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Fuzzy-Feedback Linearization Control of
a MIMO Nonlinear System.

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ABSTRACT

Input-Output Feedback Linearization is currently adopted as a powerful technique to obtain linear state models of nonlinear MIMO system dynamics. In this project, Input Output Feedback Linearization is applied to control the Permanent Magnet Synchronous Machine. The objectives are to control the machine speed, currents, and torque to meet the desired performances.

To model and reduce the effects of uncertainties on the system behavior, an Interval Type-2 Fuzzy Logic Controller is designed and applied. The efficiency of the used approach is illustrated by system simulations.
DEDICATION

To my beloved mother, I dedicate this work.
ACKNOWLEDGEMENTS

I would like to thank my supervisor, Ms. F. KESSAL for her considerable efforts in making this possible.
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<th>Description</th>
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<tr>
<td>PMSM</td>
<td>Permanent magnet synchronous machine.</td>
</tr>
<tr>
<td>d, q.</td>
<td>Park components of machine current; longitudinal and quadratic.</td>
</tr>
<tr>
<td>NLC</td>
<td>Nonlinear control.</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulation.</td>
</tr>
<tr>
<td>t.</td>
<td>time (s).</td>
</tr>
<tr>
<td>I_s</td>
<td>stator current (A).</td>
</tr>
<tr>
<td>V_s</td>
<td>stator voltage (V).</td>
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<td>V_a, V_b, V_c</td>
<td>phase voltages (V).</td>
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<tr>
<td>Φ_s</td>
<td>stator flux (Wb).</td>
</tr>
<tr>
<td>R_s</td>
<td>stator resistance (Ω).</td>
</tr>
<tr>
<td>L_{ss}</td>
<td>stator inductance (H).</td>
</tr>
<tr>
<td>p.</td>
<td>number of pole pairs.</td>
</tr>
<tr>
<td>f.</td>
<td>coefficient of viscous friction (Nm/rad/s).</td>
</tr>
<tr>
<td>J.</td>
<td>moment of inertia (Kg.m^2).</td>
</tr>
<tr>
<td>Ω</td>
<td>mechanical speed (rad/s).</td>
</tr>
<tr>
<td>Ω_{ref}</td>
<td>reference speed (rad/s).</td>
</tr>
<tr>
<td>P</td>
<td>Park matrix.</td>
</tr>
<tr>
<td>C_e</td>
<td>electromagnetic torque (Nm).</td>
</tr>
<tr>
<td>C_r</td>
<td>reluctant torque (Nm).</td>
</tr>
<tr>
<td>Θ.</td>
<td>actual position.</td>
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<td>Φ_d, Φ_q</td>
<td>Park fluxes (W).</td>
</tr>
</tbody>
</table>
\( \text{L}_{d}, \text{L}_{q} \) Park Inductances \([\text{H}]\).

\( \phi_{sf} \) Permanent magnets flux \([\text{Wb}]\).

\( x \) state vector.

\( u \) Control vector.

\( X \) Vector representing electrical quantities.

\( X_0 \) homopolar part of \( X \).

\( \theta_m \) rotor mechanical position.

\( T_i \) and \( T_{i'} \) transistors.

\( \text{IOFL} \) Input Output Feedback Linearization.

\( \text{IT-2 FLS.} \) Interval type-2 fuzzy logic system.

\( \text{FLS} \) Fuzzy logic set.

\( \text{FLC} \) Fuzzy logic controller.

\( \text{FOU} \) Footprint of uncertainty.

\( \text{UMF} \) Upper membership function.

\( \text{LMF} \) Lower membership function.

\( \text{MF} \) Membership function.

\( \text{T-1 FL} \) Type-1 fuzzy logic.

\( \text{AC} \) Alternative current.

\( \text{PWM} \) Pulse width modulation.

\( \text{DFIG} \) Doubly fed induction generator.

\( \text{DC} \) Direct current.

\( \text{SM} \) Synchronous machine.

\( \text{PM} \) Permanent magnet.

\( \text{VSI} \) Voltage source inverter.

\( \text{IO} \) Input output.

\( \text{KM} \) Karnik and Mendel.

\( \text{EKM} \) Enhanced Karnik and Mendel algorithm.
GENERAL INTRODUCTION

Nonlinear control theory have been greatly developed and applied to a wide variety of nonlinear systems. This scientific revolution has brought up many efficient and powerful techniques. One of these, the Input-Output Feedback Linearization method has attracted a great deal of research and practical application especially on MIMO systems. This approach relies on transforming the nonlinear system dynamics into a decoupled set of single variable blocks.

Although nonlinear control is exact and efficient, still it requires a background on the sophisticated differential geometry that is critically important in modeling and analysis of nonlinear systems. The lately introduced Fuzzy Logic Theory provides a solution to this problem by adopting linguistic terms and human expertise to model and control systems with unknown or hard to obtain models. In an Interval Type-2 Fuzzy Logic Controller, the system uncertainties and external perturbations are treated through the use of fuzzy sets with fuzzy membership functions.

These two mentioned approaches will be applied to control the Permanent Magnet Synchronous Machine and to stabilize its behavior. The PMSM, known for its nonlinear model, has been the subject of many control applications to handle and improve its performance.

The work flow in this project will be:

**The first chapter:** a brief introduction to the concept of Input-Output Feedback Linearization approach is performed along with the main mathematical tools required to understand the applied technique.

**The second chapter:** a presentation of Fuzzy Logic Control theory with the advantages and benefits of Fuzzy Logic Controllers are stated. Then the different components of a Type-2 FLC are introduced and explained.

**The third chapter:** presents the Permanent Magnet Synchronous Machine and its main characteristics. The derivation of the machine mathematical model is explained along with describing simulations of its behavior.
The fourth chapter: an application of the Input-Output Feedback Linearization technique on the nonlinear model of the PMSM is illustrated, the model is linearized, and simulated. The results of simulation assert the efficiency of the method being applied.

The fifth chapter: introduces the Interval Type-2 Fuzzy Logic Controller to the feedback linearized system in aim of reducing the effects of uncertainties and enhancing the PMSM performance.
CHAPTER ONE

INPUT-OUTPUT FEEDBACK LINEARIZATION
1.1 INTRODUCTION

Feedback linearization is an approach to nonlinear control design which has attracted a great deal of research interest in recent years. The main concept of this approach is to algebraically transform a nonlinear system dynamics into (fully or partly) linear ones, so that linear control techniques can be applied. In its simplest form, feedback linearization cancels the nonlinearities in a nonlinear system so that the closed-loop dynamics is in a linear form.

This differs entirely from conventional linearization (i.e., Jacobian linearization) in that feedback linearization is achieved by exact state transformations and feedback, rather than by linear approximations of the dynamics.

Feedback linearization has been used successfully in a variety of practical control problems. These include the control of helicopters, high performance aircraft, industrial robots, and biomedical devices. In this project, we will use it to control a permanent magnet synchronous machine [1].

1.2 MATHEMATICAL TOOLS

Some mathematical concepts required to understand the following sections are introduced below:

1.2.1 GRADIENT AND JACOBIAN

• Given a smooth scalar function \( h(x) \) of the state \( x \), the gradient of \( h \) is denoted by:

\[
\nabla h = \frac{\partial h}{\partial x} = \left[ \frac{\partial h}{\partial x_1}, \frac{\partial h}{\partial x_2}, \ldots, \frac{\partial h}{\partial x_n} \right]
\]  

(1.1)

• Given a vector field \( f(x) \), the Jacobian of \( f \) is denoted by:

\[
\nabla f = \frac{\partial f}{\partial x^2}
\]  

(1.2)
1.2.2 LIE DERIVATIVE

Given a scalar function $h(x)$ and a vector field $f(x)$ we define a new scalar function $L_f h$, called the Lie derivative (or the derivative of $h$ with respect to $f$).

**Definition:** Let $h: \mathbb{R}^n \to \mathbb{R}$ be a smooth scalar function, and $f: \mathbb{R}^n \to \mathbb{R}^n$ be a smooth vector field on $\mathbb{R}^n$ then the Lie derivative of $h$ with respect to $f$ is a scalar function defined by:

$$L_f h = \nabla h \cdot f.$$

Thus, the Lie derivative $L_f h$ is simply the directional derivative of $h$ along the direction of the vector $f$.

Repeated Lie derivatives can be defined recursively as:

$$
\begin{cases}
L_0^f h = h \\
L_i^f h = \nabla (L_{i-1}^f h) \cdot f & \text{For } i = 1, 2, \ldots
\end{cases}
$$

Similarly, if $g$ is another vector field, then the scalar function $L_g L_f h$ is:

$$L_g L_f h = \nabla (L_f h) \cdot g$$

1.2.3 LIE BRACKETS

**Definition:** Let $f$ and $g$ be two vector fields on $\mathbb{R}^n$. The Lie bracket of $f$ and $g$ is a third vector field defined by:

$$[f, g] = \nabla g \cdot f - \nabla f \cdot g$$

The lie bracket $[f, g]$ is commonly written as $ad_f g$ (where ad stands for "adjoint"). Repeated Lie brackets can then be defined recursively as

$$
\begin{cases}
ad_f^0 g = g \\
ad_f^i g = [f, ad_f^{i-1} g] & \text{for } i = 1, 2, \ldots
\end{cases}
$$
Properties: Lie brackets have the following properties:

a- Linearity:

\[
[\alpha_1 f_1 + \alpha_2 f_2, g] = \alpha_1 [f_1, g] + \alpha_2 [f_2, g]
\]
\[
[f, \alpha_1 g_1 + \alpha_2 g_2] = \alpha_1 [f, g_1] + \alpha_2 [f, g_2]
\]

Where \(f, f_1, f_2, g, g_1,\) and \(g_2\) are smooth vector fields, and \(a_1\) and \(a_2\) are constant scalars.

b- Skew-commutativity:

\[
[f, g] = -[g, f]
\]

c- Jacobi identity:

\[
L_{[ad_f, g]} h = L_f L_g h - L_g L_f h
\]

Where \(h(x)\) is a smooth scalar function of \(x\) [5].

1.3 INPUT-OUTPUT FEEDBACK LINEARIZATION OF MIMO SYSTEMS

We consider a multi-input multi-output (MIMO) system as follows:

\[
\dot{x} = f(x) + g(x)u
\]
\[
y = h(x)
\]

(1.6)

Where, \(x\) is the state vector, \(u\) the control input vector, \(y\) the output vector, \(f\) and \(g\) are smooth vector fields, and \(h\) is a smooth scalar function.

An approach to obtain the input-output linearization of this MIMO system is to differentiate the output \(y\) of the system until at least one of the inputs appears. By differentiating (1.6), we obtain:

\[
\dot{y}_i = L_f \ h_i + \sum_{i=1}^{m} (L_{g_i} h_i) u_i
\]

(1.7)
Where: \( L_f \ h \) and \( L_g \ h \) represent the lie derivatives of \( h(x) \) with respect to \( f(x) \) and \( g(x) \), respectively. If \( L_{gi} \ h_i(x) = 0 \) for all \( i \), then no input appeared and we have to repeat differentiation as:

\[
y_i^{ri} = L_i^{ri} \ h_i + \sum_{i=1}^{m} \left( L_{gi} L_i^{ri-1} h_i \right) u_i \tag{1.8}
\]

With: \( L_{gi} L_i^{ri-1} h_i(x) \neq 0 \) for at least one \( i \). If we perform the above procedure for each output \( y_i \), we can obtain a total of \( m \) equations in the above form, which can be written compactly as:

\[
\begin{bmatrix}
 y_1^{(r_1)} \\
 \vdots \\
 y_i^{(r_i)} \\
 \vdots \\
 y_m^{(r_m)}
\end{bmatrix}
= \begin{bmatrix}
 L_{f}^{r_1} h_1(x) \\
 \vdots \\
 \vdots \\
 L_{f}^{r_m} h_m(x)
\end{bmatrix}
+ E^{-1}(x) \begin{bmatrix}
 u_1 \\
 \vdots \\
 u_i \\
 \vdots \\
 u_m
\end{bmatrix} \tag{1.9}
\]

Where the \( m \times m \) matrix \( E(x) \) is defined as:

\[
E(x) = \begin{bmatrix}
 L_{g1} \times L_{f}^{r_1} h_1 & \ldots & L_{gm} \times L_{f}^{r_1} h_1 \\
 L_{g1} \times L_{f}^{r_m} h_m & \ldots & L_{gm} \times L_{f}^{r_m} h_m
\end{bmatrix} \tag{1.10}
\]

The matrix \( E(x) \) is called the decoupling matrix for the MIMO system. If \( E(x) \) is non-singular, then the input transformation can be obtained as:

\[
v = -E^{-1}(x) \times \begin{bmatrix}
 L_{f}^{r_1} h_1(x) \\
 L_{f}^{r_m} h_m(x)
\end{bmatrix}
+ E^{-1}(x) \times \begin{bmatrix}
 u_1 \\
 u_m
\end{bmatrix} \tag{1.11}
\]

Substituting (1.9) into (1.11), results in a linear differential relationship between the output \( y \) and new input vector \( v \).

\[
\begin{bmatrix}
 y_1^{(r_1)} \\
 \vdots \\
 y_i^{(r_i)} \\
 \vdots \\
 y_m^{(r_m)}
\end{bmatrix}
= \begin{bmatrix}
 v_1 \\
 \vdots \\
 v_i \\
 \vdots \\
 v_m
\end{bmatrix} \tag{1.12}
\]

We notice that the above input – output relationship has decoupled the system into \( m \) single-input single-output linear systems [2].
1.4 CONCLUSION

Feedback linearization is now widely used in the design of advanced controllers in industry. It is based on the idea of transforming a nonlinear dynamics into a linear form using state feedback. This approach is used both for stabilization and tracking control problems and for different types of systems and has been successfully applied to a number of practical nonlinear control problems, both in system analysis and controller design.
CHAPTER TWO

INTERVAL TYPE-2 FUZZY LOGIC CONTROL
2.1 INTRODUCTION

In this chapter we introduce the general concepts and main aspects of Fuzzy Logic Systems focusing on Interval Type-2 Fuzzy Logic Controllers as a new human experts approach applied to control various types of nonlinear systems. This type of nonlinear control systems is used to overcome parameter uncertainties and unknown dynamic model characteristics generally faced with conventional nonlinear controllers.

The reasons of adopting and even favoring this type of control systems are first discussed then a short representation of the different building blocks of an IT-2 FLS is introduced.

2.2 BENEFITS OF APPLYING FUZZY LOGIC CONTROL

The primary aspect of this new control approach is using human control operator's knowledge and experience to intuitively construct controllers so that the resulting control system can emulate human control behavior to a certain extent. Compared to the traditional control methods, the advantages of fuzzy control paradigm are twofold. First, a mathematical model of the system to be controlled is not required, and, second, a satisfactory nonlinear controller can often be developed empirically in practice without complicated mathematics.

Unlike the conventional mathematical model-based controller design methodology, fuzzy control does not need an explicit system model. Rather, a system model is implicitly built into fuzzy rules, fuzzy logic operation, and fuzzy sets in a vague manner in a sense that it combines the system modeling task and the system control task into one task [8].

2.3 APPLICABILITY OF FUZZY LOGIC CONTROLLERS

Literally a countless number of different types of systems exist in practice. Hence, as with any control technology, the applicability of fuzzy control must be well defined, which apparently relates to the strengths and limitations of fuzzy control. Fuzzy control is most desirable if (1) the mathematical model of the system to be controlled is unavailable but the system is known to be significantly nonlinear, time-varying, or have time delay, and/or (2) conventional control cannot generate satisfactory system performance [8].
2.4 INTERVAL TYPE-2 FUZZY LOGIC SYSTEMS

Fuzzy set theory has found wide applications in many fields as well as in control systems. Fuzzy logic control has emerged as a practical alternative to the conventional control techniques since it provides a decision making mechanism which allows the designer to put expert knowledge into the controller. However, the classical fuzzy logic systems (type-1 fuzzy logic systems) cannot fully handle the linguistic, measurement and parameter uncertainties. In order to reduce the effects of uncertainties, a new class of fuzzy logic systems—type-2 fuzzy logic systems—was recently introduced.

Type-2 Fuzzy Logic Systems (FLSs) are again expressed by IF-THEN rules but, their consequent and/or antecedent sets are type-2 fuzzy sets. These sets are fuzzy sets whose membership grades are not crisp values; instead, they are type-1 fuzzy sets. Type-2 fuzzy sets are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set; hence they are useful for dealing with uncertainties [9].

2.5 STRUCTURE OF AN INTERVAL TYPE-2 FUZZY LOGIC SYSTEM

The basic configuration of a fuzzy logic system consists of a fuzzifier, a fuzzy rule base, a fuzzy inference engine and a defuzzifier. The structure of a type-2 FLS is similar to type-1 counterpart, the major difference being that at least one the fuzzy set in the rule base is type-2. Figure (2.1) shows an example of general type-2 FLC.

![Fig 2.1: General scheme of an IT-2 FLC.](image)
2.5.1 INTERVAL TYPE-2 FUZZY LOGIC SETS

In a fuzzy logic system, membership functions have different shapes such as Trapezoidal, Triangular, Gaussian and etc. The parameters of membership functions can be designed by experts or tuned using optimization methods. T-2FSs were developed as a powerful alternative to model and reduce the effects of uncertainties. The main disadvantage of T-2FLSs is their computational burden. This is because of the type-reducer computations. Using IT-2FLSs reduces computations to a reasonable volume that makes the implementation of IT-2FLC easier.

A type-2 fuzzy set is characterized by membership functions that are themselves fuzzy. The key concept is footprint of uncertainly (FOU), which models the uncertainties in the shape and position of the type-1 fuzzy set. Figure (2.1) illustrates a type-2 fuzzy MF with FOU shown as shaded area. The output of inference engine for a type-2 FLS is a type-2 set, hence a type-reducer is needed to convert it into a type-1 set before defuzzification can be carried out. IT-2FSs are bounded from up and down with two T-1MFs that are called upper membership function (UMF) and lower membership function (LMF), respectively. The area between UMF and LMF is called footprint of uncertainty of the IT-2FS [9].

![Fig 2.2: Fuzzy logic sets; (a) Type-1 FLS, and (b) Interval Type-2 FLS.](image)
2.5.2 FUZZIFIER

In this part, we first define the fuzzy sets of all inputs of the system. Those memberships can contain one or several type-2 fuzzy sets. Second, the fuzzifier maps inputs into the associated fuzzy sets to determine the degree of membership of each input variable.

2.5.3 INference Engine

This block expresses the relationship that exists between the input variables (expressed as linguistic variables) and the output variables (also expressed as linguistic variables). As in type-1 fuzzy logic, in the design of a type-2 fuzzy logic we generally have IF-THEN rules. The formulation of rules is the same. The only distinction between type-1 and type-2 is associated with the nature of the membership functions. The inference engine combines rules and makes a combination between input type-2 fuzzy sets and output type-2 fuzzy sets. This is ensured by searching unions and intersections of type-2 sets, as well as compositions of type-2 relations [10].

2.5.4 TYPE REDUCER AND DEFUZZIFIER

In a type-1 fuzzy logic system, the output of the inference engine corresponding to each fired rule is a type-1 set. The defuzzifier combines those output sets to obtain a single output set. In a type-2 fuzzy logic system, since we deal with type-2 sets, then it is necessary to have a type reducer block to map a T2 FS into a T1 FS, and then defuzzification, as usual, maps that T1FS into a crisp number. The defuzzification block of a T1 fuzzy logic is replaced by the output processing block in a T2 fuzzy logic. That block consists of type-reducer followed by defuzzification [10].

2.6 OPERATION OF INTERVAL TYPE-2 FLC

The interval type-2 FLC works as follows: the crisp inputs from the input sensors are first fuzzified into input type-2 fuzzy sets. The input type-2 fuzzy sets then activate the inference engine and the rule base to produce output type-2 fuzzy sets. The type-2 FLC rules will remain the same as in a type-1 FLC but the antecedents and/or the consequents will be represented by interval type-2 fuzzy sets.
The inference engine combines the fired rules and gives a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. The type-2 fuzzy outputs of the inference engine are then processed by the type reducer which combines the output sets and performs a calculation which leads to type-1 fuzzy sets called the type-reduced sets. After the type-reduction process, the type-reduced sets are defuzzified to obtain crisp outputs that are sent to the actuators [7].

2.7 CONCLUSION

In this chapter, we have introduced briefly the Interval Type-2 Fuzzy Logic Systems, their advantages, applicability, and structure. An explanation of the roles of the main parts of an IT-2 FLC was performed including the major differences between type-1 and type-2 FLSs.

The benefits of using IT-2 fuzzy logic sets over their type-1 counterparts were mentioned along with the computational burden resulting from applying such FLSs. The Interval Type-2 Fuzzy Logic Control technique will be applied to enhance the performance of the Permanent Magnet Synchronous Machine in a chapter to follow.
CHAPTER THREE

MODELING OF PERMANENT MAGNET SYNCHRONOUS MACHINE FED BY A VOLTAGE SOURCE INVERTER
3.1 INTRODUCTION

Voltage Source Inverter fed AC machines have been increasingly used as variable speed operators in industry for their high performance and reliability. Amongst them, the Permanent Magnet Synchronous Machine has attracted more attention for its fast response to input voltage and load torque variations due to its low inductance. Another reason for favoring the PMSM is its high torque/weight ratio.

The PWM voltage source inverter with high sampling frequencies is used in controlling the PMSM to reduce the effects of torque and current fluctuations.

In this chapter, both theory and mathematical model of PMSM are presented along with a brief description of the concept of two-level VSIs. The behavior of the VSI-PMSM system is then simulated using MATLAB/SIMULINK environment.

3.2 THE PERMANET MAGNET SYNCHRONOUS MACHINE

A PMSM is an electrical machine in which the rotor excitation field is provided by permanent magnets instead of being induced by feeding current through windings. The stator is constructed in a similar way as in AC induction machines.

In later years the use of PMSM’s have become more prevalent due do their high efficiency and robustness. Since they do not need a system to provide a rotor current they require less maintenance than, e.g. doubly fed induction generators, DFIGs. Furthermore, PMSM’s can be used without a gearbox, resulting in lighter total hub weight and higher dependability.

Since the PMSM has a constant magnetic field, the frequency of induced voltage is equal to the electrical speed of the machine. In order to produce torque, the stator currents must also have the same frequency as the electrical speed of the machine.

There are three main ways of mounting the permanent magnets on the rotor as shown in figure 3.1. These three methods all influence machine behavior differently; a machine with surface mounted permanent magnets will have a comparatively simple construction and will in principle be non-salient, i.e. equal d and q inductance. However, the magnets will not be protected against mechanical stresses. One way to reduce these stresses is to use an interior mounting. This will protect the magnets but results in higher leakage flux. The middle way is to use insets to mount the magnets which will lower the leakage flux and increase the
maximum torque capabilities, but with more torque oscillation when compared to interior mounting. Another advantage of surface mounted magnets is that air gap reluctance will be uniform since the reluctance of the magnets is almost equal to that of air. This is not the case for most rotor materials, which means the magnetic coupling will be dependent on rotor position for inset and interior mounting, resulting in cogging torque if not accounted for [3].

![Fig 3.1: Different ways of mounting the permanent magnets on a PMSM.](image)

### 3.3 MODELING THE PMSM

In order to achieve an efficient control of the PMSM in both transient and steady states, it is necessary to formulate its behavior in a mathematical model. To do so, four main simplifying assumptions have to be made:

- The magnetic circuit saturation, machine hysteresis, and Foucault currents are all negligible.

- The windings’ resistances are constant with temperature variations and the skin effect is neglected.

- The stator e.m.fs have sinusoidal forms [2].
3.3.1 FORMULATING THE PMSM BEHAVIOR

The windings of three-phased synchronous machine are represented in the next figure:

Fig 3.2: PMSM quantities in a, b, c and d-q reference frames.

Where: the stator voltages are expressed as:

$$\begin{bmatrix} v_s \end{bmatrix} = [R_s] \cdot [i_s] + \frac{d}{dt} [\phi_s] \quad (3.1)$$

The stator fluxes as:

$$\begin{bmatrix} \phi_s \end{bmatrix} = [L_{ss}] \cdot [i_s] + [\phi_{sf}] \quad (3.2)$$

Where

$$\begin{bmatrix} v_s \end{bmatrix} = [v_a \ v_b \ v_c]^T \quad \begin{bmatrix} i_s \end{bmatrix} = [i_a \ i_b \ i_c]^T \quad \begin{bmatrix} \phi_s \end{bmatrix} = [\phi_a \ \phi_b \ \phi_c]^T$$

Represent the vectors of: stator voltages, stator currents, and stator fluxes respectively.

$$[R_s] = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix}$$

The stator resistance matrix,

$$[L_{ss}] = \begin{bmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{bmatrix}$$
The stator inductance matrix, and

\[
\begin{bmatrix}
\phi_{sf} \\
\phi_{af} \\
\phi_{bf} \\
\phi_{cf}
\end{bmatrix} = \begin{bmatrix}
\phi_{af} \\
\phi_{bf} \\
\phi_{cf}
\end{bmatrix}^T
\]

the vector of flux created by the permanent magnets through the stator windings.

Solving equations (3.1) and (3.2) analytically can be relatively tiresome regarding the large number of variable coefficients. Therefore, mathematical transformations need to be used to describe the machine behavior using constant coefficients differential equations. One of these-named Park transformation- is the one we will use to model the PMSM.

### 3.3.2 PARK TRANSFORMATION

In this section, the PMSM is modeled based on its representation in a two-phase coordinate system related to the rotor-called d-q reference system. This is done using Park transformation. Obtaining this model permits us to visualize the effects of circular magnetic fields (represented by circular vectors) on creating torque in the rotor. One of the main advantages of this transformation is the fact that in the new reference system, the electromagnetic torque is directly related to the quadratic component of stator current \( I_q \).

Figure (3.3) below illustrates Park model of the PMSM that can be seen as a synchronous machine with coiled rotor where the rotor is fed by an excitation circuit consisting of an excitation winding \( L_f \) which also represents the flux of permanent magnets.

![Fig 3.3: PMSM quantities in d-q reference system.](image-url)
We consider X as the vector representing electrical quantities. With \( X_0 \) as the homopolar part (i.e., having symmetrically distributed polarity), we define the following matrices in d-q and a, b, c reference systems:

\[
[X_{dq0}] = [X_d X_q X_0]^T, \quad [X_{abc}] = [X_a X_b X_c]^T.
\]

Let \([K_0]\) be the matrix of passing from the direct and power conserving 3/2 transformation \([X_{dq0}] = [P][X_{abc}]\) and its inverse transformation \([X_{abc}] = [P]^{-1}[X_{dq0}]\)

Where

\[
[P] = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{3.3}
\]

And

\[
[P]^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \frac{1}{\sqrt{2}} \\ \cos\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta - \frac{2\pi}{3}\right) & \frac{1}{\sqrt{2}} \\ \cos\left(\theta - \frac{4\pi}{3}\right) & -\sin\left(\theta - \frac{4\pi}{3}\right) & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{3.4}
\]

Where the electrical angle \( \theta = p\theta_m \) with \( \theta_m \) representing the mechanical position of the rotor and \( p \) is the number of pole pairs.

It can be shown that in d-q reference system with the d-axis aligned on the rotor flux, we obtain a simplified system of equation describing the synchronous machine where the voltages are expressed as:

\[
\begin{aligned}
V_d &= R_s I_d + \frac{d\varphi_d}{dt} - \omega \varphi_q \\
V_q &= R_s I_q + \frac{d\varphi_q}{dt} + \omega \varphi_d
\end{aligned} \tag{3.5}
\]

And the fluxes as:

\[
\begin{aligned}
\varphi_d &= L_d I_d + \varphi_{sf} \\
\varphi_q &= L_q I_q
\end{aligned} \tag{3.6}
\]
Where:

\( V_d, V_q \) represent the voltages, and \( \phi_d, \phi_q \) the fluxes of machine.

\( L_d, L_q \) the longitudinal and transversal synchronous inductances respectively.

\( \phi_{sf} \) is the flux created by rotor magnets in the air gap.

From equations (3.5) and (3.6), we obtain

\[
\begin{align*}
V_d &= R_s I_d + L_d \frac{dI_d}{dt} - \omega L_q I_q \\
V_q &= R_s I_q + L_q \frac{dI_q}{dt} + \omega L_d I_d + \omega \phi_{sf}
\end{align*}
\]  

(3.7)

The synchronous actuator with permanent magnets provides in the general case an electromagnetic torque given as:

\[
C_{em} = p \left[ \left( \phi_{sf} I_q + (L_d - L_q) I_d I_q \right) \right]
\]  

(3.8)

Where the term \( p(L_d - L_q) I_d I_q \) represents the reluctance torque due to machine anisotropy (direct dependence). The term \( p\phi_{sf} I_q \) represents the synchronous torque due to the flux created by the permanent magnets. The electro-mechanical equation of machine is then:

\[
C_{em} - C_r = J \frac{d\Omega}{dt} + f \Omega
\]  

(3.9)

Where: \( J \) is the moment of inertia of the armature, \( f \) the viscous friction coefficient, \( C_r \) is resistant torque, and \( \Omega \) is mechanical speed.

### 3.3.3 STATE REPRESENTATION

When combining equations (3.6) and (3.7), we obtain the state space representation for the PMSM as:

\[
\frac{d}{dt} \begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} R_s & \omega \cdot L_q \\ L_d & L_d & L_d & R_s \\ L_q & -L_q & L_q \end{bmatrix} \begin{bmatrix} I_d \\ I_q \end{bmatrix} + \begin{bmatrix} \frac{V_d}{L_d} \\ \frac{V_q - \omega \cdot \phi_{sf}}{L_q} \end{bmatrix}
\]  

(3.10)

In figure (3.4) below, the functional block diagram for Park model of the permanent magnet synchronous machine is illustrated:
3.4 MODELING THE VOLTAGE SOURCE INVERTER

Voltage source inverters are now widely used in controlling AC machines since they provide AC voltages with variable frequencies and amplitudes required for machine operation. They however, have the disadvantage of introducing higher order harmonics that affect the machine dynamic behavior by causing abrupt changes in torque and current.

Pulse width modulation techniques are used to reduce the effects of converters. The VSI converts a set of DC voltages with different values into an AC voltage with varying amplitude and frequency. A two-level VSI consists of three arms, each arm with two electronic switches and freewheeling diodes (whose type and number are chosen based on the design specifications). When the VSI is connected to the PMSM, the system illustrated in next diagram (figure 3.5) is obtained.

Where Ti and Ti’ (i = a, b, c) are transistors (assumed to be ideal), we set logic controls Si as

If Si = 1, then switch Ti is closed and Ti’ is open.
If Si = 0, then switch Ti is open and Ti’ is closed [1].
3.4.1 PULSE WIDTH MODULATION (PWM)

The output of the VSI serves to obtain the desired currents and voltages at the machine input terminals. The amplitude and frequency control is done by adjusting the durations of switch closing and opening over the signal period. The concept of PWM is to convert a modulated signal (reference voltage) into a variable-width-square signal (at the output of VSI) by comparing it to a higher frequency triangular signal (the carrier). The resulting signal describes the commutation of the VSI switches as shown next (figure 3.6).

![Diagram of Pulse Width Modulation (PWM)](image)

**Fig 3.6:** Generating a pulse width modulated signal.

The source to the VSI is assumed to be ideal and composed of two e.m.f generators each of E/2 connected through a common point [4].

3.4.2 FORMULATING THE VSI

At the VSI terminals, the voltages are given as:

\[
\begin{align*}
V_{ab} &= V_{ao} - V_{bo} \\
V_{bc} &= V_{bo} - V_{co} \\
V_{ca} &= V_{co} - V_{ao}
\end{align*}
\]  

(3.11)
The voltages $V_{an}$, $V_{bn}$, $V_{cn}$ form a balanced three phase system, so

$$V_{aN} + V_{bN} + V_{cN} = 0$$

Then

$$V_{aN} = V_{ao} + V_{oN}$$
$$V_{bN} = V_{bo} + V_{oN} \quad \Rightarrow \quad (3.12)$$
$$V_{cN} = V_{co} + V_{oN}$$

This leads to:

$$V_{aN} = \frac{1}{3}(V_{ao} + V_{bo} + V_{co}) \quad (3.13)$$

From (3.12) and (3.13), we obtain:

$$\begin{bmatrix}
V_{ao} \\
V_{bo} \\
V_{co}
\end{bmatrix} = \frac{1}{3}
\begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
V_{aN} \\
V_{bN} \\
V_{cN}
\end{bmatrix} \quad (3.14)$$

In each of the VSI arms, the static switch ($k = a, b, c$) can take on values +1 or -1 based on the following:

- If $(V_{ar} \geq V_t)$ then $S_a = +1$, otherwise $S_a = -1$
- If $(V_{br} \geq V_t)$ then $S_b = +1$, otherwise $S_b = -1$
- If $(V_{cr} \geq V_t)$ then $S_c = +1$, otherwise $S_c = -1$

Where $V_t$ stands for the reference voltage amplitude and $V_t$ for the triangular carrier signal. The arm $V_{ko}$ ($k = a, b, c$) can be expressed as a function of the switches $S_k$ as:

$$V_{ko} = S_k \cdot \left(\frac{E}{2}\right) \quad (3.15)$$

Also, it can be represented by a connection matrix as:

$$\begin{bmatrix}
V_{aN} \\
V_{bN} \\
V_{cN}
\end{bmatrix} = \frac{E}{6}
\begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
S_a \\
S_b \\
S_c
\end{bmatrix} \quad (3.16)$$
3.5 SIMULATION RESULTS

We have simulated the PMSM model with its input voltages provided by the VSI inverter in MATLAB/SIMULINK environment to examine the system behavior. The simulations were done for a starting without load and reference speed of 100 rad/s then a load torque of 5 Nm is applied at t = 0.2 s.

![Fig 3.7: The PMSM current components; (a) longitudinal $I_d$, (b) quadratic $I_q$.](image)

![Fig 3.8: The PMSM characteristics; (a) speed, and (b) torque.](image)

3.6 DISCUSSION

When operated (in simulation), the PMSM fed with a VSI exhibits good tracking for reference speed with a considerable overshooting and the response time to load torque is small as well as for the current $I_q$ when a load perturbation is applied. The longitudinal current
component $I_d$ however remains very large due to the nonlinear nature of the system being modeled.

3.7 CONCLUSION

In this chapter, we have introduced the Permanent Magnet Synchronous Machine; its advantages, characteristics, and model, based on the adopted simplifying assumptions. We have also used Park transformation to obtain a simplified and linearizable model of our system.

The two-level Voltage Source Inverter has been introduced with its advantages, operation concept, along with the Pulse Width Modulation technique.

In next chapter, nonlinear feedback linearization will be applied to control and improve the PMSM behavior and to obtain high performance operation.
CHAPTER FOUR

NONLINEAR INPUT-OUTPUT FEEDBACK CONTROL OF PERMANENT MAGNET SYNCHRONOUS MACHINE
4.1 INTRODUCTION

The huge developments in differential geometry in 1980s as well as Isidori’s researches have made a very powerful technique in nonlinear control ie.Input-Output feedback linearization. When applied, this method has proved its effectiveness to a wide range of nonlinear systems. This is especially true when appropriate choice of nonlinear state-feedback control law is performed. It allows the transformation (either complete or partial) into input – output or input-state linearized systems.

Amongst other control methods, input-output feedback linearization has been developed to deal with problems encountered with classic linear controllers. It provides well treatment of parameter uncertainties and relatively low robustness characteristics in nonlinear systems. In addition to reducing the effects of torque and flux fluctuations, input-output feedback linearization technique makes the dynamic system behavior less sensitive to perturbations in load torque and source currents. A brief description of its concept along with some tools in differential geometry is firstly introduced in this chapter.

This method is then applied to control the PMSM for both speed and current and a Similink model is constructed and simulated to prove the usefulness of the technique used.

4.2 VOLTAGE-CONTROLLED PMSM I-O FEEDBACK LINEARIZATION

In order to perform voltage control of permanent magnet synchronous machine, the state vectors are adopted so as to construct a complete model for the machine in the rotor based reference system d-q:

\[
\begin{align*}
\mathbf{x} &= [x_1 \ x_2 \ x_3]^T = [I \ I_q \ \Omega]^T \\
\mathbf{u} &= [u_1 \ u_2]^T = [u \ u_d]^T
\end{align*}
\]

(4.1)

The general equation governing nonlinear multi-input multi-output system models is:

\[
\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x}) \mathbf{u}
\]

(4.2)

Where:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{\Omega}
\end{bmatrix} = \begin{bmatrix}
\frac{dI_d}{dt} \\
\frac{dI_q}{dt} \\
\frac{d\Omega}{dt}
\end{bmatrix} = \begin{bmatrix}
f_1(x) \\
f_2(x) \\
f_3(x)
\end{bmatrix} + \begin{bmatrix}
g_{11} & 0 \\
0 & g_{22}
\end{bmatrix} \begin{bmatrix}
u_d \\
u_q
\end{bmatrix}
\]

(4.3)
f(x) is a smooth vector field of order n = 3 as:

\[
f(x) = \begin{bmatrix} h_1(x) \\ h_2(x) \\ h_3(x) \end{bmatrix} = \begin{bmatrix} \frac{R}{L_d} x_1 + p \frac{L_q}{L_d} x_2 x_3 \\ -p \frac{L_d}{L_q} x_1 x_3 - \frac{R}{L_q} x_2 - p \frac{\Phi_f}{L_d} x_3 \\ -p \frac{(L_d - L_q)}{J} x_1 x_2 + p \frac{\Phi_f}{J} x_2 - f x_3 - e_j \end{bmatrix}
\] (4.4)

g is a constant (3x2) matrix given by:

\[
g = \begin{bmatrix} g_1 \\ 0 \\ g_2 \\ 0 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 0 \\ \frac{1}{L_d} \\ 0 \\ \frac{1}{L_q} \\ 0 \end{bmatrix}^T
\] (4.5)

### 4.3 OUTPUT QUANTITIES

Our objective is to control the machine speed in tracking a reference value, maintain the longitudinal component of stator current Is,Id at zero, while obtaining maximum torque at the machine output. Hence the output vector is chosen as:

\[y = h(x) = [x_1 \ x_2]^T = [I \ \Omega]^T \] (4.6)

### 4.4 RELATIVE DEGREE COMPUTATION

Before input–output feedback linearization approach is applied to the system, we need first to verify whether the system is input-output linearizable by computing its total relative degree. The relative degree for a MIMO system, is computed for each output \(y_i\), as the number of times we should derive \(y_i\) for at least one control law \(u_j\) to appear.

#### 4.4.1 RELATIVE DEGREE FOR CURRENT \(I_d\)

We take first derivative of \(y_1\) as:

\[
\dot{y}_1(x) = f_1(x) = L_f h_1(x) + L_g h_1(x) u
\] (4.7)

With,

\[L_f h_1(x) = f_1(x)\]

\[L_g h_1(x) = [g_1 \ 0]\]

Hence,
\[ \dot{y}_1 = -\frac{R}{L_{dq}} I_d + p \frac{L_q}{L_{dq}} \Omega I_{ql} + \frac{1}{L_{dq}} u_d \]  

(4.8)

Input \( u_1 = u_d \) has appeared, then the relative degree of \( y_1 \) is \( r_1 = 1 \).

### 4.4.2 RELATIVE DEGREE FOR MECHANICAL SPEED \( \Omega \)

We take first derivative of output \( y_2 \) as:

\[ \dot{y}_2(x) = h'_2(x) = L_f h_2(x) \]

\[ = f_3(x) \]  

(4.9)

None of the input variables has appeared in the expression of \( dy_2 \), hence we take the second derivative as:

\[ \ddot{y}_2(x) = h''_2(x) = L_f^2 h_2(x) + L_g L_f h_2(x) u_d \]

\[ = f_1(x) p \left( \frac{1}{L_q} - \frac{1}{L_{dq}} \right) x_2 + f_2(x) \left( \frac{p}{L_q} \cdot x_1 + p \frac{\Phi_f}{L_{dq}} \right) - f_3(x) \frac{f}{j} \]

\[ + g_3 \left( \frac{1}{L_q} - \frac{1}{L_{dq}} \right) x_2 u_d + g_2 \left( \frac{p}{L_q} \cdot x_1 + p \frac{\Phi_f}{L_{dq}} \right) u_q \]  

(4.10)

Both \( u_d \) and \( u_q \) appeared, hence the relative degree of \( y_2 \) is \( r_2 = 2 \). The system total relative degree is then: \( r = r_1 + r_2 = 3 \)

Therefore, the system is exactly linearizable \((r = n = 3) \) [5].

### 4.5 LINEARIZING THE SYSTEM

In applying input–output linearization to the system dynamics, only the output derivatives are considered along with the feedback control law as:

\[ \begin{bmatrix} \dot{f}_1(x) \\ \dot{f}_2(x) \end{bmatrix} = A(x) + E(x) \begin{bmatrix} u_d \\ u_q \end{bmatrix} \]

\[ A(x) = \begin{bmatrix} f_1(x) p \left( \frac{1}{L_q} - \frac{1}{L_{dq}} \right) x_2 + f_2(x) \left( \frac{p}{L_q} \cdot x_1 + p \frac{\Phi_f}{L_{dq}} \right) - f_3(x) \frac{f}{j} \\ g_1 \frac{1}{L_q} - \frac{1}{L_{dq}} x_2 + g_2 \left( \frac{p}{L_q} \cdot x_1 + p \frac{\Phi_f}{L_{dq}} \right) \end{bmatrix} \]

\[ E(x) = \begin{bmatrix} g_1 \frac{1}{L_q} - \frac{1}{L_{dq}} x_2 \\ g_2 \left( \frac{p}{L_q} \cdot x_1 + p \frac{\Phi_f}{L_{dq}} \right) \end{bmatrix} \]  

(4.11)

E(x) is the decoupling matrix with \( \det(E(x)) \neq 0 \) for the permanent magnet SM. Hence E(x) is invertible.
Then the state feedback control law is given by:

$$[u_d] = E^{-1}(x) \left( -A(x) + [v_1]ight)$$  \hspace{1cm} (4.12)

Where \( v = [v_1 \ v_2] \) is the new input vector. By applying the control law obtained in (4.12) on the nonlinear system in (4.11), we end up with two linear and decoupled single-variable subsystems as [5]:

$$\dot{h}_1(x) = v_1$$  \hspace{1cm} (4.13)
$$\dot{h}_2(x) = v_2$$

### 4.6 SPEED AND CURRENT CONTROL

#### 4.6.1 INTERNAL CONTROL LAW

In order to obtain satisfying control of speed and current with respect to their reference values \( \Omega_{\text{ref}} \) and \( I_{\text{ref}} \) respectively, we use the following system of equations to get the internal inputs \( v_1 \) and \( v_2 \) as [4]:

$$v_1 = k_{ld}(i_{d\text{ref}} - i_t) + \frac{d}{dt} i_{d\text{ref}}$$
$$v_2 = k_{\Omega 2} (\Omega_{\text{ref}} - \Omega) + k_{\Omega 1} \left( \frac{d}{dt} \Omega_{\text{ref}} - \frac{d}{dt} \Omega \right) + \frac{d^2}{dt^2} \Omega_{\text{ref}}$$  \hspace{1cm} (4.14)

These equations result in the error for our model as:

$$\frac{d}{dt} e_1 + k_{ld} e_1 = 0$$
$$k_{\Omega 2} e_2 + k_{\Omega 1} \frac{d}{dt} e_2 + \frac{d^2}{dt^2} e_2 = 0$$  \hspace{1cm} (4.15)

Where \( e_1 \) and \( e_2 \) represent the tracking errors for speed and current as:

$$e_1 = I_{\text{ref}} - I$$
$$e_2 = \Omega_{\text{ref}} - \Omega$$  \hspace{1cm} (4.16)

The coefficients \( K_{ld}, \ k_{\Omega 1}, \) and \( k_{\Omega 2} \) are chosen such that \( K_{ld} + s \) and \( K_{\Omega 2} + K_{\Omega 1} s + s^2 \) are Hurwitz polynomials (having roots with negative real parts) and the coefficient are calculated for pole placement.
4.6.2 PHYSICAL CONTROL LAW

Based on equation (4.12), the state feedback nonlinear control law is given by:

\[
\begin{bmatrix}
    u_d \\
    u_q
\end{bmatrix} = E^{-1}(\chi) \left( -A(\chi) + \left[ k_{id}(i_{dref} - i_d) \right] \right)
\]

\[
\left( \frac{d}{dt} I_{ref} = \frac{d}{dt} \Omega_{ref} = \frac{d^2}{dt^2} \Omega_{ref} = 0 \right)
\]

The next figure (fig 4.1) summarizes the whole procedure of input–output feedback linearization approach applied on the PMSM:

**Figure 4.1:** input-output feedback linearization of PMSM [5].
Chapter Four  
Nonlinear input-output feedback control of PMSM

4.7 SYSTEM SIMULATION

We have simulated the PMSM model fed by VS inverter in MATLAB/ SIMULINK environment to examine the system behavior.

The simulations were done for a starting without load and reference speed of 100 rad/s then a load torque of 5 Nm is applied at $t = 0.2$ s.

**Fig 4.2:** The PMSM current components; (a) longitudinal $I_d$, (b) quadratic $I_q$.

**Fig 4.3:** The PMSM characteristics; (a) speed, and (b) torque.
4.8 DISCUSSION

The obtained simulation results were satisfactory for the different characteristics of PM machine and the input-output feedback linearization technique efficiency was proved.

For speed control, the reference speed tracking was exact and without overshooting. At the load perturbation (t = 0.2 s), the machine response was almost instantaneous and only a small deviation of 2.5% has occurred.

The current longitudinal component $I_d$ was null as desired while both the quadratic component $I_q$ and the output torque showed objective fulfillment behavior.

4.9 CONCLUSION

In this chapter, we have applied the input-output feedback linearization approach to control the nonlinear MIMO system model of the permanent magnet synchronous machine. The system was exactly linearized and the nonlinear control law was calculated to perform state feedback.

The whole system consisting of nonlinear controller and PM machine was then simulated in MATLAB/ SIMULINK and the results were illustrated and discussed. The results of the I-O feedback nonlinear controller were almost satisfactory except some perturbations that will be treated and compensated for using the interval type-2 fuzzy controller in next chapter.
CHAPTER FIVE

INTERVAL TYPE-2 FUZZY LOGIC CONTROL OF
PERMANENT MAGNET SYNCHRONOUS MACHINE
5.1 INTRODUCTION

In this chapter, the Interval Type-2 Fuzzy Logic Controller will be applied on the feedback linearized model of the Permanent Magnet Synchronous Machine in the aim of handling the system uncertainties and reducing their effects on performance. The Interval Type-2 Fuzzy Logic sets, characterized by their Footprint Of Uncertainty, will be used to fuzzify the crisp inputs.

The considered variables are the machine speed and the longitudinal current $I_d$ errors and their time derivatives. Each of the two will be subjected to an IT-2 FLC. The system is simulated in the MATLAB Interval Type-2 Toolbox and the results are depicted and discussed.

5.2 COMPONENTS OF THE INTERVAL TYPE-2 FLC

The fuzzy logic system building blocks including the fuzzifier, fuzzy rule base, fuzzy inference engine, and defuzzifier will be designed in this section.

**Fig 5.1:** General scheme of an IT-2 FLC.
5.2.1 INTERVAL TYPE-2 FUZZY LOGIC SETS

In our IT-2 FL controller, we have adopted fuzzy sets of triangular shape. This type of fuzzy sets is characterized by its simplicity and ease of calculation when applying defuzzification and type reduction procedures. For each input variable, we have assigned seven fuzzy sets on the universe of discourse ranging from -1 to +1.

These sets are:
- NL: for a large negative error.
- NM: for a medium negative error.
- NS: for a small negative error.
- Z: for zero error.
- PS: for a small positive error.
- PM: for a medium positive error.
- PL: for a large positive error.

5.2.2 FUZZIFIER

In previous section, we have defined the fuzzy sets for the system inputs. Those memberships contain seven interval type-2 fuzzy sets. Second, the fuzzifier maps the crisp inputs into the associated fuzzy sets to determine the degree of membership of each input variable. The next figure (5.1) illustrates the shapes and values for the fuzzy sets.

![Fig 5.2: Associated Interval Type-2 Fuzzy sets.](image-url)
5.2.3 INFERENCE ENGINE

This block expresses the relationship that exists between the input variables and the output variables. In the design of interval type-2 fuzzy logic controllers, IF-THEN rules are used to determine this relationship. The inference engine combines rules and makes a combination between input interval type-2 fuzzy sets and output interval type-2 fuzzy sets. This is ensured by searching unions and intersections of type-2 sets, as well as compositions of type-2 relations. The following table summarizes the rule set used in our IT-2 FL controller.

**Table 5.1:** rule firing determination for input variables e(t) and de(t).

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5.2.4 TYPE REDUCER AND DEFUZZIFIER

In this interval type-2 fuzzy logic system, we deal with interval type-2 sets, then it is necessary to have a type reducer block to map a T-2 FS into a T-1 FS, and then defuzzifier, as in a T-1 FL system, maps that T-1FS into a crisp number i.e. defuzzification block of a T-1 fuzzy logic is replaced by this output processing block in an IT-2 fuzzy logic system.

The type reduction method used in this controller is the Enhanced Karnik- Mendel algorithm EKM since it has the benefits of the original KM approach but with reduced computation time.

For the two outputs, the same collection of fuzzy sets was used as in inputs. The output sets, however, are IT-2 singleton fuzzy sets with their values being taken as intervals; \([\underline{u}; \bar{u}]\).
5.3 SIMULATION RESULTS

The whole system, after applying the IT-2 FL controller on the already decoupled and linearized model of the PMSM, was simulated in Matlab Interval Type-2 Toolbox to visualize its behavior and observe the effect and performance of the IT-2 FL controller. A general scheme of the system is shown in figure (5.3) below.

Fig 5.3: the combined control system- feedback linearization, interval type-2 fuzzy logic.

When the system was simulated, the objectives were achieved only with the PMSM torque and quadratic current component $I_q$ whose responses were almost perfect except some oscillations at the system start. The next figure (5.4) depicts the graphs of these two signals.

The other two outputs, the machine speed and longitudinal current $I_d$, were greatly distorted and indicated a response largely diverging from the desired values. However, this does not doubt the efficiency and applicability Interval type-2 fuzzy logic controller as it emphasizes on the necessity of a powerful trial and error examination of the system behavior.

Trial and error effort with system simulation and manipulating the gains, fuzzy sets and membership functions will overcome the lack of system expert and the effect of large number of system parameters.
5.3 CONCLUSION

In this chapter, we have applied the Interval Type-2 Fuzzy Logic Controller to the feedback linearized model of the PMSM. This has the objective of handling the system uncertainties and reducing their effects on performance.

The behavior of the system controlled with this IT-2 FL technique was simulated and the obtained results were depicted and discussed.

Fig 5.4: the IT-2 FL controlled machine response; (a) current $I_q$, (b) torque.
GENERAL CONCLUSION

In this project, the main parts of work were focused on studying two effective and applicable nonlinear control approaches; the Input-Output Feedback Linearization technique and the Interval Type-2 Fuzzy Logic method.

At first step, a general view on nonlinear feedback control was given including some important mathematical operations from differential geometry. This is in addition to a brief explanation of IOFL technique and a detailed procedure for its application on a MIMO nonlinear system.

This was followed by introducing Fuzzy Logic Control method while focusing on IT-2 FLCs and presenting its main building blocks. The major differences between type-1 and type-2 fuzzy sets and the latter strengths were pointed to.

On the nonlinear model of the Permanent Magnet Synchronous Machine, firstly feedback linearization was applied to obtain decoupled linear model dynamics, the uncertainties were then treated using the IT-2 FLC to handle their effects.

The performed simulations have illustrated the achievement of control objectives and asserted the applicability of the used techniques on physical systems with multi-variable nonlinear models.

In controlling a nonlinear system, not only on linearizing the model dynamics will be our focus, the effects of system’s different types of uncertainties and load perturbations must be taken into account and treated. The enhancement can be done by modifying the original controller or combining two different approaches in order to meet the desired performances. This was performed in our project by subjecting the PMSM model controlled by input-output feedback linearization to an interval type-2 fuzzy controller.
APPENDIX

PARAMETERS:

The Permanent Magnet Synchronous Machine parameters and used in simulation are:

- \( P_n = 3\) KW.
- \( V = 220\) V.
- \( \Omega = 230\) rad/s.
- \( R_S = 1.4\) \(\Omega\).
- \( L_D = 0.0066\) H.
- \( L_Q = 0.0058\) H.
- \( P_H = 0.1546\) H
- \( p = 3\).
- \( \Phi_{sf} = 0.1546\) Wb.
- \( J = 0.000176\) Kg.m\(^2\).
- \( F_R = 0.00038\) Nm/rad/s.
REFERENCES


