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Markovian Approach of Safety Engineering of a Machines System

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Abstract

To manage exploitation of a system of machine intended to complete a given work is to guarantee the possibility of having this system in operating condition for one determined duration before preventive repair with a given reliability. It is also to have with the availability of the repair service the quantity of parts necessary for this repair in order to the minimum reduce the time of the forced stopping of the engines. The number of machines (elements) depends especially of the repair quality, the strategy of maintenance and of the repair shop importance.

The purpose of the study consists, by using the theory of renewal, the graphs of Markov, and that of the queues while passing from a state to another, to find the dependence between the number of machines to be put in redundancy and the importance of the workshop (stands of repair) and thus to have the sufficient number in order to have an availability guaranteeing the correct operation of the company.

Keywords: Reliability, Renewal, Queuing theory, Repair, Materials

Introduction

Let's consider a system made up of N machines in simultaneous exploitation. During their exploitation these elements can have failures that require repairs in a workshop. To have a continuity of the exploitation (production) during the immobilization of the failing elements we need other elements to replace them. The system thus requires a certain number of elements that must be in reserve (redundancy) in order to ensure exploitation continues and of which it is necessary to determine the number.

The maintenance of a material under satisfactory operating conditions thus supposes that very component which is not in conformity any more with its specifications, either in consequence of an accident, or because of wear pulled by operation, or because the age modified its characteristics, must be replaced by a new element, or repaired in order to find the entirety of the initial characteristics. The identity of the spare part compared to the initial component is not limited to its geometry, but includes all the characteristics: geometrical, matter used, heat treatments, surface quality identical, etc.

The company user must thus have a sufficient number of components in stock in conformity with the specifications of the manufacturer enabling him to carry out their replacement as soon as possible.

In this list of the components likely to undergo failures, one will be able to then select the list of the components which it is necessary to have in stock to be able to ensure, with a satisfactory probability, permanence of the "necessary function" and for each one of them the necessary number. For that, one will base oneself in particular on the laws of appearance of failures, but also on the experiment of the users and the persons in charge for maintenance. C E stock Re in general presents a significant investment, for which it is necessary to proceed to the time of the manufacture of the good or the construction of the unit of exploitation (or of production), if one wants to avoid significant overcosts or excessive times (due for example to the cost or the times of the handing-over in manufacture). This stock must be managed all with long lifetime of the good, by respecting an always difficult balance between the financial constraint, which would like to minimize the value of this stock and the technical constraint, which seeks the highest possible availability, which results on the contrary in raising this stock.

Mechanical, electric or electronic components of a material it's subjected during its operation to constraints of various types:

- Mechanical constraints of a component subjected to a whole of forces of traction, inflection, torsion…;
- Electric constraints which apply to an insulator in an engine or an alternator;
- Thermal stresses in a material where reign of the very high temperatures… Or very low;

Chemical or electrochemical constraints in a material working in a corrosive condition (sea air) or treating corrosive substances (pumps in the chemical industry).

If the apparatus is well designed, the component will have been defined to resist these various constraints which it will undergo under normal operation, and will be able to thus live for a rather long length of time, but ascribable wear with frictions and ageing due to the various constraints evoked above will finish at the end of a certain time by making the component inapt to fulfill its necessary function.

It can happen that one or more from these constraints deviate suddenly and in a random way in time of their face value while leaving the definite limits. This brutal increase in a constraint, or combination of several constraints, will be able, according to the variation with their face value, to involve the deterioration or the destruction of the element considered. We see that there are thus two modes of destruction of a component:

- Destruction by wear
- Destruction by random overload. " [1]

Let's consider a maintenance workshop made up of *n* repair channels: During the exploitation we can find among these stands 0, 1, 2, 3...... *m*....., *n* occupied channels. In the article presented we consider the determination of the number of repair channels and numbers of elements to be put in redundancy in the following cases:

1. Repair without queue (availability of the service maintenance)
2. Machine which arrives for a repair can, by finding all the channel occupied, to behave in 2 different ways:

- That is to say to remain in the file a determined time and then to leave it without being repaired. (System with abundant).
- Maybe that all the elements defective on standby will be, early or late, repaired; i.e. that they will not leave the queue.

The number of the equipment of reserve depends:

- Material and human possibilities of the repair shop;
- Organization of the service of maintenance.

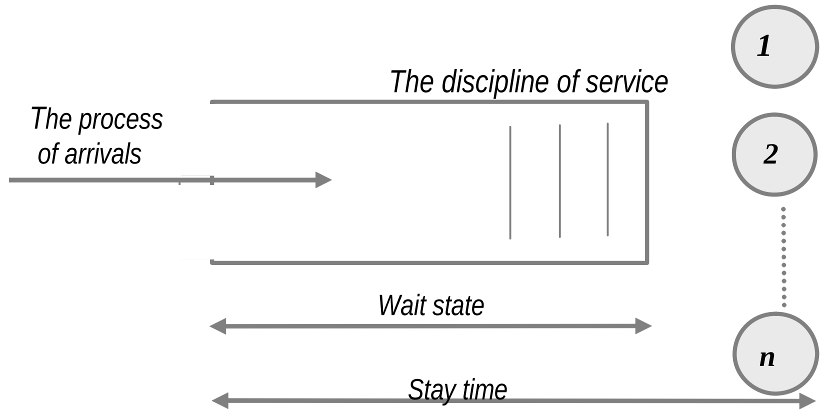
1. Théorie of the Queues

To describe a system of repair, it is necessary to be able to specify:

- The mechanism of arrival of the elements to repair in workshop (i.e., in practice, to which law the process of arrival will obey).

- The time of repair (probability distribution of its duration).
- The discipline of service (when an stand is released, which element chooses it)

The diagram of a repair shop can be represented as follows:



1.1. Process of Poisson

Let us suppose that the process of the arrivals obeys the following rules:

- The probability of an arrival in an interval *[t, t + Δt]* does not depend on what occurred before the moment *t*. It is the property known as " without memory ".
- The probability of appearance of a request is proportional to Δt, the probability of more than one event being "negligible" (infinitely small of a higher nature). The proportionality factor is noted λ (intensity of the process).

The probability of observing K arrived in an interval length t is worth:

$$Pk(t) = \frac{(\lambda t)^k}{k!} . e^{-\lambda t} \quad (1)$$

1.2. The Services Process

The service process could be of an extreme complexity, but we generally restricts oneself to suppose that each working life is independent of the different one, and that they all obey the same law of distribution: we speaks about independent and identically distributed variables (i.i.d.). We will describe this law by his probability distribution:

$$F(T) = P\ \{time\ of\ repair \leq T\}$$

1. 3. The Exponential Law

The most popular law of the service is the exponential law, which it is traditional to write by using like "service rate" the letter: μ

$$F(t) = P(time\ of\ \ repair \leq t) \ = \ 1 - e^{-\mu t} \quad (2)$$

Has corresponding density is:

$$f(t) = \mu e^{-\mu t} \quad (3)$$

The exponential law owes a good part of its prestige with the property "without memory", already met: to know that the service lasted already some duration not gives any precision on its nearest end. It will be noticed indeed that, for the exponential law, the density is worth:

$$f(t / T) = \frac{f(t+T)}{R(T)} = \frac{\mu e^{-\mu(t+T)}}{e^{-\mu T}} = \mu e^{-\mu t} = f(t) \quad (4)$$

, independently of *T*. The application of this "mental blank" shows that the probability of an end of service in the moment which comes (in the interval *[t, t+ dt]*) is μdt, whatever the age of the service.

1.4. Laws of Erlang

Let us suppose that the system of repair is composed of a whole of K exponential and identical elementary channel (i.e., of the same parameter μ), and independent from each other. The time of the service is the sum of the times spent in each channel.

Let us suppose $K = 2$. Let us note T total time, t_1 and t_2 the two times durations services; the F(t) distribution is given by *the convolution* of F_1 and F_2:

$$P\{T \leq t\} = P\{t_1 + t_2 \leq t\} = \int_{u=0}^{t} F_1(t-u)\,dF_2(u) \tag{5}$$

Obviously, F_1 and F_2 are identical, and correspond to the exponential one.

$$P\{T \leq t\} = \int_{u=0}^{t} \left[1 - e^{-\mu(t-u)}\right] \mu e^{-\mu t} du = 1 - e^{-\mu t} - \mu t e^{-\mu t} \tag{6}$$

We can shows that the startup of K channels of exponential repair with the same parameter μ led to a distribution:

$$F(t) = P(t_1 + t_2 + \ldots + t_k \leq t) = 1 - \sum_{j=0}^{k} \frac{(\mu t)^j}{j!} . e^{-\mu t} \tag{7}$$

This distribution is called distribution *of Erlang - k*, and the law of probability *(7)* is the “law of Erlang-k”. Since it is about a sum of independent random variables, average and variance are obtained easily, as the sum of the average and the variance of each exponential variable:

- Average of the variable: k/μ.
- Variance of the variable: $k/(\mu^2)$
- The coefficient of variation is $1/\sqrt{k}$

2. Safety Engineering of a Repair Shop

That is to say MTTR (Mean Time To Repairs) the average time of stay of an element on a repair channel; if the MTTR increases with the intensity of breakdown λ (t) then the number elements of redundancy in a depot increases; it can also increase if the repair possibilities of the workshop are rather weak (lack of repair channels; lack qualified personnel etc.)

It is considered the exponential distribution of the repair and operating time of these elements.

Necessary condition in order to ensure the correct operation of a repair shop.

$$\Lambda \leq \beta \tag{8}$$

Where:

$$\Lambda = N.\,\lambda = \frac{N}{MTBF} \tag{9}$$

$$\beta = N.\,\mu = \frac{n}{MTTR} \tag{10}$$

With:

Λ- Intensity of breakdown of all the elements; β - repair intensity of the workshop.

N - the total number of identical elements in exploitation; n - the number of simultaneous repairs (a number of repair channel); λ - Intensity of breakdowns of an element; μ - its intensity of repair; *MTTR* - Mean Time To Repair; *MTBF* - Mathematical expectation of the operating time .

From there we can determine the necessary number of repair channel R:

$$n \geq \frac{N.MTTR}{MTBF} \tag{11}$$

If one indicates by the letter α the number of breakdowns for N elements planned for the average repair period (MTTR);

$$\alpha = \frac{N.MTTR}{MTBF} = \frac{N.\lambda}{\mu} \quad (12)$$

We can write n roughly $\cong\alpha$ (n whole positive higher than α)

During the failure of machine the requirement in reserve element will be felt in order not to penalize the operating system. If element in the reserve deposit is missing, in this case the failing element will be on standby of repair. After repair he will regain the depot and the repair channel will be free.

The number of elements waiting repair is indicated by K. To evaluate the service of maintenance it would be necessary to determine availability A of this one.

$$A = 1 - Pz \quad (13)$$

Where

$$Pz = P\, n + k \quad (14)$$

Pz can be regarded as the probability of having the entire defective machine either in *(n)* repair, or waiting repair *(k)*.

To ensure an availability of the elements of reserve we require an availability factor *Ar* such as:

$A \geq A_r$

The required value of *Ar* depends on the economic considerations and is related to the generated costs:

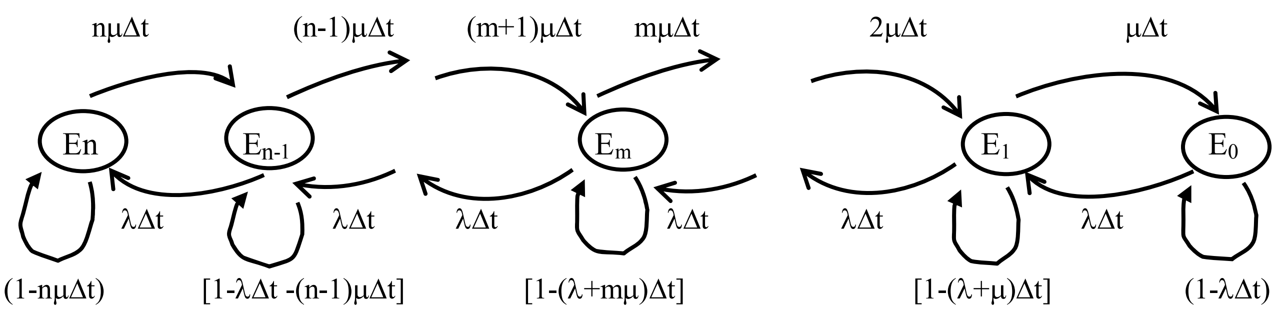
- Of dimensioned by the purchase of a great quantity of redundant machines; what increases the reliability of operation of the system but risk to cause a dead stock;
- Other dimensioned, by a lack of redundant machines from where a reduction in expenditure but represents a risk of immobilization (increase the unavailability of the element)

2.1. Study of the Case of Repair without Queue

The defective machine, finding all the repair channels occupied, leaves the queue after certain duration of waiting without being repaired.

($0 < m < n$; $k = 0$). There is no queue.

By using the Markov chain we can represent the states graph and write the equations for the various states of the system:



Graph of the states **Benmounah Amar**

Let's indicates by Ei the state where i repair channels are occupied. (Example E_n - there is n occupied channels; (all are occupied); E_0 – it there no occupied channel; all are free and can receive each one an order among K on standby in the file.

Differential equations for the probabilities $P_1(t), P_2(t) .... P_m(T), ... P_N(t)$ are written:

$$\begin{cases} \frac{dP_0(t)}{dt} = -\lambda P_o(t) + \mu P_1(t); \\ \dots\dots\dots \\ \frac{dP_m(t)}{dt} = \lambda P_{m-1}(t) - (\lambda + m\mu)P_m(t) + (m+1)\mu P_{m+1}(t) \qquad 0 < m < n; \\ \dots\dots\dots \\ \frac{dP_n(t)}{dt} = \lambda P_{n-1}(t) - n\mu P_n(t) \end{cases} \tag{15}$$

The matrix equation for n = 2 will be:

$$\begin{vmatrix} 1-2\mu\Delta t & \lambda\Delta t & 0 \\ 2\mu\Delta t & 1-(\lambda+\mu)\Delta t & 0 \\ 0 & \mu\Delta t & 1-\lambda\Delta t \end{vmatrix} x \begin{vmatrix} P_2(t) \\ P_1(t) \\ P_0(t) \end{vmatrix} = \begin{vmatrix} P_2(t+\Delta t) \\ P_1(t+\Delta t) \\ P_0(t+\Delta t) \end{vmatrix}$$

2.1.1. Stationary Availability

If $t \to \infty$ we will have $P_o(t), P_1(t), P_2(t) \dots\dots P_n(t) = const \to (P_0; P_1; P_2; \dots P_n)$

Then the entire derivatives are null; we can thus write the algebraic equations.

$$\begin{cases} -\lambda P_o + \mu P_1 = 0; \\ -(\lambda+\mu)P_1 + 2\mu P_2 + \lambda P_o = 0 \\ \dots\dots\dots \\ -(\lambda + m\mu)P_m + (m+1)\mu P_{m+1} + \lambda P_{m-1} = 0 \qquad (0 \prec m \prec n) \\ \dots\dots\dots \\ -[\lambda + (n-1)\mu]P_{n-1} + n\mu P_n + \lambda P_{n-2} = 0; \\ -n\mu P_n + \lambda P_{n-1} = 0 \end{cases} \tag{16}$$

With: $\sum_{m=0}^{n} P_m = 1$

2.1.2. Results

In the case of the exponential distribution (λ and μ constant), we can calculate the stationary probabilities using the Markov chains:

$$Pm = \frac{\prod_{j=0}^{m-1} s_j}{\prod_{j=1}^{m} d_j}.P_0 \tag{17}$$

Where: S_J = intensité of breakdown; D_J = intensité of repair

m = 1, ..., n; In our case: $s_j = \lambda = const$ *et* $dj = j\mu$

For very $m \le n$ by solving the system below we obtains the expression of P_m:

$$P_m = \frac{\lambda^m}{m!\,\mu^m}.P_0 \tag{18}$$

In addition: $\frac{\lambda}{\mu} = \rho$ Then: $P_m = \frac{\rho^m}{m!}.P_0$ (19)

By holding account that: $\sum_{m=0}^{n} P_m = P_0 \sum_{i=0}^{n} \frac{\rho^m}{m!} = 1$ (20)

We obtains: $P_0 = \frac{1}{\sum_{m=0}^{n} \frac{\rho^m}{m!}}$ (21)

We can write the probability having *m* occupied channel, among *n* existing, by the formula of Erlang for $0 \leq m \leq n$:

$$P_m = \frac{\frac{\rho^m}{m!}}{\sum_{m=0}^{n} \frac{\rho^m}{m!}} \quad (22)$$

λ = const - failure intensity of machine; μ = const - its intensity of repair.

The failure of the system occurs when all the stands are occupied. The probability of finding *n* occupied repair channel (i.e. $m = n$) is written:

$$P_{sf} = Pn = \frac{\frac{\rho n}{n!}}{\sum_{m=0}^{n} \frac{\rho^m}{m!}} \quad (23)$$

P_{sf} - Probability of system failure

In the case where there are only 01 repair channel (n = 1) the probability of system failure can be writhed:

$$P_{sf} = P_1 = \frac{\rho}{1+\rho} \quad (24)$$

2.2. Case of System with Queues

In this case the defective parts, finding *n* occupied channel remain in the file a certain time they leave the chain without being repaired.

Let us indicate by $\nu = 1/T_{att}$ - intensity of waiting in the chains and T_{att} – time of wait in the queue when all the repair channel(repair stands) are occupied. This duration is random and is distributed by the exponential law with a density:

$$h(t) = \nu . e^{-\nu . t}$$

The number of éléments in the file on standby for repair is equal to *K*.

The differential equations for the probable states of the system are written [1]:

$$\begin{cases} \frac{dP_0(t)}{dt} = -\lambda P_o(t) + \mu P_1(t); \\ \frac{dP_1(t)}{dt} = \lambda P_0(t) - (\lambda + \mu) P_1(t) + 2\mu P_2(t); \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{dP_m(t)}{dt} = \lambda P_{m-1}(t) - (\lambda + m\mu) P_m(t) + (m+1)\mu P_{m+1}(t); \qquad 1 \le m \le n-1; \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{dP_n(t)}{dt} = \lambda P_{n-1}(t) - (\lambda + n\mu) P_n(t) + (n\mu + \nu) P_{n+1}(t); \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{dP_{n+k}(t)}{dt} = \lambda P_{n+k-1}(t) - (\lambda + n\mu + k\nu) P_{n+k}(t) + [n\mu + (k+1)\nu] P_{n+k+1}(t); \end{cases} \tag{25}$$

$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

- In the case of stationary probability the solution of the system can be written:
- For: m ≤ n on a:

$$P_m = \frac{\rho^m}{m!}.P_o \tag{26}$$

- For: m > n (m = n + k et k ≥ 1)

$$P_{n+k} = \frac{\lambda^{n+k}}{n!\,\mu^n \prod_{j=1}^{k} (n\mu + j\nu)}.P_o \tag{27}$$

For 2 case (m ≤ n et m = n + k avec k ≥ 1) we can write

$$P_0 \left\{ \sum_{m=0}^{n} \frac{\rho^m}{m!} + \sum_{k=1}^{\infty} \frac{\lambda^{n+k}}{n!\,\mu^n \prod_{j=1}^{k} (n\mu + j\nu)} \right\} = 1 \Rightarrow P_0 = \frac{1}{\sum_{m=0}^{n} \frac{\rho^m}{m!} + \frac{\rho^n}{n!} \sum_{k=1}^{\infty} \frac{\rho^k}{\prod_{j=1}^{k} (n + j\beta)}} \tag{28}$$

With: $\frac{\lambda}{\mu} = \rho$ et $\frac{\nu}{\mu} = \beta$ and $p_{n+k} = \frac{\rho^{n+k}}{n! \prod_{j=1}^{k} (n + j\beta)}.p_o$ (29)

a) The probability of having *m* occupied channels is written: (0 ≤ m ≤ n)

$$Pm = \frac{\frac{\rho^m}{m!}}{\sum_{m=0}^{n} \frac{\rho^m}{m!} + \frac{\rho^n}{n!} \sum_{k=1}^{\infty} \frac{\rho^k}{\prod_{j=1}^{k} (n + j\beta)}} \tag{30}$$

where:

β - Number of elements leaving the queue after a certain time without being repaired;
k – Total number of elements on standby of repair.

b) The probability of having *n* occupied channels and *k* elements on standby of repair (among which a certain quantity*β*, leaves the queue without being repaired) is written:

(m = n + k; m > n; k ≥ 1).

$$Pn+k = \frac{\frac{\rho^n}{n!} \cdot \frac{\rho^k}{\prod_{j=1}^{k}(n+j\beta)}}{\sum_{m=0}^{n}\frac{\rho^m}{m!} + \frac{\rho^n}{n!}\sum_{k=1}^{\infty}\frac{\rho^k}{\prod_{j=1}^{k}(n+j\beta)}} \tag{31}$$

2.3. Case where the Wait State is not Limited.

If is considered where all the defective elements on standby will be, early or late, repaired; i.e. that they will not leave the queue, we can estimate, whereas $\beta \to 0$; then we can write:

2.3.1. For 0 ≤ m ≤ n et k = 0

(All the defective elements are been repaired: there is not queue);

$$Pm = \frac{\frac{\rho^m}{m!}}{\sum_{m=0}^{n}\frac{\rho^m}{m!} + \frac{\rho^n}{n!}\sum_{k=1}^{\infty}\frac{\rho^k}{n^k}} \tag{32}$$

By considering that

$$\sum_{k=1}^{\infty}\left(\frac{\rho}{n}\right)^k = \frac{\rho}{n-\rho} \text{ if } \rho < n \tag{33}$$

The formula (32) becomes:

$$Pm = \frac{\frac{\rho^m}{m!}}{\sum_{m=0}^{n}\frac{\rho^m}{m!} + \frac{\rho^{n+1}}{n!(n-\rho)}} \tag{34}$$

2.3.2. For m = n + k; m > n; k ≥ 1

(All channels are occupied, the queue starts to be formed) the formula (25) is written:

$$Pn+k = \frac{\frac{\rho^{n+k}}{n!\,n^k}\ .}{\sum_{m=0}^{n}\frac{\rho^m}{m!} + \frac{\rho^{n+1}}{n!(n-\rho)}} \tag{35}$$

Writing Pz = P n + k and while having:

$A = 1 - Pz \simeq A_r$. Probability that, at least, in any moment one element will be available i.e. that at least one repair channel will be free. In this case:

$$Pz = 1 - A_r = P_{n+k} = \frac{\frac{\rho^{n+k}}{n!.n^k}}{\sum_{m=0}^{n}\frac{\rho^m}{m!} + \frac{\rho^{n+1}}{n!(n-\rho)}} \tag{36}$$

Where: $Z = n + k$ the necessary number of reserve elements .

3. Numbers of Elements in Reserve

The problem consists in finding number of reserve elements (redundancy passivates) to have in the deposit waiting repair of those sent in the workshop and to thus avoid the prolonged immobilization of system.

The practical formula for the determination of Z, obtained after transformation of the formula *(36)*, will be as follows [4]:

$$Z = n + \frac{ln\left[\frac{n!}{\rho^n}\left(\sum_{m=0}^{n}\frac{\rho^m}{m!}+\frac{\rho^{n+1}}{n!(n-\rho)}\right)(1-A_r)\right]}{ln\left(\frac{\rho}{n}\right)} = n + k \qquad (37)$$

Where: $$\frac{ln\left[\frac{n!}{\rho^n}\left(\sum_{m=0}^{n}\frac{\rho^m}{m!}+\frac{\rho^{n+1}}{n!(n-\rho)}\right)(1-A_r)\right]}{ln\left(\frac{\rho}{n}\right)} = k \qquad (38)$$

The expectation m_K; numbers of request on standby, can be obtained from the formula (22)

$$m_k = \sum_{k=1}^{\infty} k.P_{n+k} = \frac{\frac{\rho^n}{n!}\cdot\sum_{k=1}^{\infty}\frac{k.\rho^k}{\prod_{j=1}^{k}(n+j\beta)}}{\sum_{m=0}^{n}\frac{\rho^m}{m!}+\frac{\rho^n}{n!}\sum_{k=1}^{\infty}\frac{\rho^k}{\prod_{j=1}^{k}(n+j\beta)}} \qquad (39)$$

For $\beta \to 0$ (all the defective machines are in repair):

$$m = \frac{\frac{n.\,\rho^{n+1}}{n!(n-\rho)^2}\;.}{\sum_{a=0}^{n}\frac{\rho^a}{a!}+\frac{\rho^{n+1}}{n!(n-\rho)}} \qquad (40)$$

Example

For a workshop made up of 2 repair channels receiving elements having MTBF = 7500 hours; repair duration of an element 132 hours; the number of elements N in the system 80, the parameters of this repair shop and the number of elements to be had in reserve are:

Solution:

N	*MTBF (hours)*	*MTTR (hours)*	*Ar*	α	*N*	*K*	*Z*
80	*7500*	*132*	*0,99*	*1,40*	*2*	$\cong 8$	*10*

1) The average of failure which occur during the time of repair.
   $\rho = 1,4;$ considering that $\rho < n$ we can say that the stationary regime is established.
2) Probabilities: $P_0 = 17,64\%;\ P_1 = 24,70\%;\ P_2 = 17,30\%$
3) The reliability of the system i.e. probability of not to have failures during repair.
   $R(MTTR) = e^{-\rho} = e^{-1,4} = 24,46\ \%$

4) Probability of existence of a queue: 1 - (0,1764 + 0,247 + 0,173) = 40,36%
   The average length of the queue is equal to: (ρ = 1,4 et n = 2) m = 1,345

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