

A VARIABLE STRUCTURE CONTROL SYSTEM FOR A 6 DOF PUMA 560 ROBOT MANIPULATOR

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This paper deals with the application of a variable structure observer developed for a class of nonlinear systems to solve the trajectory tracking problem for rigid robot manipulators. The considered observer design approach proposes a simple design methodology for systems having completely observable linear parts and bounded nonlinearities and/or uncertainties. The considered observer is basically the conventional Luenberger observer with an additional switching term that is used to guarantee robustness against modeling errors and system uncertainties. We propose to modify this observer structure and use the system nonlinearities in the observer structure under the Lipschitz condition. To solve the tracking problem, we have used a control law developed for robot manipulators in the full information case. The closed loop system is shown to be globally asymptotically stable based on Lyapunov arguments. Simulation results on a six D-O-F robot manipulator show the asymptotic convergence of the observation and tracking error vectors.

Keywords: Variable Structure Observers, Switching-type observers, 6 DOF PUMA560 robot manipulator, Exponential stability, Tracking control.

1. INTRODUCTION

The control of rigid robot manipulators was solved by several classical and robust efficient methods, and it is shown that each control strategy ensures the stability of the trajectory tracking error in some suitable sense. One basic assumption to these methods is that the full state information is available for feedback. In fact, for robotic systems, a state feedback control is based on the exact knowledge of both the position and velocity vectors. Unfortunately, the velocity vector cannot generally be available for feedback for several reasons. A solution to this is the design of nonlinear observers that give the reconstruction of the missing velocity signal. Due to the nonlinear and coupled structure of the robot dynamical model, the problem of designing observers for robots is a very complex one.

For nonlinear systems, several approaches have been presented in literature [13] to solve the nonlinear observer design problem. The first possibility consists to transform a nonlinear problem into a linear one by the extended linearization technique [3] or by the pseudo-linearization method ([15], [21]), which yields constant eigenvalues of the reconstruction error dynamics when linearized about any fixed equilibrium point. We also have the exact linearization technique [14] that consists of transforming the nonlinear system into a linear system with an output injection to apply the linear observation theory. A second possibility consists of designing an observer with a nonlinear observation error dynamics. To this fact, some techniques are established in the initial state coordinates [11], and others in the observable canonical form ([9], [10], [6]). All these methods are available

for nonlinear systems without uncertainties or disturbances in their dynamic equations (for a survey on nonlinear observers, the reader is asked to consult [22], [16], [20], [13]).

Motivated by this, the control problem of robots using partial knowledge of the state variable (only joint measurements) has attained an increasing interest. A straightforward approach to this problem goes along a two steps design: first, construct a nonlinear observer driven by the available inputs and outputs, which reconstructs the lacking velocity signal, second, design a state feedback controller and replace the actual velocity by the one reconstructed from the observer. Indeed, based on this procedure a number of conceptually different methods for both regulation and tracking control of robots equipped with only position sensors have been developed. ([17], [4], [5], [12]). These observers guarantee the exponential and asymptotic stability of the observation error, but do not take into consideration system uncertainties, even though several studies have shown that under some conditions, some of them present robustness properties, especially those based on the passivity approach ([2], [4]). A solution to this is the design of robust observers.

The design of observers that take into consideration system uncertainties have taken the interest of many people ([4], [7], [8], [16], [19], [20], [22]).

In [21] a variable structure observer for a class of nonlinear systems is presented. The authors propose a simple design methodology for systems having completely observable linear parts and bounded nonlinearities or uncertainties. A minimum estimate for the rate of convergence of the observer error was also given. This observer is basically the conventional Luenberger observer with an additional switching term that is used to guarantee robustness against modeling errors and system uncertainties. Due to this supplementary switching term, this observer suffers from the chattering usually associated to Variable Structure Systems. To deal with this problem, the original observer is modified and a boundary layer approach is considered. However, with this modification, the asymptotic stability aspect of the observation error dynamics is lost, and only global uniform ultimate boundedness stability of the observation error is obtained. In [8], extension to the above variable structure scheme is proposed, and a continuous observer was used to ensure the global exponential stability of the observation error system.

In [1], an application of the above variable structure observer to the class of robot manipulators is presented with an application to a 2 DOF rigid robot system. The exponential convergence of the observation error was shown. But, a difficulty in tuning observer gains was raised. This is due to the large nonlinearities contained in the system, which will induce large values of the nonlinearities upper bound to be used in the additive switching term, and as a consequence, high chattering is produced.

In this paper, we propose to modify the variable structure observer described in [22], by including the system nonlinearities in the observer structure, under the global Lipschitz condition, and apply it to the system of n -DOF robot manipulators to solve the tracking control problem with only position measurements. The exponential stability of the observation error is shown under the condition that system uncertainties and external disturbances are bounded, which is generally guaranteed for this class of systems.

The estimated velocity vector will be used in a trajectory tracking control law proposed in [18], which guarantees the global asymptotic stability of the tracking error for the manipulator control system. Keeping in mind that no separation principle exists for nonlinear systems, the study of the closed loop stability is performed using a Lyapunov function that contains two terms, one for the tracking error and the second for the estimation error. The asymptotic stability of the closed loop system is shown, under a suitable choice of the observer and controller gains.

This paper is organized as follows; we first consider from literature the variable structure observer design method. Then, we apply this observer to the class of rigid robot manipulators and show that under some assumptions, the exponential convergence of the observation error is guaranteed. Section 4 is devoted to closed loop control, where we use Lyapunov arguments to prove the closed loop stability. Finally, simulation results of the proposed scheme on a 6-DOF robot manipulator are illustrated.

2. OBSERVER DESIGN

Consider the following nonlinear system;

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x, u, t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the output vector and $u(t) \in \mathbb{R}^m$ is the control input. The vector $f(\cdot, \cdot, \cdot)$, assumed continuous in $x(t)$, is used to represent the nonlinearities and/or uncertainties in the plant. The problem is to design an observer with inputs $y(t)$ and $u(t)$, whose output $\hat{x}(t)$ is the estimated state that is ensured to converge in a finite time to the real state. Before we give the observer structure, the following assumptions should be made.

Assumption 1 - The pair (A, C) is detectable, i.e., there exists a matrix L of appropriate dimensions such that the spectrum of $A_o = A - LC$ is completely contained in the open left half-plane.

Assumption 2 - There exist a positive definite matrix $Q \in \mathbb{R}^{n \times n}$ and a function h where $h(\cdot, \cdot): \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}^p$, such that the following matching conditions hold

$$f(t, x) = P^{-1}C^T h(t, x) \quad (2)$$

where P is the unique positive definite solution to the Lyapunov equation

$$A_o^T P + P A_o = -Q \quad (3)$$

Assumption 3 - There exists a non-negative function $\rho(\cdot, \cdot): \mathbb{R}_+ \times \mathbb{R}^m \rightarrow \mathbb{R}_+$, such that

$$\|h(t, x, u)\| \leq \rho(t, u), \quad (4)$$

$$\forall x \in \mathbb{R}^n, \forall u \in \mathbb{R}^m \text{ and } t \in \mathbb{R}_+$$

If assumptions **1-3** are satisfied, then the proposed observer is described by the following differential equations

$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x}) + \nu_0(t, \hat{x}, y) \quad (5)$$

where

$$\nu_0(t, \hat{x}, y) = \begin{cases} \frac{-P^{-1}C^T C e}{\|C e\|} \rho(t, u) & \forall \|C e\| \neq 0 \\ 0 & \forall \|C e\| = 0 \end{cases} \quad (6)$$

and L is a positive diagonal design matrix.

Let the observation error be defined as, $e = \hat{x} - x$. The observation error system will then be described by

$$\dot{e} = A_o e + \nu_0(t, \hat{x}, y) - f(t, x, u) \quad (7)$$

The exponential convergence of the estimation error is stated by the following theorem.

Theorem 1 - Given the nonlinear system described by (1) and the observer governed by (5)-(6), if assumptions 1-3 are satisfied, then the observation error $e = \hat{x} - x$ is globally exponentially stable.

The poof of this theorem can be found in [22]. It can be seen that this observer is the conventional Luenberger observer with the additional switching term $\nu_0(t, \hat{x}, y)$, which ensures robustness against system nonlinearities. Unfortunately, this discontinuous term will cause the undesirable phenomenon of “chattering”. Hence, it is advantageous to design a gain law that is continuous in the error and ensures that the estimated state will converge at least asymptotically to some arbitrary small neighborhood of the real state.

To satisfy these requirements, a boundary layer strategy that offers a continuous gain function is proposed in [22]. This is done, by replacing the discontinuous term given by (6) by the continuous term given by

$$\bar{\nu}_0(t, \hat{x}, y) = \begin{cases} \frac{-P^{-1}C^T C e}{\|C e \rho\|} \rho^2 & \text{if } \|C e \rho\| > \varepsilon \\ \frac{-P^{-1}C^T C e}{\varepsilon} \rho^2 & \text{if } \|C e \rho\| \leq \varepsilon \end{cases} \quad (8)$$

with $\varepsilon > 0$. With the observer (5) (8), the error system obeys to

$$\dot{e} = A_o e + \bar{\nu}_0(t, \hat{x}, y) - P^{-1}C^T h(t, x, u) \quad (9)$$

It can easily be shown, that the error signal is globally uniformly ultimately bounded.

3. APPLICATION TO ROBOTIC SYSTEMS

In this section, we consider a modification of the above observer, and apply it to the system of n-DOF robot manipulators. Instead of using an upper bound of system nonlinearities and external disturbances, as specified by assumption 3, we consider only the upper bound of external disturbances, friction terms and possible system uncertainties, and we use system nonlinearities in the observer structure. This will prevent high values of the upper bound that causes considerable chattering, as can be seen from (6).

In order to apply this scheme to robot manipulators, we consider the dynamics of a n-DOF robot manipulator given in [23], written in the following state space representation

$$\begin{cases} \dot{x} = Ax + f(x, u, t) + \eta_d(t) \\ y = Cx \end{cases} \quad (10)$$

$$\text{with } x = \begin{pmatrix} q \\ \dot{q} \end{pmatrix}, A = \begin{pmatrix} 0 & I_n \\ 0 & 0 \end{pmatrix}, C = \begin{pmatrix} I_n & 0 \end{pmatrix} \quad (11.a)$$

$$f(x, u, t) = \begin{pmatrix} 0 \\ -M^{-1}(q)(C(q, \dot{q})\dot{q} + G(q) - u) \end{pmatrix} \quad (11.b)$$

$$u = \tau \quad (11.c)$$

where $q \in \mathbb{R}^n$ is the vector of joint angular positions, $M(q) \in \mathbb{R}^{n \times n}$ is the symmetric positive definite inertia matrix, $C(q, \dot{q})\dot{q} \in \mathbb{R}^n$ is the Coriolis and centrifugal torque vector, $G(q) \in \mathbb{R}^n$ is the gravity vector and $\tau \in \mathbb{R}^n$ is the vector of applied joint torque. $\eta_d(t)$ is the vector representing external disturbances, friction terms and system uncertainties. The vector $f(x, u, t)$ is assumed to satisfy the following assumption.

Assumption 4:- There exists a constant $\kappa > 0$ such that for all x_1 and $x_2 \in D_0 \subset \mathbb{R}^{2n}$, then

$$\|f(x_1) - f(x_2)\| \leq \kappa \|x_1 - x_2\| \quad (12)$$

where $D_0 = \left\{ x \in \mathbb{R}^{2n} / \|\dot{q}(t)\| < V_M \right\}$, with $V_M > 0$, and κ is the Lipschitz constant.

The first step to be considered in the design of the variable structure observer for robot manipulators is to satisfy assumptions 1-3. From expressions (11), assumption 1 can always be satisfied since the matrix $A_0 = A - LC$ can be selected to be a stable matrix for any positive gain matrix L , so $P = P^T > 0$ is the unique solution to the Lyapunov equation given in (3).

In addition, by exploiting the structural properties of rigid robot manipulators given in [23], we can always verify that;

$$\eta_d(t) = P^{-1}C^T w(t) \quad (13)$$

$$\text{where } \|w(t)\| \leq \rho(t) \quad (14)$$

and $w(t)$ is a parameterization of the disturbance vector $\eta_d(t)$. We can notice that we have considered only an upper bound $\rho(t)$ of friction and disturbance terms, which is different from assumption 3, where an upper bound of system nonlinearities is considered.

The observer is then given by

$$\dot{\hat{x}} = A\hat{x} + f(\hat{x}, u, t) + L(y - C\hat{x}) + \nu_0(t, \hat{x}, y) \quad (15)$$

with $\nu_0(t, x, y)$ defined as in (6). The observation error system is obtained as

$$\dot{e} = A_o e + \tilde{f}(\hat{x}, x) + \nu_0(t, \hat{x}, y) - \eta_d(t) \quad (16)$$

with $e = \hat{x} - x = \begin{pmatrix} e_1 & e_2 \end{pmatrix}^T$ is the observation error vector and $\tilde{f}(\hat{x}, x) = f(\hat{x}, u, t) - f(x, u, t)$. We can see from (16) that the additional switching term $\nu_0(t, \hat{x}, y)$ is used in the observer structure to cope with the effects of uncertainties in the plant model and input disturbances.

To show the exponential convergence of the observation error, we consider the following Lyapunov function candidate;

$$V = \frac{1}{2} e^T P e \quad (17)$$

which time derivative evaluated along the error dynamics (16) yields

$$\dot{V} = -\frac{1}{2} e^T Q e + e^T P \left(\tilde{f}(\hat{x}, x) + \nu_0(t, \hat{x}, y) - P^{-1} C^T + w(t) \right) \quad (18)$$

If we consider assumptions 1-2 with equations (6) and (13), and assumption 4, this last expression can be bounded as

$$\dot{V} \leq -\left(\frac{1}{2} \lambda_{\min}(Q) - \kappa \lambda_{\max}(P) \right) \|e\|^2 - \|C e\| \rho - e^T C^T w(t) \quad (19)$$

where $\lambda_{\min}(\cdot), \lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalue of its argument respectively. Using (14) we can finally write

$$\dot{V} \leq -\left(\frac{1}{2} \lambda_{\min}(Q) - \kappa \lambda_{\max}(P) \right) \|e\|^2 < 0 \quad (20)$$

Therefore, the time derivative of the Lyapunov function candidate is negative definite iff;

$$\kappa < \frac{\lambda_{\min}(Q)}{2 \lambda_{\max}(P)} \quad (21)$$

which implies that, under the above condition, the error converges exponentially to zero. Furthermore, from bounds on the Lyapunov function, we can write

$$\frac{1}{2} \lambda_{\min}(P) \|e\|^2 \leq V(e) \leq \frac{1}{2} \lambda_{\max}(P) \|e\|^2 \quad (22)$$

and

$$\dot{V} \leq -\left(\frac{1}{2} \lambda_{\min}(Q) - \kappa \lambda_{\max}(P) \right) \|e\|^2 \quad (23)$$

Then, we can write

$$\frac{\dot{V}(e)}{V(e)} \leq -\frac{(\lambda_{\min}(Q) - 2\kappa \lambda_{\max}(P))}{\lambda_{\max}(P)} = \varepsilon_1 \quad (24)$$

or

$$V(e(t)) \leq V(e(0))e^{-\varepsilon_1 t} \quad (25)$$

Hence, the rate at which the error converges to zero is determined as

$$\|e(t)\|^2 \leq \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \|e(0)\|^2 e^{-\varepsilon_1 t} \quad (26)$$

This shows the exponential convergence of the observation error signal, and the rate of convergence. Again, the switching term in the observer will cause “chattering”.

4. TRAJECTORY TRACKING CONTROL DESIGN

In order to use the above observer for the tracking problem of robot manipulators, we consider the trajectory tracking controller proposed in [18], with the real velocity state vector replaced with the estimated one. We have the control law is given by

$$\tau = M(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + G(q) - K_v(\dot{\tilde{q}} - \dot{q}_d) - K_p\tilde{q} \quad (27)$$

where $\tilde{q} = q - q_d$ defines the position tracking error, K_p and K_v are positive design controller gains. We should make the assumption that the desired velocity vector is bounded as; $\|\dot{q}_d\| \leq V_p$, which is reasonable from the implementation point of view. Using the robot dynamics, the closed loop system is governed by

$$M(q)\ddot{\tilde{q}} + C(q, \dot{q})\dot{\tilde{q}} - C(q, \dot{q})\dot{q}_d = -K_p\tilde{q} - K_v(\dot{\tilde{q}} + e_2) \quad (28)$$

where $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$ is the velocity tracking error and $e_2 = \dot{\tilde{q}} - \dot{q}$ is the velocity observation error. Using the structural properties of the Coriolis and centrifugal torque vector [23], we can write

$$C(q, \dot{q})\dot{\tilde{q}} - C(q, \dot{q})\dot{q}_d = C(q, \dot{q})\dot{\tilde{q}} - C(q, \dot{q}_d)e_2 \quad (29)$$

According to this, consider the following Result:

Given the control law given in (27), and the observer (15) with (6), if assumptions 1-2-4 and relations (13)-(14) are satisfied, then the closed loop system described by (16) and (28) is globally asymptotically stable. To investigate the stability of this closed loop dynamics, consider the Lyapunov function candidate

$$V(e, \tilde{q}, \dot{\tilde{q}}, t) = \frac{1}{2}e^T P e + \frac{1}{2}\dot{\tilde{q}}^T M(q)\dot{\tilde{q}} + \frac{1}{2}\tilde{q}^T K_p \tilde{q} \quad (30)$$

The time derivative of this Lyapunov function evaluated along the trajectories of the error dynamics (16) and (28) and using relations (13)-(14), is obtained directly as

$$\dot{V} = -\frac{1}{2}e^T Q e + e^T P (\tilde{f}(\hat{x}, x) + \nu_0 - P^{-1}C^T w(t)) - \dot{\tilde{q}}^T K_v \dot{\tilde{q}} - \dot{\tilde{q}}^T K_v e_2 + \dot{\tilde{q}}^T C(q, \dot{q}_d) e_2 \quad (31)$$

This can be bounded as, using the structural properties of the Coriolis and centrifugal torque vector [23];

$$\dot{V} \leq -\left(\frac{1}{2}\lambda_{\min}(Q) - \kappa\lambda_{\max}(P)\right)\|e\|^2 - K_{v,m}\|\dot{\tilde{q}}\|^2 + \|\dot{\tilde{q}}\|\|e_2\|(K_{v,M} + C_M V_p) \quad (32)$$

with $K_{v,m}$ and $K_{v,M}$ denote the minimum and maximum eigenvalues of matrix K_v . Knowing that $\|e_2\| \leq \|e\|$, we can write

$$\dot{V} \leq - \begin{pmatrix} \|\dot{q}\| \\ \|e\| \end{pmatrix}^T \begin{pmatrix} K_{v,m} & -\frac{1}{2}(K_{v,M} + C_M V_P) \\ -\frac{1}{2}(K_{v,M} + C_M V_P) & \left(\frac{1}{2}\lambda_{\min}(Q) - \kappa\lambda_{\max}(P)\right) \end{pmatrix} \begin{pmatrix} \|\dot{q}\| \\ \|e\| \end{pmatrix} \quad (33)$$

The matrix in right hand side of the above inequality is positive if

$$\left(\frac{1}{2}\lambda_{\min}(Q) - \kappa\lambda_{\max}(P)\right) > \frac{(K_{v,M} + C_M V_P)^2}{4K_{v,m}} \quad (34)$$

Under this condition and using Barballat's Lemma, we can conclude the asymptotic stability of the equilibrium point $(\tilde{q}, \dot{\tilde{q}}, e_1, e_2) = (0, 0, 0, 0)$.

Note that the above stability condition can always be satisfied if matrices K_v and Q are properly selected. In all cases, Q should be maximized.

5. SIMULATION RESULTS

In order to test the validity of our design, we have considered a 6 DOF PUMA 560 robot manipulator. The objective of our simulation work is to show that the tracking objective is achieved when the robustly estimated velocity vector is used in the tracking control law.

Due to the complexity of the control system, the control system gains should be carefully selected. The controller gains are selected to be high enough such that the tracking controller ensures the asymptotic convergence of the tracking error in the case of full state information. We have encountered several problems during observer gains tuning and have noticed that if the gain matrix L is fixed, increasing matrix Q will give large solutions for matrix P , which will cause high gain switching term, and if matrix P is fixed, increasing matrix Q will lead to use high observer gain matrix L . In both situations, the system will be more sensitive to measurement noise and high frequency unmodeled dynamics. Moreover, the observer gains should be selected according to condition (36).

The obtained results from the MATLAB simulation of the proposed scheme with a 6 DOF robot manipulator along a trajectory of order 5, and uncertainties/disturbances upper bound estimated at 32.5, are shown below.

Figures 1 show the velocity observation errors of the six axes, where we can see the convergence of the error signals and the high frequency oscillations caused by the switching term. To solve this problem, we can consider the boundary layer approach to eliminate chattering. Figures 2-3 show the position and velocity tracking errors, respectively, of the six axes, when the robustly estimated velocity vector is used in the tracking control law (29), and the asymptotic convergence is guaranteed.

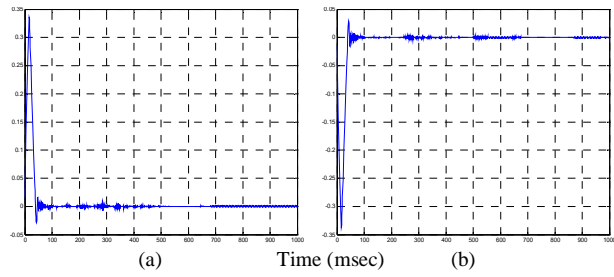


Figure 1. Velocity observation error of : (a) axis 1, (b) axis 2.

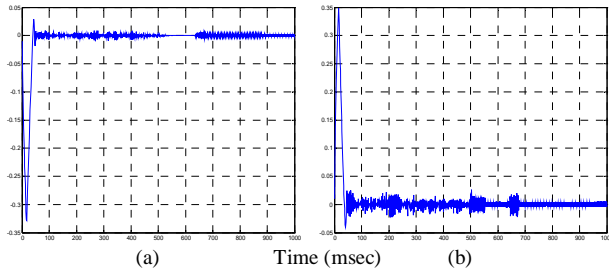


Figure 2. Velocity observation error of : (a) axis 3, (b) axis 4.

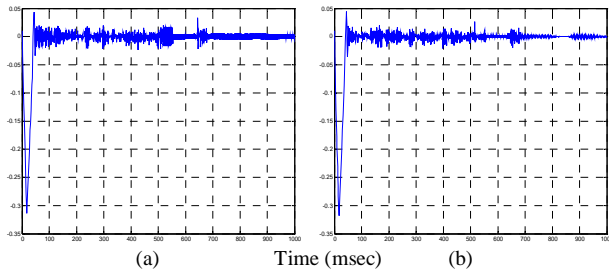


Figure 3. Velocity observation error of : (a) axis 5, (b) axis 6.

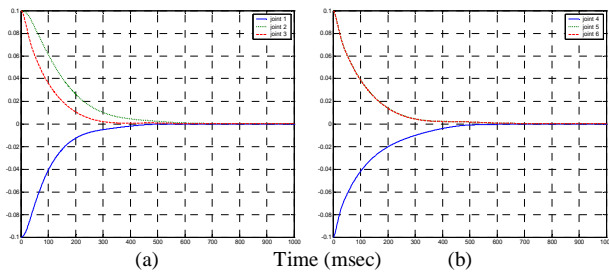


Figure 4. Position Tracking errors of :
(a) axes 1-2-3, (b) axes 4-5-6.

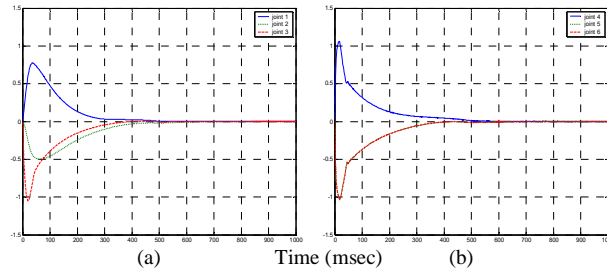


Figure 5. Velocity tracking errors of :
(a) axes 1-2-3, (b) axes 4-5-6.

6. CONCLUSION

In this paper, we have proposed a variable structure observer to the class of rigid robot manipulators in order to solve the trajectory tracking problem with only position measurements. The considered observer is basically a classical nonlinear observer with an additive switching term used to cope with the external disturbances and/or uncertainties. The design of the robust observer is based on the assumption that the linear part of the nonlinear system is completely observable, the nonlinear function is Lipschitz, and external disturbances and/or system uncertainties are upper bounded and satisfy some matching conditions. One drawback of this design is that the presence of the switching term causes the “chattering”. To solve this problem, the use of a boundary layer is a solution. Another solution is to use a continuous term that guarantees the global exponential stability of the observation error just as done in [8].

The robustly estimated states are then used in a control loop with a trajectory tracking control law, which ensures the global asymptotic stability of the system in the full information case, that is, both velocity and position vectors are available for feedback. Under the assumption that the desired velocity vector is bounded, the extended error vector is proved to be globally asymptotically stable, under the condition that the desired velocity vector is bounded. Through simulations on a 6-DOF PUMA 560 robot manipulator, we have illustrated the feasibility of the designed control system.

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