Abstract

A vertex *v* of a graph $G=(V,E)$$G=(V,E)$ is said to *ve*-dominate every edge incident to *v*, as well as every edge adjacent to these incident edges. A set $S\subseteq V$$S\subseteq V$ is a vertex-edge dominating set (or simply, a *ve*-dominating set) if every edge of *E* is *ve*-dominated by at least one vertex of *S*. The minimum cardinality of a *ve*-dominating set of *G* is the vertex-edge domination number $\gamma_{ve}(G)$.$\gamma_{ve}(G)$. A *ve*-dominating set is said to be total if its induced subgraph has no isolated vertices. The minimum cardinality of a total *ve*-dominating set of *G*is the total vertex-edge domination number $\gamma_{tve}(G)$.$\gamma_{vet}(G)$. In this paper we initiate the study of total vertex-edge domination. We show that determining the number $\gamma_{tve}(G)$$\gamma_{vet}(G)$ for bipartite graphs is NP-complete. Then we show that if *T* is a tree different from a star with order *n*, ℓ leaves and *s* support vertices, then $\gamma_{tve}(T)\leq(n-\ell+s)/2$.$\gamma_{vet}(T)\leq(n-\ell+s)/2$. Moreover, we characterize the trees attaining this upper bound. Finally, we establish a necessary condition for graphs *G* such that $\gamma_{tve}(G)=2\gamma_{ve}(G)$$\gamma_{vet}(G)=2\gamma_{ve}(G)$ and we provide a characterization of all trees *T* with $\gamma_{tve}(T)=2\gamma_{ve}(T)$$\gamma_{vet}(T)=2\gamma_{ve}(T)$.