c_i z_i - z_{i-1} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} - w_i^T \hat{\theta} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i$$

$$+ \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_i z_k + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_r^{(k-1)}} x_r^{(k)}, \qquad i = 1, \dots, m \qquad (A.2)$$

The derivative becomes

$$\dot{z}_i = z_{i+1} - c_i z_i - z_{i-1} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} - w_i^T \hat{\theta} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_i z_k$$

$$+ \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_r^{(k-1)}} x_r^{(k)} + \varphi_i^T \hat{\theta} + \varphi_i^T \tilde{\theta} - \dot{\alpha}_{i-1}$$

$$\dot{z}_i = z_{i+1} - c_i z_i - z_{i-1} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} - w_i^T \hat{\theta} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_i z_k$$

$$+ \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_r^{(k-1)}} x_r^{(k)} + \varphi_i^T \hat{\theta} + \varphi_i^T \tilde{\theta} - \dot{\alpha}_{i-1} + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_{i-1}^T \tilde{\theta} - \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_{i-1}^T \tilde{\theta}$$

$$= z_{i+1} - c_i z_i - z_{i-1} + (\varphi_i^T - w_i^T) \hat{\theta} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_i z_k$$

$$+ \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_r^{(k-1)}} x_r^{(k)} + \left(\varphi_i^T - \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_{i-1}^T \right) \tilde{\theta} + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_{i-1}^T \tilde{\theta} - \dot{\alpha}_{i-1}$$

$$= z_{i+1} - c_i z_i - z_{i-1} + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \varphi_j \hat{\theta} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_i z_k$$

$$+ \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_r^{(k-1)}} x_r^{(k)} + \left(\varphi_i^T - \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_{i-1}^T \right) \tilde{\theta} + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_{i-1}^T \tilde{\theta} - \dot{\alpha}_{i-1} \quad \text{(A.3)}$$

We note that

$$\varphi_i - w_i = \varphi_i - \varphi_i + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \varphi_j = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \varphi_j \quad \text{(A.4)}$$

$$\dot{z}_i = z_{i+1} - c_i z_i - z_{i-1} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_i z_k$$

$$+ \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_r^{(k-1)}} x_r^{(k)} + w_i^T \tilde{\theta} + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_{i-1}^T (\tilde{\theta} + \hat{\theta}) - \dot{\alpha}_{i-1}$$

$$\dot{z}_i \quad = z_{i+1} - c_i z_i - z_{i-1} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_i z_k$$

$$+ \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_r^{(k-1)}} x_r^{(k)} + w_i^T \tilde{\theta} + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_{i-1}^T \theta - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_n + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_n - \dot{\alpha}_{i-1}$$

$$\dot{z}_i \quad = z_{i+1} - c_i z_i - z_{i-1} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma(\tau_i - \tau_n) + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_i z_k + w_i^T \tilde{\theta} \tag{A.5}$$

we know that

$$\tau_i - \tau_n = - \sum_{k=i+1}^{n} w_k z_k \tag{A.6}$$

then

$$\dot{z}_i \quad = z_{i+1} - c_i z_i - z_{i-1} - \sum_{k=i+1}^{n} - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_k z_k + \sum_{k=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_i z_k + w_i^T \tilde{\theta}$$

Finally the error system for the $(z_1, \dots, z_{m-1})$-subsystem is

$$\dot{z}_i = -c_i z_i - z_{i-1} + z_{i+1} - \sum_{k=2}^{i-1} \sigma_{ki} z_k + \sum_{k=i+1}^{n} \sigma_{ik} z_k + \tilde{\theta}^T w_i$$

$$i = 1, \dots, m-1 \tag{A.7}$$

- The error system of the m-th step :

The error variable of the m-th order is

$$z_m = x_m - \alpha_{m-1} - x_r^{(m-1)} \tag{A.8}$$

Its derivative is

$$\dot{z}_m = \dot{x}_m - \dot{\alpha}_{m-1} - x_r^{(m)}$$

$$= b_m \beta(\bar{x}) x_{m+1} + \varphi_m^T \hat{\theta} + \varphi_m^T \tilde{\theta} - \dot{\alpha}_{m-1} - x_r^{(m)}$$

$$= b_m \beta(\bar{x}) (z_{m+1} + \alpha_m + \frac{\hat{\varrho}}{\beta(\bar{x})} x_r^{(m)}) + \varphi_m^T \hat{\theta} + \varphi_m^T \tilde{\theta} - \dot{\alpha}_{m-1} - x_r^{(m)}$$

$$= b_m\beta(\bar{x})(z_{m+1} + \alpha_m) - b_m\tilde{\varrho}x_r^{(m)} + \varphi_m^T\hat{\theta} + \varphi_m^T\tilde{\theta} - \dot{\alpha}_{m-1} - x_r^{(m)} \quad (A.9)$$

We note that

$$\tilde{\varrho} = \varrho - \hat{\varrho} \quad then \quad b_m\tilde{\varrho} = b_m\varrho - b_m\hat{\varrho} = 1 - b_m\hat{\varrho} \quad (A.10)$$

The Eq.(A.9) is now written as

$$\dot{z}_m = b_m\beta(\bar{x})\left(z_{m+1} + \frac{\hat{\varrho}}{\beta(\bar{x})}\bar{\alpha}_m\right) - b_m\tilde{\varrho}x_r^{(m)} + \varphi_m^T\hat{\theta} + \varphi_m^T\tilde{\theta} - \dot{\alpha}_{m-1}$$

$$= b_m\beta(\bar{x})z_{m+1} + \bar{\alpha}_m - b_m\tilde{\varrho}\bar{\alpha}_m - b_m\tilde{\varrho}x_r^{(m)} + \varphi_m^T\hat{\theta} + \varphi_m^T\tilde{\theta} - \dot{\alpha}_{m-1} \quad (A.11)$$

with replacing α_i by its equation from (A.2) the derivative becomes

$$\dot{z}_m = b_m\beta(\bar{x})z_{m+1} - c_m z_m - z_{m-1} + w_m^T\hat{\theta} + \sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{m-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_{m-1}}{\partial x_r^{(k-1)}}x_r^{(k)}\right) + \frac{\partial\alpha_{m-1}}{\partial\hat{\theta}}\Gamma\tau_m$$

$$+ \sum_{k=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma w_i z_k - b_m\tilde{\varrho}(\bar{\alpha}_m + x_r^{(m)}) + \varphi_m^T\hat{\theta} + \varphi_m^T\tilde{\theta} - \dot{\alpha}_{m-1}$$

$$\dot{z}_m = b_m\beta(\bar{x})z_{m+1} - c_m z_m - z_{m-1} + w_m^T\hat{\theta} + \sum_{k=1}^{m-1}\left(\frac{\partial\alpha_{m-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_m}{\partial x_r^{(k-1)}}x_r^{(k)}\right) + \frac{\partial\alpha_{m-1}}{\partial\hat{\theta}}\Gamma\tau_m$$

$$+ \frac{\partial\alpha_{m-1}}{\partial\hat{\theta}}\Gamma\tau_n - \frac{\partial\alpha_{m-1}}{\partial\hat{\theta}}\Gamma\tau_n + \sum_{k=2}^{m-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma w_m z_k - b_m\tilde{\varrho}\left(\bar{\alpha}_m + x_r^{(m)}\right)$$

$$+ \sum_{k=2}^{m-1}\frac{\partial\alpha_{m-1}}{\partial x_k}\varphi_{m-1}^T\tilde{\theta} - \sum_{k=2}^{m-1}\frac{\partial\alpha_{m-1}}{\partial x_k}\varphi_{m-1}^T\tilde{\theta} + \varphi_m^T\hat{\theta} + \varphi_m^T\tilde{\theta} - \dot{\alpha}_{m-1}$$

$$= b_m\beta(\bar{x})z_{m+1} - c_m z_m - z_{m-1} + \sum_{k=1}^{m-1}\left(\frac{\partial\alpha_{m-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_m}{\partial x_r^{(k-1)}}x_r^{(k)}\right) + \frac{\partial\alpha_{m-1}}{\partial\hat{\theta}}\Gamma(\tau_m - \tau_n)$$

$$+ \frac{\partial\alpha_{m-1}}{\partial\hat{\theta}}\Gamma\tau_{m-1} + \sum_{k=2}^{m-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma w_m z_k - b_m\tilde{\varrho}\left(\bar{\alpha}_m + x_r^{(m)}\right) + \sum_{k=2}^{m-1}\frac{\partial\alpha_{m-1}}{\partial x_k}\varphi_{m-1}^T(\tilde{\theta} + \hat{\theta})$$

$$+ \left(\varphi_m^T - \sum_{k=2}^{m-1}\frac{\partial\alpha_{m-1}}{\partial x_k}\varphi_{m-1}^T\right)\tilde{\theta} - \dot{\alpha}_{m-1} \quad (A.12)$$

Noting that

$$\tau_m - \tau_n = -\sum_{k=m+1}^{n} w_k z_k \tag{A.13}$$

the derivative is

$$\dot{z}_m = b_m\beta(\bar{x})z_{m+1} - c_m z_m - z_{m-1} + w_m^T\tilde{\theta} + \sum_{k=m+1}^{n} -\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma w_k z_k$$

$$+\sum_{k=2}^{m-1}\frac{\partial\alpha_{m-1}}{\partial\hat{\theta}}\Gamma w_m z_k - b_m\left(\bar{\alpha}_m + x_r^{(m)}\right)\tilde{\varrho}$$

Finally , the error system for the m-th step is written as :

$$\dot{z}_m = -c_m z_m - z_{m-1} + \hat{b}_m\beta(\bar{x})z_{m+1} - \sum_{k=2}^{m-1}\sigma_{km}z_k + \sum_{k=m+1}^{n}\sigma_{mk}z_k + w_m^T\tilde{\theta}$$

$$-b_m\left(\bar{\alpha}_m + x_r^{(m)}\right)\tilde{\varrho} + \tilde{b}_m\beta(\bar{x})z_{m+1} \tag{4.14}$$

- The error system of the (m+1)-th step

The error variable of the (m+1)-th order is

$$z_{m+1} = x_{m+1} - \frac{\hat{\varrho}}{\beta(\bar{x})}x_r^{(m)} - \alpha_m \tag{A.15}$$

the derivative of (A.15) is :

$$\dot{z}_{m+1} = \dot{x}_{m+1} - \frac{d}{dt}\left(\frac{\hat{\varrho}}{\beta(\bar{x})}x_r^{(m)}\right) - \dot{\alpha}_m$$

$$= x_{m+2} + \varphi_{m+1}^T\theta - \frac{\dot{\hat{\varrho}}}{\beta(\bar{x})}x_r^{(m)} - \hat{\varrho}\frac{d}{dt}\left(\frac{1}{\beta(\bar{x})}\right)x_r^{(m)} - \frac{\hat{\varrho}}{\beta(\bar{x})}x_r^{(m+1)} - \dot{\alpha}_m$$

$$= z_{m+2} + \frac{\hat{\varrho}}{\beta(\bar{x})}x_r^{(m+1)} + \alpha_{m+1} + \varphi_{m+1}^T\theta - \frac{\dot{\hat{\varrho}}}{\beta(\bar{x})}x_r^{(m)} - \hat{\varrho}\frac{d}{dt}\left(\frac{1}{\beta(\bar{x})}\right)x_r^{(m)}$$

$$-\frac{\hat{\varrho}}{\beta(\bar{x})}x_r^{(m+1)} - \dot{\alpha}_m$$

$$= z_{m+2} + \frac{\hat{\varrho}}{\beta(\bar{x})}x_r^{(m+1)} - c_{m+1}z_{m+1} - \hat{b}_m\beta(\bar{x})z_m + \sum_{k=1}^{m-1}\frac{\partial\alpha_m}{\partial x_k}x_{k+1} + \hat{b}_m\frac{\partial\alpha_m}{\partial x_m}\beta(\bar{x})x_{m+1}$$

$$- w_{m+1}^T\hat{\theta} + \frac{\partial\alpha_m}{\partial\hat{\theta}}\Gamma\tau_{m+1} + \left(\frac{x_r^{(m)}}{\beta(\bar{x})} + \frac{\partial\alpha_m}{\partial\hat{\varrho}}\right)\dot{\hat{\varrho}} + \sum_{k=2}^{m}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma w_{m+1}z_k + \sum_{k=1}^{m}\frac{\partial\alpha_m}{\partial x_r^{(k-1)}}x_r^{(k)}$$

$$+ \hat{\varrho}(\sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)\left(\varphi_{k+1}^T\tilde{\theta} + \varphi_{k+1}^T\hat{\theta}\right)x_r^{(m)} + \sum_{\substack{k=1\\k\neq m}}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)x_{k+1})$$

$$+ \frac{\partial}{\partial x_m}\left(\frac{1}{\beta(\bar{x})}\right)\hat{b}_m\hat{\varrho}x_r^{(m)}\beta(\bar{x})x_{m+1} + \varphi_{m+1}^T\theta - \frac{\dot{\hat{\varrho}}}{\beta(\bar{x})}x_r^{(m)} - \hat{\varrho}\frac{d}{dt}\left(\frac{1}{\beta(\bar{x})}\right)x_r^{(m)}$$

$$- \frac{\hat{\varrho}}{\beta(\bar{x})}x_r^{(m+1)} - \dot{\alpha}_m \qquad \text{(A.16)}$$

$$\dot{z}_{m+1} = z_{m+2} - c_{m+1}z_{m+1} - \hat{b}_m\beta(\bar{x})z_m - w_{m+1}^T\hat{\theta} + \sum_{k=1}^{m-1}\frac{\partial\alpha_m}{\partial x_k}x_{k+1} + \hat{b}_m\frac{\partial\alpha_m}{\partial x_m}\beta(\bar{x})x_{m+1}$$

$$+ \frac{\partial\alpha_m}{\partial\hat{\theta}}\Gamma\tau_{m+1} + \frac{\partial\alpha_m}{\partial\hat{\varrho}}\dot{\hat{\varrho}} + \sum_{k=2}^{m}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma w_{m+1}z_k + \sum_{k=1}^{m}\frac{\partial\alpha_m}{\partial x_r^{(k-1)}}x_r^{(k)}$$

$$+\hat{\varrho}(\sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)\left(\varphi_{k+1}^T\tilde{\theta} + \varphi_{k+1}^T\hat{\theta}\right)x_r^{(m)} + \sum_{\substack{k=1\\k\neq m}}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)x_{k+1})$$

$$+ \frac{\partial}{\partial x_m}\left(\frac{1}{\beta(x)}\right)\hat{b}_m\hat{\varrho}x_r^{(m)}\beta(\bar{x})x_{m+1} + \varphi_{m+1}^T\tilde{\theta} + \varphi_{m+1}^T\hat{\theta}$$

$$-\hat{\varrho}\sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)\left(x_{k+1} + \varphi_{k+1}^T\tilde{\theta} + \varphi_{k+1}^T\hat{\theta}\right)x_r^{(m)} - \dot{\alpha}_m \qquad \text{(A.17)}$$

With noting that

$$\frac{d}{dt}\left(\frac{1}{\beta(\bar{x})}\right) = \frac{d}{dx}\left(\frac{1}{\beta(\bar{x})}\right) \times \frac{dx}{dt} = \sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)\dot{x}_k = \sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)\left(x_{k+1} + \varphi_{k+1}^T\tilde{\theta} + \varphi_{k+1}^T\hat{\theta}\right)$$

$$\text{(A.18)}$$

Equ.(A.17) is now written as

$$\dot{z}_{m+1} = z_{m+2} - c_{m+1}z_{m+1} - \hat{b}_m\beta(x)z_m - w_{m+1}^T\hat{\theta} + \sum_{k=1}^{m-1}\frac{\partial\alpha_m}{\partial x_k}x_{k+1} + \hat{b}_m\frac{\partial\alpha_m}{\partial x_m}\beta(x)x_{m+1}$$

$$+\frac{\partial\alpha_m}{\partial\hat{\theta}}\Gamma\tau_{m+1} + \frac{\partial\alpha_m}{\partial\hat{\varrho}}\dot{\hat{\varrho}} + \sum_{k=2}^{m}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma w_{m+1}z_k + \sum_{k=1}^{m}\frac{\partial\alpha_m}{\partial x_r^{(k-1)}}x_r^{(k)} + \varphi_{m+1}^T\tilde{\theta}$$

$$+\hat{\varrho}(\sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)\left(\varphi_{k+1}^T\tilde{\theta} + \varphi_{k+1}^T\hat{\theta}\right)x_r^{(m)} + \sum_{\substack{k=1\\k\neq m}}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)x_{k+1}) + \varphi_{m+1}^T\hat{\theta}$$

$$-\hat{\varrho}\sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)(\varphi_{k+1}^T\tilde{\theta} + \varphi_{k+1}^T\hat{\theta})\,x_r^{(m)} - \hat{\varrho}\sum_{\substack{k=1\\k\neq m}}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)x_{k+1} - \dot{\alpha}_m$$

then,

$$\dot{z}_{m+1} = z_{m+2} - c_{m+1}z_{m+1} - \hat{b}_m\beta(\bar{x})z_m - w_{m+1}^T\hat{\theta} + \hat{b}_m\frac{\partial\alpha_m}{\partial x_m}\beta(\bar{x})x_{m+1}$$

$$+\frac{\partial\alpha_m}{\partial\hat{\theta}}\Gamma(\tau_{m+1} - \tau_n) + \sum_{k=2}^{m}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma w_{m+1}z_k + \varphi_{m+1}^T\tilde{\theta} - \sum_{k=1}^{m-1}\frac{\partial\alpha_m}{\partial x_k}\varphi_{k+1}^T\theta$$

$$+\hat{\varrho}(\sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)\left(\varphi_{k+1}^T\tilde{\theta} + \varphi_{k+1}^T\hat{\theta}\right)x_r^{(m)} + \sum_{\substack{k=1\\k\neq m}}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)x_{k+1})$$

$$+\varphi_{m+1}^T\hat{\theta} - \hat{\varrho}\sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)(\varphi_{k+1}^T\tilde{\theta} + \varphi_{k+1}^T\hat{\theta})\,x_r^{(m)} - \hat{\varrho}\sum_{\substack{k=1\\k\neq m}}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)x_{k+1} \qquad (A.19)$$

Finally the error system for the (m+1)-th step is

$$\dot{z}_{m+1} = -c_{m+1}z_{m+1} - \sum_{k=2}^{m}\sigma_{k,m+1}z_k - z_{m+2} - \hat{b}_m z_m + \sum_{k=m+2}^{n}\sigma_{m+1,k}z_k$$

$$+w_{m+1}^T\tilde{\theta} + \tilde{b}_m(\frac{\partial}{\partial x_m}\left(\frac{1}{\beta(\bar{x})}\right)\hat{\varrho}x_r^{(m)} - \frac{\partial\alpha_m}{\partial x_m})\beta(\bar{x})x_{m+1} \qquad (A.20)$$

- Calculate of the error system for $(z_{m+2}, \ldots, z_n)$-subsystem

The error variable is

$$z_j = x_j - \alpha_{j-1} - \frac{\hat{\varrho}}{\beta(\bar{x})} x_r^{(j-1)} \qquad (A.21)$$

The derivative is

$$\dot{z}_j = \dot{x}_j - \frac{d}{dt}\left(\frac{\hat{\varrho}}{\beta(\bar{x})} x_r^{(j-1)}\right) - \dot{\alpha}_{j-1}$$

$$= x_{j+1} + \varphi_j^T \theta - \frac{\dot{\hat{\varrho}}}{\beta(\bar{x})} x_r^{(j-1)} - \hat{\varrho}\frac{d}{dt}\left(\frac{1}{\beta(\bar{x})}\right) x_r^{(j-1)} - \frac{\hat{\varrho}}{\beta(\bar{x})} x_r^{(j)} - \dot{\alpha}_{j-1}$$

$$= z_{j+1} + \alpha_j + \varphi_j^T \theta - \frac{\dot{\hat{\varrho}}}{\beta(\bar{x})} x_r^{(j-1)} - \hat{\varrho}\frac{d}{dt}\left(\frac{1}{\beta(\bar{x})}\right) x_r^{(j-1)} - \dot{\alpha}_{j-1} \qquad (A.22)$$

with replacing α_j by its equation

$$\begin{aligned}
\bar{\alpha}_j \quad &= -c_j z_j - z_{j-1} - w_j^T \hat{\theta} + \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \tau_j + \sum_{\substack{k=1 \\ k \neq m}}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_k} x_{k+1} + \hat{b}_m \beta(x) \frac{\partial \alpha_{j-1}}{\partial x_m} x_{m+1} \\
&+ \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_r^{(k-1)}} x_r^{(k)} + \sum_{k=2}^{j-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma w_i z_k + \frac{\partial \alpha_{j-1}}{\partial \hat{b}_m} \gamma \pi_j + \left(\frac{x_r^{(j-1)}}{\beta(x)} + \frac{\partial \alpha_{j-1}}{\partial \hat{\varrho}}\right) \dot{\hat{\varrho}} \\
&+ \hat{\varrho}\left(\sum_{k=1}^{n} \frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)\left(\varphi_{k+1}^T \tilde{\theta} + \varphi_{k+1}^T \hat{\theta}\right) x_r^{(j-1)} + \sum_{\substack{k=1 \\ k \neq m}}^{n} \frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right) x_{k+1}\right) \\
&+ \frac{\partial}{\partial x_m}\left(\frac{1}{\beta(x)}\right) \hat{b}_m \hat{\varrho} x_r^{(m)} \beta(x) x_{m+1} + \sum_{k=m+1}^{j-1} \frac{\partial \alpha_{k-1}}{\partial \hat{b}_m} \gamma \frac{\partial \alpha_{j-1}}{\partial x_m} x_{m+1} z_k,
\end{aligned}$$

$$j = m+2, \dots, n \qquad (A.23)$$

The Eq.(A.22) becomes

$$\begin{aligned}
\dot{z}_j = \; & z_{j+1} - c_j z_j - z_{j-1} - w_j^T \hat{\theta} + \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \tau_j + \sum_{\substack{k=1 \\ k \neq m}}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_k} x_{k+1} + \hat{b}_m \beta(\bar{x}) \frac{\partial \alpha_{j-1}}{\partial x_m} x_{m+1} \\
&+ \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_r^{(k-1)}} x_r^{(k)} + \sum_{k=2}^{j-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma w_i z_k + \frac{\partial \alpha_{j-1}}{\partial \hat{b}_m} \gamma \pi_j + \left(\frac{x_r^{(j-1)}}{\beta(\bar{x})} + \frac{\partial \alpha_{j-1}}{\partial \hat{\varrho}}\right) \dot{\hat{\varrho}}
\end{aligned}$$

$$+\frac{\partial}{\partial x_m}\left(\frac{1}{\beta(\bar{x})}\right)\hat{b}_m\hat{\varrho}x_r^{(m)}\beta(\bar{x})x_{m+1}+\sum_{k=m+1}^{j-1}\frac{\partial\alpha_{k-1}}{\partial\hat{b}_m}\gamma\frac{\partial\alpha_{j-1}}{\partial x_m}x_{m+1}z_k$$

$$+\varphi_j^T\theta-\frac{\dot{\hat{\varrho}}}{\beta(\bar{x})}x_r^{(j-1)}-\hat{\varrho}\frac{d}{dt}\left(\frac{1}{\beta(\bar{x})}\right)x_r^{(j-1)}-\dot{\alpha}_{j-1}$$

$$\dot{z}_j = z_{j+1}-c_jz_j-z_{j-1}-w_j^T\hat{\theta}+\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\Gamma\tau_j+\sum_{\substack{k=1\\k\neq m}}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_k}x_{k+1}+\hat{b}_m\beta(\bar{x})\frac{\partial\alpha_{j-1}}{\partial x_m}x_{m+1}$$

$$+\hat{\varrho}(\sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)\left(\varphi_{k+1}^T\tilde{\theta}+\varphi_{k+1}^T\hat{\theta}\right)x_r^{(j-1)}+\sum_{\substack{k=1\\k\neq m}}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)x_{k+1})+\sum_{k=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_r^{(k-1)}}x_r^{(k)}$$

$$+\sum_{k=2}^{j-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma w_iz_k+\frac{\partial\alpha_{j-1}}{\partial\hat{b}_m}\gamma\pi_j+\frac{\partial\alpha_{j-1}}{\partial\hat{\varrho}}\dot{\hat{\varrho}}+\sum_{k=m+1}^{j-1}\frac{\partial\alpha_{k-1}}{\partial\hat{b}_m}\gamma\frac{\partial\alpha_{j-1}}{\partial x_m}x_{m+1}z_k+\varphi_j^T\tilde{\theta}+\varphi_j^T\hat{\theta}$$

$$-\hat{\varrho}\sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)(\varphi_{k+1}^T\tilde{\theta}+\varphi_{k+1}^T\hat{\theta})x_r^{(j-1)}-\hat{\varrho}\sum_{\substack{k=1\\k\neq m}}^{n}\frac{\partial}{\partial x_k}\left(\frac{1}{\beta(\bar{x})}\right)x_{k+1}-\dot{\alpha}_{j-1}$$

$$\dot{z}_j \quad = -c_jz_j-\sum_{k=2}^{j-1}\sigma_{kj}z_k-z_{j-1}+z_{j+1}+\sum_{k=j+1}^{n}\sigma_{jk}z_k+w_j^T\tilde{\theta}$$

$$-\tilde{b}_m\left(\frac{\partial}{\partial x_m}\left(\frac{1}{\beta(\bar{x})}\right)\hat{\varrho}x_r^{(j-1)}-\frac{\partial\alpha_{j-1}}{\partial x_m}\right)\beta(\bar{x})x_{m+1}, \qquad i = m+2,\dots,n-1 \qquad (A.24)$$

- The error system for the n-th step

The error variable of the (m+1)-th order is

$$z_n = x_n-\alpha_{n-1}-\frac{\hat{\varrho}}{\beta(\bar{x})}x_r^{(n-1)} \qquad (A.25)$$

after calculus the derivative is written as

$$\dot{z}_n = -c_j z_j - \sum_{k=2}^{n-1} \sigma_{kj} z_k - z_{n-1} + \sum_{k=j+1}^{n} \sigma_{jk} z_k + w_j^T \tilde{\theta} - b_n(\alpha_n + \frac{\hat{\varrho}}{\beta(x)} x_r^{(n)})\hat{\lambda}$$

$$-\tilde{b}_m \left(\frac{\partial}{\partial x_m} \left(\frac{1}{\beta(\bar{x})} \right) \hat{\varrho} x_r^{(j-1)} - \frac{\partial \alpha_{j-1}}{\partial x_m} \right) \beta(\bar{x}) x_{m+1} , \tag{A.26}$$

where σ_{ik} is defined for $k = i + 1, \dots, n$ as

$$\sigma_{ik} = \begin{cases} 0, \quad i = 1 \\ -\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_k, \qquad i = 2, \dots, m + 1 \\ -\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma w_k + \frac{\partial \alpha_{i-1}}{\partial \hat{b}_m} \gamma \frac{\partial \alpha_{k-1}}{\partial x_m} x_{m+1} \qquad i = m + 2, \dots, n - 1 \end{cases} \tag{A.27}$$