The objective of this work is to employ spin-polarized density functional theory (sDFT) calculations for the exploration of ultrathin magnetic films with large magnetic moments and a strong perpendicular anisotropy. Monolayer films of Fe$_{1-x}$Co$_x$ (with $x=0, 0.25, 0.5, 0.75,$ and $1$) on Rh(001) were addressed to study their magnetic properties using the all-electron full-potential linearized augmented plane wave (FLAPW) method in film geometry. We studied the magnetic order of these films including structural relaxations of the topmost layers. Fe$_{1-x}$Co$_x$ monolayer films were found to be ferromagnetic (FM) in a broad range of Co content $x$ with a maximum magnetic moment of $2.8 \, \mu$B and of an out-of-plane magneto-crystalline anisotropy of $0.25 \, $meV per magnetic atom at $x=0.5$. The sDFT results were mapped onto a classical Heisenberg model, demonstrating FM Fe-Co and Co-Co couplings, while the Fe-Fe interaction is antiferromagnetic on Rh(001). The ordering temperature of the FeCo film was estimated to be well above room temperature (482 K).