3D NUMERICAL STUDY OF MAGNETOHYDRODYNAMIC INSTABILITY IN LIQUID METAL TAYLOR-COUETTE FLOW

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Abstract

This purpose is about a 3D study of magnetohydrodynamic (MHD) instability in liquid matal Taylor-Couette flow, this problem is receiving more and more research interest due to its application in the engineering, oceanography and the astrophysical research The Taylor-Couette system consists of two coaxial cylinders in differential rotation, which is considered as a hydrodynamic model system, allowed researchers to progress in understanding the laminar-turbulent transition phenomena. A set of states found in narrow gap of Taylor-Couette systems where the outer cylinder is held fixed and the inner cylinder speed increased. The symmetry breaking parameter is the Taylor number *Ta* that gives a measure of the ratio of centrifugal forces to viscous forces. When the liquid is replaced by an electrically conducting fluid and an external magnetic field is applied, this leads to MHD Taylor-Couette flow. Additional body force, Lorentz force, acting on the fluid arises. Lorentz force is in the direction perpendicular to both magnetic and electric fields. The behaviour of flow depends on strength and geometry of applied field, magnetic and electric properties of the liquid, cylinders and endplates. In this work, the MHD instability Taylor-Couette flow is considered for liquid sodium with its magnetic Prandtl number $Pm<1$. The results of pressure and angular momentum in the Taylor-Couette flow under the effect of an external uniform axial magnetic field B= 4 Tesla are investigated numerically for the different cases of electrically conducting or insulating walls at the Ekman cell, at the middle of the first Taylor-votex flow (TVF) and between two cells.

Keywords: MHD, instability, Taylor-Couette, magnetic field

1 Introduction

Taylor-Couette flow is that which develops in the annular space between two coaxial relatively rotating cylinders was studied by Newton [1]. Stokes [2] suggested that the rotation of the inner cylinder would produce the least of stable state; it created a force centrifugal which ejects the fluid towards the outer wall, thereby to destabilize the flow. Rayleigh [3] has developed a first theoretical formalization for inviscid fluids. The motion of an incompressible, viscous fluid confined between tow coaxial rotating cylinders has been originally investigated by Couette [4] and Mallock [5]. Taylor [6] combined theoretical and experimental approaches. Velikhov [7] and Chandrasekhar [8] have studied the axial magnetic field applied to a Taylor-Couette flow. Balbus and Hawley. [9], [10] have found that magnetohydrodynamic instability can play a role in wide of astrophysics. Hollerbach and Rüdiger, [11] studied the effect of combined axial and azimuthal magnetic fields on stability of cylindrical Taylor-Couette flow. Szklarski and Rüdiger [12] studied the magnetic effects induced by rigidly rotating plates enclosing a cylindrical magnetohydrodynamic Taylor-Couette flow, and then they found that the instability has essentially a centrifugal character as Rayleigh criterion is locally violated. Shalybkov [13] has studied the combination effect of unstable rotation and an unstable azimuthall magnetic field on stability of Taylor-Couette flow.

In the present study, three-dimensional computational results are performed to analyse the effect of magnetic field on the pressure and the angular momentum in the system of Taylor-Couette sodium liquid flow for different situations of insulating and conducting boundaries.

2. Main problem and modelling

The liquid sodium is contained in the gap between a stationary outer cylinder of radius $Ro = 28.5$ *mm* and an inner cylinder of radius $Ri = 23.65$ *mm*. $H = 155$ *mm* is the height of the device. The aspect ratio $\Gamma = H / d$, where d is the gap between the cylinders is defined as $d = Ro - Ri$. The inner cylinder is rotating at angular velocity Ω corresponding a Taylor number $Ta = Re\ .\ \delta^{1/2}$ with $Re = Ri\ \Omega\, d\ /\ \nu$ and δ is the gap ratio given by $\delta = d / Ri$, ν is the kinematic viscosity. Two main cases are considered, the first one without magnetic fields, the second one with magnetic field B corresponding to a Hartmann number $Ha = B.(d\ Ri\ /\mu_0 \rho\ \nu\ \eta)^{1/2}$ where μ_0 denotes vacuum permeability, ρ and η density and magnetic diffusivity of the fluid. Different cases are studied when magnetic field is applied for insulating and conducting walls as resumed in table1

Case0	Without magnetic field		
With magnetic field			
	Inner cylinder	Outer cylinder	Upper and down boundaries
Case 1	Insulating	Insulating	Insulating
Case 2	Conducting	Insulating	Insulating
Case 3	Insulating	Conducting	Insulating
Case 4	Conducting	Conducting	Insulating
Case 5	Insulating	Insulating	Conducting
Case 6	Conducting	Conducting	Conducting

Tab. 1. Different studied cases

The governing equations of continuity, momentum, magnetic field for this problem of velocity V, pressure P can be written as,

$$\nabla \cdot V = 0 \tag{1}$$

$$\rho \frac{\partial V}{\partial t} + \rho (V \cdot \nabla) V = -\nabla P + \rho \nu \nabla^2 V + J \times B \tag{2}$$

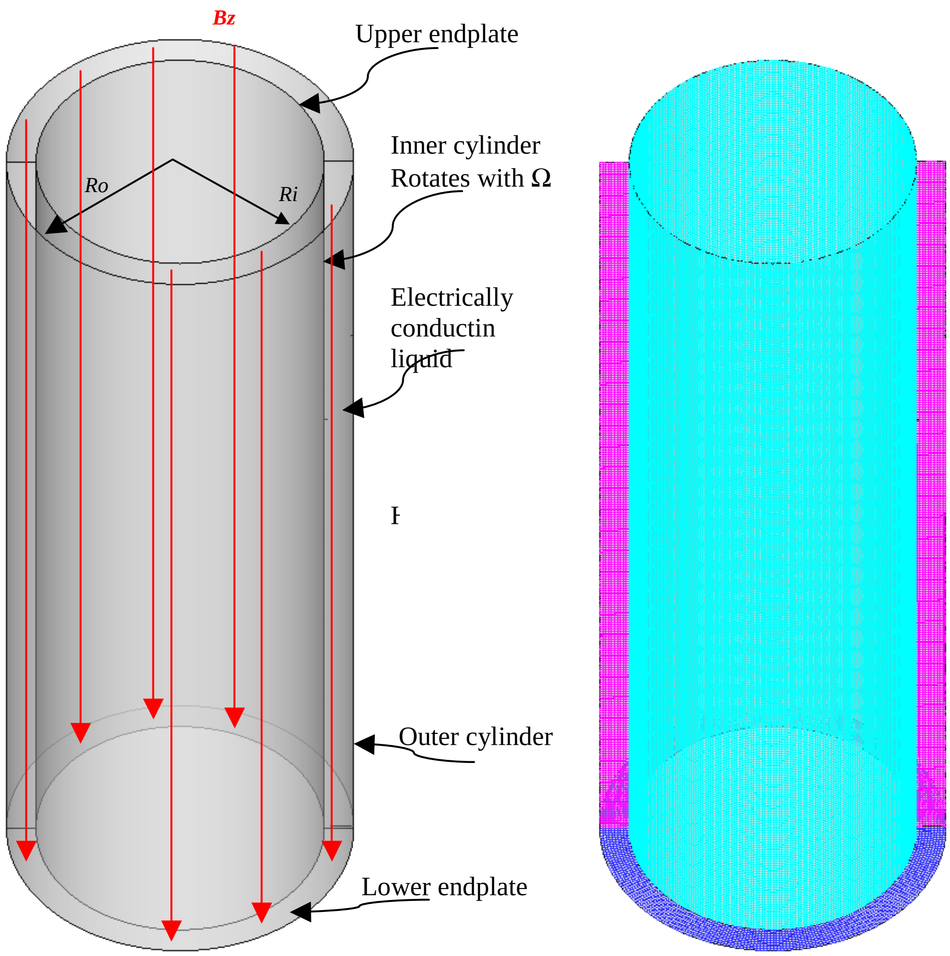
$$\nabla \cdot J = 0 \tag{3}$$

$$J = (E + V \times B)\sigma \tag{4}$$

$$E = \nabla \Phi \tag{5}$$

Where B, J, E and Φ are magnetic flux density, electric current density, electric field and electric potential, respectively. In the Navier-Stokes equations including the Lorentz body force. Non-slip boundary conditions are applied at inner and outer cylinders and endplates for radial and axial velocity components. The angular velocity matches the rotation rates of the inner cylinder.

The governing equations and the boundary conditions for the flow and electromagnetic fields were discretized and solved numerically using the finite volume package Fluent with double precision, the PISO (Pressure-Implicit with Splitting of Operators) is based on higher degree of the approximate relation between corrections for pressure and velocity coupling and the PRESTO (PRE-Staggering Option) scheme for discrete continuity balance, and quick scheme for spatial discretization of momentum equations. By carrying out the convergence test of grid, we choose a structured and hexahedral grid of about one million nodes as shown in Figure 1. (b)



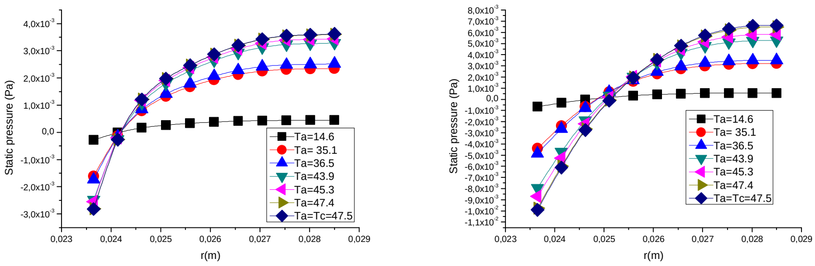
(a) **(b)**

Figure 1: (a) Geometry of Taylor Couette system, (b) Grid used in simulation *1 094 720* nodes, *992 000* hexahedral cells.

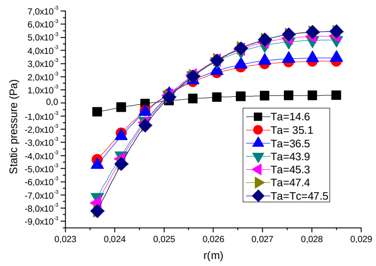
3. Results and discussion

We consider the flow of an electrically conducting fluid as liquid sodium of density $\rho = 920\,kg/m^2$ between two finite cylinders. Let us start at $Z = 153.7$ *mm*, the pressure increases exponentially from the inner rotating cylinder to the exterior fixed one. This radial pressure gradient is induced by the rotation of the inner cylinder and the shear stress induced close to the upper boundary. It is responsible of the appearance of Taylor-vortex flow TVF. Due to the weak rotation rate the pressure is almost uniform for small Taylor number values ($Ta = 14.7$) with respect to the other Taylor numbers. All pressure plots have an intersection point at $r=24.2$ mm which can be taken as a critical radius. For $Ta = 47.5$ a large radial pressure gradient is obtained indicating that the first critical instability is reached for this system. At $Z = 149$ *mm* and at $Z =$

146.7 mm which corresponds to the middle of the first (TVF), we notice almost the same evolution of the pressure field. However the intersection point moves radially outward, as shown in Figure 2c. Computations were conducted for seven cases for insulating and conducting boundaries as shown in Table1. At *Z = 146.7 mm* (the middle of first TVF) the pressure field appears to be uniform for the case 0 compared to the presence of magnetic field, figure 3.

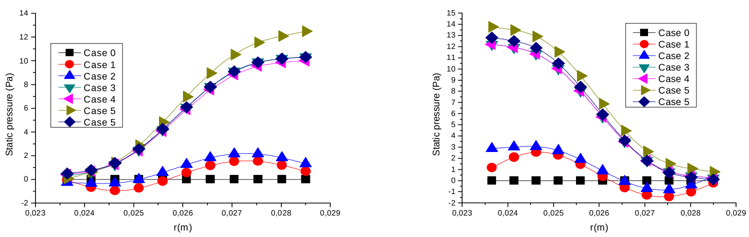


(A) Z = 153.7 mm *(B) Z =149 mm*

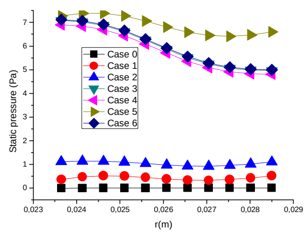


(C) Z =146,7 mm

Figure 2: Pressure versus radius for different Taylor number values in the case 0



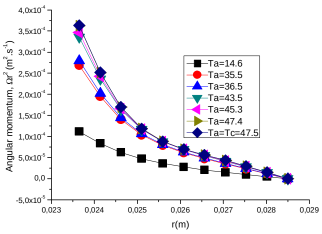
(A) Z = 153.7 mm *(B) Z =149 mm*



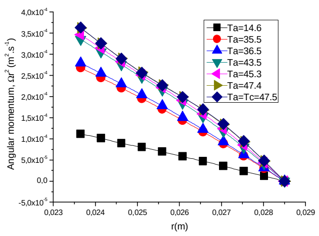
(C) Z =146.5mm

Figure 3 : Pressure versus radius for *Ta = Tc = 47, 5*

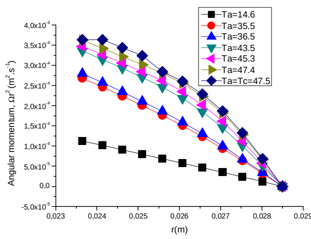
For the case of insulating boundaries (case 1) pressure values are very close to the case 0. No great changes are noticed when only the inner cylinder is conducting (case 2). Whereas for the cases 3,4,5 and 6 the pressure changes considerably. At Z=153.7 mm corresponding to the middle of Ekman cell, the decrease of the radial angular momentum gradient is larger in the vicinity of the inner cylinder, than close to outer cylinder. Away from the upper boundary and placing between the Ekman cell and TVF, at *Z = 149 mm* we note that the effect of the rotating inner cylinder relatively dominant over braking fixed boundary $\partial\Omega(r) / \partial r$ and it decreases rapidly to become low in the vicinity of the inner cylinder, outwardly $\partial\Omega(r) / \partial r$ it becomes relatively large due to the effect of the outer cylinder which is at rest. In the middle of the first TVF at Z= 146.7 mm, we note that all the curves tend to be linear, and then the profiles are centrifugally instable with angular momentum $r^2\ \Omega(r)$ decreases radially outward. The radial angular momentum gradient in the middle of the Ekman cell at *Z = 153.7 mm* (case 6), is the most important which means that the effect of magnetic fields is huge in the case with the conducting walls, also in the case 5 with conducting upper and down boundaries.



(A) Z=153.7 mm

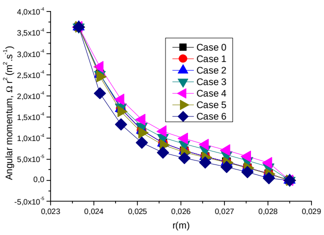


(B) Z=149 mm

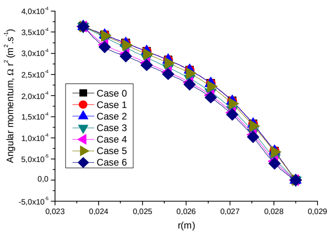


(C) Z=146,7 mm

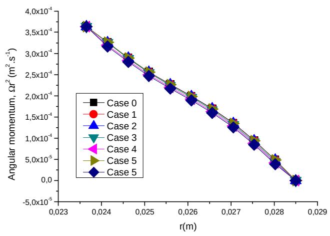
Figure 4: Angular momentum versus radius for different Taylor numbers for case 0.



(A) Z=153.7 mm



(B) Z=149 mm



(C) Z = 146,7 mm

Figure 5: Angular momentum versus radius for different cases of conducting walls for *Ta = Tc1*

The magnetic field does not have a remarkable effect for the case with insulating boundaries, thus destabilizes the flow field at the Ekman cell in the case when the upper and down conducting boundaries. Between Ekman cell and the first TVF, we note that the magnetic field stabilizes the flow better when all walls are conductive, a lesser effect of stabilization in the case when the inner and outer cylinder are conducting. While, when just the boundaries are conducting the magnetic effect is relatively low. The effect of magnetic field does not appear in the case when all walls are insolating and even in the case when the inner cylinder is conducting. While in case of the two inner and outer cylinders are conducting the magnetic fields effect is relatively large than effect in the case when only the outer cylinder is conducting. At the first TVF the case 6, 4 and 5, we notice a weak stabilization of the flow.

Concluding remarks

Numerical computations are conducted to study the magnetohydrodynamic instability with insulating and conducting walls. The main effect of axial magnetic field in Ekman cell is when upper and down boundaries are conducting. Also, we found that the magnetic field effect is not the same in all liquid metal annulus. Each region is affected differently, and it depends to conducting or insulating walls. Under the effect of a strong uniform vertical magnetic field (B=4 Tesla) corresponding to Hartmann number Ha=2400 the pressure field and angular momentum have been analyzed.

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