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ELECTROSTATIC SOLITARY STRUCTURES IN A NONTHERMAL PLASMA

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Abstract

An investigation on the effect of dust temperature and dust streaming on dust acoustic solitary waves in unmagnetized dusty plasma, whose constituents are inertial charged dust grains, non-thermal electrons and ions has been conducted. Sagdeev pseudo potential method which takes into account the full nonlinearity of plasma equations is used here to study solitary wave solution.

Introduction

Dusty plasma are responsible for introduction of a new ultraslow frequency domain of wave modes and instabilities, the most popular of these being the dust acoustic wave (DAW) first reported by Rao *et al*. **[1]**. Many authors have adopted non-thermal velocity distribution in nonlinear plasma studies. Cairns *et al.* **[2]** proposed that the non-thermal electrons in a two component model comprising Boltzmann ions as well, could support the simultaneous existence of solitary waves with both positive and negative potential. Mamun *et al*. **[3]** employed a model comprising non-thermal ions and negatively charged dust and reported that solitary waves with positive and negative potential can coexist. K. Annou and R. Annou **[4]** employed a model comprising non-thermal ions distribution with inertial dust fluid and Boltzmann distribution electrons, leads to the possibility of co-existence of large amplitude compressive as well as refractive dust acoustic solitary. In this work, we consider dust temperature and streaming and study their effect on the properties of dust acoustic waves in nonthermal plasma.

Formulation

We consider three-component dusty plasma with extremely massive, micron-sized, negatively charged dust grains, nonthermal distributed electrons and ions. The quasi-neutrality at equilibrium is written as, $N_{i0} = N_{e0} + Z_d N_{d0}$ where, N_{i0}, N_{e0} and N_{d0} are the unperturbed ion, electron and dust densities respectively, and Z_{d0} being the number of elementary charges residing on the dust grain. The electrons and ions are nonthermal there distributions are given by

$$N_e = N_{e0}\left(1-\beta_e \frac{e\phi}{T_e}+\beta_e\left(\frac{e\phi}{T_e}\right)\right)\exp\left(\frac{e\phi}{T_e}\right) \quad \textbf{(1)}$$

$$N_i = N_{i0}\left(1+\beta_i \frac{e\phi}{T_i}+\beta_i\left(\frac{e\phi}{T_i}\right)\right)\exp\left(-\frac{e\phi}{T_i}\right) \quad \textbf{(2)}$$

where, $\beta_{e(i)} = \frac{4\alpha_{e(i)}}{1+3\alpha_{e(i)}}$ $\alpha_{e(i)}$ define the population of non-thermal electrons (ions)

Arbitrary amplitude solitary structures

In this section we will be looking for arbitrary large amplitude solutions of the nonlinear equations system. The normalised governing equations of the plasma evolution are given by,

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d u_d)=0 \quad \textbf{(3)}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}=\frac{\partial \Phi}{\partial x}-\frac{\sigma}{n_d}\frac{\partial P}{\partial x} \quad \textbf{(4)}$$

$$\frac{\partial P}{\partial t}+u_d\frac{\partial P}{\partial x}+3P\frac{\partial u_d}{\partial x}=0 \quad \textbf{(5)}$$

$$\frac{\partial^2\Phi}{\partial x^2}=n_d+\frac{n_{e0}}{Z_d n_{d0}}\left(1-\beta_e\delta+\beta_e\delta^2\Phi^2\right)\exp(\delta\Phi)-\frac{n_{i0}}{Z_d n_{d0}}\left(1+\beta_i+\beta_i\Phi^2\right)\exp(-\Phi) \quad \textbf{(6)}$$

where, $\delta = T_i/T_e$ is the normalized electron temperature, $\sigma = T_d/Z_d T_i$ is the normalized dust temperature, n_d is the dust density normalized by n_{d0} and $n_e(n_i)$ are the electron and ion number densities both normalized by $Z_d n_{d0}$, u_d is the dust fluid velocity normalized by the dust acoustic speed $C_d = (Z_d k_B T_i/m_d)^{1/2}$, P is the dust pressure normalized by $n_{d0}k_B T_d$, ϕ is the electrostatic wave potential normalized by $k_B T_i/e$, the space variable is normalized by $\lambda = \left(k_B T_i/4\pi Z_d^2 e^2 n_{d0}\right)^{1/2}$ and time normalized by inverse dust plasma frequency $\omega_{pd}^{-1} = (m_d/4\pi e^2 Z_d^2 n_{d0})$. For solitary wave solutions, we transform to the stationary frame $\xi = x - Mt$ where M is the Mach number (the solitary wave velocity normalized by dust acoustic speed C_d)

We have imposed appropriate boundary conditions for the localized disturbances, viz., $\Phi \to 0, u_d \to u_{d0}$ where u_{d0} is the equilibrium dust drift speed, $P \to 1$ and $n_d \to 1$ at $|\xi| \to \pm\infty$, equations (3) and (4) can be integrated to give

$$n_d = \frac{u_{d0}-M}{u_d - M} \quad \textbf{(7)}$$

$$P = n_d^3 \quad \textbf{(8)}$$

Combining (3), (4), (7) and (8), one gets

$$\left[3\sigma n_d - (u_{d0}-M)^2 n_d^{-3}\right]\frac{\partial n_d}{\partial \xi}=\frac{\partial \Phi}{\partial \xi} \quad \textbf{(9)}$$

Integrating (9), and using the same boundary conditions at $|\xi| \to \pm\infty$, we obtain a quadratic equation in n_d^2,viz.

$$3\sigma n_d^4 - \left[(M-u_{d0})^2 + 3\sigma + 2\Phi\right] n_d^2 + (M-u_{d0})^2 = 0 \qquad \textbf{(10)}$$

The solution for n_d is given by,

$$n_d = \frac{1}{\sqrt{2}} \frac{\sigma_1}{\sigma_0} \times \sqrt{1 + \frac{2\Phi}{(M\text{-}u_{d0})^2 \sigma_1^2} - \sqrt{\left(1 + \frac{2\Phi}{(M\text{-}u_{d0})^2 \sigma_1^2}\right)^2 - 4\frac{\sigma_0^2}{\sigma_1^4}}} \qquad \textbf{(11)}$$

where, $\sigma_0 = \sqrt{\dfrac{3\sigma}{(M-u_{d0})^2}}$ and $\sigma_1 = \sqrt{1+\sigma_0^2}$

The nature of solutions of this equation is seen by introducing Sagdeev potential. This (6) can be rewritten in the form

$$\frac{1}{2}\left(\frac{\partial \Phi}{\partial \xi}\right)^2 + \mathrm{V}(\Phi) = 0 \qquad \textbf{(12)}$$

where the Sageedv potential $\mathrm{V}(\Phi)$ is given by

$$\mathrm{V}(\Phi) = \mathrm{I}_e(\Phi) + \mathrm{I}_i(\Phi) + \mathrm{I}_d(\Phi) \qquad \textbf{(13)}$$

where,

$$\mathrm{I_e}(\Phi) = \frac{n_{e0}}{Z_d n_d} \frac{1}{\delta}\left[(1+3\beta_e)(1-\exp(\delta\Phi)) - (-3\beta_e \delta\Phi + \beta_e \delta^2 \Phi^2)\exp(\delta\Phi)\right]$$

$$\mathrm{I_i}(\Phi) = \frac{n_{i0}}{Z_d n_d}\left[(1+3\beta_i)(1-\exp(-\Phi)) - (3\beta_i \Phi + \beta_i \Phi^2)\exp(-\Phi)\right]$$

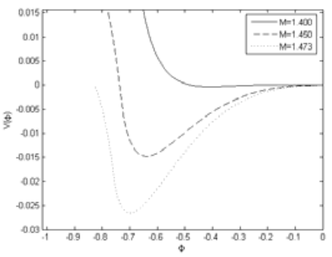
$$\mathrm{I_d}(\Phi) = (M-u_{d0})^2 \frac{\sigma_1}{\sqrt{2}}\left[\left\{1+\sqrt{1-4\frac{\sigma_0^2}{\sigma_1^4}}\right\}^{1/2} - \left\{\Psi(\Phi) + \sqrt{(\Psi(\Phi))^2 - 4\frac{\sigma_0^2}{\sigma_1^4}}\right\}^{1/2}\right] + 2\sqrt{2}\frac{\sigma}{\sigma_1^3} \times \left[\left\{1+\sqrt{1-4\frac{\sigma_0^2}{\sigma_1^4}}\right\}^{-3/2} - \left\{\Psi(\Phi) + \sqrt{(\Psi(\Phi))^2 - 4\frac{\sigma_0^2}{\sigma_1^4}}\right\}^{-3/2}\right]$$

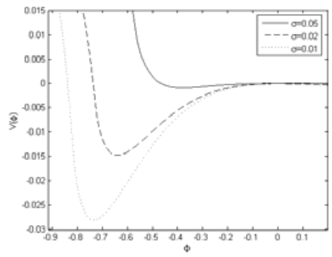
and $\Psi(\Phi) = 1 + \dfrac{2\Phi}{(M-u_{d0})^2 \sigma_1^2}$.

The solitary wave solutions of **(13)** exist if we have (*i*) $(\mathrm{d}^2\mathrm{V}/\mathrm{d}\Phi^2)_{\Phi=0} < 0$ so that the fixed point at the origin is unstable, (*ii*) $\mathrm{V}(\Phi) < 0$ when $0 < \Phi < \Phi_{\max}$ for positive solitary waves and $\Phi_{\min} < \Phi < 0$ for negative solitary waves where $\Phi_{\max(\min)}$ is the maximum (minimum) value Φ for which $\mathrm{V}(\Phi) = 0$ and (*iii*) $(\mathrm{d}^3\mathrm{V}/\mathrm{d}\Phi^3)_{\Phi=0} > 0 (< 0)$ for solitary waves with $\Phi > 0 (< 0)$.

Results

The Sagdeev potential $V(\Phi)$versus Φ for different values of M using equation **(13)** is plotted. It is seen that a potential well exists on the side $\Phi \le 0$ for $\sigma = 0.02$ and $1.400 \le M \le 1.473$. This indicates that solitary wave solution exists for this rang of Mach number. It is shown that when we have dust streaming we need a higher value of critical Mach number to have soliton. It is clearly seen from Fig2, that the width of the soliton increase and the amplitude decrease as the dust temperature increases. It is also shown that for ($\alpha_e = 0, \alpha_i = 0$) and ($\alpha_e = 0.25, \alpha_i = 0$) soliton don't propagate. Furthermore, it is seen that the potential well is deeper for smaller value of dust temperature σ.





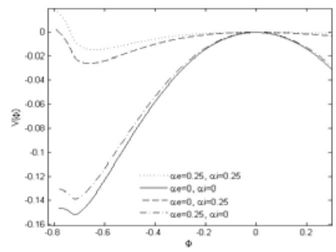


Fig.1 : $V(\Phi)$ vs Φ for $\sigma = 0.02$ **Fig.2** : $V(\Phi)$vs for $M = 1.4$: **Fig.3**: $V(\Phi)$vs for $M = 1.450$

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