

New approach for robust multi-objective optimization of turning parameters using probabilistic genetic algorithm

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Abstract In this paper, a contribution to the determination of reliable cutting parameters is presented, which is minimizing the expected machining cost and maximizing the expected production rate, with taking into account the uncertainties of uncontrollable factors. The concept of failure probability of stochastic production limitations is integrated into constrained and unconstrained formulations of multi-objective optimization problems. New probabilistic version of the nondominated sorting genetic algorithm P-NSGA-II, which incorporates the Monte Carlo simulations for accurate assessment of cumulative distribution functions, was developed and applied in two numerical examples based on similar and anterior work. In the first case, it is a question of the search space that is completely ‘closed’ by high natural variability related to the multi-pass roughing operation: in this case, the failure risk of technological limitations are considered as objectives to minimize with economic objectives. The second case is related to deformed search space due to the uncertainties specific to finishing operation; therefore, the economic objectives are minimized under imposed maximum probabilities of failure. In both situations, the efficiency and robustness of optimal solutions

generated by the P-NSGA-II algorithm are analysed, discussed and compared with sequence of unconstrained minimization technique (SUMT) method.

Keywords Failure probability · Monte Carlo simulations · Pareto optimal solutions · Optimization · NSGA-II · Reliable machining parameters

Nomenclature

R_{\max}	The maximum roughness (μm)
T_{\max}	The maximum tool life (min)
F_{\max}	The maximum cutting force (kg)
P_{\max}	Power on the spindle (kW)
L	Length to be machined for a single pass (mm)
d	Diameter of workpiece (mm)
t_m	Cutting time (min)
t_l	Nonproductive and handling time (min)
t_r	Tool changing time (min/edge)
a	Depth of cut (mm)
t	Total depth of metal to be cut in roughing operation (mm)
n	Number of passes in rough machining (an integer)
p_0	Direct labour cost added to overhead (paise/min)
$p_0^*t_m$	Machining cost by actual time in cut (paise/pc)
$p_0^*t_l$	Machine idle cost due to loading and unloading operations (paise)
$p_1^*(t_m/T)$	Tool cost per unit piece (paise/pc)
p_1	The cost of a cutting edge (paise/edge)
$p_0^*(t_r^*t_m/T)$	Tool replacement cost (paise/pc)
r_ε	Nominal nose radius (mm)
η	Nominal machine efficiency (%)
E	The expectation measurement

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X_x	Random decision variables
$f(x+X_x)$	Random « objective » function
$g(x+X_x)$	Random « constraint » function
$g_{r_j}(x)$	Robust « constraint » function
U_a	Uniform distribution of (a) uncertainty
U_{r_ε}	Uniform distribution of (r_ε) uncertainty
\bar{R}	Random roughness in finishing (μm)
\bar{P}_{r_s}	Random production rate in finish machining (pc/min)
\bar{P}_{r_r}	Random production rate in rough machining (pc/min)
\bar{C}_s	Random total cost in finish machining (paise)
\bar{C}_r	Random total cost in rough machining (paise)
\bar{T}	Random tool life (min)
\bar{F}	Random cutting force (kg)
\bar{P}	Random cutting power (kW)
K_t	Constant in the Taylor's equation for the tool life
p, q, r	Exponents in the Taylor's equation for the tool life
$\beta_0, \beta_1, \beta_2,$ β_3	Constants depending on cutting tool and workpiece
PDF	Probability density function
CDF	Cumulative distribution function
Φ	Failure probability (%)
F	The reliability level (%)
γ	Target failure probability (%)
SUMT	Sequence of unconstrained minimization technique
NSGA-II	Nondominated sorting genetic algorithm

1 Introduction

Generally, the classical deterministic optimization does not explicitly consider the uncertainties of decision variables. These uncertainties are implicitly considered through the introduction of safety coefficients to ensure the reliability of an optimal design. In machining, the optimization of cutting conditions with consideration of uncontrollable factors is always specific to certain cutting methods [1], to a few uncertain factors [2], and to the high-speed machining process [3]. Thenceforward, neglect of these uncertain parameters, in order to simplify the mathematical models and reducing the prohibitive calculation time, alters the accuracy and reliability of the expected results. Also, the respect of the constraints related to machined workpieces quality, the guarantee of productivity, the operator safety, as well as the machining equipment longevity, requires a mastery of uncertain factors such as the power consumed by the machine, tool wear and tool deflection.

One of the predominant error sources affecting the cutting process is the tool deflection [4, 5]. For conventional milling and turning, this defect depends essentially on the rigidity of

the used tool. The error magnitude order of the machined surface is 25 to 30 μm [6], considering a slender end mill (6 mm in diameter and 60 mm in length) and by applying the finishing conditions on steel. In the worst case, the error values may be higher than 90 μm . In other scales, typical parameters of micro-milling cause cutting forces on the order of 30–100 mN, which can lead to a tool deviation of 3 μm . The tool wear is another important error source, increasing the cutting forces and produces a variation in tool dimensions [7, 8] and causing a loss of precision [9]. The nose radius of the straight end mills and the turning inserts become more rounded because of the wear, causing a decrease of the diameter or the tool length compared to the nominal dimensions. Moreover, the ball-end mills lose their initial radius, particularly, in the case of the hard materials machining, such as the tempered and hardened steels to over 50 HRC [4]. In much optimization work of cutting conditions, the machine efficiency is taken, falsely, as a constant. It was also verified that machine efficiency depends on cutting regime [10]. Exceptionally, in robust optimization problems under uncertainties, Hati and Rao in [11] have taken into account the variability of the machine power.

The reliable optimization is closely linked to respect constraints or limitations imposed by the machinist. Of mathematical viewpoint, an optimum is robust or reliable if and only if it satisfies the constraint functions subjected to natural variability or error propagation from uncontrollable factors which enter their formulation. Moreover, an efficient optimum is defined as the global minimum of the objective function. Thus, upon the resolution of robust optimization problems, particularly in [12, 13], the use of safety coefficients under another name (penalty factors) is taken up in the penalistic formulation of constraint functions. But, the penalty factor is not related directly to the requirements of a reliable machining. As stated in [13], overestimation of the penalty factor can lead to less efficient solutions of the optimization problem, and even that there would be no solutions because of the closure or distortion of search space. To remove this ambiguity, an approach to determine the optimum cutting conditions with consideration of uncertainty, called 'probabilistic optimization', has emerged. This last introduces a more practical concept for considering natural variability of constraints, which is the probability of failure or the reliability of limitations. Indeed, the failure probability of a constraint is the risk to which this constraint may be violated or unsatisfied because of the natural variability of the system.

Several techniques have been used to solve probabilistic optimization problems with single objective in machining, inter alia; the sequence of unconstrained minimization technique (SUMT) in [11], SUMT method combined with the Newton-Raphson method in [14] and dynamic objective-particle swarm optimization (DO-PSO) that has been used recently in [15]. The genetic algorithm (GA) has been a long

time applied effectively to solve deterministic optimization problems in different machining processes, such as multi-pass turning operation in [16] and end milling operation in [17]. In a comparative study, Khan et al. [18] have shown that the genetic algorithm is very efficient compared to other methods based on the gradient information, such as SUMT, Box's Complex Search, Hill Algorithm. Moreover, the nondominated sorting genetic algorithm NSGA-II, originally developed by Deb and Sirinivas [19, 20], is an excellent tool to solve constrained multi-objective optimization problems. Indeed, it is able to maintain a better dissemination of nondominated solutions and it converges better in obtaining the Pareto front, compared to other elitist methods, such as Pareto-Archived Evolution Strategy (PAES) and Strength-Pareto Evolution Strategy (SPES). Yusoff et al. [21] have realized an exhaustive review of work involving NSGA-II in optimization of cutting conditions, relating to traditional and modern machining operations. In some of these works, the NSGA-II was compared to other optimization techniques, such as multi-objective differential evolution (MODE), quadratic programming (QP), GA, PSO, scatter search (SS), ACO and differential evolution (DE). It was found that the NSGA-II is more efficient than other methods in terms of solutions number and ratio of nondominated individuals. Yusoff et al. [21] concluded that the NSGA-II is a reliable and efficient technique for optimizing simultaneously many interest variables in machining.

In this contribution, a proposed new version of the nondominated sorting genetic algorithm P-NSGA-II is used for multi-objective probabilistic optimization of cutting conditions. This new algorithm incorporates the Monte Carlo simulations for accurate assessment of failure probabilities of technological constraints. The limitations related to the minimum and maximum tool life, the cutting temperature, the cutting force, the power consumption and the surface roughness are the considered constraints. The developed algorithm is implemented in two numerical applications based on a similar anterior work [11]. In the first case, it is a question of the search space that is completely 'closed' by high natural variability related to the multi-pass roughing operation: in this case, the failure risk of technological limitations are considered as objectives to minimize with economic objectives. The second case is related to deformed search space due to the uncertainties specific to finishing operation; therefore, the economic objectives are minimized under imposed maximum probabilities of failure.

In the next section, a brief review on the probabilistic optimization in machining is exposed. Then, the Section 3 is dedicated to the development of mathematical models for multi-objective optimization in multi-pass turning operation, namely, the 'objective' functions and the different constraint of production. In Section 4, two probabilistic formulations of robust multi-objective optimization problem for two different

cases are presented, particularly, when the search space is closed and deformed. A diagram explaining the operation of the developed nondominated sorting genetic algorithm P-NSGA-II is presented in Section 5. In section 6, two numerical illustrative applications based on similar work [11] in multi-pass roughing and finishing operations are proposed. Finally, the results generated by the genetic algorithm P-NSGA-II of different probabilistic formulations will be discussed, interpreted and compared with SUMT method.

2 A succinct review on probabilistic optimization in machining

Based on the results of Kyparissis [22] for the geometric programming problems, Dupocova et al. [23] have proposed statistical sensitivity analysis of the optimal machining conditions, the minimal value of the total machining costs in a single pass and single tool turning operation. Their study is concentrated on the case of random parameters only in the objective function, assuming that the only source of uncertainty stems from Taylor's equation. Similarly, Szantai et al. [24] have minimized the total machining time taking into account the stochastic nature of the tool life via the probability model of Rosetto-Levi [25], while considering a probabilistic constraint for log-normally distributed tool life. His approach is insufficient for an exhaustive probabilistic optimization of cutting conditions where the probabilistic nature of the other cutting parameters and some constraint of production must be considered. In contrast, Iwata et al. [14] have proposed an analytic method based on the chance-constrained programming concept as an effective technique for determining the optimum cutting conditions in relation to the probabilistic nature of the objective function and constraints.

Iakovou et al. have presented models for simultaneously determining the optimal cutting speed and the optimal tool replacement policy in cost minimization machining economics [26]. It is shown that when the tool lives follow a certain class of phase type distributions, including Gamma, the objective function is separable and they have exploited this structure to develop efficient solution procedures. Sheikh et al. in [27] attempted to determine analytically the optimal cutting conditions and tool replacement policies in a machining economics system, but the resulting equations could be solved only by numerical methods. Furthermore, the optimal cutting speed and feed were determined sequentially instead of being determined simultaneously. One of the variables (speed of feed) had to be preselected, and then the optimal value of the other variable was determined [28]. Another limitation of the Sheikh's approach is that it is applicable only to unconstrained machining economics problems, while many real life machining economics problems are constrained by maximum allowable feed, or by available machine horsepower and/or by

surface finish requirements. However, Koulamas et al. in [28] have developed an analytical model for the simultaneous determination of the optimal machining conditions and the optimal tool replacement policies with stochastic tool lives. The constraints on the optimal values of the cutting speeds, feeds and/or tool life fractiles can be handled in the geometric programming formulation through the optimization of the dual objective function. Hati and Rao in [11] have developed a probabilistic model for the optimization of the production cost per piece, of the production rate and of the profit by using the method of sequence of unconstrained minimization technique (SUMT). They have assumed that if the uncertain parameters follow normal distributions, similarly, the constraint functions will follow the same distribution type. This is not necessarily correct, given the nonlinearity of constraints functions. They have also considered too many uncertain factors in their study, which led to the closing of the feasible space. Similarly to Iwata [14], they studied and considered different predetermined probability levels to satisfy the stochastic constraints. Recently, Hippalgaonkar and Shin [15] have used a modified version of the particle swarm optimization algorithm (dynamic objective PSO), for the minimization of the production cost while considering the tool life uncertainty in multi-pass turning. The decision variables include not only the machining parameters but also the tool replacement time. Unusually, they have integrated the concept of tool failure cost in calculation of total production cost.

3 Models for the considered multi-pass optimization problem

For comparison reasons, we take as references in the following, the models considered by Haiti and Rao [11] to determining optimal cutting parameters minimizing the expected production cost and production time under uncertainty of uncontrollable factors associated with multi-pass roughing and finishing operations. The stochastic constraints related to the surface roughness, the cutting power, the tool life, the cutting force and temperature are also taken into account.

3.1 Objective functions

O.1 The production rate

The total production time can be decomposed into production time in multi-pass roughing and single pass in finishing:

$$T_u = T_{u_s} + T_{u_r} \quad (1)$$

- For the single pass finishing operation:

$$T_{u_s} = t_l + t_m + t_r \left(\frac{t_m}{T} \right) \quad (2)$$

$\left(\frac{t_m}{T} \right)$ is the number of tool changes per piece.

Then the production rate for finishing operation is calculated as the following:

$$P_{r_s} = 1 / T_{u_s} \quad (3)$$

- For the multi-pass roughing operation:

$$T_{u_r} = t_l + (t_{ln}(n-1)) + n t_m + t_r \left(\frac{n t_m}{T} \right) \quad (4)$$

And the production rate for roughing operation is calculated as the following:

$$P_{r_r} = 1 / T_{u_r} \quad (5)$$

The cutting time t_m per single pass and the tool life T in [29], for turning process, are given by:

$$t_m = \frac{\pi L d}{1000 f V} \quad (6)$$

$$T = \frac{K_t}{V^p f^q a^r} \quad (7)$$

where p, q, r and K_t are characteristic positive constants of the tool and the workpiece material.

O.2 The production cost

The production cost is the total manufacturing cost of a complete product; it is linked to the cutting parameters by the lifetime T and the production time T_u . The global equation of the production costs C is given by [11]. The total production cost can be decomposed into production cost in multi-pass roughing C_r and finishing C_s :

$$C = C_s + C_r \quad (8)$$

Then, we have for a single pass in finishing operation:

$$C_s = p_0 t_l + p_0 t_m + p_l \left(\frac{t_m}{T} \right) + p_0 \left(t_r \frac{t_m}{T} \right) \quad (9)$$

And for a multi-pass roughing operation:

$$C_r = p_0 \{ t_l + [t_{ln}(n-1)] \} + p_0 n t_m + p_l \left(\frac{n t_m}{T} \right) + p_0 \left(t_r \frac{n t_m}{T} \right) \quad (10)$$

3.2 Constraints set

C.1 The workpiece quality

The surface roughness directly reflects the machining quality. Many factors influence the workpiece roughness (wear, vibrations, deflection), but the most preponderant is the tool feed and the nose radius [30]. Therefore, the following relationship allows respecting a certain level of roughness R_{max} :

$$1000 \frac{f^2}{8r_e} \leq R_{max} \tag{11}$$

C.2 The chip-tool interface temperature

To reduce the dimensional and geometric errors due to thermal stress, the temperature at the chip-tool interface T_e should be less than a maximum allowable temperature denoted $T_{e,max}$. It is given by [31]:

$$T_e = \beta_0 V^{\beta_1} f^{\beta_2} a^{\beta_3} \leq T_{e,max} \tag{12}$$

C.3 The tool life

During machining, we should not exceed the maximum lifetime T_{max} fixed for a given tool to avoid all unforeseen breakage and we must not use the tool below a minimum time T_{min} , in order to avoid high production time.

$$T_{min} \leq T \leq T_{max} \tag{13}$$

C.4 The cutting force

This constraint is imposed to restrict the deflection of tool and the workpiece and keep the machine stable. The expression for the cutting force is given by [32].

$$F = af(28.10 V^{0.07} - 0.525 V^{0.5}) \left[1.590 + 0.946 \left(\frac{1+e}{\sqrt{(1-e)^2 + e}} \right) \right] \leq F_{max} \tag{14}$$

where e is a variable dependent on the cutting speed V and the feed rate f .

C.5 The machine motor power

During machining, it is imperative that the cutting power does not exceed the power of machine P_{max} :

$$P = \frac{0.746 FV}{4500\eta} \leq P_{max} \tag{15}$$

4 Formulation of optimization problem

4.1 Deterministic formulation

The deterministic optimization problem without uncertainties consideration can be expressed by:

$$\begin{cases} \text{minimize } f(x) \\ \text{Such as } g_j(x) \leq 0 & j = 1, \dots, n. \\ x_{min} \leq x \leq x_{max} \end{cases} \tag{16}$$

where x is the design variables vector x_{min} and x_{max} are the lower and upper bounds of the search space. $f(x)$ and $g_j(x)$ are respectively the objective functions and the n constraint functions.

4.2 Proposed probabilistic-robust formulation

By taking into account the uncertainties of uncontrollable factors, a major change in the formulation of constraint functions is proposed in this approach (Eq. 17).

$$g_{r_j}(x) = \Phi_{g_j}(x, X_x) \leq \gamma_j \tag{17}$$

The failure probability $\Phi_{g_j}(x, X_x)$ allows quantifying the risk in terms of probabilities that a limitation g_{max} would be suddenly exceeded due to the errors and uncontrollable factors (Fig. 1). Therefore, the reliability level of a constraint is defined as such (Eq. 18):

$$F_g(x, X_x) = 1 - \Phi_g(x, X_x) \tag{18}$$

The calculation of the robustness measurement related to constraint is realized by the Monte Carlo simulations of the cumulative distribution function CDF in optimization algorithm. After CDF simulation, the failure probability Φ (Eq. 19) is estimated in the point g_{max} (e.g. maximum cutting force, maximum roughness and spindle power) and then, it is compared with a target failure probability γ_j imposed a priori (Fig. 1).

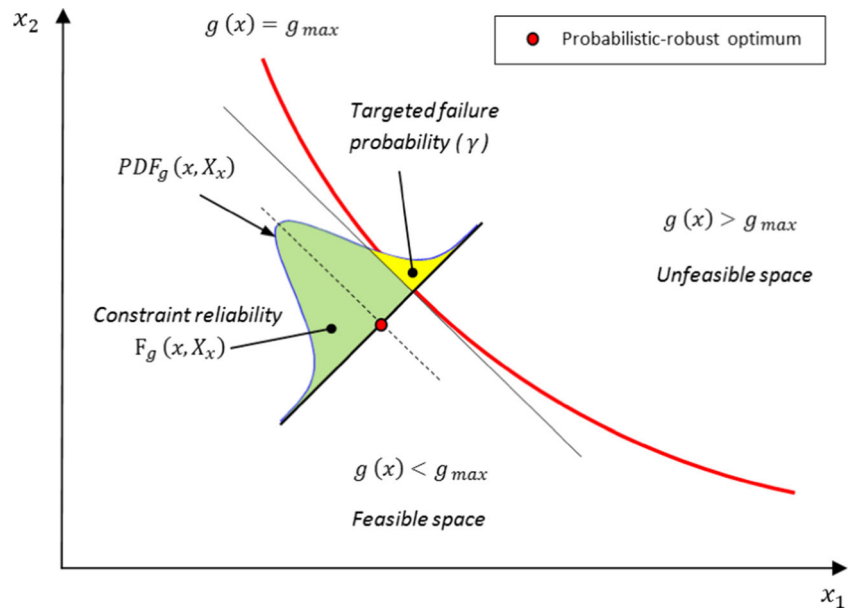
$$\Phi_g(x, X_x) = 1 - \text{CDF}[g(x + X_x) \leq g_{max}] \tag{19}$$

Numerically, the CDF is calculated as the following:

$$\text{CDF}[g(x + X_x) \leq g_{max}] = \frac{1}{N} \sum_{r=1}^N I[g(x + X_{x_i}^r) \leq g_{max}] \tag{20}$$

Where N is the number of Monte Carlo samples and I is the index function (Eq. 21):

Fig. 1 Robust optimum in the feasible space for a target failure probability (γ)



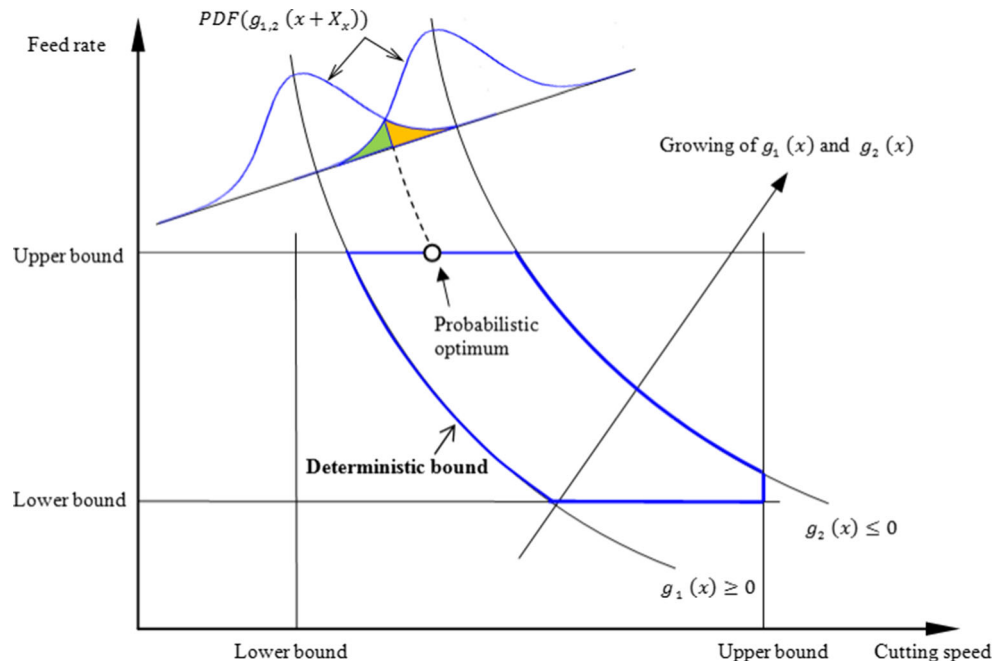
$$I[g(x + X_{x_i}^r) \leq g_{\max}] = \begin{cases} 1 & \text{if } g(x + X_{x_i}^r) \leq g_{\max} \\ 0 & \text{if } g(x + X_{x_i}^r) > g_{\max} \end{cases} \quad (21)$$

Analytically, the CDF is given as the following (Eq. 22):

$$\text{CDF}[g(x + X_x) \leq g_{\max}] = \int_{g(x+X_x) \leq g_{\max}} \text{PDF}_g(x, X_x) dX_x \quad (22)$$

We present in this section two typical problems related to the constraints variability (Figs. 2 and 3). For purposes of illustration, the variability of constraint functions is assumed to follow normal distributions.

Fig. 2 The closing of the search space due to the constraints dispersions $g_i(x)$

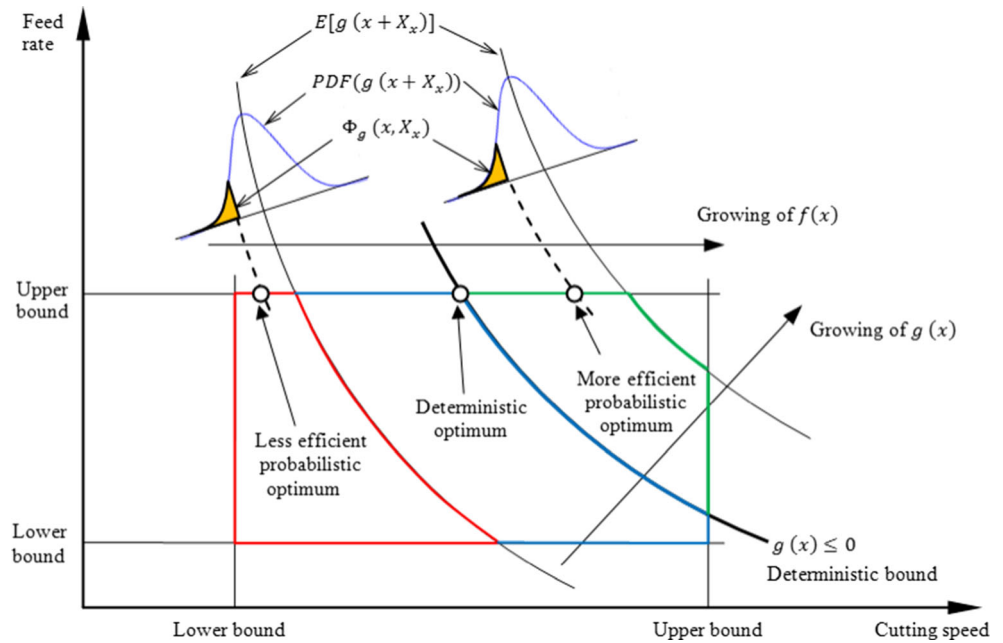


4.2.1 The closing of search space

The first is when the search space is completely or partially closed by the dispersions of limitations; this can occur when a large number of uncertain factors exist with high variability. Therefore, it is impossible to satisfy the imposed constraints using a penalistic formulation [13] or even with a probabilistic formulation. So, the solution to this problem is impossible (Fig. 2). In this case, a compromise must be performed to determine the robust solutions satisfying all the stochastic constraints that delimit the search space.

For example on the Fig. 2 above, the two constraints bounding the deterministic feasible space, $g_1(x)$ and $g_2(x)$ are

Fig. 3 The narrowing and expanding of the robust feasible space



subject to a high variability. At this point, we cannot satisfy a constraint without violating the other; therefore, we can say that $g_1(x)$ and $g_2(x)$ are a conflictual nature. Accordingly, this problem must be resolved through an unconstrained multi-objective formulation, where the failure probabilities of deterministic limitations $\Phi_{g_j}(x, X_x)$ will be considered as objectives to minimize with initial objectives $f(x + X_x)$ of the problem (Eq. 23):

$$\begin{cases} \text{minimize } E[f(x + X_x)] \\ \text{minimize } \Phi_{g_j}(x, X_x) \\ x_{\min} \leq x \leq x_{\max} \end{cases} \quad (23)$$

4.2.2 The deformation of the search space

It sometimes happens, depending on the distribution law of the uncertain parameter and of uncertainty propagation towards the interest variables, that the mean (expectation) of disturbances $E[g(x + X_x)]$ will be completely shifted from the original and deterministic function $g(x)$, as it is the case of constraint functions. Therefore, two possible cases could be distinguished, according to the narrowing or expanding of the feasible space (Fig. 3).

Using the probabilistic formulation of the problem under uncertainties, we obtain robust and reliable solutions in both situations, i.e. narrowing or expanding of feasible space. But the efficiency of solutions depends on nature of the set objective $f(x)$. For example (Fig. 3), if we wanted to maximize the material removal rate, the solutions corresponding to the

search space expansion will be more efficient than those obtained by the deterministic formulation and vice versa.

When the search space is distorted, the probabilistic-robust optimization problem (Eq. 24) can be expressed as follows:

$$\begin{cases} \text{minimize } E[f(x + X_x)] \\ \text{Such as } \Phi_{g_j}(x, X_x) - \gamma_j \leq 0 & j = 1, \dots, m \\ x_{\min} \leq x \leq x_{\max} \end{cases} \quad (24)$$

5 The proposed new algorithm P-NSGA-II

In this section, the major modification of the Nondominated Sorting Genetic Algorithm NSGA-II [19, 20] is presented by incorporating the concept of failure probability of constraints in order to solve an optimization problem under uncertainties. The proposed Probabilistic Genetic Algorithm P-NSGA-II contains the same steps as the deterministic genetic algorithm [19, 20]. Therefore, we will process here only the essential step relating to the posed problem, namely the evaluation step or fitness. The following sub-steps illustrate the conduct of the P-NSGA-II based on MC evaluations:

- Step 1: Random generation of N samples for each individual of the uncertain variable (Xx'_j) according to a distribution law
- Step 2: Calculation of stochastic functions (Xf'_j) and (Xg'_j) for each N sample value of uncertain variable
- Step 3: Evaluation of the expectation ($E[Xf]$)
- Step 4: Evaluation of the cumulative distribution function of constraints violation (CDF[Xg_j, g_{jmax}]) at the point (g_{jmax}) for each sample
- Step 5: Calculation of failure probability ($\Phi_{g_j}(x, X_x)$) for the m constraints

Step 6: Replacement of constraint functions by $(\Phi_{g^j}(x, X_x) \leq \gamma_j)$ and the objective functions by $(E(Xf))$ for unconstrained problem. For unconstrained problem, replacement of objective functions by $(E(Xf))$ and $\Phi_{g^j}(x, X_x)$

Step 7: Evaluation of chromosomes by robust adaptation function

6 Numerical applications

In order to evaluate the methodology, two distinct numerical applications, inherent in multi-pass roughing and finishing operations are proposed: (i) The first application is characteristic of a search space completely closed by high variability, as is the case with the problem posed by Hati and Rao in [11]. (ii) The second application is related to a deformed search space, as is the case in finishing operation. In this case, the geometrical and dimensional errors, mainly caused by tool deflection and wear of nose radius, propagate to the dependent variables such as surface roughness and tool life. For both numerical applications, mild steel is taken as the workpiece material and a carbide tool for cutting.

6.1 When the search space is obturated

The considered numerical case is based on the example treated by Hati and Rao [11]. The numerical data (Table 1) and the used equations in [11] are reconsidered in the mathematical formulation of our unconstrained multi-objective problem.

6.1.1 Uncertain factors

The numerical values of uncertain factors used in [11] are given in Table 2.

* U_{it3} and U_{it4} are the uncertainty of handling time, respectively, for $n=3$ and $n=4$.

Table 1 The values of the used constants [11]

$a_{\min}=1.2$	$a_{\max}=2.75$	$f_{\min}=0.3$	$f_{\max}=0.75$
$V_{\min}=50$	$V_{\max}=400$	$t=5$	$K_r=6 * 10^{11}$
$p=5$	$q=1.75$	$r=0.75$	$F_{\max}=85$
$T_{\min}=25$	$T_{\max}=45$	$P_{\max}=2.5$	$T_{e_{\max}}=1000$
$\eta=90\%$	$t_l=1$	$t_{l_n}=0.2$	$t_r=0.5$
$p_1=50$	$p_0=10$	$d=100$	$L=1000$
$\beta_0=132$	$\beta_1=0.4$	$\beta_2=0.2$	$\beta_3=0.105$
$r_\epsilon=1.2$	$R_{\max}=10$		

Table 2 The numerical values and distributions of uncertain factors

$U_p=N(5, 0.05)$	$U_r=N(0.75, 0.075)$	$U_{\beta_1}=N(0.4, 0.02)$
$U_{\beta_2}=N(0.2, 0.01)$	$U_{T_{\max}}=N(45, 3)$	$U_{T_{\min}}=N(25, 1)$
$U_{P_{\max}}=N(2.5, 0.25)$	$U_{t_r}=N(0.5, 0.1)$	$U_{it3}=N(1.4, 0.2)^*$
$U_{it4}=N(1.6, 0.2)^*$	$U_{c0}=N(10, 0.05)$	$U_{c_t}=N(50, 5)$
$U_{T_{e_{\max}}}=N(1000, 50)$	$U_{T_{e_{\max}}}=N(1000, 50)$	$U_{F_{\max}}=N(85, 5)$

As we have previously stated, the probabilistic optimization problem posed by [11] comprising single objective and five constraints related to failure probabilities is converted into unconstrained multi-objective optimization problem. Only the constraints related to upper and lower bounds of the decision variables are taken into account. The cost or production time are minimized simultaneously with failure probabilities of production limitations $\Phi T_{\max}, \Phi T_{\min}, \Phi P_{\max}, \Phi F_{\max}, \Phi T_{e_{\max}}$. So, our problem (Eq. 25) is formulated as follows:

$$\left\{ \begin{array}{l} \text{minimize } E(\overline{C_r}) \text{ or maximize } E(\overline{P_r}) \\ \text{minimize } \Phi_T \left[\overline{T} \leq T_{\max} \right] \\ \text{minimize } \Phi_T \left[\overline{T} \geq T_{\min} \right] \\ \text{minimize } \Phi_F \left[\overline{F} \leq F_{\max} \right] \\ \text{minimize } \Phi_P \left[\overline{P} \leq P_{\max} \right] \\ \text{minimize } \Phi_{Te} \left[\overline{T_e} \leq T_{e_{\max}} \right] \\ V_{\min} \leq V \leq V_{\max} \\ f_{\min} \leq f \leq f_{\max} \\ a_{\min} \leq a \leq a_{\max} \end{array} \right. \quad (25)$$

The unconstrained multi-objective problem above is implemented in the Matlab™ and resolved by P-NSGA-II. The parameter values of which are given in Table 3 below.

6.1.2 Results and discussion

After presentation and critic of probabilistic solutions obtained by the SUMT method in [11], a comparison with the solutions generated by P-NSGA-II is presented below.

The optima obtained by SUMT [11] First, the optima presented by Hati and Rao [11] are not reliable because of the dissatisfaction of failure probabilities limitations (Table 4). Indeed, due to obturation of the search space, there is no solutions to probabilistic optimization problem, as formulated by Hati and Rao [11] for a target probability of failure $\gamma_j=2.5\%$. Such as we can see it on Fig. 4, the specific feasible space to a depth of cut $a=1.25$ ($n=4$) is completely obturated by the constraint dispersions, particularly those related

Table 3 The parameter values of P-NSGA-II

Population size	Number of Monte Carlo samples	Selection	Crossover rate	Mutation rate	Maximum number of generations
100	10,000	By tournament	0.8	0.01	300

to maximum cutting temperature, to minimum and maximum tool life. The high fluctuations of cited constraints is necessarily due to the large number of uncertain parameters considered simultaneously in the study, and it may be to overestimation of their standard deviations. The solutions, given by the SUMT method, provide practically a high probability of failure ΦT_{\min} (Table 5) followed by maximum cutting temperature $\Phi T_{e_{\max}}$ for both objectives, i.e. production cost and production rate, for all possible depths of cut ($n=2, 3, 4$).

The optima obtained by P-NSGA-II The multi-objective optimization problem (Eq. 25) is resolved by P-NSGA-II. The optimal solutions generated by P-NSGA-II are represented under Pareto's abacus form of nondominated solutions. This arrangement of solutions, also called Pareto's front of nondominated solutions, is based on the principle stated by Vilfredo Pareto [33]. The optima that we selected in each Pareto front (Figs. 5, 6, 7, 8, 9, 10) for the pass numbers ($n=2, 3, 4$) are shown in Tables 4 and 5. Unlike Hati and Rao [11], we were able to obtain solutions for a number of passes $n=2$ (Table 4), using P-NSGA-II and the unconstrained multi-objective formulation. The optima obtained by the multi-objective probabilistic approach are less efficient compared to those of deterministic case that are not represented here. This fact is the characteristic of

Table 4 Representation of results given by the P-NSGA-II and SUMT methods for number of pass $n=2$

Pass number	$n=2$ and $a=2.5$	
Algorithm	SUMT	P-NSGA-II
Objectives	No feasible optimum was obtained	$E(\bar{C}_r) = 176.25$ $E(\bar{P}_r) = 0.06764$
ΦP_{\max}		6.55 12.38
ΦF_{\max}		3.31 22.95
ΦT_{\max}		19.87 9.92
ΦT_{\min}		9.24 18.52
$\Phi T_{e_{\max}}$		8.33 8.99
V		148.58 144.45
f		0.3160 0.3405
Time taken		4 min 27 s 4 min 24 s

the robust optimization, which is observed in other works as [11, 13, 14].

It is observed on Pareto fronts that is impossible to attain simultaneously a probability of failure lower or equal to 2.5 % for all the technological limitations, contrary to what is stated in [11].

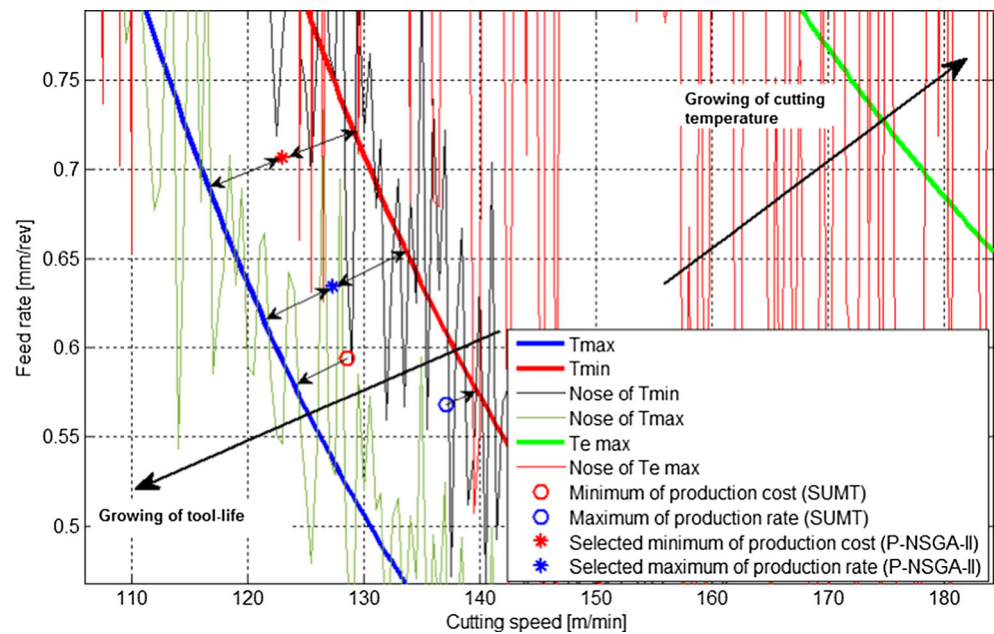
We notice for all passes ' n ' that the selected optimal solutions involve a risk of 8 to 9 % for which the maximum cutting temperature (1000 °C) can be exceeded. Because of high dispersion in constraint related to cutting temperature $T_{e_{\max}}$, it influences substantially on the robust-probabilistic solutions, despite it is not involved in the delimitation of deterministic feasible space (Fig. 4). The failure probability of the constraint related to maximum cutting power ΦP_{\max} is somewhat high for $n=2$ than for depths of cut $n=3$ and $n=4$ (Table 5). Practically, for number of passes $n=3$ and $n=4$, the risk to generate a superior cutting power to that available on spindle ΦP_{\max} is acceptable (Figs. 7, 8, 9, 10). Generally, this risk increases when the depth of cut increases. The failure probability of maximum cutting force is proportional to the feed rate, for two and three cutting passes, but not for four cutting passes. This is due to the fact that the maximum cutting force is not involved in the demarcation of classical feasible space for $n=4$. Therefore, we can say that the risk to generate a superior cutting force to 85 kg decreases when the depth of cut decreases.

The contradictory nature between objectives ΦT_{\min} and ΦT_{\max} can be observed on all Pareto fronts (Figs. 5, 6, 7, 8, 9, 10). We also note that the failure probability of the maximum tool life ΦT_{\max} is proportional to production cost. In fact, the Iwata [14] and Hati [11] findings, for which the machining cost is inversely proportional to the reliability level of constraints, are verified only for the maximum tool life in the roughing case but not for all constraints.

In addition, the production cost decreases when the failure probability of minimum tool life ΦT_{\min} increases. However, the production rate is inversely proportional to the failure probability of the maximum tool life ΦT_{\max} .

In terms of robustness, the solutions can be selected a posteriori, according to the constraint reliability levels desired by the manufacturer. The minimum production cost and maximum production rate generated by the

Fig. 4 Probabilistic optima for the minimum of production time and cost obtained by SUMT [11] and NSGA-II for $n=4$



developed algorithm P-NSGA-II and the unconstrained multi-objective formulation (Figs. 7, 8, 9, 10) provide more efficiency ($E(\bar{C}_r) = 176.55$ paise/piece; $E(\bar{P}_{r_r}) = 0.06232$ piece/min), ($E(\bar{C}_r) = 191.25$ paise/piece; $E(\bar{P}_{r_r}) = 0.05557$ piece/min), respectively, for $n=3$ and $n=4$, compared with the solutions offered by SUMT method.

Using Monte Carlo simulations by P-NSGA-II for estimating failure probabilities resulted to the increase of the calculation time approximately two times greater than that taken by SUMT method [11]. The time taken P-NSGA-II to resolve the problem is always acceptable, since the genetic algorithm is not very affected by the choice of the input vector, unlike SUMT method which takes longer to converge, if the starting vector is far from optimal vector.

6.2 When the search space is distorted

In the finishing operation optimization, wear and tool deflection mainly affect the variability of constraints related to tool life and the machined surface roughness. So, these uncontrollable factors are at origin of the deformation of search space. Later, we study the reliability level influence of stochastic constraints on the probabilistic optima.

6.2.1 Uncertain factors

We distinguish two geometric dispersion types in turning (Fig. 11):

- Systematic dispersion: during cutting, the tool tip declines gradually from a distance U_{r_ϵ} due to wear of nose radius. In addition, it becomes larger r'_ϵ relative to the nominal nose radius r_ϵ . The uncertainty on the nose radius is assumed to 5 % on the machining tolerance basis fixed by the standards [34, 35], which makes a variation range of the nose radius $U_{r_\epsilon} = 0.06 \mu\text{m}$ (Table 6).
- Random dispersion: the error on the depth of cut is caused by the random displacements of the tool D_r in the radial direction (assumed at $10 \mu\text{m}$). The tool deflection D_r combined with nose radius wear U_{r_ϵ} , out comes the following equation (Eq. 26):

$$U_a = U_{r_\epsilon} + D_r \tag{26}$$

On one hand, the radial displacement affects strongly the profile height of the machined surface [36]. Moreover, the wear of the cutting edge generates an uncontrollable surface state and first-order dimensional errors. In this part, the uncertainty of the machine efficiency is taken into account. Based on other optimization works, it has found that the efficiency varies between 55 % to 85 % for the same machined material and therefore, nominal machine efficiency for the deterministic case is taken to 70 %.

The parameters $\tilde{a}_p, \tilde{r}_\epsilon, \tilde{\eta}$ are the uncertain variables of this problem. The numerical values of uncertain factors

Table 5 The solutions given by the P-NSGA-II and SUMT methods for number of pass $n=3$ and $n=4$

Pass number	$n=3$ and $a=1.667$		$n=4$ and $a=1.25$	
	SUMT	P-NSGA-II	SUMT	P-NSGA-II
Objectives	$E(\bar{C}_r) = 194.60$	$E(\bar{P}_{r_r}) = 0.0579$	$E(\bar{C}_r) = 206.202$	$E(\bar{P}_{r_r}) = 0.0555$
ΦP_{max}	0.5	0.3	10^{-2}	10^{-2}
ΦF_{max}	10^{-3}	10^{-5}	10^{-2}	10^{-4}
ΦT_{max}	0	0	3	3
ΦT_{min}	32.49	34.13	31.12	31.12
$\Phi T_{e_{max}}$	6.48	6.64	6.88	6.88
V	144.656	147.887	137.08	127.31
f	0.4322	0.4082	0.5940	0.7066
Time taken	3 min	2 min 38 s	2 min 50 s	4 min 17 s
		$E(\bar{C}_r) = 176.55$		$E(\bar{P}_{r_r}) = 0.0555$
		2.80		10^{-2}
		25.76		10^{-2}
		9.03		17.28
		16.21		7.62
		9.49		4.88
		134.03		128.591
		0.5448		0.5319
		4 min 29 s		4 min 26 s

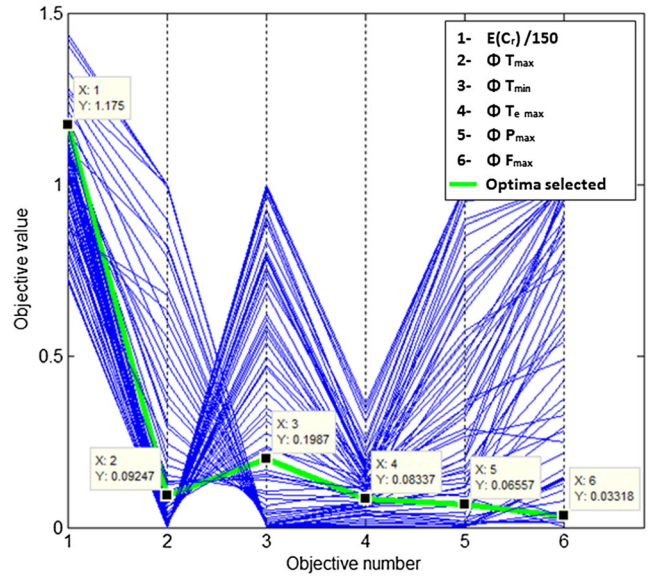


Fig. 5 The Pareto abacus of minimum production cost and minimum failure probabilities, for two cutting passes

are given in Table 6. The parameters not included in Table 6 are considered deterministic, and their numerical values are given in Table 1.

The constrained multi-objective problem below is implemented in the Matlab™ and resolved by probabilistic nondominated sorting genetic algorithm P-NSGA-II which presented before. The parameter values of used algorithm are already given in Table 3. In this section, the levels of failure probabilities have been fixed as $\gamma_j=2.5, 5, 10, 20\%$.

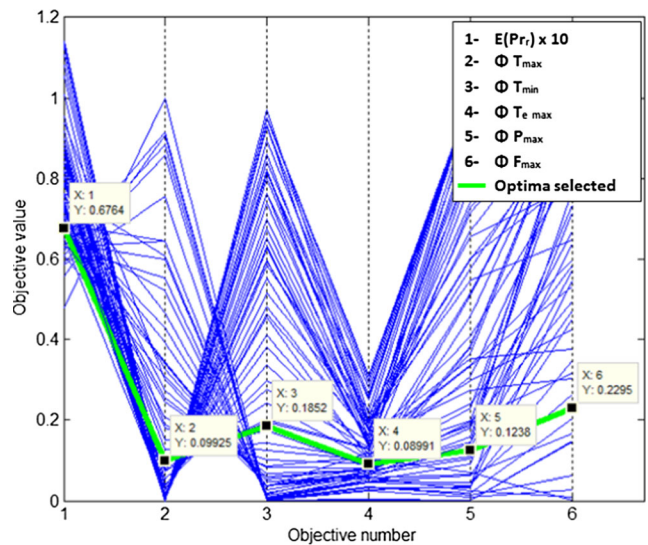


Fig. 6 The Pareto abacus of minimum production rate and minimum failure probabilities, for two cutting passes

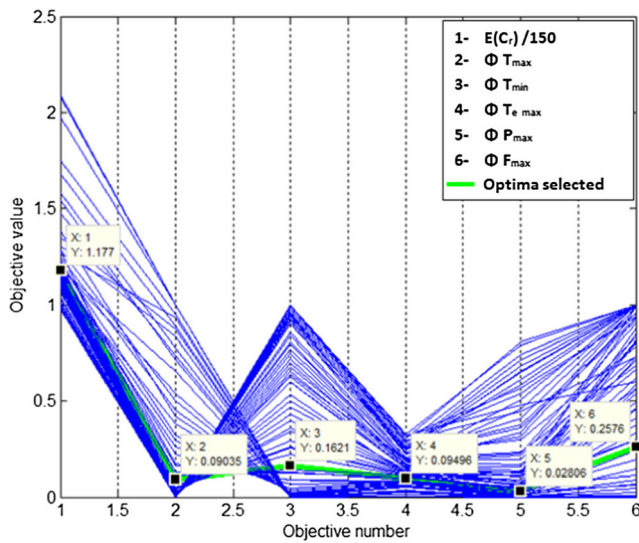


Fig. 7 The Pareto abacus of minimum production cost and minimum failure probabilities, for three cutting passes

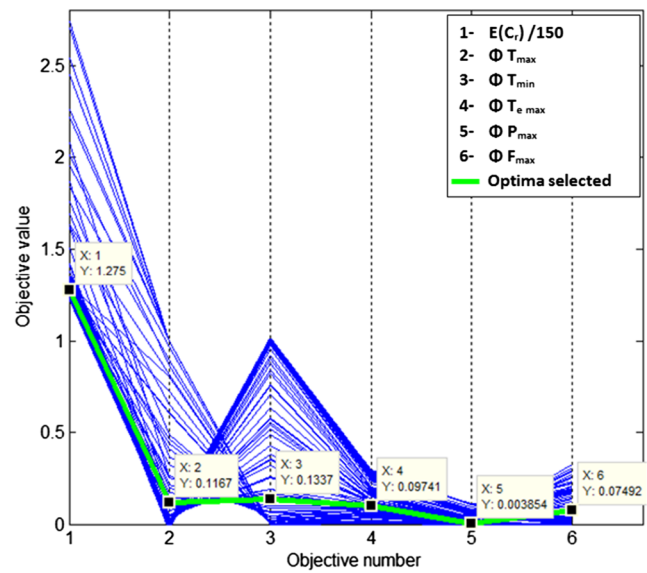


Fig. 9 The Pareto abacus of minimum production cost and minimum failure probabilities, for four cutting passes

$$\left\{ \begin{array}{l} \text{maximize } E(\bar{P}_{r_s}) \\ \text{minimize } E(\bar{C}_s) \\ \text{such as} \\ \Phi_T \left[\begin{array}{l} \bar{T} \leq T_{\max} \\ \bar{T} \geq T_{\min} \end{array} \right] \leq \gamma_T \\ \Phi_R \left[\bar{R} \leq R_{\max} \right] \leq \gamma_R \\ \Phi_P \left[\bar{P} \leq P_{\max} \right] \leq \gamma_P \\ \Phi_{T_e} \left[\bar{T}_e \leq T_{e_{\max}} \right] \leq \gamma_{T_e} \\ V_{\min} \leq V \leq V_{\max} \\ f_{\min} \leq f \leq f_{\max} \\ a = 1 \text{ mm} \end{array} \right. \quad (27)$$

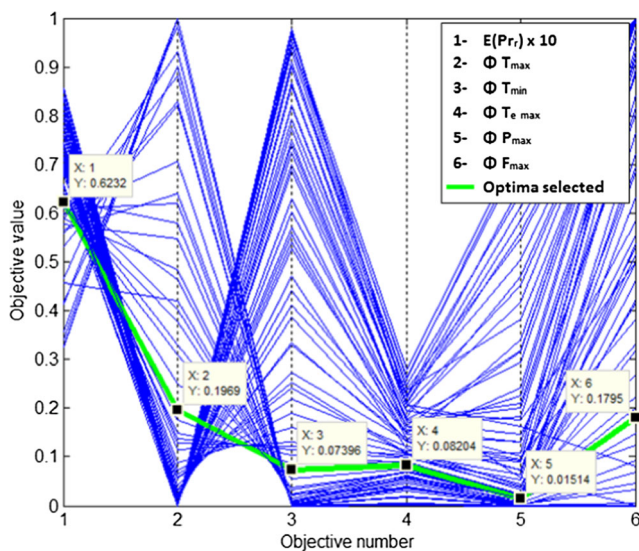


Fig. 8 The Pareto abacus of minimum production rate and minimum failure probabilities, for three cutting passes

6.2.2 Results and discussion

In finishing, we can choose and compare between solutions that reduce simultaneously the time and cost of production, with and without consideration of the uncertainties (Table 7). In this optimization case, the limitation related to the surface quality and tool life (R_{\max} , T_{\max} , T_{\min}) influences the probabilistic and deterministic solutions. As stated previously in section (§4.2.2), the average of the three constraints function $E[R_{\max}, T_{\max}, T_{\min}]$ is shifted compared to the original function. And the robust feasible space, defined by the dispersion, is deformed (Fig. 12).

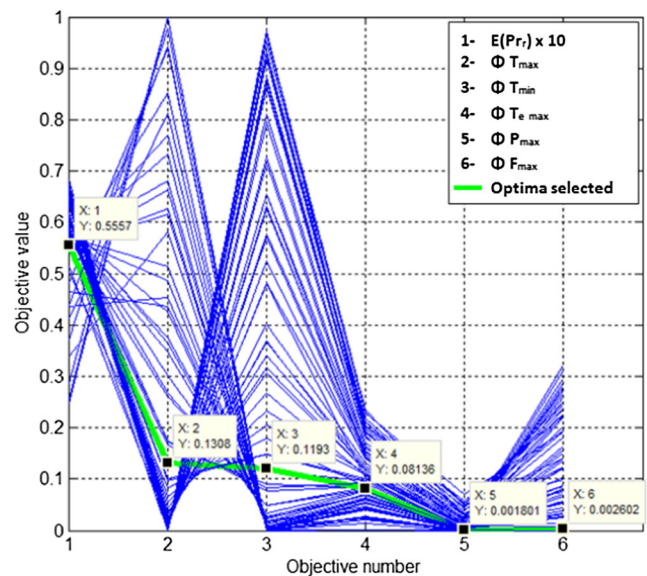
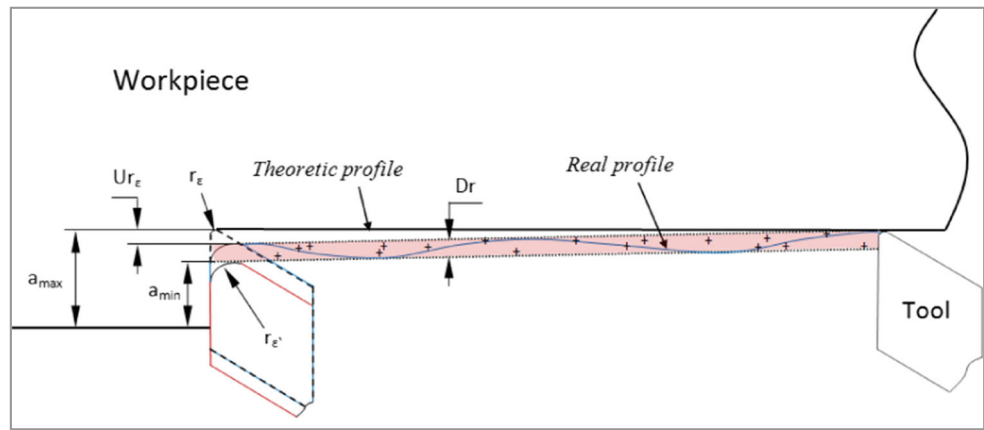


Fig. 10 The Pareto abacus of minimum production rate and minimum failure probabilities, for four cutting passes

Fig. 11 The uncertainty of depth of cut caused by the tool deflection D_r and the wear of nose radius U_{r_ϵ}



In this case, the deterministic formulations provide ‘reliable’ solutions with respect to imposed constraints, after considering the variability of the cutting process. The failure probabilities associated to the minimum tool life and maximum surface roughness are acceptable, respectively, 1.70 and 1.45 %. This is due to the shift of disturbances upwards (expansion of the feasible space) (Fig. 12). On the contrary, if the feasible space had shrunk, these reliable solutions would be infidels and unusable because of the significant failure probability.

The generated optimal cutting parameters by P-NSGA-II algorithm satisfy the different targets failure probabilities, γ_j (2.5, 5, 10 to 20 %), as shown in the table below. For example, if we agreed to take a risk of 8.6 and 9.33 % for having respectively a machined surface roughness upper than 10 μm , and a tool life lower than 25 min, the production cost and production rate will be more efficient than those obtained by the deterministic and classical formulation. With the evaluations based on Monte Carlo simulations, we find that the average evaluation time required for solving probabilistic problem is approximately eight times greater than time required for solving deterministic problem (Table 7).

The effect of failure probability levels The effect of failure probability level on the optimum cutting conditions is illustrated on Fig. 13. Table 7 gives the list of optimum

Table 6 Uncertain factors and distribution law

Uncertain factors	Distribution law	Value
Depth of cut a	Uniform	$U[a - 0.07; a]$
Tool deflection D_r	Uniform	$U[1; 10]$
Machine efficiency η	Uniform	$U[55; 85]$
Wear of nose radius r_ϵ	Uniform	$U[r_\epsilon; r_\epsilon + 0.06]$

cutting conditions for different predetermined levels of failure probability γ_j , are 2.5, 5, 10 and 20 %. In this case study, the production cost is proportional to reliability level of constraints and it is inversely proportional to failure probability of constraints (Fig. 13). In addition, production rate increases when the failure probability level increases and therefore, it decreases when reliability level increases. It can be said also that the efficiency of objectives and the robustness (reliability) of constraints are conflicting concepts, where a compromise must be achieved. In the case studied, the optimal cutting parameter values, corresponding to failure probability of 2.5 %, are efficient and robust solutions in the same time.

The new probabilistic formulation makes possible, a priori, a compromise between reliable cutting conditions ensuring failure risk controls of machining equipment and efficient cutting conditions minimizing the time and costs of machining. A future alleviation of probabilistic formulation by removing the constraints with low variability will reduce systematically the calculation times.

7 Conclusion

In this paper, two probabilistic multi-objective optimization problems in of multi-pass roughing and finishing operations are studied. In the first, the failure probabilities of technological limitations are considered as objectives to optimize with economic objectives, i.e. production cost and production rate. In the other, probabilities of failure are considered as constraints to satisfy. The new developed genetic algorithm P-NSGA-II was used to determine the optimum cutting parameters minimizing the machining cost and maximizing the production rate.

Table 7 The probabilistic and deterministic optima in finishing operation for $a=1$ mm

Formulations	Variables of decision		Objectives (O.1, O.2)		Constraints (C.1, C.3)		
	V	f	C_{r_s}	P_{r_s}	ΦR_{\max}	ΦT_{\min}	Time taken
Det	179.534	0.3098	78.90	0.1479	1.45	1.70	30.15 s
Pro ($\gamma_j = 2.5\%$)	179.490	0.3097	78.57	0.1479	0.96	0.62	3 min 52 s
Pro ($\gamma_j = 5\%$)	179.547	0.3101	78.52	0.1481	3.88	4.14	4 min 6 s
Pro ($\gamma_j = 10\%$)	179.574	0.3105	78.45	0.1482	8.60	9.33	3 min 53 s
Pro ($\gamma_j = 20\%$)	179.629	0.3113	78.34	0.1486	17.33	19.61	4 min 35 s

Det deterministic, *Pro* probabilistic

The findings from the two studies are:

- The presented solutions by Hati and Rao [11] are unreliable and probably unusable, because of target failure probability that cannot be satisfied for all constraints, due to obturation of search space.
- The optimums obtained by developed algorithm P-NSGA-II and the unconstrained multi-objective formulation showed more efficiency ($E(\bar{C}_r) = 176.55$ paise/piece, $E(\bar{P}_{r_r}) = 0.06232$ piece/min), ($E(\bar{C}_r) = 191.25$ paise/piece, $E(\bar{P}_{r_r}) = 0.05557$ piece/min) respectively for three and four cutting passes, compared with solutions generated by SUMT method for the same case studied in [11].
- It was found that the reliabilities of limitations related to minimum and maximum tool life are conflictual. The production cost decreases when failure probability of the minimum tool life increases. However, the production rate is inversely proportional to the violation risk of maximum tool life.

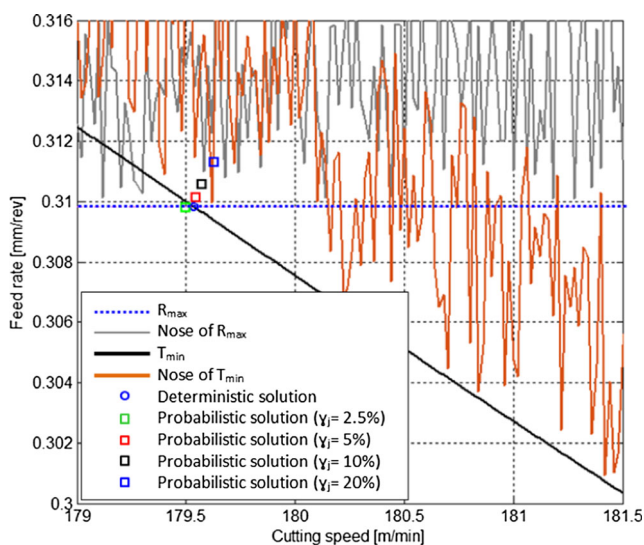


Fig. 12 Probabilistic and deterministic optima in the feasible space for $a=1$ mm

- When the feasible space is deleted, the Iwata [18] and Hati and Rao [11] findings, for which the machining cost is inversely proportional to the reliability level of constraints, are verified only for the maximum tool life but not for all constraints.
- The unconstrained multi-objective optimization formulation has provided various solutions in the form of Pareto fronts, i.e. efficient and robust solutions. These solutions can be chosen a posteriori according to reliability level of each of the considered technological limitations or according to the production cost and time that the machinist desires.
- In the case relating to the finishing operation where the search space is deformed, the efficient and robust obtained solutions are contradictory and a compromise can be achieved.
- The computation time taken by the P-NSGA-II algorithm for probabilistic formulation is about eight times greater than that taken by NSGA-II algorithm for the deterministic formulation. However, to improve the processing time, the parallelization of computations or replacement

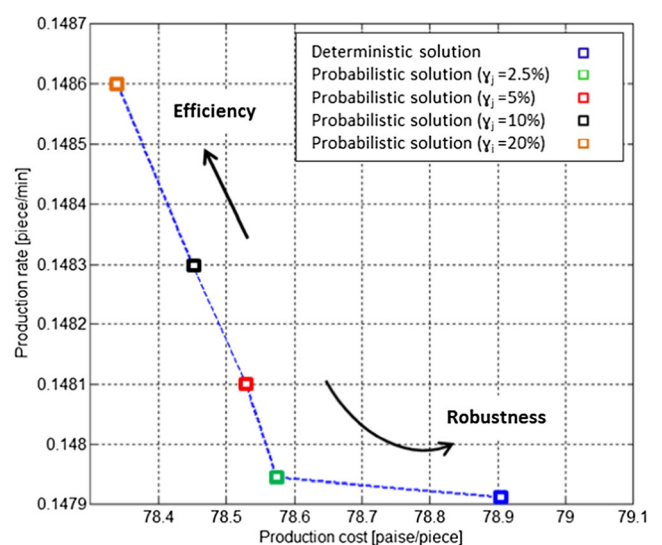


Fig. 13 Evolution of solutions based on failure probability of constraints

of Monte Carlo simulations by other sampling methods such as importance sampling and directional sampling are potential solutions.

- The flexibility of the P-NSGA-II resides in the efficient evaluations of failure probability, by the Monte Carlo simulations for any combination of the uncertain parameters distribution laws. In the case where the search space is closed because of uncertainties, the constrained optimization problem can be converted into an unconstrained multi-objective optimization problem and thus obtain compromising solutions. Finally, P-NSGA-II algorithm can be applied to the cutting conditions optimization for other conventional and modern machining processes, which represent high natural variability or many uncontrollable factors.

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