The box counting method for evaluate the fractal Dimension in radiographic images

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Abstract : - Since the end of the Seventies, following work of the French mathematician Mandelbrot, We are witnessing the true exploitation of the interest expressed by the scientific community for the fractals objects. The aim of this paper is to introduce the fractal theory by the calculation of the fractal dimension in the radiographic images. We implement for that, the box counting method for the segmentation of the images. This method will be presented, as well as a study of the effect of the change of the rang of the box sizes (rmin and rmax) on fractal dimension is carried out.

Key-Words: - Box counting method, fractal, fractal dimension, radiographic images, side length, self-similarity.

1 Introduction

One names fractal, a curve or surface of irregular form or parcelled out which is created by following deterministic or stochastic rules. The term “fractal” is a neologism created by Benoît Mandelbrot in 1974 starting from the Latin root “fractus”, which means broken. This term was at the beginning an adjective: the fractals objects.

Historically it is in work of the mathematicians Cantor and Peano, at the end of the XIXth century, which one finds the first references to sets often considered as pathological, whose geometry particularly complex and is structured.

Very quickly the community of the mathematicians of that time realizes that a description of such sets cannot be defined as being dimension corresponding to the number of co-ordinates necessary to characterize the position of a point in this set.

In 1919, Hausdorff proposes a new definition of the dimension of a set which can take non-integer values and which makes it possible to account for the degree of irregularity of these objects. One of the greatest merits of Mandelbrot is to have known to know that “the exception is often the rule” and to have shown that these fractured structures (singular) are in fact very present in nature. The profiles of our mountains, various the ramified geometries that the trees constitute, the rivers or the overlaps of the bronchi in the lungs are as many examples as one can apprehend within the definite federator framework by Mandelbrot.

A fractal is a geometrical figure or a natural object which combines the following characteristics:

1 - Its parts have the same form or structure that the whole, with this close which they are on a different scale and can be slightly deformed.

2 - Its form, either extremely irregular, or extremely is stopped or split up with the remainder, in any scale of examination.

3 - It contains “distinctive elements” whose scales are very fast varied and cover a large range.

2 Concept of fractal dimension

A usual method to measure a length, a surface or a volume, consists in covering these sets with paving stone (pertaining to the space in which the unit is plunged) of which the length, surface or volume is taken as measuring unit.

If ε is the side (length standard) of a paving stone, measurement obtained is [1]:

$$M = N \cdot \varepsilon^{d} = N \mu \qquad (1)$$

Where μ is the unit of measure (length, surface or volume).

Mandelbrot postulates that there are curves of intermediate size between 1 and 2 of surfaces of size higher than 2, and that these objects precisely have the property to have no length or a precise surface, not more than one volume does not have a surface or a square does not have a length. This dimension, intermediary between the integer values, was baptized neologism “fractal” so that no confusion is made between a traditional surface (of D=2 dimension) [2].

One is brought to believe that a geometrical object, about scale, can also generate as well the small as the big details. Such an object will be known as to have an internal homothety, or to be self-similar. It is known that if one transforms a line by a homothety of arbitrary ratio, whose centre belongs to it, one finds this same line, and it is the same for any plane and entire Euclidean space. One can generalize even for non-integer dimensions, as this definition indicates it:

If a fractal object S is divided in N objects similar to S in a homothety 1/r, the dimension of homothety or fractal dimension is the ratio:

$$D = \frac{\log N}{\log \frac{1}{r}} \qquad (2)$$

It should be checked that the curves do not have double points. It is not the same with other curves which have a double infinity of points. It follows that for them, the concept of paving changes significance and that the definition of the homothety dimension becomes debatable.

3 The box counting method

Natural surfaces, in general, do not have a deterministic self-similarity. However, they show a statistical self-similarity and the preceding definition remains applicable. If the definition of fractal dimension by the self-similarity is direct, it is often difficult to directly estimate it starting from the data of an image, such as for example the radiographic images. For this reason a suitable method is appropriate to estimate fractal dimension, these images known as “non-linear”.

Among the techniques discussed by Mandelbrot [3], the box counting method is found like the most adapted for the estimate of fractal dimension [4]. Voss, Keller and Sarkar [5 6 7] carried out a box counting method, the purpose of which is to consider the average number, noted N (r), of cubic boxes with fixed side length r, necessary to cover the image, considered as a surface in R^3 space. For that we estimates P (m, r), the probability that one box of size r, centred on an arbitrary point of surface, contains m points of the set. We have thus [8]:

$$\forall r, \sum_{m=1}^{Np} P(m,r) = 1 \qquad (3)$$

Where Np is the number of possible points in the cube.

The estimate of the average number of disjoined boxes necessary to cover surface is :

$$N(r) = \sum_{m=1}^{Np} N(m,r) = \sum_{m=1}^{Np} \frac{P(m,r)}{m} \qquad (4)$$

The estimate by the least squares method of the slope of the group of dots (log (r), - log (N (r))), obtained with boxes of increasing size r, gives the estimate of fractal dimension. The algorithm 1 [8] presents this calculation:

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Initialisation:
FOR r =1 to rmax and m = 1 to r^3 DO
        P(m,r) = 0
FOR any site s of the image DO
        BEGIN
        For r = 1 to rmax DO
                BEGIN
                  - Center a cube with
                    dimension r on [s, A[s]]
                  - Count the number m of
                    pixels of the image
                    which belong to this cube
                  -  Increment P (m,r) of 1
                END
        END
FOR  r = 1 to rmax DO
```

$$N(r) = \sum_{m=1}^{Np} \frac{P(m,r)}{m}$$

Estimate by the method of least squares the slope D of the curve (log (r), -log (N(r)))

Algorithm. 1. The box counting method algorithm.

Recent work on the fractal dimension using the box counting method in the radiographic images were made are: Imai, Ikeda, Enchi and Niimi [9], witch have conducted a fractal analysis (with the box-counting method for a binary images) of low-dose digital chest phantom radiographs and calculate fractal-feature distance using the fractal dimension, This method uses a lot more materials and methods than offer.

We have also, Podsiadlo, Dahl, Englund, Lohmander and Stachowiak [10], have studied Differences in trabecular bone texture between knees with and without radiographic osteoarthritis detected by fractal methods, A newly developed augmented Hurst orientation transform (HOT) method was used to calculate texture parameters for regions selected in X-ray images of non-OA and OA tibial bones. This method produces a mean value of fractal dimensions, FDs in the vertical (FD_V) and horizontal (FD_H) directions and along a direction of the roughest part of the tibial bone (FD_{Sta}), fractal signatures and a texture aspect ratio (Str).

Fig. 1. radiographic image of the hand

In what follows we will apply the method for the image X-ray test (figure 1) of size 256 X 256 grayscale, and see the results of the calculation of fractal dimension. For that we propose to plot curves giving the number of boxes versus their side length r, then the straight regression line which estimates as well as possible the log (N(r)) versus log (r); the plot will be done on all the points and outdistances it between the size of boxes is equal to 1.

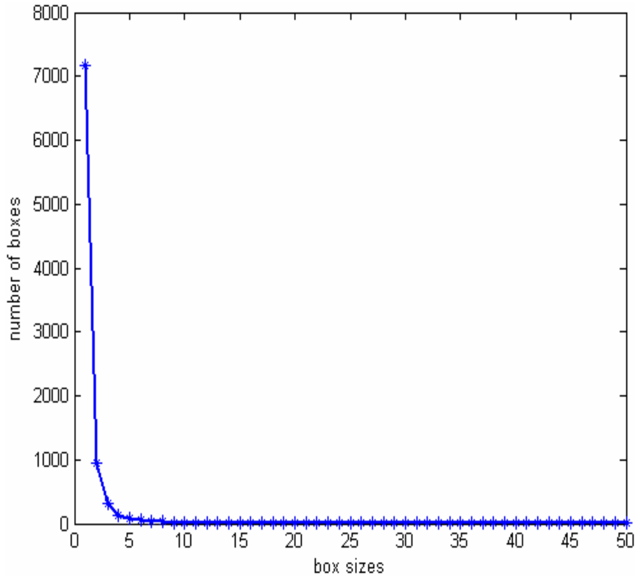

Fig. 2. The plot of the number of boxes versus the side length r for the radiographic image.

The Figure 2 represents the number of boxes according to their sizes (r), in this case r = 1 to 50. We notice that more the size is small more the number of boxes is large, and inversely, more we increased the size of boxes, more the number of these last tends towards 0. In addition we see the brutal fall of the number of boxes after the side length r = 1, this explains the existence of a significant number of boxes if we fix the side length r at 1, but we calculate badly the number of these boxes beyond 2 or 3, for that we give a table of some values to see what happens for boxes witch side length r = 1 to 50 (Table I).

Table 1.
Some values of a number of boxes according to the side length r

r	N(r)
1	7181
2	951
3	136
5	74
10	20
15	6
30	3
40	2
50	1

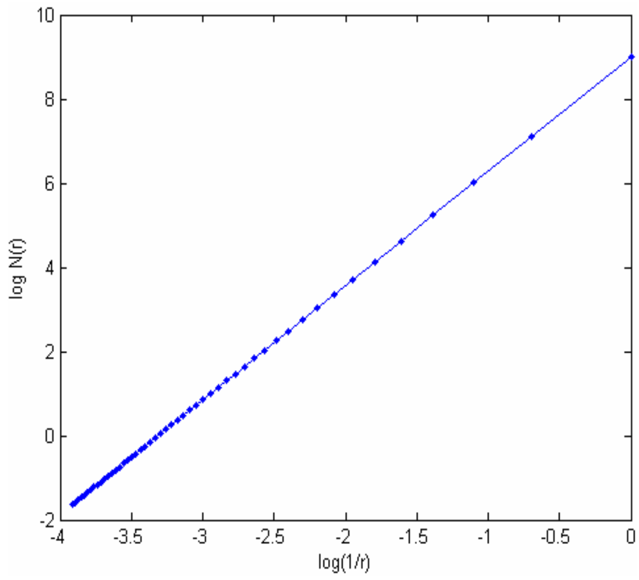

Fig. 3. The plot regression of the curve log N(r) versus log (1/r) by the method of least squares.

For this experimentation we obtained a dimension D = 2.72, for r = 1 to 50, the only parameter which can influence the calculation of fractal dimension is well the rang of the sizes of the windows (boxes), this point will be exposed in the following section.

4 Influence of box sizes

To find fractal dimension, it is not easy to choose the sizes of the boxes for complex images like the radiographic images, we carried out several tests on different sizes and we obtained various values of fractal dimension.

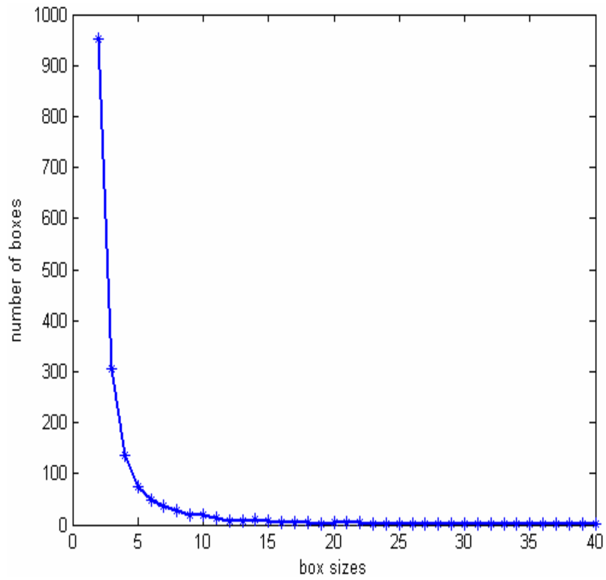

Fig. 4. The box counting method with rmin =2, rmax = 40.

An application of a rang of side length of boxes is illustrated in figure 4, we notice that the number of boxes change.

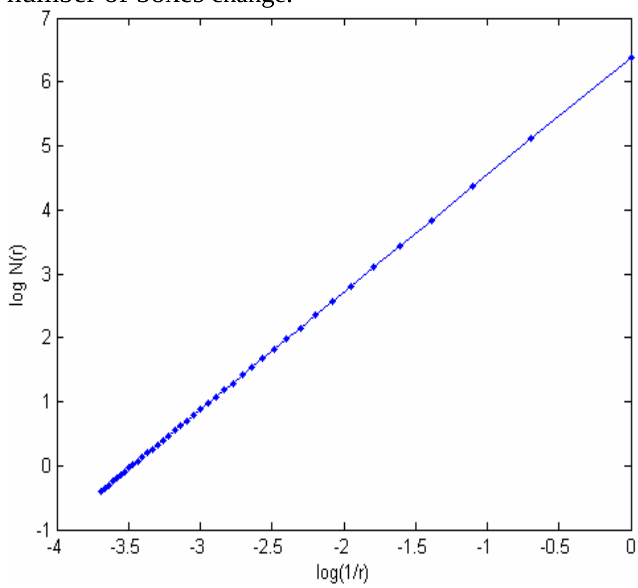

Fig. 5. The regression of the curve log N(r) by the least squares method.

The change of the rang of the box sizes gave us a different result of first for the same image, The fractal dimension in this case is D = 1.84 and this for rmin =2 and rmax = 40, Therefore we can conclude that there are 4 possibilities:

- rmin small, rmax small
- rmin small, rmax large
- rmin large, rmax small
- rmin large, rmax large

of course it is necessary that they are sufficiently isolated to avoid the effect of overlapping.

We give in table 2 certain values of fractal dimension corresponding to the rang of box sizes:

Table 2.
Some examples of the fractal dimension.

rmin - rmax	Fractal dimension
1 – 50	2.72
1 – 40	2.66
2 – 40	1.84
2 – 50	2.03
3 – 50	1.48

This last informs us about the influence of the rang of box size on the calculation of fractal dimension (Table 2).

5 Choice of box sizes

If we see the table 2 we notice a variability of the values of fractal dimension, an improvement of the original method can be implemented, it is the choice of rmin and rmax.

A priori it is difficult to choose the side length r of the boxes considering the complexity of the images, but we can always take a compromise between rmin and rmax, these two parameters not only influential one on the other but more especially on the calculation of the fractal dimension.

For the choice of rmin and after several tests carried out, we can say that we can choose it from 2 and this to be able to have a probability of find at least one box; so, for rmin = 1, we are confronted with the problem of the pixel size. Therefore, for rmin = 1, one true box of side length r cannot be centred on a pixel.

Another problem which does not make it possible to begin the processing of rmin = 1, is the insulated pixels, we should not take them in consideration, so rmin = 2, is a good choice to be able to carry out one box and to detect pixels going up to 4 (2x2).

For rmax, it is fixed in the following way:

As soon as the N(r) approach or overlap too to each other, we stop the process; from there, an extraction of the maximum value of r will be done. we can also fix it from the moment when the number of windows (boxes), tends towards 0 (this number must stop at 1); from this moment it is not necessary to repeat iterations for larger sizes, it is a waste of time, from where the optimization of the computing time.

In addition rmax should not exceed a certain value under penalty of exceed the framework of the image, where:

- If the horizontal size of the image (x) ≤ to the vertical size of the image :

$$r\max < \frac{x}{2} \tag{5}$$

- If the vertical size of the image (y) ≤ to the horizontal size of the image :

$$r\max < \frac{y}{2} \tag{6}$$

6 Conclusion

Throughout of this paper we studied the fractal dimension for the radiographic images with two dimension, after an introduction on the theory of the fractals and its dimension, we said that these images was complex and non-linear, for the calculation of their fractal dimension it would have been necessary found a method that can be freed from the existence of the phenomenon of the statistical random elements that these images present. For that the box counting method is up to be to this task and it gave a good results.

We didn't calculate only the fractal dimension, we saw also that the rang of the box sizes influenced much the calculation of this dimension, for that we presented some calculations of fractal dimension corresponding to certain rang of r. Finally a choice of the size of these boxes is worked out for better calculating fractal dimension.

References:

[1] X1. J.F Gouyet, Physique et structures fractales, Masson paris, 1992.

[2] X2. H. Maître, Le traitement des images, Lavoisier paris, 2003.

[3] X3. B.B Mandelbrot, The fractal geometry of nature, freeman, new york, 1983.

[4] X4. K. Fouroutan-pour, P. Dutilleul, D.L. Smith, Advances in the implementation of the box-counting methode of fractal dimension estimation, *Applied Mathematics and computation 105*, 1999 pp.195-210, .

[5]X5. R.Voss - Random fractals: charaterization and measurement, in Scaling phenomena and disordred systems, *in R. Pynn & A. Skjeltorp eds, plenum press*, *New york*, 1986, pp.1-11.

[6] X6. J.M.Keller, S.Chen, R.M.Crownover - Texture description and segmentation through fractal geometry, *CVGIP*, *45*, 1989, pp.150-166.

[7] X7. N.Sarkar, B.B.Chaudhuri - An efficient approch to estimate fractal dimension of textural images, *Pattern Recognition, vol. 25, n°9*, 1992, pp.1035-1041.

[8] X8. J.P. Cocquerez, S. Philipp, Analyse d'images: filtrage et segmentation, Masson paris, 1995.

[9] X9. K. Imai, M. Ikeda, Y. Enchi, T.Niimi, Fractal-Feature Distance Analysis of Radiographic Image, *Academic Radiology, Volume 14, Issue 2*, 2007, pp.37-143.

[10] X10. P. Podsiadlo, L. Dahl, M. Englund, L.S. Lohmander and G.W. Stachowiak, Differences in trabecular bone texture between knees with and without radiographic osteoarthritis detected by fractal methods, *Osteoarthritis and Cartilage, In Press, Corrected Proof, Available online 6 September 2007.*