Design of Optimal Decentralized Controller Using Overlapping Decomposition for Smart Building System

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Abstract. Many industrial systems are known to have complex structure with large dimension variables. For such type of complexities, it is generally preferable to evade the design of centralized controller because of dimensionality augmentation in the step of implementation. Many research studies have been focused on designing decentralized controller for large scale systems. The aim of this paper is not just designing high dimension decentralized controller but also increase the robustness and improve systems' performance, the optimality of these systems has been considered and discussed in the frame work of mathematical development of inclusion-contraction principle and overlapping decomposition. Furthermore, the proposed control strategy has been applied to a smart building system in order to minimize the damage caused by earthquake; the obtained results allow us to conclude that the proposed control strategy can be so useful for constructing smart cities.

Keywords: Optimal decentralized control · Smart building system · Overlapping decomposition · Overlapping decomposition · Smart cities

1 Introduction

The mechanical systems behavior is in many cases very complex in a way the mathematical model that describes its dynamic can be very dimensional [1]. Thus, stability analysis and controller design of such type of systems become more difficult; because these steps will require much more computational efforts and the control tasks with required performance cannot achieved easily. The latter imposes the development of new strategies by beneficiating from the mathematical structure of the system in order to decompose it into smaller and low dimension subsystems easy to deal with; the subsolutions of each subsystem can be then joined together with respect to the interaction constraints to construct a solution of the original system [2, 3]. The main objective of

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this manuscript is to design a decentralized optimal controller for smart building system by using the overlapping decomposition strategy and extension-contraction principle; this work is development of (*L.Bakule and J.Rodellar 1995*) study by improving the performance of responses using optimization technique given by [4, 5].

2 System Description

Engineering is a large domain that gathers many disciplines, which are all unified in a principle of application of science for practical reasons, it can be said the applied science and engineering are almost equivalent. Civil engineering is one of the applied sciences in which people have constructed many important things such buildings, dams, canals, roads, bridges. It is the scientific vision of the construction who improved the science of modern civil engineering [6]. The combination of practical knowledge of materials and construction with mathematics and science has accelerated the development of building toward smart cities [1].Consider the mechanical second order building system shown in Fig. 1; the building is composed of six floors (Fig. 1.b).

The mathematical model that describes the system shown in Fig. 1 is given as:

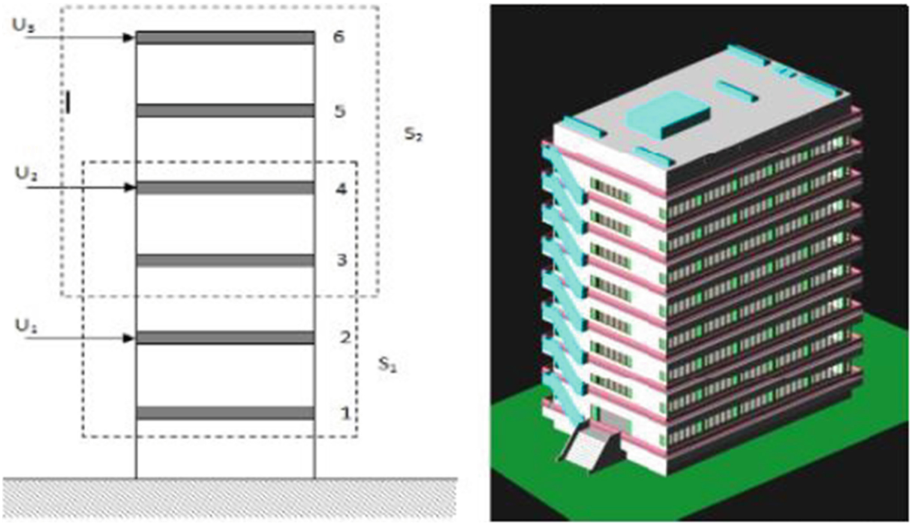

Fig. 1. a) Schema highlighting the overlapping structure of building system, **b)** Figure of Real Building System.

$$\begin{cases} M\ddot{q} + D\dot{q} + Sq = Bu \\ y = Cq \\ v = V\dot{q} \end{cases} \tag{1}$$

$M_{6\times6}$: is the mass matrix, it symmetric and definite positive,

$D_{6\times6}$: is the damping coefficients matrix,

$S_{6\times6}$: is the stiffness coefficients matrix,

$q_{6\times1}$: is the displacement vector, represents the degree of freedom of the system,

$B_{6\times3}$: is the input matrix, represents locations of actuators in the floors of the smart building,

$u_{3\times1}$: is the input signal, it is sinusoidal signal for this model.

Equation (1) is designated to represent the response of smart building system to continuous earthquake disturbances; Fig. 2 demonstrated a real example of response failure of building system without actuator to earthquake [6, 7]. Equation (1) can rewritten in the following state space form.

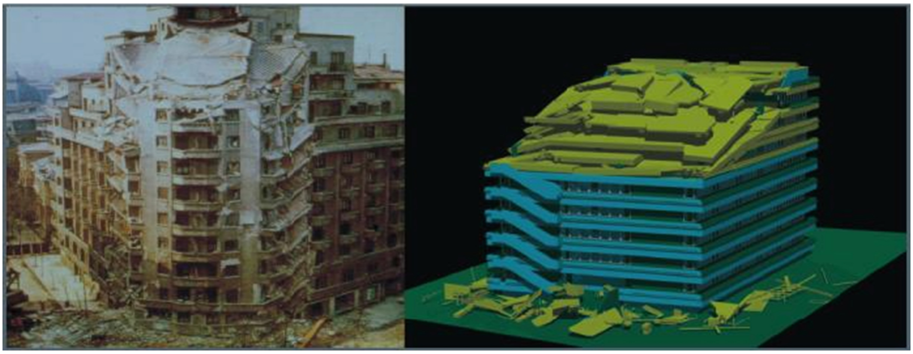

Fig. 2. Example of failure response of a building system to earthquake disturbances [3].

$$\begin{cases} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \\ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \\ \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} V_{11} & 0 & 0 \\ 0 & V_{22} & 0 \\ 0 & 0 & V_{33} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \end{cases} \tag{2}$$

The dashed lines in Eq. (2) defines the subsystems, for building system composed of six floors we have two subsystems: S_1 (floors 1, 2, 3, and 4) and S_2 (floors 3, 4, 5, and 6), the commons floors are 3 and 4, and the shared information is $(M_{22}, D_{22}, S_{22}, B_{22}, C_{22}, V_{22})$.

3 Expansion–Contraction Principle

The original system given by Eq. (1) is named the overlapping system; it can be transformed into expanded system described by the following equation

$$S_e : \begin{cases} M_e\ddot{q}_e + D_e\dot{q}_e + S_e q_e = B_e u_e \\ y_e = C_e q_e \\ v_e = V_e\dot{q}_e \end{cases} \tag{3}$$

To achieve this objective, transformation matrices are proposed between the overlapping and the expanded systems

$$\begin{cases} q_e = Tq \\ u_e = U^I u \\ y_e = Gy \\ v_e = Hv \end{cases} \quad \text{or} \quad \begin{cases} q = T^I q_e \\ u = Uu_e \\ y = G^I y_e \\ v = H^I v_e \end{cases} \tag{4}$$

Where $T^I T = I_n$, $UU^I = I_m$, $G^I G = I_p$, and $H^T H = I_r$, T^I, U^I, G^I, H^I: are the pseudo-inverse of T, U, G, H respectively. One can say that the system represented by Eq. (3) is an expansion of the system given by (1) (reversely (1) is contraction of the system in (3)) if transformation T, U, G and H can be found and satisfies conditions in Eq. (4) for any initial states $(q_e(0), \dot{q}_e(0))$ and for any input $u_e(t) \in R^m$, $t \geq 0$ [8, 9].

$$\begin{cases} q_e(0) = Tq(0) \\ \dot{q}_e(0) = T\dot{q}(0) \\ u(t) = Uu_e(t) \end{cases} \Rightarrow \begin{cases} q_e(t) = Tq(t) \\ \dot{q}_e(t) = T\dot{q}(t) \\ v_e(t) = Hv(t) \end{cases} \tag{5}$$

Theoretically, there exist two main methods to the necessary and sufficient conditions for expansion principle:

A. First Method

This method necessitates working on the matrices of the second order system directly for the original system and expanded system as well. It means the use of the matrices M and M_e is mandatory.

B. Method Two

In this method, a first order equivalent system should be obtained from the second order system, thus, it requires working with the inverse matrices M^{-1} and M_e^{-1}. Consider the original system given by Eq. (1) and its expansion given by Eq. (3); where the state vectors x, x_e are defined as: $x = \left(q^T, \dot{q}^T\right)^T$ [2, 9], these equations can be rewritten as shown in Eqs. (6) and (7).

$$S_x : \begin{cases} \dot{x} = A_x x + B_x u \\ y_x = C_x x \end{cases} \tag{6}$$

$$S_{ex} : \begin{cases} \dot{x}_e = A_{ex}x_e + B_{ex}u_e \\ y_{ex} = C_{ex}x_e \end{cases} \tag{7}$$

Where $A_x, B_x,$ and C_x, $A_{ex}, B_{ex},$ and C_{ex} are defined as follows:

$$\begin{cases} A_x = \begin{bmatrix} 0_{6\times 6} & I_6 \\ -M^{-1}S & -M^{-1}D \end{bmatrix} \\ B_x = \begin{bmatrix} 0_{6\times 3} \\ M^{-1}B \end{bmatrix} \\ C_x = diag(C, V) \end{cases} \text{and} \begin{cases} A_{ex} = \begin{bmatrix} 0_{8\times 8} & I_8 \\ -M_e^{-1}S_e & -M_e^{-1}D_e \end{bmatrix} \\ B_{ex} = \begin{bmatrix} 0_{8\times 4} \\ M_e^{-1}B_e \end{bmatrix} \\ C_{ex} = diag(C_e, V_e) \end{cases}$$

By defining the transformation matrices T, U, G that satisfy Eq. (5) for the original system given by Eq. (6); we can find the transformation matrices of the expanded system (7) as: $T_d = diag(T, T)$; $C_d = diag(G, H)$, this equation means that:

$$\begin{cases} x_e(0) = T_d x(0) \\ u(t) = Uu_e(t) \end{cases} \Rightarrow \begin{cases} x_e(t) = T_d x(t) \\ y_{ex}(t) = C_d x(t) \end{cases} \tag{8}$$

C. Theorem One

The system S_{ex} given by Eq. (7) is an expansion of the system S given by Eq. (6) or equally S is the contraction of S_e, if and only of there exists full rank transformation matrices T, U, G and H such that

$$\begin{cases} M_e^{-1}S_eT = TM^{-1}S \\ M_e^{-1}D_eT = TM^{-1}D \\ M_e^{-1}B_e = TM^{-1}BU \\ GC = C_eT \\ HV = V_eT \end{cases} \tag{9}$$

Equation (9) is obtained by converting the systems (1) and (3) into state space model and it be can be rewritten as follows:

$$\begin{cases} M_e^{-1} = TM^{-1}T^I + M_{cq} \\ S_e = TST^I + S_{cq} \\ D_e = TDT^I + D_{cq} \\ B_e = TBU + B_{cq} \\ C_e = GCT^I + G_c \\ V_e = HVT^I + V_c \end{cases} \tag{10}$$

The matrices M_e, S_e, D_e, B_e, C_e and V_e are given in the following form

$$M_e = \begin{bmatrix} M_{11} & M_{12} & 0 & M_{13} \\ M_{21} & M_{22} & 0 & M_{23} \\ M_{21} & 0 & M_{22} & M_{23} \\ M_{31} & 0 & M_{32} & M_{33} \end{bmatrix}, \quad D_e = \begin{bmatrix} D_{11} & D_{12} & 0 & D_{13} \\ D_{21} & D_{22} & 0 & D_{23} \\ D_{21} & 0 & D_{22} & D_{23} \\ D_{31} & 0 & D_{32} & D_{33} \end{bmatrix},$$

$$S_e = \begin{bmatrix} S_{11} & S_{12} & 0 & S_{13} \\ S_{21} & S_{22} & 0 & S_{23} \\ S_{21} & 0 & S_{22} & S_{23} \\ S_{31} & 0 & S_{32} & S_{33} \end{bmatrix}, \quad B_e = \begin{bmatrix} B_{11} & 0 & 0 & 0 \\ 0 & B_{22} & B_{22} & 0 \\ 0 & B_{22} & B_{22} & 0 \\ 0 & 0 & 0 & B_{33} \end{bmatrix},$$

$$C_e = \begin{bmatrix} C_{11} & 0 & 0 & 0 \\ 0 & C_{22} & 0 & 0 \\ 0 & 0 & C_{22} & 0 \\ 0 & 0 & 0 & C_{33} \end{bmatrix}, \text{ and } V_e = \begin{bmatrix} V_{11} & 0 & 0 & 0 \\ 0 & V_{22} & 0 & 0 \\ 0 & 0 & V_{22} & 0 \\ 0 & 0 & 0 & V_{33} \end{bmatrix}$$

$M_{qc}, S_{qc}, D_{qc}, B_{qc}, C_{qc}$ and V_{qc} are complementary matrices calculated in a way to respond to the necessary and sufficient conditions of extension-contraction principle given by theorem two [8].

D. Theorem Two [6]

If theorem one is satisfied, we can say that system (7) is an expansion of the system (6) if condition given by Eq. (11) is verified.

$$\begin{cases} M_{qc}T = 0 \\ K_{qc}T = 0 \\ D_{qc}T = 0 \\ B_{qc} = 0 \\ C_{qc}T = 0 \\ V_{qc}T = 0 \end{cases} \tag{11}$$

One of the appropriate choices of the complementary matrices is given by Eq. (12), this form of complementary matrices will guarantee the verification of condition in Eq. (11).

$$[\cdot]_{qc} = \begin{bmatrix} 0 & \frac{1}{2}[\cdot]_{12} & -\frac{1}{2}[\cdot]_{12} & 0 \\ 0 & \frac{1}{2}[\cdot]_{22} & -\frac{1}{2}[\cdot]_{22} & 0 \\ 0 & -\frac{1}{2}[\cdot]_{22} & \frac{1}{2}[\cdot]_{22} & 0 \\ 0 & -\frac{1}{2}[\cdot]_{32} & \frac{1}{2}[\cdot]_{32} & 0 \end{bmatrix} \tag{12}$$

4 Contraction Principle of Controllers

In order to discuss the contractibility of controller, consider the control law, given by Eq. (13), applied to building system.

$$u = Fy + Lv + w \tag{13}$$

Consider, also, the control law given by Eq. (14), applied to expanded system in (7).

$$u_e = F_e y_e + L_e v_e + w_e \tag{14}$$

w and w_e denote external inputs which represent the signal of earthquake in this study [10, 11].

A. Theorem Three

The control law (14) is contractible to the control law (13) if and only if

$$\begin{cases} FC = UF_eGC \\ LV = UL_eHV \end{cases} \tag{15}$$

B. Theorem Four

If Eq. (2) is an extension of Eq. (1) and if Eq. (2) is stable (respectively asymptotically stable) then Eq. (1) is stable (respectively asymptotically stable) [5].

5 Decentralized Control Design

A. Problematic Description

Consider the original system described by Eq. (6), the objective of control design is to find the gain matrix for the output feedback control law $u = Ky_x$ that minimizes the following cost function:

$$J = \int_{-\infty}^{+\infty} \left(x^T Q x + u^T R u\right) dt \tag{16}$$

So that the closed loop system given by Eq. (17)

$$S_c = \begin{cases} \dot{x} = (A + B_x K C_x)x \\ y_x = C_x x \end{cases} \tag{17}$$

Will be asymptotically stable [13].

B. Proposed Solution

The expanded system of building system can be seen as combination of two subsystems, they can be decoupled into two separate low dimension systems each preserve the effect of interactivity between them. The two subsystems' models are given by Eq. (18.a) and Eq. (18.b) respectively.

$$\begin{cases} M_1\ddot{q}_{e1} + D_1\dot{q}_{e1} + S_1 q_{e1} = B_1 u_{e1} \\ y_{e1} = C_1 q_{e1} \\ v_{e1} = V_1 \dot{q}_{e1} \end{cases} \tag{18.a}$$

$$\begin{cases} M_2\ddot{q}_{e2} + D_2\dot{q}_{e2} + S_2 q_{e2} = B_2 u_{e2} \\ y_{e2} = C_2 q_{e2} \\ v_{e2} = V_2 \dot{q}_{e2} \end{cases} \tag{18.b}$$

The state space representation of the decoupled subsystems is given by Eq. (19.a) and Eq. (19.b) respectively.

$$S_1 : \begin{cases} \dot{x}_{e1} = A_{ex1} x_{e1} + B_{ex1} u_{e1} \\ y_1 = C_{ex1} x_{e1} \end{cases} \tag{19.a}$$

$$S_2 : \begin{cases} \dot{x}_{e2} = A_{ex2} x_{e2} + B_{ex2} u_{e2} \\ y_2 = C_{ex2} x_{e2} \end{cases} \tag{19.b}$$

Where the matrices $A_{ex1}, B_{ex1},$ and $C_{ex1}, A_{ex2}, B_{ex2},$ and C_{ex2} are defined as follows:

$$\begin{cases} A_{ex1} = \begin{bmatrix} 0_{(2+2)(2+2)} & I_{(2+2)(2+2)} \\ -M_1^{-1}S_1 & -M_1^{-1}D_1 \end{bmatrix} \\ B_{ex1} = \begin{bmatrix} 0_{(2+2)(2+2)} \\ -M_1^{-1}B_1 \end{bmatrix} \\ C_{ex1} = diag(C_1, V_1) \end{cases} \begin{cases} A_{ex2} = \begin{bmatrix} 0_{(2+2)(2+2)} & I_{(2+2)(2+2)} \\ -M_2^{-1}S_2 & -M_2^{-1}D_2 \end{bmatrix} \\ B_{ex2} = \begin{bmatrix} 0_{(2+2)(2+2)} \\ -M_2^{-1}B_2 \end{bmatrix} \\ C_{ex2} = diag(C_2, V_2) \end{cases},$$

The optimal output feedback of each systems is $u_i = K_i y_i$; $i = 1, 2$, and the performance index for each subsystem is defined as:

$$J_i = \int_{-\infty}^{+\infty} \left(x_i^T Q_i x_i + u_i^T R_i u_i \right) dt, \ i = 1, 2 \tag{20}$$

By respecting the demonstrated theorems, the necessary and sufficient conditions for each subsystem are obtained as given in Eq. (21).

$$\begin{cases} \phi_i^T P_i + P_i \phi_i + Q_i + C_{xi}^T K_i^T R_i K_i C_{xi} = 0 \\ K_i = -R_i^{-1} B_{xi}^T P_i L_i C_{xi}^T \left(C_{xi} L_i C_{xi}^T \right)^{-1} \\ \phi_i L_i + L_i \phi_i^T + X_{0i} = 0 \end{cases} \tag{21}$$

The matrix is $\phi_i = A_i + B_{xi}K_iC_{xi}$, and the initial state vector $X_{0i} = x_{0i}x_{0i}^T$; in this study we took the initial state as an identity vector $x_{0i} = I$. The optimal cost is the trace of the resultant matrix as given in the following equation:

$$J_i = \frac{1}{2} \times trac(P_iX_{0i})$$

The value of the optimal cost corresponds to the optimal control law as $u_i = K_iy_i$ [11].

The form of the optimal gain matrix is $K_i = \begin{bmatrix} K_{11}^i & K_{12}^i & K_{13}^i & K_{14}^i \\ K_{21}^i & K_{22}^i & K_{23}^i & K_{24}^i \end{bmatrix}$.

The output feedback control law for expanded system in (18) is given by the following equation:

$$K_i = \begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 & 0 & K_{13}^1 & K_{14}^1 & 0 & 0 \\ K_{21}^1 & K_{22}^1 & 0 & 0 & K_{23}^1 & K_{24}^1 & 0 & 0 \\ 0 & 0 & K_{11}^2 & K_{12}^2 & 0 & 0 & K_{13}^2 & K_{14}^2 \\ 0 & 0 & K_{21}^2 & K_{22}^2 & 0 & 0 & K_{23}^2 & K_{24}^2 \end{bmatrix}$$

The contracted form of the controller can now be obtained by applying the contraction principle to the expanded controller, the contracted controller is expressed by Eq. (22).

$$K = \begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 & K_{13}^1 & K_{14}^1 & 0 \\ K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 & K_{21}^1 & K_{24}^1 + K_{13}^2 & K_{14}^2 \\ 0 & K_{21}^2 & K_{22}^2 & 0 & K_{23}^2 & K_{24}^2 \end{bmatrix} \tag{22}$$

In order to implement the controller in Eq. (22) for smart building system, it is necessary to write it in the following form:

$$u = Fy + Lv + w$$

w is the external earthquake input signal to the six-floor building system [12]. The controller of the system described by Eq. (1) is given by the following equations

$$K = [F, L] \text{ with } F = \begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 \\ K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 \\ 0 & K_{21}^2 & K_{22}^2 \end{bmatrix}, \; L = \begin{bmatrix} K_{13}^1 & K_{14}^1 & 0 \\ K_{23}^1 & K_{24}^1 + K_{13}^2 & K_{14}^2 \\ 0 & K_{23}^2 & K_{24}^2 \end{bmatrix}$$

Finally, the implementation of the designed controller in the original building system gives us the following closed loop form for the smart building system:

$$\begin{cases} M\ddot{q} + (D + BLV)\dot{q} + (K + BFC)q = Bw \\ y = Cq \\ v = V\dot{q} \end{cases} \tag{23}$$

6 Results Discussion

All results presented in this paper are obtained under Matlab environment, where different controllers have been simulated starting from centralized feedback controller without optimality constraints, to decentralized output feedback controller with optimization algorithms, for the six floors of the smart building system. Figure 3 shows the results of the designed feedback centralized controller for floor 2, 4, and 6 without any optimization condition, the response of the smart building system has been compared to open loop system which conventional building without any actuator. It is clear that the smart building response to earthquake is better than the open loop response (Fig. 3 a and b) this may minimize the effect of disasters of earthquake, however in floor 6 even with actuator the response of the smart building system in this floor is considerable and it may create some damage. For this reason, it is necessary to improve the robustness and performance of smart building for the worst case where we have quick disturbances with large amplitudes.

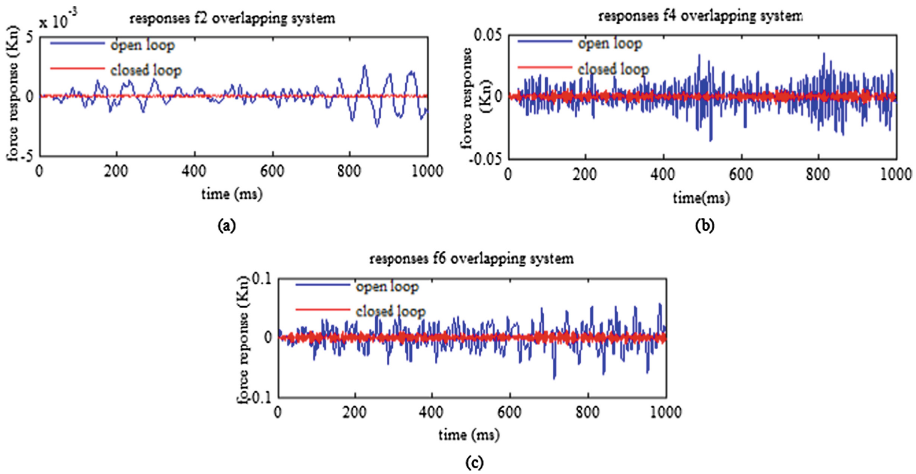

Fig. 3. Non-optimal centralized control system; **a)** Floor 2, **b)** Floor 4, **c)** Floor 6

As an improvement of the obtained results from the centralized controller, we proposed the use of decentralized controller in order to increase the performance of the system to the structured and unstructured external disturbances as shown in Fig. 4.

It is noticeable that in the floor 4 which is the common floor between, the two subsystems (Fig. 4 b and d), the response of smart building system in closed loop form is very considerable with high amplitudes; this is due to the effect of interconnection terms between subsystem 1 (floors 1, 2, 3, and 4) and subsystem 2 (floors 3, 4, 5, and 6). The actuator in this study are placed in floors 2, 4, and 6 respectively.

The interaction terms have been considered in the following results, in which an optimization algorithm has been integrated with overlapping decomposition strategy in order to improve the robustness of controller and performance of the smart building

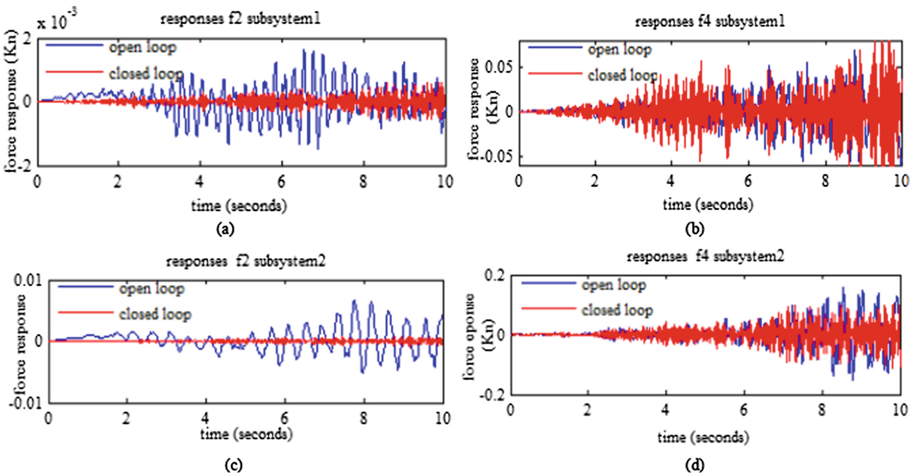

Fig. 4. Non-optimal decentralized control; **a)** subsystem 1 Floor 2, **b)** subsystem 1 Floor 4, **c)** subsystem 2 Floor 2, **d)** subsystem 2 Floor 4

systems especially for the actuator of the fourth floor [10]. Figure 5 shows the results of centralized controller with an optimization function however Fig. 6 shows the results of decentralized controller with the optimization technique.

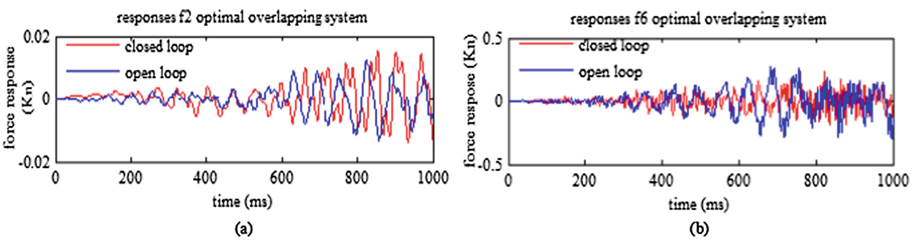

Fig. 5. Response of smart building system with centralized controller and optimization algorithm: **a)** Floor 2, **b)** Floor 6

We can notice from Fig. 5 that even with integrating the optimization algorithm the responses of the floors 2 and 6 is still considerable, this might damages the structure of the smart building, the cost function value found for this controller is equal to $J_t = 7.31 * 10^3$, it represents the optimal total energy consumed to generate dynamic counter force that should be applied by the actuators in order to absorb the oscillations smoothly without creating damages to the building. Figure 6 shows the responses of the floors 2, 4, and 6 with decentralized controller designed by using overlapping decomposition and optimization algorithm.

Figure 6 demonstrates the effectiveness of the designed controller based on overlapping decomposition strategy, indeed, the response of the common floor has been minimized (Fig. 6b and d), this means that the performance of the smart building

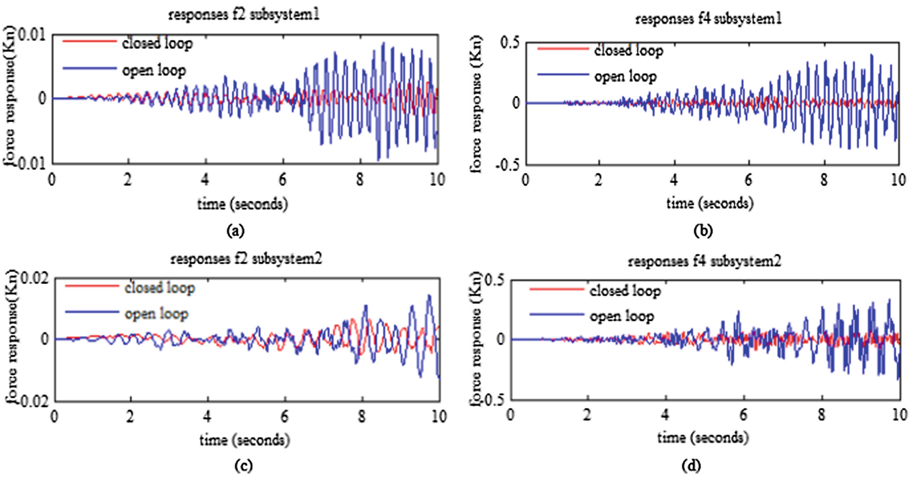

Fig. 6. Response of smart building system with decentralized controller and optimization algorithm: **a)** Floor 2 of subsystem 1, **b)** Floor 4 of subsystem 1, **c)** Floor 2 of subsystem 2, **d)** Floor 4 of subsystem 2

system has been improved, the cost functions of subsystems 1 and 2 are respectively $J_1 = 2.47 * 10^3$ and $J_2 = 1.41 * 10^3$, these values indicates that the overlapping decomposition strategy has not just improved the robustness of the controller but also optimized the value of the energy that should be applied to protect the building in the presence of harmful earthquake disturbances.

7 Conclusion

A new decomposition strategy has been proposed in this study, the mathematical development of the overlapping technique is detailed with a focus on inclusion-contraction principle. The development of decentralized controller is based on the necessary and sufficient conditions of the principle summarized in some theories. In order to introduce the optimization aspect of the proposed strategy, the application of inclusion-contraction for the cost function has been intensively discussed. The contractibility of output feedback controller has been proved for smart building system composed of six floors and with two degrees of freedom. A comparison between centralized and decentralized output feedback controllers allowed us to prove the effectiveness of the decomposition strategy to improve the performance of smart building system and to increase the robustness of its controllers connected directly to the actuators. Furthermore, the optimality of the control law has been also examined; the cost function for smart building system represents the optimal energy that should be generated in order to distribute counter forces that mitigate smoothly the oscillations without damaging the building. it is concluded, through the obtained results, that the proposed decomposition techniques does not just increase the performance of smart building system but also minimize the necessary energy consumed by the actuators to ensure that. Thus, it is highly recommended to use the proposed technique to design building of next smart cities with minimized and renewable energies.

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