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Controller Canonical Form Based Design: An Airplane Case Study

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Abstract

The state feedback multivariable control design based on eigenvalues assignment is reviewed and is employed to develop a systematic design procedure to meet the lateral handling qualities design objectives of a fighter aircraft over a single flight condition.

The objective in this project is to investigate state feedback multivariable control design based on similarity transformations in terms of feedback gain, robustness and effect of similarity transformations. The desirable design can be made using two main transformations block controller form and general controller form. The block controller, observer and diagonal forms are used among an infinite number of choices of assigning a set of solvents. In addition to those forms, the general controller form is used directly to find the resulting state feedbacks.

The similarity transformation provides significant insight into the design process and plays a pivotal role in the design of state feedback gain magnitude according to the specified criteria of robustness, sensitivity and time specifications of the feedback system.

Through this project we would like to give the designer many possibilities to select the most suitable design depending on the specified need for the flight condition of a fighter aircraft.

Dedication

It is my genuine gratefulness and warmest regard that I dedicate this work to my parents, my brother and my sister the hidden strength behind my every success behind my every success.

To everyone who have a highest credit of my achievements from primary school till this graduation time.

To all friends with whom I spent nice moments.

SEHILI LAID

First and foremost, all the praises and thanks be to Allah, my Creator and my Master.

I want to dedicate this work to:
To my beloved parents for their unconditional love, and for their endless support and encouragement throughout my life.
To all my brothers and sisters, a huge thanks for Zouina and Lounis, their families and the rest of my family.
To my best friend Lakhdar Mamouri.
To all my friends who have always been a constant source of support and encouragement during the challenges of my whole college life. I will never forget their beautiful actions toward me, especially, Mounir Lakhal.
Finally, I would like to dedicate this thesis to every single person who had given me a hand to achieve my goal. Shout outs to you.

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Nomenclature

SISO	Single-Input Single-Output
MIMO	Multi-input Multi-Output
SSD	State Space Description
RMFD	Right Matrix Fraction Description
LMFD	Left Matrix Fraction Description
x	Vector of states
$\dot{x}$	Derivative of the state x
u	Vector of inputs
y	Vector of Outputs
A	System matrix
B	Input matrix
C	Output matrix
D	Feed forward matrix
$R(A, B)$	Reachability matrix
$P(A, B)$	Reachability base matrix
T_c	Controller similarity transformation matrix
λ_i	i^{th} eigenvalue of the system
K	State feedback gain matrix
$D(\lambda)$	Characteristic matrix polynomial
I_m	Identity matrix $m \times m$
λ_0	Latent root

$D_L(S)$	Left denominator matrix
$D_R(S)$	Right denominator matrix
$A_R(X)$	Right matrix polynomial associated with λ- matrix
$S(\lambda_i)$	i^{th} individual eigenvalue sensitivity
$S(V)$	Overall eigenvalue sensitivity
M_2	Stability measure 2
M_3	Stability measure 3
r_i	Relative change measure
p	Roll rate, deg/sec
$\dot{\psi}$	Yaw rat, deg/sec
β	Sideslip, deg
Φ	Bank angle, deg
δ_a	Aileron angular deflection, deg
δ_r	Rudder angular deflection, deg
a_y	Lateral acceleration, m/sec^2 (ft/sec^2)
AOA	Angle of Attack
T	Matrix transpose
-1	Inverse matrix
$\Vert\ \Vert$	Matrix or vector norm
$\vert\ \vert$	Absolute value
EV	Eigenvalue

Chapter 1 Multivariable control systems

1.1 Introduction

This chapter is about reviewing the main concepts of multivariable control systems and the necessary mathematical fundamentals for the control design and analysis.

1.2 Basic concepts

A system is called a SISO system if it has only one input and output terminals. Starting from that concept systems with more than one input and/or more than one output are known as MIMO systems.

Figure 1-1 MIMO system representation

Here are a few examples of multivariable processes:

- A heated liquid tank where both the level and the temperature shall be controlled.
- A distillation column where the top and bottom concentration shall be controlled.
- A robot manipulator where the positions of the manipulators (arms) shall be controlled.
- A chemical reactor where the concentration and the temperature shall be controlled.

• A head box (in a paper factory) where the bottom pressure and the paper mass level in the head box shall be controlled.

Any linear multivariable control system can be described in one of the two forms:

✓ Internal description: State space description.

✓ External description: Transfer function description.

In this project, the system under study will be described using state-space description.

1.3 State-space description

A linear MIMO system can be described using a state space description (SSD). This form is very useful to describe the internal dynamics of a multivariable system through the concept of state.

The state space description of the system provides a complete picture of the system structure showing how all of the internal variables $X_i(t)$ ($i = 1,2,\dots, n$) interact with one another, how the inputs $U_k(t)$ ($k = 1,2,\dots, m$) affect the system states $X_i(t)$, and how the outputs $Y_j(t)$ ($j = 1,2,\dots, p$) are obtained from various combinations of the state-variables $X_i(t)$ and the inputs $U_k(t)$.

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{1.1}$$

A linear state model is formed by a set of first order linear differential equations with constant coefficient ($\dot{x}(t)$) and a set of linear equations ($y(t)$).

The state at the initial time $t_0 = 0$ is $x_0 = x(t_0)$

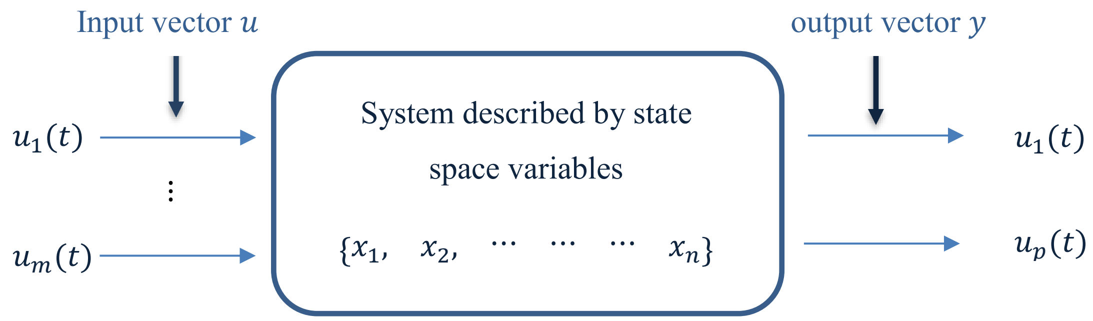

Figure 1-2 System inputs and outputs

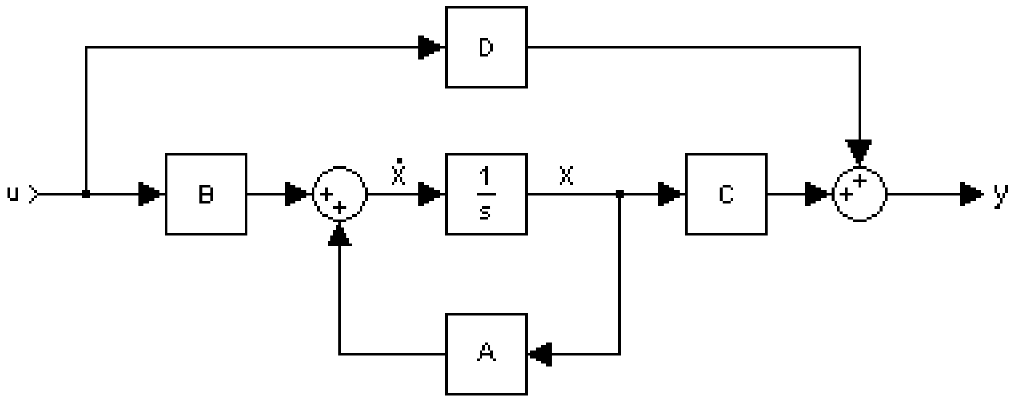

Figure 1-3 Block diagram representation of the linear state space equations.

where

$x(t) = [x_1(t), \ . \ . \ . \ , x_n(t)]^T$ is the state vector and $x_i(t), i = 1,2,\ldots,n$ are the state variables.

$u(t) = [u_1(t), \ . \ . \ . \ , u_m(t)]^T$ it the input vector. m refers to the number of inputs.

$y(t) = [y_1(t), \ . \ . \ . \ , y_p(t)]^T$ is the output vector. p refers to the number of outputs.

and the system matrices *(A, B, C, D)* are real, constant, and with dimensions $n{\times}n$, $n{\times}m$, $p{\times}n$, and $p{\times}m$, respectively.

In the above model, from Eq. (1.1) $\dot{x}(t)$ is called the "dynamic equation," which describes the "dynamic part" of the system and how the initial system state x(0) and system input u*(t)* will determine the system state x*(t)*. Hence matrix *A* is called the "dynamic matrix" of the system, and from (1.1), *y(t)* describes how the system state x*(t)* and system input u(t) will instantly determine system output $\mathbf{y}(t)$. This is the "output part" of the system and is static (memoryless) as compared with the dynamic part of the system.

From the definition of eq (1.1), parameters m and p represent the number of system inputs and outputs, respectively. If $m > 1$, then we call the corresponding system "multi-input." If $p > 1$, then we call the corresponding system "multi-output." A multi-input or multi-output system is also called a "MIMO system." On the other hand, a system is called "SISO" if it is both single-input and single-output. [1]

Definition 1.1 [2]

The state of a system at time t_0 is the amount of information at t_0 that, together with $[t_0 , \infty]$determines uniquely the behavior of the system for all $t \geq t_0$.

1.4 Reachability and Controllability

Definition 1.2 [3]

Reachability is the ability of the control input to drive the state $x(t)$ from any initial condition $x(t_o)$ to any final value $x(t_f)$.

Theorem 1.1

The system described in (1.1) is said to be fully reachable if and only if:

rank ($\boldsymbol{R}(A, B)$) = n.

where $\boldsymbol{R}$ (A, B) is the reachability matrix and it is given by:

$$\boldsymbol{R}(A,B) = [B \quad AB \quad A^2B \quad \quad A^{n-1}B \ ] \tag{1.2}$$

Proof: [3]

Definition 1.3 [4]

The dynamic system described by (1.1) or the pair (A,B) is said to be controllable , if there exists an input $u_{[0,t]}$ which transfers the initial state $x(0) = x_0$ to the zero state $x(t_1) = 0$ in a finite time t_1, the state x_0 is said to be controllable.

If all initial states are controllable the system is said to be completely controllable.

The solution of (1.1) is:

$$x(t) = e^{At}x_0 + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau \tag{1.3}$$

If the system is controllable, i.e., there exists an input to make $x(t_1) = x_1 = 0$ at a finite time $t = t_1$, then after pre-multiplying by e^{-At_1} yields:

$$x_0 = \int_0^{t_1} e^{-A\tau}Bu(\tau)d\tau \tag{1.4}$$

Thus, any controllable state satisfies (1.3), and for a completely controllable system, every state $x_0 \in R^n$ satisfies $t_1 > 0$ and $u_{[0,t]}$.

It is found that complete controllability of a system depends on matrix *A* and *B* and is independent of the output matrix *C*.

Theorem 1.2 [2]

The n dimensional linear time invariant state equation in (1.1) is controllable if and only if any of the following equivalent conditions is satisfied:

a. All rows of $e^{-A\tau}B$ are linearly independent on $[0, \infty]$ over the field of complex numbers.
b. $w(0, t_1) = \int_0^{t_1} e^{-At\tau}BB^T e^{-A^T t}dt$ is nonsingular for any $t_1 > 0$.
c. The $n \times mn$ controllability matrix $\Phi = [\, B\ AB\ A^2B\ , \dots , A^{n-1}B]$ has rank n.

Proof: [2]

Remarks

- Since the state value *x(t)* depends on the eigenvalues (modes) of the system, we can rephrase reachability also as the ability of the control input *u(t)* to drive the eigenvalues from any location to any other location.

- The system is completely (totally, fully) reachable i.e. rank ($\boldsymbol{R}(A, B)$) = n, meaning that all the eigenvalues of the system can be relocated by state feedback using the input.
- If rank ($\boldsymbol{R}(A, B)$) < n the system is said to be partially reachable and so only some eigenvalues can be relocated.
- Reachability implies controllability but the inverse is not correct.

1.5 Observability

Definition 1.4 [4]

The dynamical system described by the equations (1.1) or the pair *(A,B)* is said to be observable if, for any $t_1 > 0$, the initial state $x(0) = x_0$ can be determined from the time history of the input *u(t)* and the output *y(t)* in the interval of [0; t_1]. Otherwise, the system is said to be unobservable.

The output of the system (1.1) is given by:

$$y(t) = Ce^{At}x_0 + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(\tau) \tag{1.5}$$

Theorem 1.3: [2]

The *n* dimensional linear time invariant dynamical equation in (1.1) is observable if and only if any of the following equivalent conditions are satisfied:

a. All columns of Ce^{At} are linearly independent on $[0, \infty]$ over the field of complex numbers.
b. $w(0, t_1) = \int_0^{t_1} e^{A^T t} C^T C e^{At} dt$ is nonsingular for any $t_1 > 0$.
c. The $np \times p$ observability matrix $\Phi_o = [\, C\ CA\ CA^2, \ldots, CA^{n-1}]^T$ has rank n.

Proof: [2]

1.6 MIMO Canonical Forms

A reachable (observable) system can be transformed to canonical representations pointing out this property. These representations will be useful in illustrating some control properties, as will be seen later in control design on chapter 5.

For MIMO systems there is possibility of defining some canonical forms. The most well-known are the following: [3]

1.6.1 Block Controller Form

Definition 1.5: The system is block controllable of index l if the matrix

$w_c = [\,B\ AB\ A^2B\ ,\dots,A^{l-1}B]$ has full rank.

where l is the ratio of $\frac{n}{m}$, n is the number of columns of the matrix A and m is the number of columns of the matrix B.

The system (1.1) can be transformed into block controller form if the following conditions are satisfied:

a. The number $\frac{n}{m} = l$ is an integer.
b. The system is controllable of index l.

Let $w_c = [\,B\ AB\ A^2B\ ,\dots,A^{l-1}B]$; the system is controllable if rank $(w_c) = n$.

Then we make a change of coordinates

$$x_c = T_c x \tag{1.6}$$

where

$$T_c = \begin{bmatrix} T_{c1} \\ T_{c1}A \\ . \\ . \\ . \\ T_{c1}A^{l-2} \\ T_{c1}A^{l-1} \end{bmatrix} \tag{1.7}$$

and

$$T_{c1} = [\, 0_m \; 0_m \, \dots . \, I_m][\, B \; AB \;\; \dots . \, A^{l-1}B]^{-1} \tag{1.8}$$

Then, (1.1) becomes

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c u(t) \\ y(t) = \; C_c x_c(t) + Du(t) \end{cases} \tag{1.9}$$

where $A_c = T_c A T_c{}^{-1}$, $B_c = T_c B$ and $C_c = C T_c{}^{-1}$

or

$$A_c = \begin{bmatrix} 0_m & 0_m & & . \; . \; . & 0_m \\ 0_m & I_m & & . \; . \; . & 0_m \\ . & . & & . \; . \; . \; . & . \\ . & . & & . \; . \; . \; . & . \\ 0_m & 0_m & & . \; . \; . & I_m \\ -A_l & -A_{l-1} & & . \; . \; . & -A_1 \end{bmatrix}, B_c = \begin{bmatrix} 0_m \\ 0_m \\ . \\ . \\ . \\ I_m \end{bmatrix} \text{ and } C_c = [\, C_l \; C_{l-1} \dots . \, C_1].$$

0_m the null and I_m the identity matrices are both $m \times m$, A_i and $C_i (i = 1,2, \dots, l)$ are block elements.

1.6.2 Block Observer Form

Definition 1.6: The system is block observable of index q if the matrix

$w_o = [\, C \; CA \; CA^2 \,, \dots . , CA^{q-1}]^T$ has full rank.

where q is the ratio of $\frac{n}{p}$, n is the number of columns of the matrix A and p is the number rows of the matrix C.

The system (1.1) can be transformed into block observer form if the following conditions are satisfied:

a. The number $\frac{n}{p} = q$ is an integer.
b. The system is observable of index q.

Let $w_o = \begin{bmatrix} C \\ CA \\ . \\ . \\ . \\ CA^{q-1} \end{bmatrix}$; the system is observable if rank $(w_o) = n$.

Then we make a change of coordinates

$$x_o = T_o^{-1} x \tag{1.10}$$

where

$$T_o = [\, T_{o1} \;\; AT_{o1} \;\; \ldots . A^{q-2}T_{o1} \;\; A^{q-1}T_{o1}] \tag{1.11}$$

and

$$T_{o1} = \begin{bmatrix} C \\ CA \\ . \\ . \\ . \\ CA^{q-1} \end{bmatrix}^{-1} \begin{bmatrix} O_p \\ O_p \\ . \\ . \\ . \\ I_p \end{bmatrix} \tag{1.12}$$

Then (1.1) becomes

$$\begin{cases} \dot{x}_o(t) = A_o x_o(t) + B_o u(t) \\ y(t) = C_o x_o(t) + Du(t) \end{cases} \tag{1.13}$$

where $A_o = {T_o}^{-1} A T_o$, $B_o = {T_o}^{-1} B$ and $C_o = C T_o$

or

$$A_o = \begin{bmatrix} 0_p & 0_p & . & . & . & 0_p & -A_q \\ 0_p & I_p & . & . & . & 0_p & -A_{q-1} \\ . & . & & . & . & . & . \\ . & . & & . & . & . & . \\ 0_p & 0_p & . & . & . & 0_p & -A_2 \\ 0_p & 0_p & . & . & . & I_p & -A_1 \end{bmatrix}, B_o = \begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ . \\ B_q \end{bmatrix} \text{and } C_o = [\, 0_p \; 0_p \ldots . I_p].$$

1.6.3 General Controller form

To convert a MIMO system into general controller form, we must find another similarity transformation using the reachability matrix $R(A,B) = (B \; AB \ldots \; A^{n-m})$, where m is the rank of B. If $R(A,B)$ is full rank, the system is said to be fully reachable.

Now $B = [b_1 \; b_2 \ldots b_m]$ then

$$R(A,B) = (b_1 \; b_2 \ldots b_m, Ab_1 \; Ab_2 \ldots Ab_m, \ldots, A^{n-m} \; b_1 \; A^{n-m} b_2 \ldots A^{n-m} b_m) \qquad (1.14)$$

In the reachability matrix $R(A,B)$ we look for linearly independent vectors corresponding to b_i, we then record the number of these linearly independent vectors and are referred to as reachability indices K_i or Kronecker indices.

K_i is the number of linearly independent vectors corresponding to b_i. $i = 1, 2, \ldots, m$.

If the sum of the reachability indices equals to the n, the reachability base matrix $P(A,B)$ can be constructed. where

$$P(A,B) = [b_1, Ab_1, A^2 b_1, \ldots., A^{k_1 - 1} b_1, b_2, Ab_2, A^2 b_2, \ldots., A^{k_2 - 1} b_2, \ldots \ldots, b_m, Ab_m, A^2 b_m, \ldots., A^{k_m - 1} b_m] \qquad (1.15)$$

now a similarity transformation T_c can be constructed, such that

$$x_c = T_c x \qquad (1.16)$$

$$T_c = [p_1, p_1 A, p_1 A^2, \ldots., p_1 A^{k_1 - 1}, b_2, Ap_2, p_2 A^2, \ldots., p_2 A^{k_2 - 1}, ., p, p_m A, p_m \, A^2, \ldots., p_m A^{k_m - 1}]^T \qquad (1.17)$$

where $p_i = \sigma_i^{th}$ row of $P(A,B)^{-1}$, $i = 1,2,\ldots,m$ and $\sigma_i = \sum_{j=1}^{i} K_j$

Then (1.1) becomes (1.9)

where $$A_c = T_cAT_c{}^{-1},\ B_c = T_cB \text{ and } C_c = CT_c{}^{-1}$$

Except matrices are in the forms:

$$A_c = T_cAT_c^{-1} = \begin{array}{c} \\ K_1\left\{\begin{array}{c} \\ \\ \\ \end{array}\right. \\ \\ \\ \\ K_m\left\{\begin{array}{c} \\ \\ \\ \end{array}\right. \\ \\ \end{array} \left[\begin{array}{cccc|c|cccc|cccc}
0 & 1 & \cdots & 0 & & & & & & & & & \\
\vdots & \vdots & \ddots & \vdots & \cdots & & (0) & & & & (0) & & \\
0 & 0 & \cdots & 1 & & x & x & x & x & x & x & x & x \\
x & x & \cdots & x & & & & & & & & & \\
 & & \vdots & & & & \ddots & & & & \vdots & & \\ \hline
 & & \vdots & & & & & & & & \vdots & & \\ \hline
 & & & & & & & & & 0 & 1 & \cdots & 0 \\
 & (0) & & & \cdots & & (0) & & & \vdots & \vdots & \ddots & \vdots \\
x & x & x & x & & x & x & x & x & 0 & 0 & \cdots & 1 \\
 & & & & & & & & & x & x & \cdots & x
\end{array}\right] B_c = \left[\begin{array}{cccc}
0 & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
1 & x & \cdots & x \\ \hline
0 & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 1 & \cdots & x \\ \hline
\vdots & \vdots & \vdots & \vdots \\ \hline
0 & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \cdots & 1
\end{array}\right]$$

$$\underbrace{\qquad\qquad}_{K_1} \qquad \underbrace{\qquad\qquad}_{K_{m-1}} \qquad \underbrace{\qquad\qquad}_{K_m}$$

A_c has m blocks in controller form on the diagonal of dimensions $K_i \times K_i$ each.

C_c is in trivial form.

Remark: as long as m is greater than 1, there would be m! possible permutations of the columns of the matrix B, and thus there would be a reachability base matrix for each permutation and consequently a state feedback gain. So, for m inputs there would be m! state feedback gains.

Chapter 2 Elements of Matrix Polynomial Theory

2.1 Introduction

In linear time-invariant single-input single-output systems, the transfer function is a ratio of two scalar polynomials. The system modelling of physical, linear, time-invariant multi-input multi-output control systems, results in high degree coupled differential equations, or an *n-th* degree *m-th* order differential equation in the form: [7]

$$U(t) = X^{(n)}(t) + A_1 X^{(n-1)}(t) + \dots + A_{n-1} X^{(n)}(t) + A_n X(t) \tag{2.1}$$

where $A_i \in R^{m\times m}$, $X^{(i)} \in R^{m\times 1}$ represents the *i-th* derivate of the vector $X(t)$, and $U(t) \in R^{m\times 1}$ being the input vector.
The output $y(t) \in R^{p\times 1}$ is generally given by a linear equation in the form:

$$Y(t) = C_1 X^{(n-1)}(t) + C_2 X^{(n-2)}(t) + \dots + C_{n-1} X^{(1)}(t) + C_n X(t) \tag{2.2}$$

where $C_i \in R^{p\times m}$.

The Laplace transformation of (2.1) and (2.1) with zero initial conditions results in:

$$S^n X(s) + A_1 S^{n-1} X(s) + \dots + A_n X(s) = U(s) \tag{2.3}$$

and

$$Y(s) = C_1 S^{n-1} X(s) + C_2 S^{n-2} X(s) + \dots + C_n X(s) \tag{2.4}$$

which yields,

$$Y(s) = [C_1 S^{n-1} + C_2 S^{n-2} + \dots + C_n][I_m S^n + A_1 S^{n-1} + \dots + A_n]^{-1} U(s) \tag{2.5}$$

where I_m stands for the $m \times m$ identity matrix.

Equation (2.5) can be written as:

$$Y(s) = N_R(s)D_R^{-1}(s)U(s) \tag{2.6}$$

which yields the $p \times m$ transfer function matrix,

$$H(s) = N_R(S)D_R^{-1}(S) \tag{2.7}$$

where $D_R(s)$ and $N_R(s)$ are $m \times m$ and $p \times m$ matrix polynomials also called λ-matrices, (the complex variable λ is often used instead of s for continuous time systems and z for discrete time systems), defined by:

$$D_R(S) = I_m S^n + A_1 S^{n-1} + \ldots + A_n \tag{2.8}$$

$$N_R(S) = C_1 S^{n-1} + C_2 S^{n-2} + \ldots + C_n \tag{2.9}$$

The equation (2.7) is the right matrix fraction description (RMFD), or the polynomial matrix description of MIMO system shown in (2.1 & 2.2). The matrix polynomial $D_R(s)$ in (2.7) is a right denominator matrix.

An alternative presentation of $H(s)$ is the left matrix fraction description (LMFD) defined by:

$$H(s) = D_L^{-1}(S)N_L(S) \tag{2.10}$$

where $D_L(s)$ is a $p \times p$ left denominator matrix polynomial and $N_L(s)$ is $p \times m$ left numerator matrix polynomial.

In this section, we attempt to emphasize on the latent structure of the matrix polynomials, which consists mainly of the latent roots and latent vectors as well as solvents.

Definition 2.1

The following $m \times m$ matrix:

$$A(\lambda) = \begin{bmatrix} a_{11}(\lambda) & a_{12}(\lambda) & \ldots & a_{1m}(\lambda) \\ a_{21}(\lambda) & a_{22}(\lambda) & \ldots & a_{2m}(\lambda) \\ . & . & \ldots & . \\ . & . & \ldots & . \\ . & . & \ldots & . \\ a_{m1}(\lambda) & a_{m2}(\lambda) & \ldots & a_{mm}(\lambda) \end{bmatrix} \tag{2.11}$$

is called a λ-matrix of order m, where $a_{ij}(\lambda)$ are scalar polynomials over the field of complex numbers [8].

Definition 2.2 The matrix polynomial A(λ) is called:

i. Monic if A_0 is the identity matrix.
ii. Comonic if A_n is the identity matrix.
iii. Regular if $\det(A(\lambda)) \neq 0$.
iv. Nonsingular if $\det(A(\lambda))$ is not identically zero.
v. Unimodular if $\det(A(\lambda))$ is a nonzero constant.

2.2 Latent Structure of Matrix Polynomials

Definition 2.3 [9]

- The complex number λ_0 is called a latent root of the $A(\lambda)$ if it is a solution of the scalar polynomial equation $\det(A(\lambda)) = 0$.
- The vector $X_i \in R^m$ is called a right latent vector associated with λ_i if it satisfies $A(\lambda_i)X_i = \theta$.
- The row vector $Y_i \in R^m$ is called a left latent vector associated with λ_i if it satisfies $Y_i\, A(\lambda_i) = \theta$.

2.3 Construction of solvents

2.3.1 Construction of Right Solvents

Suppose that the set $\{\lambda_1, \lambda_2, \dots, \lambda_m\}$ of m latent roots of $A(S)$ has a linear independent set of corresponding right latent vectors$\{X_1, X_2, \dots, X_m\}$. Let $M = (X_1\ X_2 \dots X_m)$ be the $m \times m$ matrix whose columns are the linearly independent right latent vectors and $M^{-1} = [Y_1\ \ Y_2\ \dots\ \ Y_m]^T$ be its inverse. The $m \times m$ matrix $R = M\Lambda M^{-1}$, where $\Lambda = \mathrm{diag}\,(\lambda_1, \lambda_2, \dots, \lambda_m)$, is a right solvent of $A(s)$. [10]

2.3.2 Construction of left solvents

In a similar manner, we will establish that a left solvent $L \in R^{m\times m}$ can be constructed from a set $\{\lambda_1, \lambda_2, \dots, \lambda_m\}$ of m latent roots and a corresponding set of m linearly independent left latent (row) vectors $\{Y_1, Y_2, \dots, Y_m\}$. The $m \times m$ matrix $L = P^{-1}\Lambda P$, where $P = [Y_1\ Y_2\ \ \dots\ \ Y_m]^T$, $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \dots, \lambda_m)$

and $P^{-1} = (X_1\ X_2 \dots\ X_m)$ is a left solvent of $A(s)$. [10]

2.4 Solvents of Matrix Polynomials

Definition 2.4

Let X be $m\times m$ complex matrix, the two matrix polynomials, defined by:

$$A_R(X) = A_0X^l + A_1X^{l-1} + \dots + A_{l-1}X + A_l \tag{2.12}$$

and

$$A_L(X) = X^lA_0 + X^{l-1}A_1 + \dots + XA_{l-1} + A_l \tag{2.13}$$

are referred to as the right and the left matrix polynomials associated with the λ-matrix $A(\lambda)$ respectively.

Definition 2.5

A right solvent R of $A(\lambda)$ is defined by

$$A(R) = A_0R^l + A_1R^{l-1} + \dots + A_{l-1}R + A_l = 0_m \tag{2.14}$$

and the left Solvent L of $A(\lambda)$ is defined by

$$A(L) = L^lA_0 + L^{l-1}A_1 + \dots + LA_{l-1} + A_l = 0_m \tag{2.15}$$

where 0_m is an $m\times m$ null matrix, and R, L *are* $m \times m$ complex matrices.

2.5 Block Companion Form

In analogy with scalar polynomials a useful tool for the analysis of matrix polynomials is the block companion form matrix. [3]

Given a λ – *matrix*

$$A(\lambda) = I\lambda^l + A_1\lambda^{l-1} + \dots + A_l \tag{2.16}$$

where $A_i \in C^{m\times m}$ and $\lambda \in C$, the *associated lower block companion* form is:

$$A_L = \begin{bmatrix} 0_m & I_m & 0_m & \dots & 0_m \\ 0_m & 0_m & I_m & \dots & 0_m \\ . & . & . & \ddots & . \\ 0_m & 0_m & 0_m & \dots & I_m \\ -A_l & -A_{l-1} & -A_{l-2} & \dots & -A_1 \end{bmatrix} \tag{2.17}$$

and the associated *right block companion form* is:

$$A_R = \begin{bmatrix} 0_m & 0_m & \dots & 0_m & -A_l \\ I_m & 0_m & \dots & 0_m & -A_{l-1} \\ 0_m & I_m & \dots & 0_m & -A_{l-2} \\ . & . & \dots & . & . \\ 0_m & 0_m & \dots & 0_m & -A_2 \\ 0_m & 0_m & \dots & I_m & -A_1 \end{bmatrix} \tag{2.18}$$

Remarks

- A_L is the block transpose of A_R.
- If λ_i is a latent root of $A(\lambda)$ and p_i and q_i are the corresponding right and left latent vectors respectively, then λ_i is an eigenvalue of A_L and of A_R defined in (2.17) and (2.18) respectively.

2.6 Block Vandermonde Matrix

The block Vandermonde matrix has a fundamental importance in the theory of matrix polynomials.

Given a set of $m \times m$ matrices $\{R_1, R_2, \dots, R_k\}$ which are a complete set of right solvents of a matrix polynomial $A(\lambda)$, the following $km \times km$ matrix

$$V(R_1, R_2, \dots, R_k) = \begin{bmatrix} I_m & I_m & \dots & I_m \\ R_1 & R_2 & \dots & R_k \\ \vdots & \vdots & \vdots & \vdots \\ R_1^{k-1} & R_2^{k-1} & \dots & R_k^{k-1} \end{bmatrix} \tag{2.19}$$

is called *the right block Vandermonde matrix* of order k, and the block transpose of *left block Vander monde matrix* of order k is a $km\times km$ matrix defined by

$$V(L_1, L_2, \dots, L_k) = \begin{bmatrix} I_m & L_1 & \dots & L_1^{k-1} \\ I_m & L_2 & \dots & L_2^{k-1} \\ \vdots & \vdots & \vdots & \vdots \\ I_m & L_k & \dots & L_k^{k-1} \end{bmatrix} \tag{2.20}$$

where $\{L_1, L_2,., L_k\}$ represents a set of $m \times m$ left solvents of a matrix polynomial $A(\lambda)$.

Remark: *Vander monde matrices* are non-singular [11]

2.7 Complete Set of Solvents

Definition 2.6 [10]

Given $A(\lambda)$, the set of $m \times m$ matrices $\{R_1, R_2, \dots, R_l\}$is called a complete set of solvents if the following conditions are met:

i- $\sigma(R_i) \cap \sigma(R_j) = \emptyset$ for $i \neq j$

ii- $\bigcup_{i=1}^{l} \sigma(R_i) = \sigma(A(\lambda))$

iii- $\det V(R_1, R_2, \dots, R_k) \neq 0$

where $\sigma(R_i)$ is the spectrum of R_i and $\sigma(A(\lambda))$ is the spectrum of $A(\lambda)$.

2.8 Matrix polynomial construction from a complete set of solvents

[12]

We want to construct the matrix polynomial defined by $D(\lambda)$ from a set of solvents or a set of desired poles which will determine the behavior of the system that we want. Suppose we have a desired complete set of solvents. The problem is to find the desired polynomial matrix or the characteristic equation of the block controller form defined by:

$$D(\lambda) = D_0\lambda^l + D_1\lambda^{l-1} + \dots + D_{l-1}\lambda + D_l \tag{2.21}$$

we want to find the coefficients D_i for $i = 1, \dots, l$

2.8.1 Matrix polynomial Construction from a complete set of right solvents

Consider a complete set of right solvents $\{R_1, R_2, ..., R_l\}$ for the matrix polynomial *D(λ),* If R_i is a right solvent of *D(λ)* so:

$$R_i^l + D_1 R_i^{l-1} + \ ... + \ D_{l-1}R + D_l = 0_m \ \Rightarrow \ D_1 R_i^{l-1} + \ ... + \ D_{l-1}R + D_l = \ -R_i^l$$

Replacing *i* from 1 to l we obtain the following:

$$[D_{dl}, D_{d(l-1)}, \dots, D_{d1}] = -[R_1^l, R_2^l, \dots, R_i^l]V_R^{-1} \tag{2.22}$$

where V_R is the right block Vander monde matrix.

2.8.2 Matrix polynomial Construction from a complete set of left solvents

Consider a complete set of left solvents $\{L_1, L_2, ..., L_l\}$ for the matrix polynomial *D(λ),* If L_i is a left solvent of *D(λ)* so:

$$L_i^l + L_i^{l-1}D_1 + \ ... + \ LD_{l-1} + D_l = 0_m \ \Rightarrow \ L_i^{l-1}D_1 + \ ... + \ LD_{l-1} + D_l = \ -L_i^l$$

Replacing *i* from 1 to l we obtain the following:

$$\begin{bmatrix} D_{dl} \\ D_{d(l-1)} \\ \vdots \\ D_{d1} \end{bmatrix} = -V_L^{-1} \begin{bmatrix} L_1^l \\ L_2^l \\ \vdots \\ L_i^l \end{bmatrix} \tag{2.23}$$

where V_L is the left block Vander monde matrix.

Chapter 3 State Feedback Design and Criteria

3.1 Introduction

One of the most popular and well-known techniques used to assign the eigenvalues of the closed-loop system to desired locations is the state feedback. In the case of multivariable systems, the feedback gain matrix permitting the assignment of the desired set of poles is not unique.

In this part of the thesis, the specifics of two multivariable control methodologies are detailed. The first is the general controller canonical form transformation design, the second is block controller canonical form transformation design. The second section deals with the design criteria used to evaluate the methodologies.

3.2 State Feedback Design Methodologies

Consider the n-dimensional linear time–invariant, multivariable dynamical system described by equation (1.1)

A linear state-feedback control signal can be applied to the system as:

$$u(t) = -Kx(t) \tag{3.1}$$

where K is a $p \times n$ real constant matrix, called the feedback gain matrix, and equation (1.1) becomes:

$$\begin{cases} \dot{x}(t) = (A - BK)x(t) \\ y(t) = (C - DK)x(t) \end{cases} \tag{3.2}$$

In the following, we will illustrate that if the dynamical (1.1) is reachable, then the eigenvalues of (*A* - *BK*) can be arbitrarily assigned by a proper choice of *K*. This will be established by using two different methods.

3.2.1 Method 1: General controller canonical form transformation

Design procedure:

After obtaining the matrix $[A_c - B_C K_C]$ in the desired form, we construct a desired matrix A_d which has its eigenvalues as the desired ones. The matrix A_d can have different blocks in companion form on the diagonal, which means that it will have in the rows $[K_1, k_1 + k_2, \ldots., \sum_{i=1}^{m} K_i]$ elements resulting from the choice of the number of blocks and their sizes. Meaning that, we can choose different blocks in the desired matrix to assign the desired eigenvalues (from 1 to m blocks) each describing a part of the set of the desired eigenvalues. Then, we equate the two matrices A_d and $[A_c - B_C K_C]$ and computing K_C by identification.

1) Transform the given system (1.1) into general controller canonical form (see section 1.6.3 of chapter 1).
2) Construct the desired matrix A_d with the desired number of blocks and the desired eigenvalues.
3) Compute *Kc* by identification from $A_d = [A_c - B_C K_C]$.
4) Compute *K* from *Kc*, such that *K=KcTc.*

3.2.2 Method 2: Block controller canonical form transformation

Design Procedure:

1) Transform the given system (1.1) to block controller form (see section 1.6.1 chapter 1).
2) Apply the control signal $u(t) = -K_c x_c(t)$

where $K = K_c T_c = \left[K_{cl}, K_{c(l-1)}, \ldots, K_{c1}\right] T_c$ and $K_{ci} \in R^{m \times m}$ for $i = 1, \ldots, l$

The resulting closed loop system is shown below:

$$\begin{aligned} \dot{x}_c &= (A_C - B_C K_C) x_c \\ y_c &= C_C x_C \end{aligned} \tag{3.2}$$

where

$$A_C - B_C K_C = \begin{bmatrix} 0_m & I_m & \cdots & 0_m \\ 0_m & 0_m & \cdots & 0_m \\ \vdots & \vdots & \ddots & \vdots \\ 0_m & 0_m & \cdots & I_m \\ -(A_l + K_{cl}) & -\left(A_{l-1} + K_{c(l-1)}\right) & \cdots & -(A_1 + K_{c1}) \end{bmatrix} \tag{3.3}$$

The characteristic matrix polynomial of this closed loop system is then:

$$D(\lambda) = I_m \lambda^l + (A_1 + K_{c1})\lambda^{l-1} + \ \ldots + (A_l + K_{cl}) \tag{3.4}$$

3) Construct the block poles (solvents) using the desired eigenvalues (see section 2.3 chapter 2).
4) Using the constructed solvents, construct the matrix coefficients of the desired characteristic matrix polynomial $D_d(\lambda)$ (see section 2.8 chapter 2).

$$D_d(\lambda) = I_m \lambda^l + D_{d1}\lambda^{l-1} + \ \ldots \ + D_{dl}$$

5) Compute K_C by equating the characteristic matrix polynomial of this closed loop system and the desired characteristic matrix polynomial $D_d(\lambda)$.
 $D_d(\lambda) = D(\lambda)$. we obtain the coefficients K_{ci} as follows:

$$K_{ci} = D_{di} - A_i \ for \ i = 1, \ldots, l \tag{3.5}$$

 where is D_{di} is obtained while Constructing the complete set of solvents (see section 2.8 chapter 2).
6) Compute the gain matrix K from K_C, such that $K = K_C T_C$.

3.3 Evaluation Criteria

The design techniques will be compared based on an evaluation of how well each method handles the specific requirements of lateral handling qualities of the fighter aircraft control design, based on the following list of evaluation criteria:

i. The gain magnitude
ii. Time domain response
iii. Robust performance
iv. Robust stability

3.3.1 Feedback Gain magnitude

The norm of a matrix can provide a scalar measure to the magnitude of the matrix.

3.3.2 Norm of a matrix

[6]

Definition 3.1

The norm is a real number, denoted as $||e||$, which satisfies the following properties:

1. Non-negative: $||e|| \geq 0$.
2. Positive: $||e|| = 0$ *iff* $e = 0$.
3. Homogenous: $||\alpha.e||= |\alpha|.||e||$.
4. Triangle inequality: $||e_1+ e_2|| \leq ||e_1||+ ||e_2||$.

where: e is a vector, and α is a scalar.

In this thesis we will consider only matrix norms(2-Norm).

Definition 3.2

A norm $||A||$ of a matrix A is a matrix norm which, in addition to the four norm properties given earlier in definition 3.1, satisfies the multiplicative property (also called the consistency condition): $||A.B|| \leq ||A|| \cdot ||B||$.

3.3.2.1 Most Common Matrix Norm Types

i. The Matrix 1-norm

It is the maximum absolute column sum.

$$\|A\|_1 = \max_{1\leq j\leq n}\left(\sum_{i=1}^{m}|a_{ij}|\right) \tag{3.6}$$

ii. The Matrix ∞-norm

It is the maximum absolute row sum.

$$\|A\|_\infty = \max_i \left(\sum_{i=1}^{m} |a_{ij}|\right) \tag{3.6}$$

iii. The Matrix Euclidian norm (also called the 2-norm)

It is the square root of the largest eigen value of $A^T.A$ or the largest singular value of A

$$\|A\|_2 = max\left\{eigenvalue(\sqrt{A^T A})\right\} \tag{3.7}$$

iv. The Matrix Frobenius-norm

$$\|A\|_F = \sqrt{traceA^T A} \tag{3.8}$$

3.3.3 Condition number

Definition 3.3

The condition number of an invertible matrix A is defined as $\chi(A) = |A||A^{-1}|$

This quantity enables to know how close is the matrix A to singularity. This affects the accuracy of computations based on the matrix A. It can also be seen as a function to a perturbed input argument.

Note that the condition number of a matrix is always greater or equal to 1.

3.3.3.1 Condition Number and Conditioning

- If $\chi(A)$ is large, A is called ill-conditioned (with respect to inversion).
- If $\chi(A)$)is small, A is called well-conditioned (with respect to inversion).

3.3.4 Time Domain Performance

Time domain criteria are often used to describe the performance of control systems. Although developed for second order systems, they can be valuable for higher order systems.

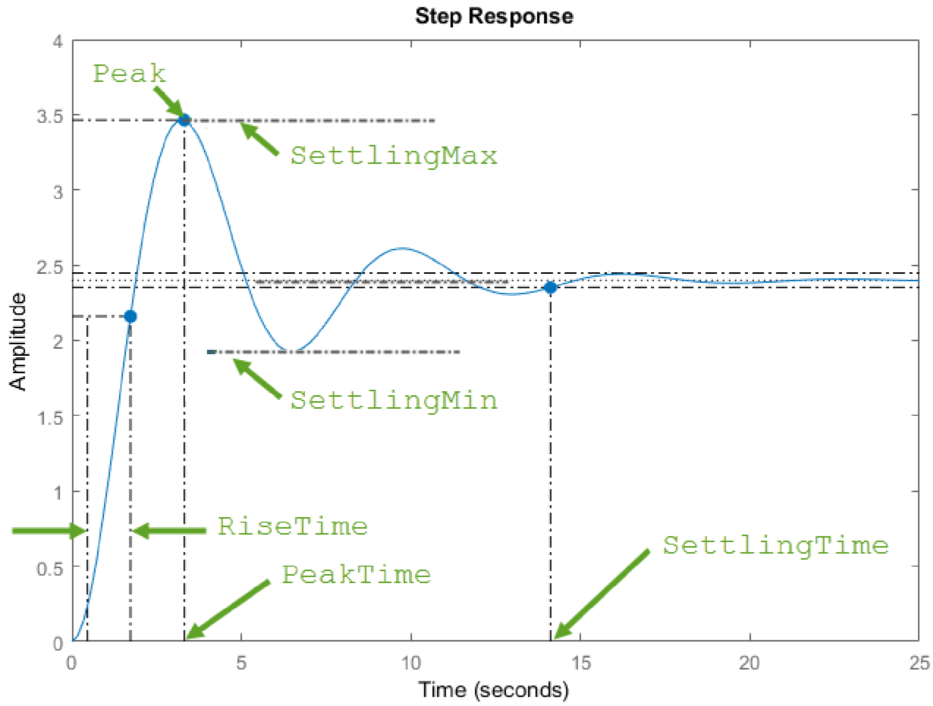

Figure 3-1 typical second-order system step response

3.3.4.1 Maximum Overshoot

The maximum overshoot is related to the maximum peak value of the response with respect to the final value.

$$M_p = y(t_p) - y(\infty) \tag{3.9}$$

$$Percent\ maximum\ oveshoot = \frac{maximum\ oveshoot}{y(\infty)} \times 100\ \% \tag{3.10}$$

3.3.4.2 Peak Time T_P

It is the time needed for the response to reach the first break of overshoot (i.e.: the peak value).

3.3.4.3 Settling Time Ts

It is the time required for the response curve to reach and stay within a range about the final value of a size specified by an absolute percentage of the final value (usually 2% or 5%).

3.3.4.4 Rise Time Tr

It is defined as the time required for the step response to reach- 10 to 90 percent of the final value.

3.3.5 Robustness and Sensitivity Analysis

3.3.5.1 The sensitivity of eigenvalues (Robust Performance)

Robust performance is defined as the low sensitivity of system performance with respect to system model uncertainty and terminal disturbance. It is well known that the eigenvalues of the dynamic matrix determine the performance of the system then from that the sensitivities of these eigenvalues determine the robustness of the system. [13]

3.3.5.1.1 Types of Robustness

1) Eigen Value Sensitivity

It is used to measure how much the system's eigenvalues are sensitive to the model uncertainties, it includes:

i. **Individual Eigen Value Sensitivity**:

The sensitivity of the i^{-th} eigen value of a matrix A to perturbations in some or all of its elements is given by the following expression:

$$s(\lambda_i) = \frac{||L_i||_2 . ||R_i||_2}{|L_i^T\ R_i|} \tag{3.11}$$

where Li and Ri are the left and right eigenvectors corresponding to eigen value λ_i, respectively.

ii. Overall Eigen Value Sensitivity:

The overall eigen value sensitivity of the matrix A, which is the condition number of the modal matrix, is defined as:

$$S(V) = ||R||_2 . ||R^{-1}||_2 \tag{3.12}$$

where R is the right eigenvector matrix of the matrix A.

2) Relative change:

It measures the relative change in eigen value λ_i following a perturbation of the system matrix A.

$$r(\lambda_i) = \frac{|\lambda_i - \lambda_i'|}{|\lambda_i|} \tag{3.13}$$

where:
λ_i is the original eigen value.
λ_i' is the new eigen value following the perturbation.

3.3.5.2 Stability Robustness

Stability is the most important property in control design; the sensitivity to such a property is called stability robustness. Basically, stability means that if every dynamic matrix eigen value has a negative real part; hence the sensitivity of these eigenvalues with respect to model uncertainties is a direct way to measure the sensitivity of the whole system stability.

Some stability robustness measures have been developed in the control literature; among these, we have the so-called M_2 and M_3 measures. [4]

3.3.5.2.1 The Robust Stability Measure M_2

It is defined as

$$M_2 = s(v)^{-1}|Re\{\lambda_n\}| \tag{3.14}$$

where $(|Re\{\lambda_n\}| \leq \cdots \leq |Re\{\lambda_1\}|)$

$|Re\{\lambda_n\}|$ is the shortest distance between the unstable region and the eigenvalue λ_i

M_2 equals this distance divided (or weighted) by the sensitivity of all the eigenvalues of the matrix. As the sensitivity goes up, M_2 goes down.

3.3.5.2.2 The Robust Stability Measure *M_3*

It is defined as:

$$M_3 = \min_{1 \le i \le n} \{s(\lambda_i)^{-1} |Re\{\lambda_i\}|\} \tag{3.15}$$

M_2 measures the likelihood margin for every eigenvalue to become unstable. It is equal to $|Re(\lambda_i)|$ divided by its corresponding sensitivity $s(\lambda_i), i = 1, \dots, n$.

Chapter 4 Aircraft Aerodynamics

4.1 Introduction

Airplanes, also known as aircrafts come in various shapes and sizes each with their own characteristics. There are number of ways to identify aircrafts by type. The primary distinction is between those that are lighter than air such as hot air balloons, airships or dirigibles and those that are heavier than air such as helicopters, gliders and airbuses. Kites also fall in the latter category

In this thesis the aircraft used to conduct the study is a fighter-aircraft which is of the type heavier than air.

4.2 Airplane Definition

Airplane, also called Aeroplan or plane, is any of a class of fixed-wing aircraft that is heavier than air, propelled by a screw propeller or a high-velocity jet, and supported by the dynamic reaction of the air against its wings. [14]

An airplane is composed of four essential parts: a wing system, an Empennage (also known as tail), a power plant and a fuselage.

- Wing system: In order to fly, one must lift the weight of the airplane itself, the fuel, the passengers, and the cargo. The wings generate most of the lift to hold the plane in the air.
- Tail: Small wings are located in the tail which serves to control and manoeuvre the aircraft. The tail usually has a fixed horizontal piece, called the horizontal stabilizer, and a fixed vertical piece, called the vertical stabilizer. The stabilizers job is to provide stability for the aircraft, to keep it flying straight.
- Power plant: the power plant provides the thrust necessary to push the vehicle through the air.

- Fuselage: The fuselage or body of the airplane, holds all the pieces together. The pilots sit in the cockpit at the front of the fuselage. Passengers and cargo are carried in the rear of the fuselage. Some aircraft carry fuel in the fuselage; others carry the fuel in the wings.

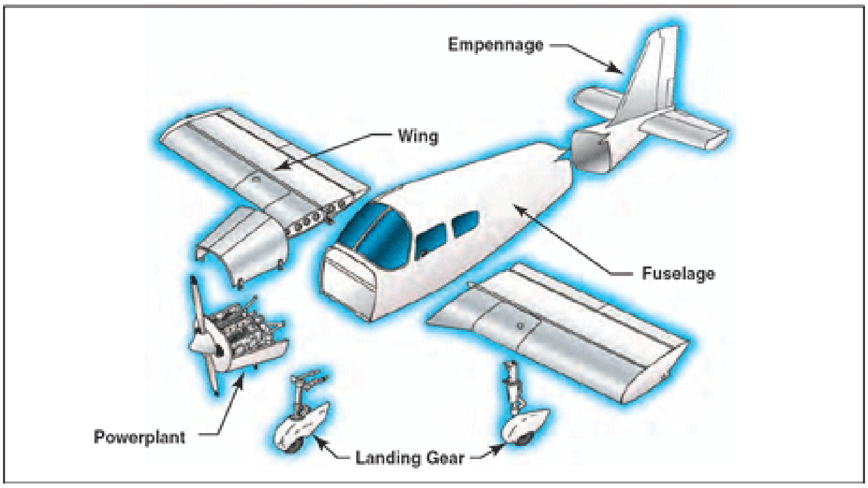

Figure 4-1 Major components of an aircraft

4.3 Axes of an Aircraft

An airplane in flight is controlled around one or more of three axes of rotation. These axes of rotation are the longitudinal, lateral, and vertical. On the airplane, all three axes intersect at the center of gravity (CG). As the airplane pivots on one of these axes, it is in essence pivoting around the center of gravity (CG). The center of gravity is also referred to as the center of rotation.

On the brightly colored airplane shown in the figure 4.1, the three axes are shown in the colors red (vertical axis), blue (longitudinal axis), and orange (lateral axis). The flight control that makes the airplane move around the axis is shown in a matching color.

The rudder, in red, causes the airplane to move around the vertical axis and this movement is described as being a yaw. The elevator, in orange, causes the airplane to move around the lateral axis and this movement is described as being a pitch. The ailerons, in blue, cause the airplane to move around the longitudinal axis and this movement is described as being a roll. [15]

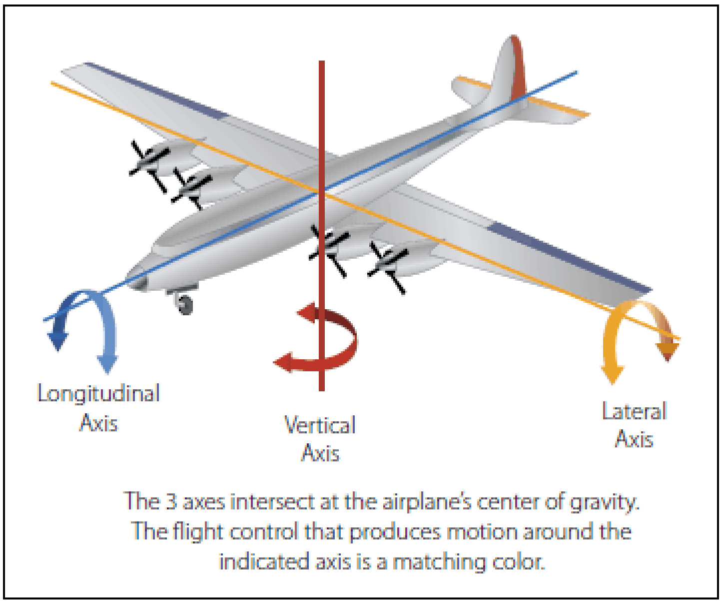

Figure 4-2 Axes of an aircraft

4.4 Aircraft Primary Flight Controls

The primary controls are the ailerons, elevator, and the rudder, which provide the aerodynamic force to make the aircraft follow a desired flightpath. In the figure 4-3 the flight control surfaces are hinged or movable airfoils designed to change the attitude of the aircraft by changing the airflow over the aircraft's surface during flight. These surfaces are used for moving the aircraft about its three axes.

Typically, the ailerons and elevators are operated from the flight deck by means of a control stick, a wheel, and yoke assembly and on some of the newer design aircraft, a joystick. The rudder is normally operated by foot pedals on most aircraft. Lateral control is the banking movement or roll of an aircraft that is controlled by the ailerons. Longitudinal control is the climb and dive movement or pitch of an aircraft that is controlled by the elevator. Directional control is the left and right movement or yaw of an aircraft that is controlled by the rudder. [16]

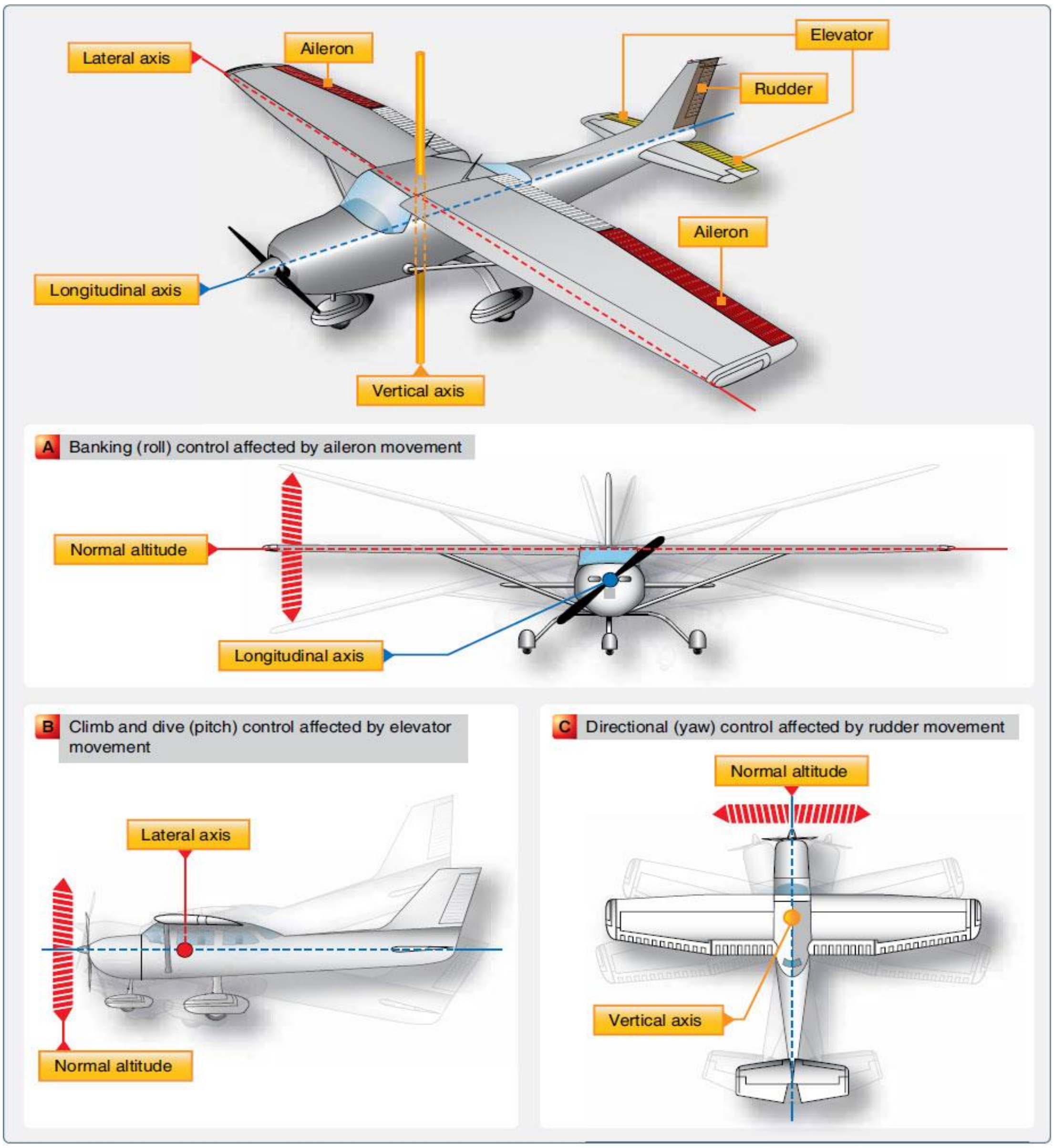

Figure 4-3 Primary flight control of an aircraft

4.5 The Forces Acting on an Aircraft

There are four forces acting on an aircraft in flight:

- Weight: Weight is a force that is always directed toward the center of the earth. The magnitude of the weight depends on the mass of all the airplane parts, plus the amount of fuel, plus any payload on board (people, baggage, freight, etc.). The weight is distributed throughout the airplane. But we can often think of it as collected and acting through a single point called the center of gravity. In flight, the airplane rotates about the center of gravity.

- Lift: To overcome the weight force, airplanes generate an opposing force called lift. Lift is generated by the motion of the airplane through the air and is an aerodynamic force. Lift is directed perpendicular to the flight direction. The magnitude of the lift depends on several factors including the shape, size, and velocity of the aircraft. As with weight, each part of the aircraft contributes to the aircraft lift force. Most of the lift is generated by the wings. Aircraft lift acts through a single point called the center of pressure. The center of pressure is defined just like the center of gravity, but using the pressure distribution around the body instead of the weight distribution.
- Drag: As the airplane moves through the air, there is another aerodynamic force present. The air resists the motion of the aircraft and the resistance force is called drag. Drag is directed along and opposed to the flight direction. Like lift, there are many factors that affect the magnitude of the drag force including the shape of the aircraft, the "stickiness" of the air, and the velocity of the aircraft. Like lift, we collect all of the individual components' drags and combine them into a single aircraft drag magnitude. And like lift, drag acts through the aircraft center of pressure.
- Thrust: To overcome drag, airplanes use a propulsion system to generate a force called thrust. The direction of the thrust force depends on how the engines are attached to the aircraft. The magnitude of the thrust depends on many factors associated with the propulsion system including the type of engine, the number of engines, and the throttle setting.

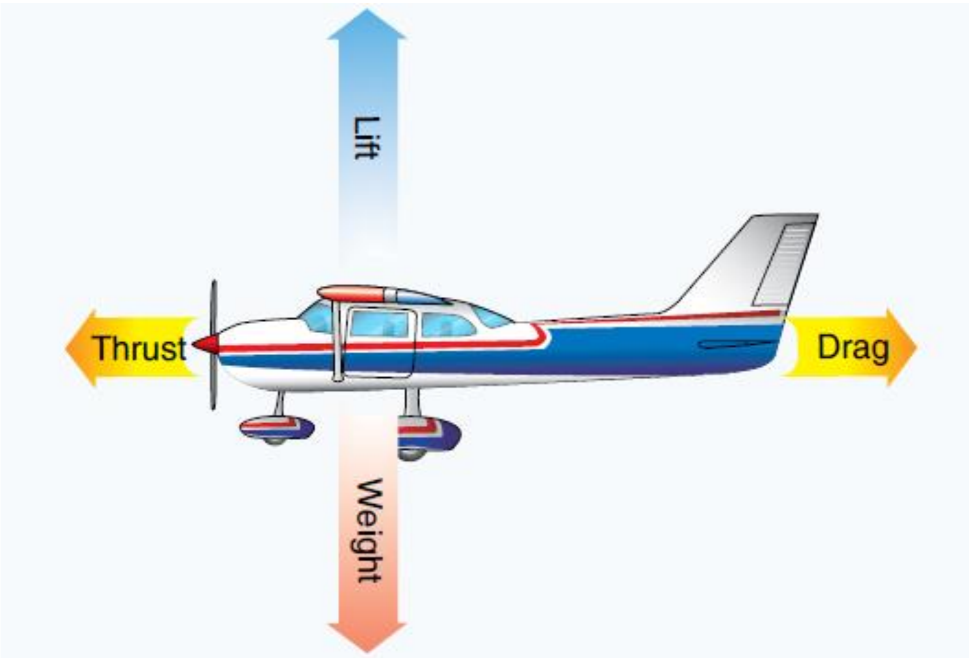

Figure 4-4 The four forces acting on an aircraft.

The motion of the airplane through the air depends on the relative strength and direction of the forces shown above. If the forces are balanced, the aircraft cruises at constant velocity. If the forces are unbalanced, the aircraft accelerates in the direction of the largest force. [17]

4.6 Stability and Control of an Aircraft

When an airplane is in straight-and-level flight at a constant velocity, all the forces acting on the airplane are in equilibrium. If that straight-and-level flight is disrupted by a disturbance in the air, such as wake turbulence, the airplane might pitch up or down, yaw left or right, or go into a roll. If the airplane has what is characterized as stability, once the disturbance goes away, the airplane will return to a state of equilibrium. Also, to achieve the best performance, the aircraft must have the proper response to the movement of the controls. Control is the pilot action of moving the flight controls, providing the aerodynamic force that induces the aircraft to follow a desired flightpath. When an aircraft is said to be controllable, it means that the aircraft responds easily and promptly to movement of the controls. Different control surfaces are used to control the aircraft about each of the three axes. Moving the control surfaces on an aircraft changes the airflow over the aircraft's surface. This, in turn, creates changes in the balance of forces acting to keep the aircraft flying straight and level. [18]

Static Stability

An aircraft is in a state of equilibrium when the sum of all the forces acting on the aircraft and all the moments is equal to zero. An aircraft in equilibrium experiences no accelerations, and the aircraft continues in a steady condition of flight. A gust of wind or a deflection of the controls disturbs the equilibrium, and the aircraft experiences acceleration due to the unbalance of moment or force.

The three types of static stability are defined by the character of movement following some disturbance from equilibrium. Positive static stability exists when the disturbed object tends to return to equilibrium. Negative static stability, or static instability, exists when the disturbed object tends to continue in the direction of disturbance. Neutral static stability exists when the disturbed object has neither tendency, but remains in equilibrium in the direction of disturbance. These three types of stability are illustrated in Figure 4.5.

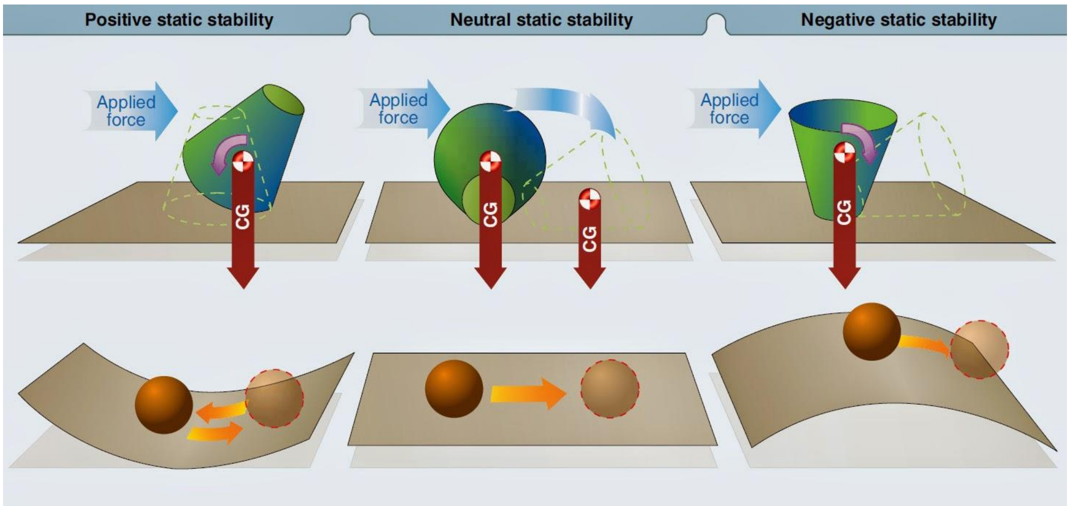

Figure 4-5 The Three types of stability of an aircraft

Dynamic Stability

While static stability deals with the tendency of a displaced body to return to equilibrium, dynamic stability deals with the resulting motion with time. If an object is disturbed from equilibrium, the time history of the resulting motion defines the dynamic stability of the object. In general, an object demonstrates positive dynamic stability if the amplitude of motion decreases with time. If the amplitude of motion increases with time, the object is said to possess dynamic instability.

Any aircraft must demonstrate the required degrees of static and dynamic stability. If an aircraft were designed with static instability and a rapid rate of dynamic instability, the aircraft would be very difficult, if not impossible, to fly. Usually, positive dynamic stability is required in an aircraft design to prevent objectionable continued oscillations of the aircraft.

Longitudinal Stability

When an aircraft has a tendency to keep a constant AOA (Angle of Attack) with reference to the relative wind (i.e., it does not tend to put its nose down and dive or lift its nose and stall); it is said to have longitudinal stability. Longitudinal stability refers to motion in pitch. The horizontal stabilizer is the primary surface which controls longitudinal stability. The action of the stabilizer depends upon the speed and AOA of the aircraft.

4.7 Motion of an Aircraft

The aircraft is assumed to be a rigid-body; the distance between any points on the aircraft do not change in flight. Thus, its motion can be considered to have six degrees of freedom. By applying Newton's Second Law to that rigid body the equations of motion

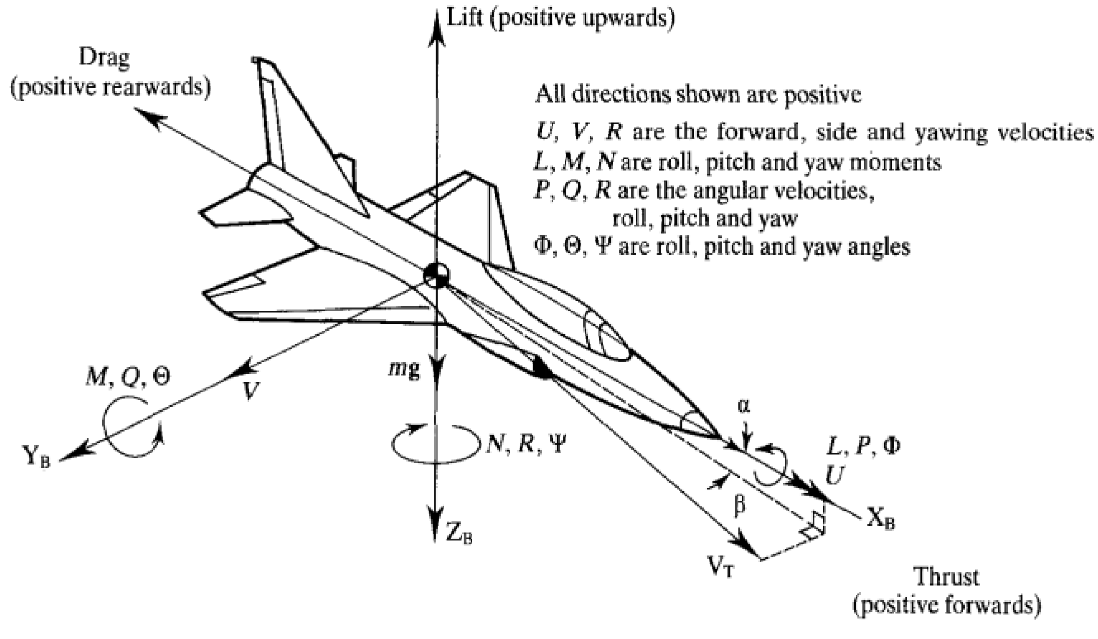

Figure 4-6 Body axis system.

can be established in terms of the translational and angular accelerations which occur as a consequence of some forces and moments being applied to the aircraft.

From Newton's Second Law:

$$F = \frac{d}{dt}(mV_T) \tag{4.1}$$

and

$$M = \frac{d}{dt}(H) \tag{4.2}$$

Where F represents the sum of all externally applied forces, M represents the sum of all applied torques, V_T the total velocity vector, and H is the angular momentum.

Chapter 5 Simulation and Results

5.1 Introduction

The nonlinear equations of motion of a typical fighter aircraft are used to generate linear perturbation models at various flight conditions [19]. Flight condition 1 represents the nominal cruise condition [Mach number, 0.67; altitude, 9096 m (20 000 ft); angle of attack, 3.45°].

The system chosen for the investigation of the state feedback multivariable control design based on similarity transformation in terms of feedback gain and robustness is obtained from the fighter aircraft state space model at flight condition 1 [19]. (NASA Technical Paper 1234)

The state space representation, referenced to the stability axes, takes the form of equation (1.1) as follows:

$$A = \begin{bmatrix} -3.79e+00 & 4.06e-01 & -5.20e+01 & 0. \\ -1.34e-01 & -3.59e-01 & 4.24e-01 & 0. \\ 6.02e-01 & -9.97e-01 & -2.72e-01 & 4.62e-02 \\ 1.00e+00 & 6.03e-01 & 0. & 0. \end{bmatrix}$$

$$B = \begin{bmatrix} 2.50e+01 & 9.83e+00 \\ 1.42e+00 & -4.20e+00 \\ 5.01e-03 & 5.03e-02 \\ 0. & 0. \end{bmatrix}$$

$$C = \begin{bmatrix} -1.25e-02 & -6.12e-02 & -3.41e+00 & -1.50e-03 \end{bmatrix}$$

$$D = \begin{bmatrix} 1.03e+00 & -2.66-01 \end{bmatrix}$$

and where $x(t) = \begin{pmatrix} p \\ \dot{\psi} \\ \beta \\ \Phi \end{pmatrix}$; $u(t) = \begin{pmatrix} \delta_a \\ \delta_r \end{pmatrix}$; $\mathrm{y(t)} = a_y$; p : Roll rate (deg); a_y: lateral acceleration(m/sec^2); δ_a: Aileron angular deflection (deg)t ; δ_r : Rudder angular deflection; $\dot{\psi}$: Yaw rate (deg/sec); β: sideslip angle (deg) and Φ : Bank angle (deg).

System eigenvalues are: $\lambda_1 = -3.70$, $\lambda_2 = -0.03$ and $\lambda_{3,4} = -0.34 \pm j2.66$.

The desired eigenvalues are: $\lambda_1 = -6.00$, $\lambda_2 = -0.01$ and $\lambda_{3,4} = -1.5 \pm j0.75$.

For more information about linearized equations of motion of an aircraft and the equations of motion in stability axis system, the reader may see [20].

5.2 State Feedback design using General Controller Canonical Form

The matrix *B* in the previous system has two columns. Thus, there are two possible permutations of the columns of *B*. Referring to the resulting matrices after permutation, B_1 and B_2, with $B_1 = (b_1\ b_2)$ and $B_2 = (b_2\ b_1)$, where b_1 is the first column of *B* and b_2 is the second column. From these two matrices and using the matrix A, there would be two reachability matrices $R_1(A, B_1)$ and $R_2(A, B_2)$.

$R_1 = (b_1\ b_2, Ab_1\ Ab_2)$ with reachability indices $K_1 = 2$ and $K_2 = 2$.

$R_2 = (b_2\ b_1, Ab_2\ Ab_1)$ with reachability indices $K_1 = 2$ and $K_2 = 2$.

5.2.1 State Feedback design using the reachability matrix R_1

First, the reachability base matrix obtained from *A* and B_1 is $P_1 = (b_1\ Ab_1\ b_2\ Ab_2)$.

Then, the similarity transformation $T_c = \begin{bmatrix} 0.0001 & -0.0008 & -0.0803 & 0.0403 \\ 0.0353 & 0.0828 & 0.0151 & -0.0037 \\ -0.0002 & 0.0021 & 0.2104 & -0.0010 \\ 0.0120 & -0.2106 & -0.0397 & 0.0097 \end{bmatrix}$

and its inverse are computed. After that, A_c, B_c and C_c are also computed, and are:

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.1872 & -3.7608 & -7.1298 & -1.3432 \\ 0 & 0 & 0 & 1 \\ -0.0808 & -0.0878 & -7.3003 & -0.6602 \end{bmatrix}$$

$$B_{1c} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C_c = [-0.4775 \quad -3.2290 \quad -16.3761 \quad -1.1432]$$

A_c is as expected in general controller form composed of two blocks in companion form of dimension 2 x 2 in the diagonal. Thus, there would be three ways of assigning the eigenvalues in the closed-loop system's matrix $[A_c - B_{1c}K_c]$ where:

$$K_c = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \end{bmatrix}$$

The three ways of assigning the eigenvalues are:

1. Putting the matrix $[A_c - B_{1c}K_c]$ in block diagonal form with the complex pair $\lambda_{3,4}$ in the upper block and the two other eigenvalues in the lower block.
2. Putting the matrix $[A_c - B_{1c}K_c]$ in block diagonal form with the complex pair $\lambda_{3,4}$ in the lower block and the two other eigenvalues in the upper block.
3. Putting the matrix $[A_c - B_{1c}K_c]$ in one companion form.

Now the eigenvalue λ_2 is the closest to the $j\omega$-axis. It's being pushed closer to and away from the imaginary axis to investigate the effect of this pole's placement.

The following tables summarizes the obtained state feedback gains from each of the previous stated ways and each with their resulting gains for each placement of the eigenvalue λ_2.

Case λ2 = -0.01:

Table 5-1 State feedback gains obtained from case λ_2 = -0.01 using R_1

	State feedback gain K_c	State feedback gain $K = K_cT_c$
1.	$K_{c1} = \begin{bmatrix} 2.6253 & -0.7608 & -7.1298 & -1.3432 \\ -0.0808 & -0.0878 & -7.2403 & 5.3498 \end{bmatrix}$	$K_1 = \begin{bmatrix} -0.0416 & 0.2024 & -1.6711 & 0.1039 \\ 0.0621 & -1.1491 & -1.7300 & 0.0565 \end{bmatrix}$
2.	$K_{c2} = \begin{bmatrix} -0.1272 & 2.2492 & -7.1298 & -1.3432 \\ -0.0808 & -0.0878 & -4.4878 & 2.3398 \end{bmatrix}$	$K_2 = \begin{bmatrix} 0.0645 & 0.4539 & -1.4023 & -0.0192 \\ 0.0256 & -0.5095 & -1.0317 & 0.0244 \end{bmatrix}$
3.	$K_{c3} = \begin{bmatrix} -0.1872 & -3.7608 & -8.1298 & -1.3432 \\ 0.08795 & 16.9953 & 13.6022 & 8.3498 \end{bmatrix}$	$K_3 = \begin{bmatrix} -0.1475 & -0.0457 & -1.6988 & 0.0017 \\ 0.6976 & -0.3221 & 2.7804 & 0.0076 \end{bmatrix}$

Case λ2 = -0.001:

Table 5-2 State feedback gains obtained from case λ_2 = -0.001 using R_1

	State feedback gain K_c	State feedback gain $K = K_cT_c$
1.	$K_{c1} = \begin{bmatrix} 2.6253 & -0.7608 & -7.1298 & -1.3432 \\ -0.0808 & -0.0878 & -7.2943 & 5.3408 \end{bmatrix}$	$K_1 = \begin{bmatrix} -0.0416 & 0.2024 & -1.6711 & 0.1039 \\ 0.0620 & -1.1473 & -1.7410 & 0.0565 \end{bmatrix}$
2.	$K_{c2} = \begin{bmatrix} -0.1812 & 2.2402 & -7.1298 & -1.3432 \\ -0.0808 & -0.0878 & -4.4878 & 2.3398 \end{bmatrix}$	$K_2 = \begin{bmatrix} 0.0641 & 0.4532 & -1.3981 & -0.0213 \\ 0.0256 & -0.5095 & -1.0317 & 0.0244 \end{bmatrix}$
3.	$K_{c3} = \begin{bmatrix} -0.1872 & -3.7608 & -8.1298 & -1.3432 \\ -0.0639 & 16.8080 & 13.5212 & 8.3408 \end{bmatrix}$	$K_3 = \begin{bmatrix} -0.1475 & -0.0457 & -1.6988 & 0.0017 \\ 0.6908 & -0.3358 & 2.7731 & 0.0022 \end{bmatrix}$

Case λ2 = -0. 1

Table 5-3 State feedback gains obtained from case λ_2 = -0.1 using R_1

	State feedback gain K_c	State feedback gain $K = K_cT_c$
1.	$K_{c1} = \begin{bmatrix} 2.6230 & -0.7608 & -7.1298 & -1.3432 \\ -0.0808 & -0.0878 & -6.7030 & 5.4398 \end{bmatrix}$	$K_1 = \begin{bmatrix} -0.0416 & 0.2024 & -1.6711 & 0.1039 \\ 0.0631 & -1.1669 & -1.6206 & 0.0568 \end{bmatrix}$
2.	$K_{c2} = \begin{bmatrix} 0.4128 & 2.3392 & -7.1298 & -1.3432 \\ -0.0808 & -0.0878 & -4.4878 & 2.3398 \end{bmatrix}$	$K_2 = \begin{bmatrix} 0.0677 & 0.4509 & -1.4443 & 0.0022 \\ 0.0256 & -0.5095 & -1.0317 & 0.0244 \end{bmatrix}$
3.	$K_{c3} = \begin{bmatrix} -0.1872 & -3.7608 & -8.1298 & -1.3432 \\ 1.6067 & 18.8682 & 14.4122 & 8.4398 \end{bmatrix}$	$K_3 = \begin{bmatrix} -0.1475 & -0.0457 & -1.6988 & 0.0017 \\ 0.7647 & -0.1855 & 2.8536 & 0.0619 \end{bmatrix}$

5.2.2 State Feedback design using the reachability matrix R_2

The previous process is repeated using the reachability matrix R_2

Case λ2 = -0.01

Table 5-4 State feedback gains obtained from case λ_2 = -0.01 using R_2

	State feedback gain K_c	State feedback gain $K = K_c T_c$
1.	$K_{c1} = \begin{bmatrix} -4.4878 & 2.3389 & -0.0808 & -0.0878 \\ -7.1298 & -1.3422 & -0.1272 & 2.2492 \end{bmatrix}$	$K_1 = \begin{bmatrix} 0.0256 & -0.5093 & -1.0316 & 0.0244 \\ 0.0645 & 0.4537 & -1.4023 & -0.0192 \end{bmatrix}$
2.	$K_{c2} = \begin{bmatrix} -7.2403 & 5.3498 & -0.0808 & -0.0878 \\ -7.1298 & -1.3422 & 2.6253 & -0.7608 \end{bmatrix}$	$K_2 = \begin{bmatrix} 0.0621 & -1.1491 & -1.7300 & 0.0565 \\ -0.0416 & 0.2023 & -1.6689 & 0.1028 \end{bmatrix}$
3.	$K_{c3} = \begin{bmatrix} -7.3003 & -0.6602 & -1.0808 & -0.0878 \\ -6.9610 & 15.7399 & 20.715 & 5.2492 \end{bmatrix}$	$K_3 = \begin{bmatrix} -0.0099 & 0.1170 & -1.4240 & -0.0421 \\ 0.3761 & -0.9114 & -3.6725 & 0.9746 \end{bmatrix}$

Case λ2 = -0.001

Table 5-5 State feedback gains obtained from case λ_2 = -0.001 using R_2

	State feedback gain K_c	State feedback gain $K = K_c T_c$
1.	$K_{c1} = \begin{bmatrix} -4.4878 & 2.3389 & -0.0808 & -0.0878 \\ -7.1298 & -1.3422 & -0.1812 & 2.2402 \end{bmatrix}$	$K_1 = \begin{bmatrix} 0.0256 & -0.5093 & -1.0316 & 0.0244 \\ 0.0645 & 0.4530 & -1.3981 & -0.0213 \end{bmatrix}$
2.	$K_{c2} = \begin{bmatrix} -7.2943 & 5.3408 & -0.0808 & -0.0878 \\ -7.1298 & -1.3422 & 2.6253 & -0.7608 \end{bmatrix}$	$K_2 = \begin{bmatrix} 0.0620 & -1.1473 & -1.7410 & 0.0565 \\ -0.0416 & 0.2023 & -1.6689 & 0.1028 \end{bmatrix}$
3.	$K_{c3} = \begin{bmatrix} -7.3003 & -0.6602 & -1.0808 & -0.0878 \\ -7.1129 & 15.5526 & 20.6343 & 5.2407 \end{bmatrix}$	$K_3 = \begin{bmatrix} -0.0099 & 0.1170 & -1.4240 & -0.0421 \\ 0.3735 & -2.8729 & -3.6907 & 0.9697 \end{bmatrix}$

Case λ_2 = -0.1

Table 5-6 State feedback gains obtained from case λ_2 = -0.1 using R_2

	State feedback gain K_c	State feedback gain $K = K_c T_c$
1.	$K_{c1} = \begin{bmatrix} -4.4878 & 2.3389 & -0.0808 & -0.0878 \\ -7.1298 & -1.3422 & -0.4128 & 2.3392 \end{bmatrix}$	$K_1 = \begin{bmatrix} 0.0256 & -0.5093 & -1.0316 & 0.0244 \\ 0.0677 & 0.4607 & -1.4443 & -0.0022 \end{bmatrix}$
2.	$K_{c2} = \begin{bmatrix} -6.7003 & 5.4398 & -0.0808 & -0.0878 \\ -7.1298 & -1.3422 & 2.6253 & -0.7608 \end{bmatrix}$	$K_2 = \begin{bmatrix} 0.0631 & -1.1669 & -1.6200 & 0.0568 \\ -0.0416 & 0.2023 & -1.6689 & 0.1028 \end{bmatrix}$
3.	$K_{c3} = \begin{bmatrix} -7.3003 & -0.6602 & -1.0808 & -0.0878 \\ -5.4423 & 17.6128 & 21.5253 & 5.3397 \end{bmatrix}$	$K_3 = \begin{bmatrix} -0.0099 & 0.1170 & -1.4240 & -0.0421 \\ 0.4015 & -3.2957 & -3.4910 & 1.0235 \end{bmatrix}$

5.2.3 Robustness and sensitivity analysis

After finding all the gain matrices, the individual and overall eigenvalue sensitivities, robust performances and stability measures M_2 and M_3 (for λ_2) are computed.

$S(\lambda_i)$ refers to the sensitivity of the eigenvalue λ_i, while *S(V)* refers to the overall EV sensitivity.

RP refers to the robust performance and EV refers to eigenvalue.

M_2 and M_3 refer to stability measures 2 and 3 respectively.

Robust Stability

Using the reachability matrix R_1 and the state feedback gain K_{c1}, the resulting closed-loop matrix is $[A - B_1K_1] = \begin{bmatrix} -3.3606 & 6..2755 & 6.7835 & -3.1532 \\ 0.1859 & -5.4728 & -0.6531 & 0.0898 \\ 0.0573 & -0.9402 & -0.1766 & 0.0428 \\ 1.000 & 0.0603 & 0 & 0 \end{bmatrix}$

The right eigenvector associated to this closed-loop matrix is

$$V = \begin{bmatrix} 0.9097 & 0.8592 & 0.8592 & 0.0017 \\ -0.3817 & 0.0282 - 0.0105i & 0.0282 + 0.0105i & -0.0426 \\ -0.0695 & -0.0018 - 0.0010i & -0.0018 + 0.0010i & 0.4481 \\ -0.1478 & -0.4548 - 0.2325i & -0.4548 + 0.2325i & 0.8930 \end{bmatrix}$$

- The norm of this eigenvector is $||V|| = 1.7071$, while the norm of the left eigenvector is

$||T|| = ||V^{-1}|| = 7.4762$. Then the overall sensitivity is equal to:

$$S(V) = ||V|| \cdot ||V^{-1}|| = 12.7625$$

- The norms of the component vectors of the right eigenvector are all equal to 1, and the component vectors of the left eigenvector have norms as follows:

$||t_1|| = 2.3197,$ $||t_2|| = 2.2170,$ $||t_3|| = 5.2496,$ $||t_4|| = 5.2496$

- The sensitivities of the individual eigenvalue are:

$s(\lambda_1) = ||v_1|| \cdot ||t_1|| = 2.3197, s(\lambda_2) = ||v_2|| \cdot ||t_2|| = 2.2170,$

$$s(\lambda_{3/4}) = ||v_{3,4}|| \cdot ||t_{3,4}|| = 5.2496$$

- The stability measures are:

  $M_2 = (S(V))^{-1}|Re\{\lambda_2\}| = 7.83\text{e} - 04$

  $M_3 = min_{0\leq i\leq 4}\left\{\left(S(\lambda_i)\right)^{-1}|Re\{\lambda_i\}|\right\} = 4.51\text{e} - 03$

Robust Performance

The following perturbation matrix is $A' = \begin{bmatrix} 0.0042 & 0.0066 & 0.0068 & 0.0066 \\ 0.0092 & 0.0004 & 0.0076 & 0.0017 \\ 0.0079 & 0.0085 & 0.0074 & 0.0071 \\ 0.0096 & 0.0093 & 0.0039 & 0.0003 \end{bmatrix}$

generated randomly using MATLAB

The new closed-loop matrix, after perturbation, is:

$$[A - B_1K_1 + A'] = \begin{bmatrix} -3.3554 & 6..2646 & 6.8984 & -3.1463 \\ 0.1947 & -5.4649 & -0.6917 & 0.0914 \\ 0.0652 & -0.9318 & -0.1687 & 0.0499 \\ 1.0096 & 0.0696 & 0.0039 & 0.0003 \end{bmatrix}$$

its eigenvalues are: $\lambda_1 = -6.0173$, $\lambda_2 = 0.0179$ and $\lambda_{3,4} = -1.4946 \pm j0.7859$

The relative change of the eigenvalues of the closed-loop matrix due to the perturbation is $r_i = \left|\frac{\lambda_i - \lambda_i{}'}{\lambda_i}\right|$ were λ_i is the eigenvalue of the closed-loop matrix and $\lambda_i{}'$ the eigenvalue of the perturbated closed-loop matrix. This lead

$r_1 = 0.0029$, $r_2 = 0.0031$ and $r_{3,4} = 0.031$

The previous computations are repeated for each case input matrix B_1 and B_2, state feedback gains, K_1, K_2 and K_3 and the three different values of λ_2. The results are summarized in the following tables.

Case $\boldsymbol{\lambda_2}$ = -0.01

Table 5-7 General controller form from R_1 results summary

Used input matrix			B1		
Gain			K_1	K_2	K_3
$\|\|K\|\|$			2.5190	1.7436	3.3370
S(V)			12.7625	12.7996	42.9231
Robust stability	$s(\lambda_i)$	-6.0	2.3197	2.3189	12.7814
		-0.01	2.2170	2.2225	2.9699
		-1.5+j0.75	5.2496	5.2434	15.2784
		-1.5-j0.75	5.2496	5.2434	15.2784
	M_2		7.8354e-04	7.812e-04	2.3297e-04
	M_3		0.00451	0.00449	0.003367
Robust performance	$r_i(\lambda_i)$	-6.0	0.0029	0.0039	0.0064
		-0.01	0.0031	0.0021	0.0035
		-1.5+0.75i	0.0031	0.0025	0.0068
		-1.5-0.75i	0.0031	0.0025	0.0068

Table 5-8 General controller form from R_2 results summary

Used input matrix			B2		
Gain			K_1	K_2	K_3
$\|\|K\|\|$			1.7436	2.5177	4.9110
S(V)			12.7781	12.9779	88.9438
Robust stability	$s(\lambda_i)$	-6.0	2.3176	2.3211	28.3074
		-0.01	2.2244	2.7170	3.0646
		-1.5+j0.75	5.2349	5.3362	30.4233
		-1.5-j0.75	5.2349	5.3362	30.4233
	M_2		7.8258-04	7.705e-04	1.1243e-04
	M_3		0.004495	0.0036	0.00326
Robust performance	$r_i(\lambda_i)$	-6.0	0.0039	0.0029	0.0365
		-0.01	0.0021	0.0031	0.0002
		-1.5+0.75i	0.0025	0.0032	0.0278
		-1.5-0.75i	0.0025	0.0032	0.0278

Case λ_2 = -0.001

Table 5-9 General controller form from R_1 results summary

Used input matrix			B1		
Gain			K_1	K_2	K_3
$\|\|K\|\|$			2.5268	1.7401	3.3306
S(V)			12.7424	12.7786	42.7129
Robust stability	$s(\lambda_i)$	-6.0	2.3201	2.3189	12.7518
		-0.001	2.2140	2.2217	2.9368
		-1.5+j0.75	5.2496	5.2434	15.2173
		-1.5-j0.75	5.2496	5.2434	15.2173
	M_2		7.8478e-05	7.8255e-05	2.3412e-05
	M_3		4.5167e-04	4.5010e-04	3.4050e-04
Robust performance	$r_i(\lambda_i)$	-6.0	0.0029	0.0039	0.0063
		-0.001	0.0031	0.0021	0.0036
		-1.5+0.75i	0.0031	0.0025	0.0067
		-1.5-0.75i	0.0031	0.0025	0.0067

Table 5-10 General controller form from R_2 results summary

Used input matrix			B2		
Gain			K_1	K_2	K_3
$\|\|K\|\|$			1.7401	2.5256	4.9026
S(V)			12.7571	12.9574	88.1914
Robust stability	$s(\lambda_i)$	-6.0	2.3176	2.3215	28.1276
		-0.001	2.2209	2.2140	3.0305
		-1.5+j0.75	5.2349	5.3362	30.1930
		-1.5-j0.75	5.2349	5.3362	30.1930
	M_2		7.8387e-05	7.7175e-05	1.1338e-05
	M_3		4.5026e-04	4.5167e-04	3.2997e-04
Robust performance	$r_i(\lambda_i)$	-6.0	0.0039	0.0029	0.0363
		-0.001	0.0021	0.0031	0.0002
		-1.5+0.75i	0.0025	0.0032	0.0276
		-1.5-0.75i	0.0025	0.0032	0.0276

Case $\lambda_2 = -0.1$

Table 5-11 General controller form from R_1 results summary

Used input matrix			B1		
Gain			K_1	K_2	K_3
$\|\|K\|\|$			2.4416	1.7784	3.4045
S(V)			12.9784	13.0239	45.1850
Robust stability	$s(\lambda_i)$	-6.0	2.3254	2.3290	13.0940
		-0.1	2.2573	2.2702	3.3552
		-1.5+j0.75	5.2496	5.2434	15.9371
		-1.5-j0.75	5.2496	5.2434	15.9371
	M_2		7.7051e-03	7.6781e-03	2.2131e-03
	M_3		0.04430	0.04404	0.02980
Robust performance	$r_i(\lambda_i)$	-6.0	0.0028	0.0039	0.0073
		-0.1	0.0031	0.0021	0.0032
		-1.5+0.75i	0.0031	0.0025	0.0077
		-1.5-0.75i	0.0031	0.0025	0.0077

Table 5-12 General controller form from R_2 results summary

Used input matrix			B2		
Gain			K_1	K_2	K_3
$\|\|K\|\|$			1.7784	2.4399	5.0149
S(V)			13.0017	13.1985	96.9013
Robust stability	$s(\lambda_i)$	-6.0	2.3276	2.3269	30.1647
		-0.1	2.2693	2.2578	3.5143
		-1.5+j0.75	5.2347	5.3367	32.8359
		-1.5-j0.75	5.2347	5.3367	32.8359
	M_2		7.6913e-03	7.5766e-03	1.0319e-03
	M_3		0.04406	0.04429	0.02845
Robust performance	$r_i(\lambda_i)$	-6.0	0.0039	0.0028	0.0382
		-0.1	0.0021	0.0031	0.0005
		-1.5+0.75i	0.0025	0.0032	0.0295
		-1.5-0.75i	0.0025	0.0032	0.0295

5.2.4 Time Response

The step response to both inputs of the system are shown in the next figures for all three cases of λ_2 and the information are show in the tables.

Remarks about the figures and tables:

- The response due to the first input (b_1) is shown on the left while the response due to the second input (b_2) is shown on the right.
- For each case of λ_2 there are six resulting state feedback gains. Thus, there are six closed loop systems. These systems are labeled as follows:

  GB1Kd1: Closed loop system obtained using the input matrix B1 and the state feedback gain K_1

  GB2Kd2: Closed loop system obtained using the input matrix B1 and the state feedback gain K_2

  GB1Kc: Closed loop system obtained using the input matrix B1 and the state feedback gain K_3

  GB2Kd1: Closed loop system obtained using the input matrix B2 and the state feedback gain K_1

  GB2Kd2: Closed loop system obtained using the input matrix B2 and the state feedback gain K_2

  GB2Kc: Closed loop system obtained using the input matrix B2 and the state feedback gain K_3

- The response for the original system is also shown with its information and is referred to as G.

- RiseTime — Time it takes for the response to rise from 10% to 90% of the steady-state response.
- SettlingTime — Time it takes for the error $|y(t) - y_{final}|$ between the response $y(t)$ and the steady-state response y_{final} to fall to within 2% of y_{final}.
- SettlingMin — Minimum value of $y(t)$ once the response has risen.
- SettlingMax — Maximum value of $y(t)$ once the response has risen.
- Overshoot — Percentage overshoot, relative to y_{final}).
- Undershoot — Percentage undershoot.
- Peak — Peak absolute value of $y(t)$

- PeakTime — Time at which the peak value occurs.

Case λ_2 = -0.01

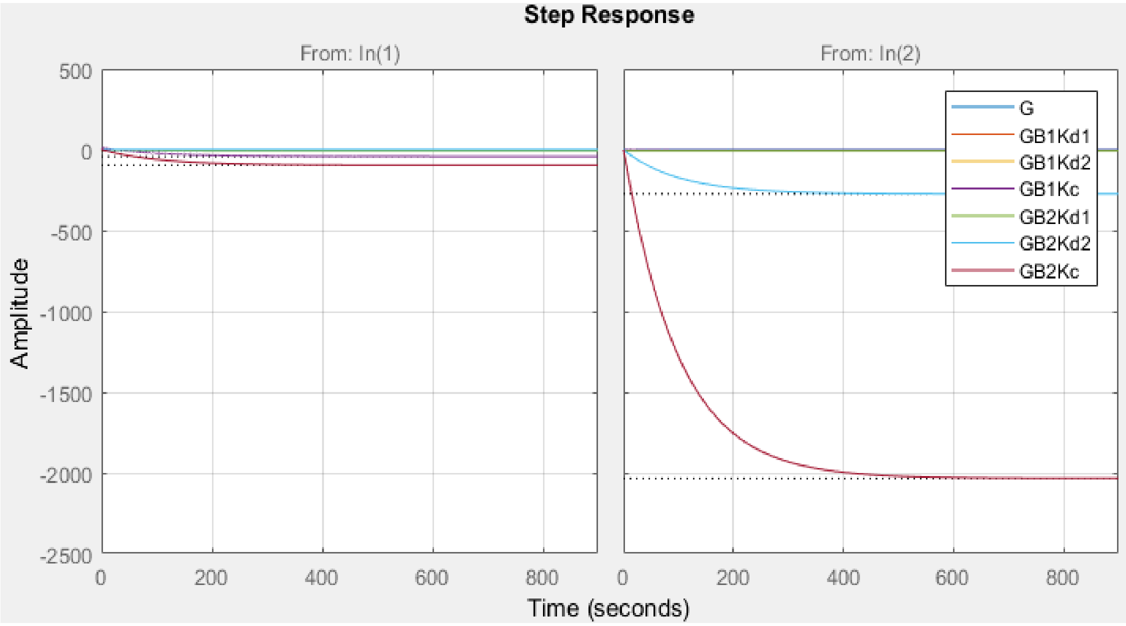

Figure 5-1 response due to both inputs Case λ_2 = -0.01

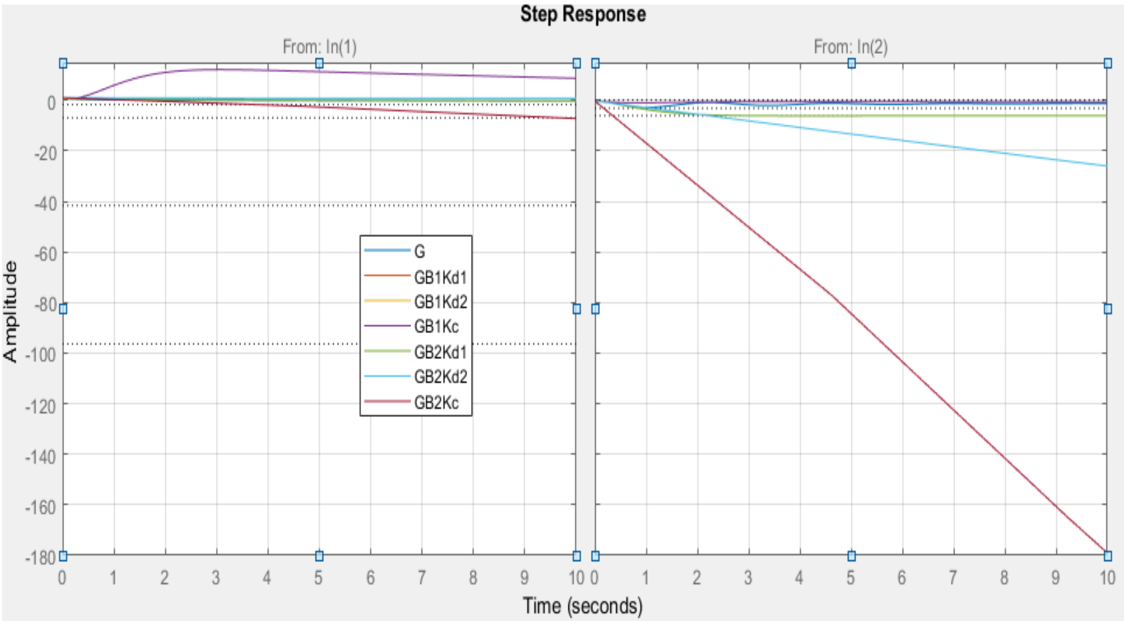

Figure 5-2 response due to both inputs (zoomed in) Case λ_2 = -0.01

Table 5-13 time response specifications Case $\lambda_2 = -0.01$

System	Step info for S (1,1)	Step info for S (1,2)
G	RiseTime: 65.3146 SettlingTime: 119.4925 SettlingMin: -1.7035 SettlingMax: -1.4317 Overshoot: 0 Undershoot: 60.4029 Peak: 1.7035 PeakTime: 235.2944	RiseTime: 73.8259 SettlingTime: 116.5274 SettlingMin: 0.1193 SettlingMax: 0.1604 Overshoot: 0 Undershoot: 1.6943e+03 Peak: 2.7468 PeakTime: 1.0448
GB1Kd1	RiseTime: 3.6182 SettlingTime: 33.8363 SettlingMin: 0.8623 SettlingMax: 0.8664 Overshoot: 18.8766 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 219.7776 SettlingTime: 391.4370 SettlingMin: -273.2897 SettlingMax: -247.1973 Overshoot: 0 Undershoot: 0 Peak: 273.2897 PeakTime: 1.0549e+03
GB1Kd2	RiseTime: 219.6539 SettlingTime: 384.2402 SettlingMin: -6.9189 SettlingMax: -6.1260 Overshoot: 0 Undershoot: 14.8828 Peak: 6.9189 PeakTime: 828.8830	RiseTime: 1.6963 SettlingTime: 2.6923 SettlingMin: -6.0996 SettlingMax: -5.5304 Overshoot: 0.1788 Undershoot: 0 Peak: 6.0996 PeakTime: 4.1140

GB1Kc	RiseTime: 219.1488 SettlingTime: 393.6142 SettlingMin: -41.6059 SettlingMax: -37.3552 Overshoot: 0 Undershoot: 29.2464 Peak: 41.6059 PeakTime: 844.8479	RiseTime: 224.0568 SettlingTime: 384.7843 SettlingMin: -3.0893 SettlingMax: -2.8076 Overshoot: 0 Undershoot: 0 Peak: 3.0893 PeakTime: 844.8479
GB2Kd1	RiseTime: 219.6539 SettlingTime: 384.2402 SettlingMin: -6.9189 SettlingMax: -6.1260 Overshoot: 0 Undershoot: 14.8828 Peak: 6.9189 PeakTime: 829.7460	RiseTime: 1.6957 SettlingTime: 2.6908 SettlingMin: -6.0997 SettlingMax: -5.5317 Overshoot: 0.1801 Undershoot: 0 Peak: 6.0997 PeakTime: 4.1153
GB2Kd2	RiseTime: 3.6118 SettlingTime: 33.8469 SettlingMin: 0.8606 SettlingMax: 0.8648 Overshoot: 19.0980 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 219.7776 SettlingTime: 391.4370 SettlingMin: -273.2897 SettlingMax: -247.1973 Overshoot: 0 Undershoot: 0 Peak: 273.2897 PeakTime: 1.0549e+03
GB2Kc	RiseTime: 220.3227 SettlingTime: 393.4654 SettlingMin: -96.2824 SettlingMax: -86.8828 Overshoot: 0 Undershoot: 1.0697 Peak: 96.2824 PeakTime: 1.0575e+03	RiseTime: 220.3172 SettlingTime: 393.0509 SettlingMin: -2.0340e+03 SettlingMax: -1.8383e+03 Overshoot: 0 Undershoot: 0 Peak: 2.0340e+03 PeakTime: 1.0575e+03

Case $\lambda_2 = -0.001$

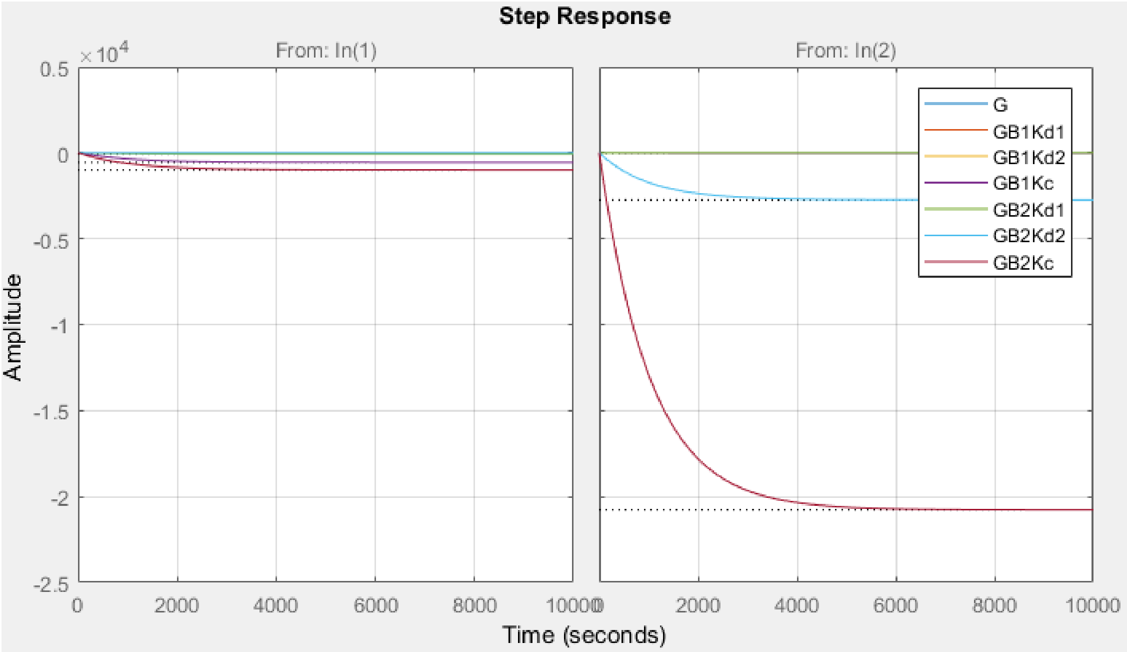

Figure 5-3 response due to both inputs case $\lambda_2 = -0.001$

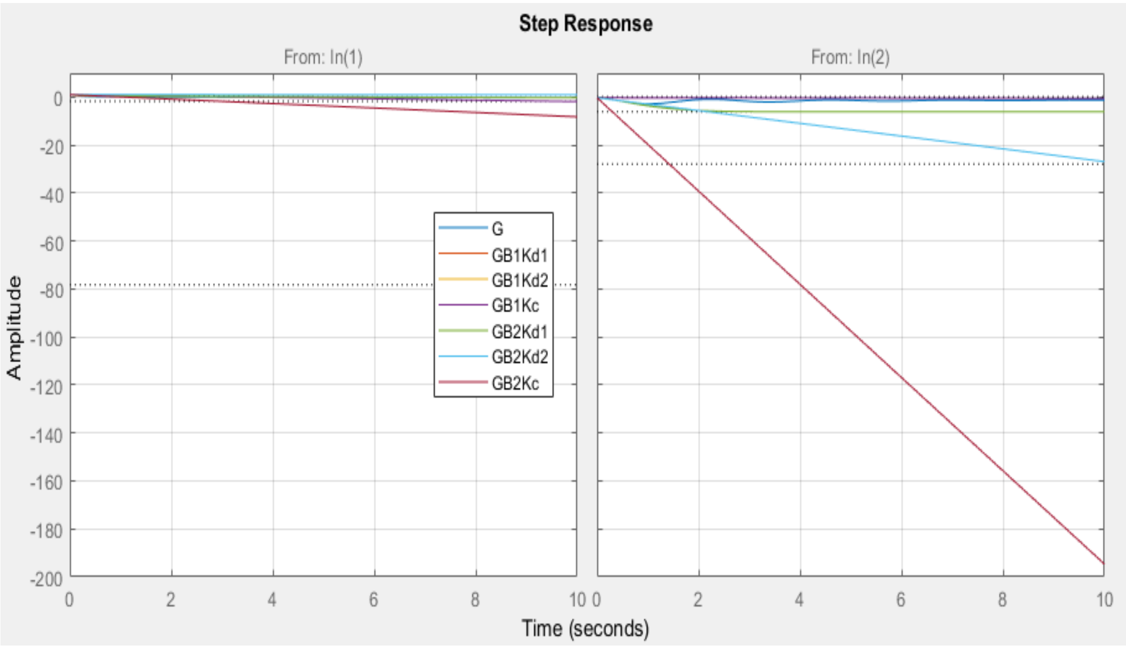

Figure 5-4 response due to both inputs (zoomed in) case $\lambda_2 = -0.001$

Table 5-14 time response specifications Case $\lambda_2 = -0.001$

System	Step info for S (1,1)	Step info for S (1,2)
GB1Kd1	RiseTime: 27.2481 SettlingTime: 2.9382e+03 SettlingMin: 0.8639 SettlingMax: 0.9076 Overshoot: 13.4868 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 2.2047e+03 SettlingTime: 3.9259e+03 SettlingMin: -2.7392e+03 SettlingMax: -2.4777e+03 Overshoot: 0 Undershoot: 0 Peak: 2.7392e+03 PeakTime: 1.0583e+04
GB1Kd2	RiseTime: 2.1902e+03 SettlingTime: 3.8930e+03 SettlingMin: -78.2311 SettlingMax: -70.3213 Overshoot: 0 Undershoot: 1.3163 Peak: 78.2311 PeakTime: 8.3546e+03	RiseTime: 1.7073 SettlingTime: 2.7012 SettlingMin: -6.0995 SettlingMax: -5.5846 Overshoot: 0.1650 Undershoot: 0 Peak: 6.0995 PeakTime: 4.0397
GB1Kc	RiseTime: 2.1407e+03 SettlingTime: 3.8337e+03 SettlingMin: -556.5080 SettlingMax: -502.3746 Overshoot: 0 Undershoot: 0.1850 Peak: 556.5080 PeakTime: 7.1335e+03	RiseTime: 2.1406e+03 SettlingTime: 3.8058e+03 SettlingMin: -27.8304 SettlingMax: -25.1058 Overshoot: 0 Undershoot: 0 Peak: 27.8304 PeakTime: 7.1335e+03

GB2Kd1	RiseTime: 2.1902e+03 SettlingTime: 3.8930e+03 SettlingMin: -78.2311 SettlingMax: -70.3213 Overshoot: 0 Undershoot: 1.3163 Peak: 78.2311 PeakTime: 8.3546e+03	RiseTime: 1.7067 SettlingTime: 2.6997 SettlingMin: -6.0996 SettlingMax: -5.5853 Overshoot: 0.1664 Undershoot: 0 Peak: 6.0996 PeakTime: 4.0397
GB2Kd2	RiseTime: 27.2481 SettlingTime: 2.9382e+03 SettlingMin: 0.8623 SettlingMax: 0.9064 Overshoot: 13.6377 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 2.2047e+03 SettlingTime: 3.9259e+03 SettlingMin: -2.7392e+03 SettlingMax: -2.4777e+03 Overshoot: 0 Undershoot: 0 Peak: 2.7392e+03 PeakTime: 1.0583e+04
GB2Kc	RiseTime: 2.2598e+03 SettlingTime: 4.0250e+03 SettlingMin: -997.1370 SettlingMax: -901.7286 Overshoot: 0 Undershoot: 0.1033 Peak: 997.1370 PeakTime: 1.0847e+04	RiseTime: 2.2598e+03 SettlingTime: 4.0246e+03 SettlingMin: -2.0784e+04 SettlingMax: -1.8798e+04 Overshoot: 0 Undershoot: 0 Peak: 2.0784e+04 PeakTime: 1.0847e+04

Case $\lambda_2 = -0.1$

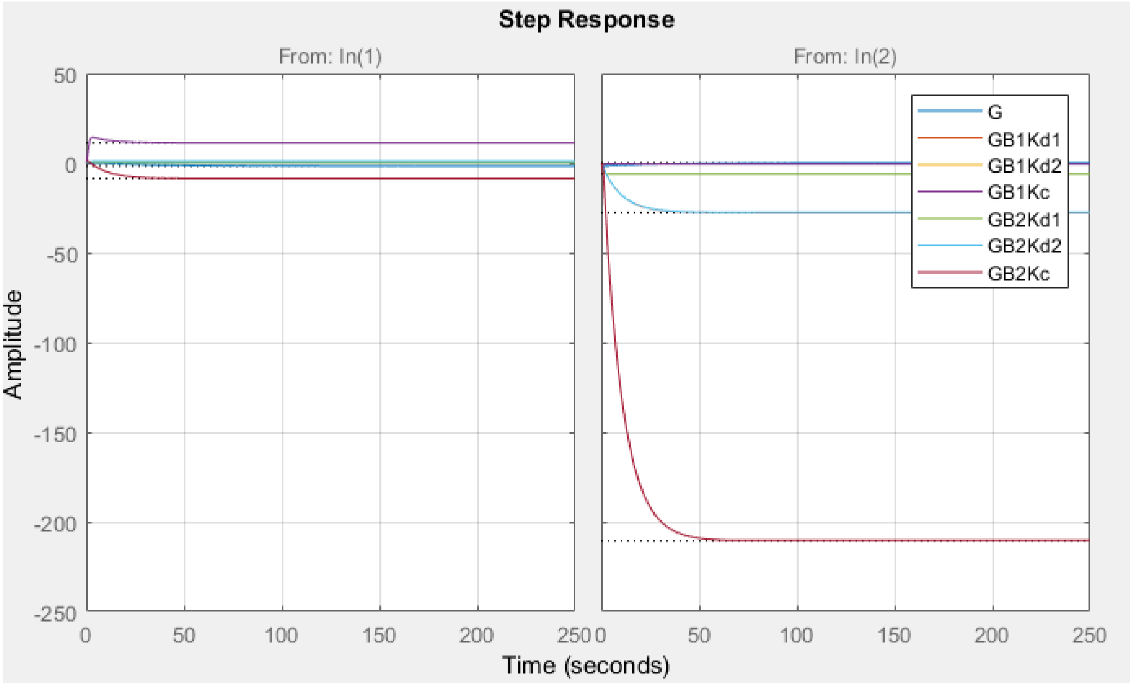

Figure 5-5 response due to both inputs case λ_2 = -0.1

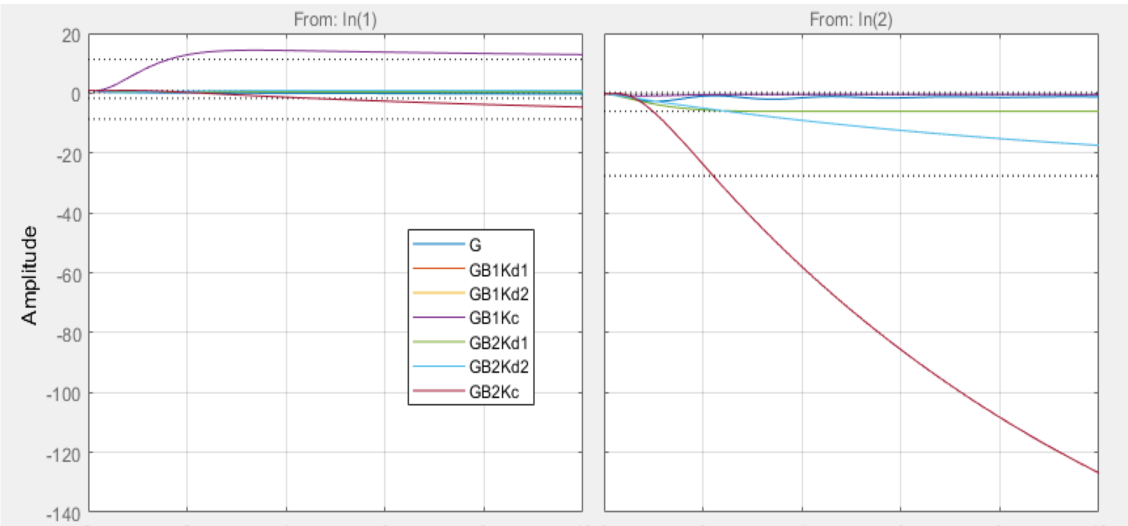

Figure 5-6 response due to both inputs (zoomed in) case λ_2 = -0.1

Table 5-15 time response specifications case $\lambda_2 = -0.1$

System	Step info for S (1,1)	Step info for S (1,2)
GB1Kd1	RiseTime: 0.0453 SettlingTime: 3.4087 SettlingMin: 0.2172 SettlingMax: 0.8634 Overshoot: 19.4416 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 22.0740 SettlingTime: 39.3992 SettlingMin: -27.6832 SettlingMax: -24.9531 Overshoot: 0 Undershoot: 0 Peak: 27.6832 PeakTime: 106.7768
GB1Kd2	RiseTime: 11.8748 SettlingTime: 27.9958 SettlingMin: 0.2351 SettlingMax: 0.3141 Overshoot: 338.8631 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 1.6957 SettlingTime: 2.6918 SettlingMin: -6.0996 SettlingMax: -5.5304 Overshoot: 0.1800 Undershoot: 0 Peak: 6.0996 PeakTime: 4.1140
GB1Kc	RiseTime: 1.0015 SettlingTime: 30.5550 SettlingMin: 10.3933 SettlingMax: 14.3456 Overshoot: 27.4039 Undershoot: 0 Peak: 14.3456 PeakTime: 3.3772	RiseTime: 0.1629 SettlingTime: 33.8965 SettlingMin: -0.9083 SettlingMax: -0.4086 Overshoot: 65.4233 Undershoot: 0 Peak: 0.9083 PeakTime: 0.7982
GB2Kd1	RiseTime: 11.8748 SettlingTime: 27.9958 SettlingMin: 0.2351 SettlingMax: 0.3141 Overshoot: 338.8631 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 1.6950 SettlingTime: 2.6902 SettlingMin: -6.0997 SettlingMax: -5.5317 Overshoot: 0.1816 Undershoot: 0 Peak: 6.0997 PeakTime: 4.1153

GB2Kd2	RiseTime: 0.0458 SettlingTime: 3.4527 SettlingMin: 0.2155 SettlingMax: 0.8616 Overshoot: 19.6710 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 21.9729 SettlingTime: 39.2194 SettlingMin: -27.5076 SettlingMax: -24.8375 Overshoot: 0 Undershoot: 0 Peak: 27.5076 PeakTime: 62.5679
GB2Kc	RiseTime: 21.9607 SettlingTime: 40.3163 SettlingMin: -8.6586 SettlingMax: -7.7103 Overshoot: 0 Undershoot: 11.8703 Peak: 8.6586 PeakTime: 63.8077	RiseTime: 21.9520 SettlingTime: 39.9084 SettlingMin: -210.1792 SettlingMax: -189.5572 Overshoot: 0 Undershoot: 0 Peak: 210.1792 PeakTime: 63.8077

5.2.5 Discussion

The discussion is based on the following criteria:

- Feedback gain magnitude.
- EV sensitivity (individual and overall).
- Stability measure M_1 and M_2.
- Relative change.

The case where the matrix $[A_c - B_{1c}K_c]$ is put in block diagonal form with the complex pair $\lambda_{3,4}$ in the upper block and the two other eigenvalues in the lower block is referred to as the upper block case from the two block form.

The case where the matrix $[A_c - B_{1c}K_c]$ is put in block diagonal form with the complex pair $\lambda_{3,4}$ in the lower block and the two other eigenvalues in the upper block is referred to as the lower block case from the two block from.

The case where matrix $[A_c - B_{1c}K_c]$ is put in one companion form is referred to as the one block form or one main block.

Case λ_2 = -0. 01

Feedback magnitude gain

The gain magnitude is found to be smaller when choosing the two blocks form (two blocks in the diagonal) rather than only one main block, since the gain ended being bigger in both permutations B1 and B2.

In the first permutation (B1), the smallest gain was K_2 = 1.7436; when the complex pair was chosen to be in the lower block of the two-block form feedback gain matrix. This smallest gain happens to be the same in the second permutation (B2), but in this case, it was K_1 = 1.7436; when the complex pair was chosen to be in the upper block of the two-block form feedback gain matrix.

Individual and Overall eigenvalue sensitivity

Here also the individual and overall EV sensitivities ended being smaller when choosing the two blocks form for feedback gain matrix.

Apart for from the EV λ_2, the smallest individual sensitivities were with upper block case in the second permutation (B2)) with S(λ_1) = 2.3176 and S($\lambda_{3,4}$) = 5.2349, while the smallest overall and the individual sensitivity of λ_2 were with the upper block case of the first permutation (B1) with S(λ) = 12.7625 and S(λ_2) = 2.2170.

Robust stability measures M_2 and M_3

The robust stability measures M_2 and M_3 were found to be bigger in the two blocks form too in both permutation B1 and B2. Also, in these measures, they were both big with upper block case. But between the two permutations, B1 happens to be the greatest with $M_2 = 7.8354e-04$ for λ_2 and M_3 = 0.00451.

Relative change

The smallest values of the relative change of the eigenvalues are mostly in the lower block case of the two blocks form, form the input matrix B1 with $r_1 = 0.0039$ being the exception, $r_2 = 0.0021$ and $r_{3,4} = 0.025$ the smallest. Meanwhile the upper block case of the two blocks from, from the input matrix B2 had mostly the smallest values; $r_1 = 0.0039$ being the exception, $r_2 = 0.0021$ and $r_{3,4} = 0.025$ the smallest.

Time Response

a. for the first input of B

The rise time was the smallest in the two blocks from mainly in the lower block case of the second permutation with RT = 3.6118s.

The settling time was the smallest in the upper block case of the first permutation with ST = 33.83.63s.

The smallest undershoot (1.0697) was obtained with the one block form in B2.

b. for the second input of B

The smallest rise time was with the upper block case in B2 with RT = 1.657s.

The settling time was the smallest with the upper block case in B2 with ST = 2.6908s.

Here, mostly in all the cases there was no overshoot nor undershoot.

Case $\lambda_2 = -0.001$

Feedback magnitude gain

The gain magnitude is found to be smaller when choosing the two blocks form (two blocks in the diagonal) rather than only one main block, since the gain ended being bigger in both permutations B1 and B2.

In the first permutation (B1), the smallest gain was K_2 with a magnitude of 1.7401. This smallest gain happens to be the same in the second permutation (B2), but in this case, it was K_1 (the upper block case).

Individual and Overall eigenvalue sensitivity

Apart for from the EV λ_2, the smallest individual sensitivities were with upper block case in the second permutation (B2) with S(λ_1) = 2.3176 and S($\lambda_{3,4}$) = 5.2349, while the smallest overall and the individual sensitivity of λ_2 were with upper block case of the first permutation (B1) with S(V) = 12.7424 and S(λ_2) = 2.2140.

Robust stability measures *M_2* and *M_3*

The robust stability measures M2 was found to be the biggest with upper block case with $M_2 = 7.84781e - 05$ for λ_2 , and M_3 = 4,5167118e-04 for both B1 and B2.

Relative change

The values of the relative change aren't that different from the previous case. Thus, the same results and comment.

Time Response

a. for the first input of B

The rise time was the smallest in the two blocks from mainly in the upper block case of B1 the lower block case of B2with RT = 27.2481s.

The settling time was the smallest in the upper block case of B1 and the lower block case of B2 with ST = 2.9382e+03.

The smallest undershoot (0.1033) was obtained with the one block form of B2.

b. for the second input of B

The smallest rise time was with the upper block case in B2 with RT = 1.7067s.

The settling time was the smallest with the upper block case in B2 with ST = 2.6997s.

Here too, in most of the cases there was no overshoot nor undershoot.

Case $\lambda_2 = -0.1$

Feedback gain magnitude

The gain magnitude is found to be smaller when choosing the two blocks form (two blocks in the diagonal) rather than only one main block, since the gain ended being bigger in both permutations B1 and B2.

In the first permutation (B1), the smallest gain was K_2 with a magnitude of 1.7784. This smallest gain happens to be the same in the second permutation (B2), but in this case, it was K_1 (the upper block case).

Individual and Overall eigenvalue sensitivity

The smallest overall and the individual sensitivities of λ_1 and $\lambda_2$were with upper block case of the first permutation (B1) with S(V) = 12.9784, S(λ_1) = 2.3254 and S(λ_2) = 2.2573. For the complex pair, the smallest individual sensitivities were with upper block case in the second permutation (B2) with S($\lambda_{3,4}$) = 5.2347.

Robust stability measures M_2 and M_3

The robust stability measures M_2 and M_3 were found to be the biggest with upper block case of B1 with $M_2 = 7.70511e - 03$ for λ_2 and M3 = 0.04430.

Relative change

Here too, the values of the relative change are approximately the same as in the previous cases.

Time Response

a. for the first input of B

The rise time was the smallest in the two blocks form, mainly in the upper block case of B1 with RT = 0.0453s.

The settling time was the smallest in the upper block case of ST = 3.4087.

The smallest undershoot (11.8703) was with the one block form in B2.

b. for the second input of B

The smallest rise time was with the single block configuration with RT = 0.16257s for the first permutation.

The settling time was the smallest with the upper block case in B2 with ST = 2.6902s.

Apart from the one block form in B1 in which there was a big overshoot of 65.4233, mostly in all the other cases there was no overshoot nor undershoot.

Comparison between the 3 cases of λ_2

In terms of gain magnitude, the smallest gain was obtained when pushing the EV λ_2 closer to the *jω-axis*, meaning that the best case was with $\lambda_2 = 0.001$.

The overall EV sensitivity was the smallest in the second case too (case $\lambda_2 = -0.001$). Also, the same for all the individual eigenvalues sensitivity, apart from the complex pair's sensitivity which was smaller in the third case (case $\lambda_2 = -0.001$).

The robust stability measures were bigger in the third case, meaning when pushing the EV λ_2 away from the *jω-axis* (away from the unstable region too).

For the time response, the settling and rise time are smaller when moving the EV λ_2 away from the *jω-axis* (case $\lambda_2 = -0.1$), but the overshoot/undershoot is smaller when pushing it toward it (case $\lambda_2 = -0.001$).

5.2.6 Conclusion

Overall the two block forms are the best choice in terms of feedback gain magnitude, individual and overall EV sensitivity, robust performance, stability measures M_2 and M_3 and time response. Thus, going for a bigger number of blocks would be a better choice for the previously stated design criteria. However, the movement of the closest eigenvalue to the *jω-axis* away from that axis, improves only the time response characteristics (in terms of rise and settling time), robust stability measures M_2 and M_3, and degrades the other characteristics. The reverse happens when moving it closer the imaginary axis instead.

5.3 State Feedback design using Block Controller Form

A controlled *MIMO* system has an infinite number of state feedback controllers that may be found which will provide the required stability characteristics. Consequently, an alternative and very powerful method for designing a state feedback controller for stabilizing systems is the right blocks/ left blocks/ right and left blocks pole placement method. The method is based on the manipulation of the equations of motion in block state space form and makes full use of the appropriate computational tools in the analytical process. The forms of block poles (Solvents) are not unique, but we restricted our study to the case of the canonical forms (diagonal, controllable and observable).

The dimension of the matrix A of our system is 4×4 and the number of inputs is 2.

The rank of the reachability matrix $w_r = [\,B\ AB]$ is 4 (full rank)

a. The number $\frac{n}{m} = \frac{4}{2} = 2$ is an integer.
b. The system is reachable of index $l = 2$.

Therefore, we can convert the system into block controller form by the following transformation matrix T_c:

$$T_c = \begin{bmatrix} 0.0002 & -0.0013 & -0.4004 & 0.0416 \\ 0.0000 & 0.0003 & 0.1025 & -0.0005 \\ 0.0171 & 0.4022 & 0.0957 & -0.0185 \\ 0.0058 & -0.1023 & -0.0245 & 0.0047 \end{bmatrix}$$

$$\text{where } T_c = \begin{bmatrix} t_{c1} \\ t_{c1}A \end{bmatrix} \ ; \ w_r = [B\ AB] \ ; t_{c1} = [0_2 \quad I_2]w_r^{-1}$$

we obtain the following:

$$A_c = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0.0665 & 7.9811 & -3.6266 & -11.8852 \\ -0.0372 & -7.4150 & -0.0429 & -0.7944 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C_c = [-0.4674 \quad -35.0101 \quad -3.2290 \quad -12.2020]$$

5.3.1 State Feedback Design Using Right Solvents in Diagonal Form

- **Construction of the feedback gain matrix:**

The desired right block poles in diagonal form are constructed as follows:

$$\Longrightarrow \qquad R1 = \begin{pmatrix} -1.5 & 0.75 \\ -0.75 & -1.5 \end{pmatrix} \qquad R2 = \begin{pmatrix} -6.00 & 0 \\ 0 & -0.01 \end{pmatrix}$$

where the desired eigenvalues are $\lambda_{1,2,3,4} = \{-6.00, -0.01, -1.5 \pm j0.75\}$

and Such that: R_1 consists of the eigenvalues: $-1.5 \pm j0.75$

and R_2 consists of the eigenvalues: $-6.00, -0.01$.

Then we have to construct the matrix coefficients of the desired characteristic matrix polynomial: $D_d(\lambda) = I\lambda^2 + D_{d1}\lambda + D_{d2}$

Such that: $[D_{d2}, D_{d1}] = -[R_1^2, R_2^2]V_R^{-1}$ where the right block Vandermonde matrix

$$V_R = \begin{bmatrix} I_2 & I_2 \\ R_1 & R_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1.5 & 0.75 & -6.0 & 0 \\ -0.75 & -1.5 & 0 & -0.01 \end{bmatrix}$$

$$\Longrightarrow \qquad [D_{d2}, D_{d1}] = \begin{bmatrix} 12.2912 & 0.0254 & 8.0485 & 2.5412 \\ -2.0385 & 0.0095 & -0.3398 & 0.9615 \end{bmatrix}$$

Then by applying equation (3.5) we obtain *Kc*:

$$K_c = \begin{bmatrix} 12.2247 & 8.0065 & 4.4219 & -9.3440 \\ -2.0758 & -7.4055 & -0.3827 & 0.1671 \end{bmatrix}$$

$$K = K_c T_c = \begin{bmatrix} 0.0230 & 2.7216 & -3.4227 & 0.3794 \\ -0.0056 & -0.1708 & 0.0318 & -0.0752 \end{bmatrix}$$

The norm of the feedback gain matrix is $||K|| = 4.3915$

Robustness:

Robust stability: Robust stability is determined using the measures defined in the previous section. First, we find the norms of the left and right eigenvectors associated to each eigenvalue. The right eigenvector matrix associated to the closed-loop system (*A-BK*) is:

$$V = \begin{bmatrix} -0.9852 & 0.0112 & -0.8574 + 0j & -0.8574 + 0j \\ -0.0485 & -0.0216 & 0.0431 + 0.0229j & 0.0431 - 0.0229j \\ 0.005 & -0.1033 & 0.0388 + 0.0331j & 0.0388 - 0.0331j \\ 0.1647 & -0.9943 & 0.4563 + 0.2272j & 0.4563 - 0.2272j \end{bmatrix}$$

$||V|| = 1.7474$

The left eigenvector matrix associated to the closed-loop system (*A-BK*) has norm equal to:

$$||T|| = ||V^{-1}|| = 95.9165$$

Then the overall sensitivity is equal to:

$$S(V) = ||V||||V^{-1}|| = 197.6044$$

- The norms of all the right eigenvectors are equal to 1, and the associated left eigenvectors have norms as follows:

$||t_1|| = 27.9395,$ $||t_2|| = 19.7933,$ $||t_3|| = 64.1534,$ $||t_4|| = 64.1534$

- The sensitivity of each individual eigenvalue is:

$s(\lambda_i)$ = {27.9395, 19.7933, 64.1534, 64.1534}

Now the stability measures are:

1) $M_2 = (S(V))^{-1}|Re\{\lambda_2\}| = 5.9664\text{e} - 05$

2) $M_3 = min_{0\leq i\leq 4}\left\{\left(S(\lambda_i)\right)^{-1}|Re\{\lambda_i\}|\right\} = 5.0522\text{e} - 04$

Robust Performance:

The following is a random small perturbation applied to state feedback matrix (*A-BK*) using MATLAB software:

$$A' = \begin{bmatrix} 0.2290 & 0.5383 & 0.1067 & 0.8173 \\ 0.9133 & 0.9961 & 0.9619 & 0.8687 \\ 0.1524 & 0.0782 & 0.0046 & 0.0844 \\ 0.8258 & 0.4427 & 0.7749 & 0.3998 \end{bmatrix}$$

Then the eigenvalues of the matrix (*A-BK+A'*) are:

$$\lambda_{1,2,3,4} = \{-5.9991, -0.0102, -1.5002 \pm j0.7503\}$$

The relative change of each eigenvalue is given below by the following:

$$r_i(\lambda_i) = \{0.0002;\ 0.0200;\ \ 2.14 \times 10^{-4};\ 2.14 \times 10^{-4}\}$$

Remark:

With the same procedure of this section, the obtained results for right solvents in controller form and observer forms are summarized in tables 5-17 & 5-18 respectively.

The right solvents in controller form are:

$$R1 = \begin{pmatrix} 0 & 1 \\ -2.8125 & -3 \end{pmatrix} \quad R2 = \begin{pmatrix} -6.01 & -0.06 \\ 1 & 0 \end{pmatrix}$$

Such that: R_1 consists of the eigenvalues: $-1.5 \pm j0.75$
and R_2 consists of the eigenvalues: $-6.00, -0.01$.

The right solvents in observer form are:

$$R1 = \begin{pmatrix} -6.01 & 1 \\ -0.06 & 0 \end{pmatrix} \quad R2 = \begin{pmatrix} 0 & -2.8125 \\ 1 & -3 \end{pmatrix}$$

Such that: R_1 consists of the eigenvalues: $-6.00, -0.01$
and R_2 consists of the eigenvalues: $-1.5 \pm j0.75$.

5.3.2 State Feedback Design Using Left solvents in Controller Form

- **Construction of the feedback gain matrix:**

The desired left block poles in controller form are constructed as follows:

$$L1 = \begin{pmatrix} 0 & 1 \\ -2.8125 & -3 \end{pmatrix} \quad L2 = \begin{pmatrix} -6.01 & -0.06 \\ 1 & 0 \end{pmatrix}$$

where the desired eigenvalues are $\lambda_{1,2,3,4} = \{-6.00, -0.01, -1.5 \pm j0.75\}$

and such that: L_1 consists of the eigenvalues: $-6.00, -0.01$
and L_2 consists of the eigenvalues: $-1.5 \pm j0.75$.

Then we have to construct the matrix coefficients of the desired characteristic matrix polynomial: $D_d(\lambda) = I\lambda^2 + D_{d1}\lambda + D_{d2}$

Such that: $\begin{bmatrix} D_{d2} \\ D_{d1} \end{bmatrix} = -V_L^{-1} \begin{bmatrix} L_1^2 \\ L_2^2 \end{bmatrix}$ where

the left block Vandermonde matrix $V_L = \begin{bmatrix} I_2 & L_1 \\ I_2 & L_2 \end{bmatrix} = \begin{bmatrix} 1.00 & 0.00 & 0 & 1.00 \\ 0.00 & 1.00 & -2.8125 & -3.00 \\ 1.00 & 0.00 & -6.01 & -0.06 \\ 0.00 & 1.00 & 1.00 & 0.00 \end{bmatrix}$

$$\Longrightarrow \quad [D_{d2}, D_{d1}] = \begin{bmatrix} 7.1998 & 1.2318 & 7.2418 & 0.2473 \\ -1.2318 & -0.1873 & -4.3873 & 1.7682 \end{bmatrix}$$

Then by applying equation (3.5) we obtain *Kc*:

$$K_c = \begin{bmatrix} 7.1333 & 9.2129 & 3.6152 & -11.6379 \\ -1.2690 & -7.6023 & -4.4302 & 0.9738 \end{bmatrix}$$

$$K = K_c T_c = \begin{bmatrix} -0.0050 & 2.6385 & -1.2814 & 0.1709 \\ -0.0701 & -1.8823 & -0.7187 & 0.0372 \end{bmatrix}$$

The norm of the feedback gain matrix is $||K|| = 3.3150$

Robustness:

Robust stability: Robust stability is determined using the measures defined in the previous section. First, we find the norms of the left and right eigenvectors associated to each eigenvalue. The right eigenvector matrix associated to the closed-loop system (*A-BK*) is:

$$V = \begin{bmatrix} -0.9833 & -0.8576 + 0j & -0.8576 + 0j & 0.0112 \\ -0.0777 & 0.0443 + 0.0010j & 0.0443 - 0.0010j & -0.0206 \\ -0.0220 & 0.0472 + 0.0207j & 0.0472 - 0.0207j & -0.1079 \\ -0.1631 & 0.4560 + 0.2279j & 0.4560 + 0.2279j & -0.9939 \end{bmatrix}$$

$||V|| = 1.7495$

The left eigenvector matrix associated to the closed-loop system (*A-BK*) has norm equal to:

$$||T|| = ||V^{-1}|| = 234.1382$$

Then the overall sensitivity is equal to:

$$S(V) = ||V||||V^{-1}|| = 409.6260$$

- The norms of all the right eigenvectors are equal to 1, and the associated left eigenvectors have norms as follows:

$||t_1|| = 48.6598,$ $||t_2|| = 58.7224,$ $||t_3|| = 158.842,$ $||t_4|| = 158.842$

- The sensitivity of each individual eigenvalue is:

$s(\lambda_i) = \{48.6598, 58.7224, 158.842, 158.842\}$

Now the stability measures are:

1) $M_2 = (S(V))^{-1}|Re\{\lambda_2\}| = 2.4413\text{e}-04$

2) $M_3 = min_{0\le i\le 4}\left\{\left(S(\lambda_i)\right)^{-1}|Re\{\lambda_i\}|\right\} = 1.7000\text{e}-04$

Robust Performance:

The previous small perturbation is applied to state feedback matrix (*A-BK*).

Then the eigenvalues of the matrix (*A-BK+A'*) are:

$$\lambda_{1,2,3,4} = \{-6.0022, -0.0102, -1.4989 \pm j0.7498\}$$

The relative change of each eigenvalue is given below by the following:

$$r_i(\lambda_i) = \{3.3333 \times 10^{-4};\ 0.0200;\ \ 6.6667 \times 10^{-4};\ \ 6.6667 \times 10^{-4}\}$$

Remark:

With the same procedure of this section, the obtained results for left solvents in diagonal form and observer forms are summarized in tables 5-16 & 5-18 respectively.

The left solvents in diagonal form are:

$$L1 = \begin{pmatrix} -1.5 & 0.75 \\ -0.75 & -1.5 \end{pmatrix} \quad L2 = \begin{pmatrix} -6 & 0 \\ 0 & -0.01 \end{pmatrix}$$

Such that: L_1 consists of the eigenvalues: $-1.5 \pm j0.75$

and L_2 consists of the eigenvalues: $-6.00, -0.01$.

The left solvents in observer form are:

$$L1 = \begin{pmatrix} -6.01 & 1 \\ -0.06 & 0 \end{pmatrix} \quad L2 = \begin{pmatrix} 0 & -2.8125 \\ 1 & -3 \end{pmatrix}$$

Such that: L_1 consists of the eigenvalues: $-6.00, -0.01$

and L_2 consists of the eigenvalues: $-1.5 \pm j0.75$.

5.3.3 State Feedback Design Using Right and Left solvents in Observer Form

- **Construction of the feedback gain matrix:**

The desired right and left block poles in observer form are constructed as follows:

$$R = \begin{pmatrix} -6.01 & 1 \\ -0.06 & 0 \end{pmatrix} \quad L = \begin{pmatrix} 0 & -2.8125 \\ 1 & -0.01 \end{pmatrix}$$

where the desired eigenvalues are $\lambda_{1,2,3,4} = \{-6.00, -0.01 - 1.5 \pm j0.75\}$

and such that: R consists of the eigenvalues: $-6.00, -0.01$

and L consists of the eigenvalues: $-1.5 \pm j0.75$.

For the case of right and left solvents, equation (3.4) must equivalent to following equation:

$$D(\lambda) = (\lambda I - L)(\lambda I - R) = I\lambda^2 + (-L - R)\lambda + LR$$

Such that: $\begin{cases} A_1 + K_{c1} = -L - R \\ A_2 + K_{c2} = LR \end{cases}$

Then we obtain Kc:

$$K_c = \begin{bmatrix} 0.1022 & 7.9811 & 2.3834 & -10.0727 \\ -5.8672 & -6.4150 & -0.9829 & 2.2056 \end{bmatrix}$$

$$K = K_c T_c = \begin{bmatrix} -0.0180 & 1.9914 & 1.2516 & -0.0912 \\ -0.0047 & -0.6156 & 1.5441 & -0.2128 \end{bmatrix}$$

The norm of the feedback gain matrix is $||K|| = 2.3920$

Robustness:

Robust stability: Robust stability is determined using the measures defined in the previous section. First, we find the norms of the left and right eigenvectors associated to each eigenvalue. The right eigenvector matrix associated to the closed-loop system (*A-BK*) is:

$$V = \begin{bmatrix} 0.9833 & -0.8576 + 0j & -0.8576 + 0j & 0.0112 \\ -0.0777 & 0.0443 + 0.0010j & 0.0443 - 0.0010j & -0.0206 \\ -0.0220 & 0.0472 + 0.0207j & 0.0472 - 0.0207j & -0.1079 \\ -0.1631 & 0.4560 + 0.2279j & 0.4560 + 0.2279j & -0.9939 \end{bmatrix}$$

$||V|| = 1.7476$

The left eigenvector matrix associated to the closed-loop system (*A-BK*) has norm equal to:

$$||T|| = ||V^{-1}|| = 244.1199$$

Then the overall sensitivity is equal to:

$$S(V) = ||V||||V^{-1}|| = 426.6152$$

- The norms of all the right eigenvectors are equal to 1, and the associated left eigenvectors have norms as follows:

$||t_1|| = 11.1127,\quad ||t_2|| = 77.4321,\quad ||t_3|| = 163.777,\quad ||t_4|| = 163.777$

- The sensitivity of each individual eigenvalue is:

$s(\lambda_i) = \{11.1127, 77.4321, 163.777, 163.777\}$

Now the stability measures are:

1) $M_2 = (S(V))^{-1}|Re\{\lambda_2\}| = 2.3440\text{e} - 05$

2) $M_3 = min_{0\le i\le 4}\left\{\left(S(\lambda_i)\right)^{-1}|Re\{\lambda_i\}|\right\} = 1.2901\mathrm{e} - 04$

Robust Performance:

The previous small perturbation is applied to state feedback matrix (*A*-*BK*).

Then the eigenvalues of the matrix (*A*-*BK*+*A'*) are:

$$\lambda_{1,2,3,4} = \{-5.9993, -0.0088, -1.5008 \pm j0.7498\}$$

The relative change of each eigenvalue is given below by the following:

$$r_i(\lambda_i) = \{1.1667 \times 10^{-4};\ 0.12;\ \ 4.91 \times 10^{-4};\ \ 4.91 \times 10^{-4}\}$$

Remark:

With the same procedure of this section, the obtained results for right and left solvents in diagonal form and controller forms are summarized in tables 5-16 & 5-17 respectively.

where the right and left solvents in diagonal form are:

$$R = \begin{pmatrix} -1.5 & 0.75 \\ -0.75 & -1.5 \end{pmatrix} \quad L = \begin{pmatrix} -6 & 0 \\ 0 & -0.01 \end{pmatrix}$$

Such that: *L* consists of the eigenvalues: $-1.5 \pm j0.75$
and *R* consists of the eigenvalues: $-6.00, -0.01$.

The right and left solvents in controller form are:

$$R = \begin{pmatrix} 0 & 1 \\ -2.8125 & -3 \end{pmatrix} \quad L = \begin{pmatrix} -6.01 & -0.06 \\ 1 & 0 \end{pmatrix}$$

Such that: *R* consists of the eigenvalues: $-6.00, -0.01$
and *L* consists of the eigenvalues: $-1.5 \pm j0.75$.

Results summary:

- **Diagonal Form**

Table 5-16 Diagonal Form Results Summary

			Diagonal form		
			2 Rights Solvents	**2 Lefts Solvents**	**Right & Left Solvents**
			K_1	K_2	K_3
$\|\|K\|\|$			4.3915	4.3387	3.8683
$S(V)$			167.6044	482.9884	142.6934
Robust stability	$s(\lambda_i)$	-6.0	27.9395	29.1998	38.5332
		-0.01	19.7933	76.0885	12.0739
		$-1.5 + j0.75$	64.1534	186.9085	52.5446
		$-1.5 - j0.75$	64.1534	186.9085	52.5446
	M_2		5.9664e-05	2.0704e-05	7.0080e-05
	M_3		5.0522e-04	1.3108e-04	8.2787e-04
Robust performance	$r_i(\lambda_i)$	-6.0	0.0002	1.16 e-04	2.33 e-04
		-0.01	0.0200	0.03	0.05
		-1.5+0.75i	2.14e-04	3.77 e-04	2.98e-04
		-1.5-0.75i	2.14 e-04	3.77e-04	2.98e-04

- **Controller Form**

Table 5-17 Controller Form Results Summary

<table>
<tr><td colspan="3" rowspan="3"></td><td colspan="3">Controller form</td></tr>
<tr><td>2 Rights Solvents</td><td>2 Lefts Solvents</td><td>Right & Left Solvents</td></tr>
<tr><td>K_4</td><td>K_5</td><td>K_6</td></tr>
<tr><td colspan="3">$\|K\|$</td><td>3.0557</td><td>3.3150</td><td>2.4059</td></tr>
<tr><td colspan="3">$S(V)$</td><td>205.7081</td><td>409.6260</td><td>95.1026</td></tr>
<tr><td rowspan="6">Robust stability</td><td rowspan="4">$s(\lambda_i)$</td><td>-6.0</td><td>5.3578</td><td>48.6598</td><td>30.0311</td></tr>
<tr><td>-0.01</td><td>35.4676</td><td>58.7224</td><td>8.0311</td></tr>
<tr><td>$-1.5 + j0.75$</td><td>79.9771</td><td>158.8421</td><td>34.2491</td></tr>
<tr><td>$-1.5 - j0.75$</td><td>79.9771</td><td>158.8421</td><td>34.2491</td></tr>
<tr><td colspan="2">M_2</td><td>4.8613e-05</td><td>2.4413e-05</td><td>1.0515e-04</td></tr>
<tr><td colspan="2">M_3</td><td>2.8195e-04</td><td>1.7000e-04</td><td>0.0012</td></tr>
<tr><td rowspan="4">Robust performance</td><td rowspan="4">$r_i(\lambda_i)$</td><td>-6.0</td><td>6.6667e-04</td><td>3.3333 e-04</td><td>3.8333 e-04</td></tr>
<tr><td>-0.01</td><td>0.0500</td><td>0.02</td><td>0.01</td></tr>
<tr><td>-1.5+0.75i</td><td>1.78 e-04</td><td>6.6666 e-04</td><td>9.0000 e-04</td></tr>
<tr><td>-1.5-0.75i</td><td>1.78 e-04</td><td>6.6666 e-04</td><td>9.0000 e-04</td></tr>
</table>

- **Observer From**

Table 5-18 Observer Form Results Summary

			Observer form		
			2 Rights Solvents	**2 Lefts Solvents**	**Right & Left Solvents**
			K_7	K_8	K_9
$\|\|K\|\|$			3.4658	3.4285	2.3920
$S(V)$			138.7501	201.9087	426.6152
Robust stability	$s(\lambda_i)$	-6.0	18.6887	45.4315	11.1127
		-0.01	28.1662	21.2768	77.4321
		$-1.5 + j0.75$	52.6906	76.5988	163.7778
		$-1.5 - j0.75$	52.6906	76.5988	163.7778
	M_2		7.2072e-05	4.9527e-05	2.3440e-05
	M_3		3.5504e-04	4.6989e-04	1.2901e-04
Robust performance	$r_i(\lambda_i)$	-6.0	2e-04	3.3333e-04	1.1667e-04
		-0.01	0.06	0.05	0.12
		-1.5+0.75i	1.3333e-04	4.8074e-04	4.91e-04
		-1.5-0.75i	1.3333e-04	4.8074e-04	4.91e-04

5.3.4 Time response results

The following part summarize the time specifications (Settling time T_S, Rise time T_r, Overshoot/Undershoot) to the step response to both inputs of the system.

Remarks about the table and figures:

- The response due to the first input (b_1) is shown first while the response due to the second input (b_2) is shown second.
- For each case of solvents there are nine resulting state feedback gains. Thus, there are nine closed loop systems. These systems are labelled as follows:

G_{Ki}: is for the closed loop system obtained using the state feedback gain Ki for i=1,..,9.

- The response for the original system is also shown with its information and is referred to as G.

Table 5-19 Time Response results

System	Step info for S (1,1)	Step info for S (1,2)
G	RiseTime: 65.3146 SettlingTime: 119.4925 SettlingMin: -1.7035 SettlingMax: -1.4317 Overshoot: 0 Undershoot: 60.4029 Peak: 1.7035 PeakTime: 235.2944	RiseTime: 73.8259 SettlingTime: 116.5274 SettlingMin: 0.1193 SettlingMax: 0.1604 Overshoot: 0 Undershoot: 1.6943e+03 Peak: 2.7468 PeakTime: 1.0448
G_{K1}	RiseTime: 0.6096 SettlingTime: 9.6108 SettlingMin: 0.2765 SettlingMax: 0.6538 Overshoot: 97.4603 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 0.3639 SettlingTime: 10.6617 SettlingMin: -5.3642 SettlingMax: -1.9108 Overshoot: 68.2580 Undershoot: 0 Peak: 5.3642 PeakTime: 1.1475

G_{K2}	RiseTime: 0.0086 SettlingTime: 7.0069 SettlingMin: 0.1387 SettlingMax: 1.0860 Overshoot: 8.9753 Undershoot: 0 Peak: 1.0860 PeakTime: 2.4451	RiseTime: 0.7751 SettlingTime: 5.8650 SettlingMin: -5.0823 SettlingMax: -3.9627 Overshoot: 2.3452 Undershoot: 0 Peak: 5.0823 PeakTime: 4.0277
G_{K3}	RiseTime: 0.0079 SettlingTime: 8.8903 SettlingMin: 0.7529 SettlingMax: 1.2830 Overshoot: 28.1312 Undershoot: 0 Peak: 1.2830 PeakTime: 1.1579	RiseTime: 0.4381 SettlingTime: 7.4176 SettlingMin: -7.1861 SettlingMax: -3.5689 Overshoot: 55.4968 Undershoot: 0 Peak: 7.1861 PeakTime: 1.2991
G_{K4}	RiseTime: 0.5449 SettlingTime: 8.9079 SettlingMin: -0.3766 SettlingMax: 0.3261 Overshoot: 1.0782e+03 Undershoot: 430.7843 Peak: 1.0300 PeakTime: 0	RiseTime: 0.2668 SettlingTime: 10.1253 SettlingMin: -3.4592 SettlingMax: -0.9319 Overshoot: 85.5690 Undershoot: 0 Peak: 3.4592 PeakTime: 0.9717
G_{K5}	RiseTime: 0.1932 SettlingTime: 7.0555 SettlingMin: -0.4448 SettlingMax: 0.6261 Overshoot: 122.2883 Undershoot: 95.9887 Peak: 1.0300 PeakTime: 0	RiseTime: 0.8437 SettlingTime: 7.9500 SettlingMin: -3.3381 SettlingMax: -2.2514 Overshoot: 0 Undershoot: 0 Peak: 3.3381 PeakTime: 11.8714
G_{K6}	RiseTime: 33.2665 SettlingTime: 43.2508 SettlingMin: -0.4604 SettlingMax: -0.3137 Overshoot: 0 Undershoot: 518.3220 Peak: 2.3990 PeakTime: 1.6956	RiseTime: 0.1347 SettlingTime: 43.5421 SettlingMin: -6.9652 SettlingMax: -1.0335 Overshoot: 540.1970 Undershoot: 0 Peak: 6.9652 PeakTime: 1.6064

G_{K7}	RiseTime: 4.5001 SettlingTime: 8.5128 SettlingMin: 2.4641 SettlingMax: 2.6213 Overshoot: 0 Undershoot: 0 Peak: 2.6213 PeakTime: 15.2464	RiseTime: 3.9951 SettlingTime: 7.7838 SettlingMin: -9.8086 SettlingMax: -8.8604 Overshoot: 0 Undershoot: 0 Peak: 9.8086 PeakTime: 15.2464
G_{K8}	RiseTime: 0.0056 SettlingTime: 10.3681 SettlingMin: 0.6783 SettlingMax: 1.8510 Overshoot: 77.7232 Undershoot: 0 Peak: 1.8510 PeakTime: 1.1647	RiseTime: 0.4019 SettlingTime: 10.2685 SettlingMin: -8.0714 SettlingMax: -3.2769 Overshoot: 73.0877 Undershoot: 0 Peak: 8.0714 PeakTime: 1.3644
G_{K9}	RiseTime: 0.6714 SettlingTime: 9.5657 SettlingMin: -1.1323 SettlingMax: -0.0309 Overshoot: 168.5211 Undershoot: 244.2667 Peak: 1.1323 PeakTime: 1.5011	RiseTime: 0.0020 SettlingTime: 10.3762 SettlingMin: -0.9174 SettlingMax: 0.9284 Overshoot: 734.8625 Undershoot: 359.8365 Peak: 2.1541 PeakTime: 0.7119

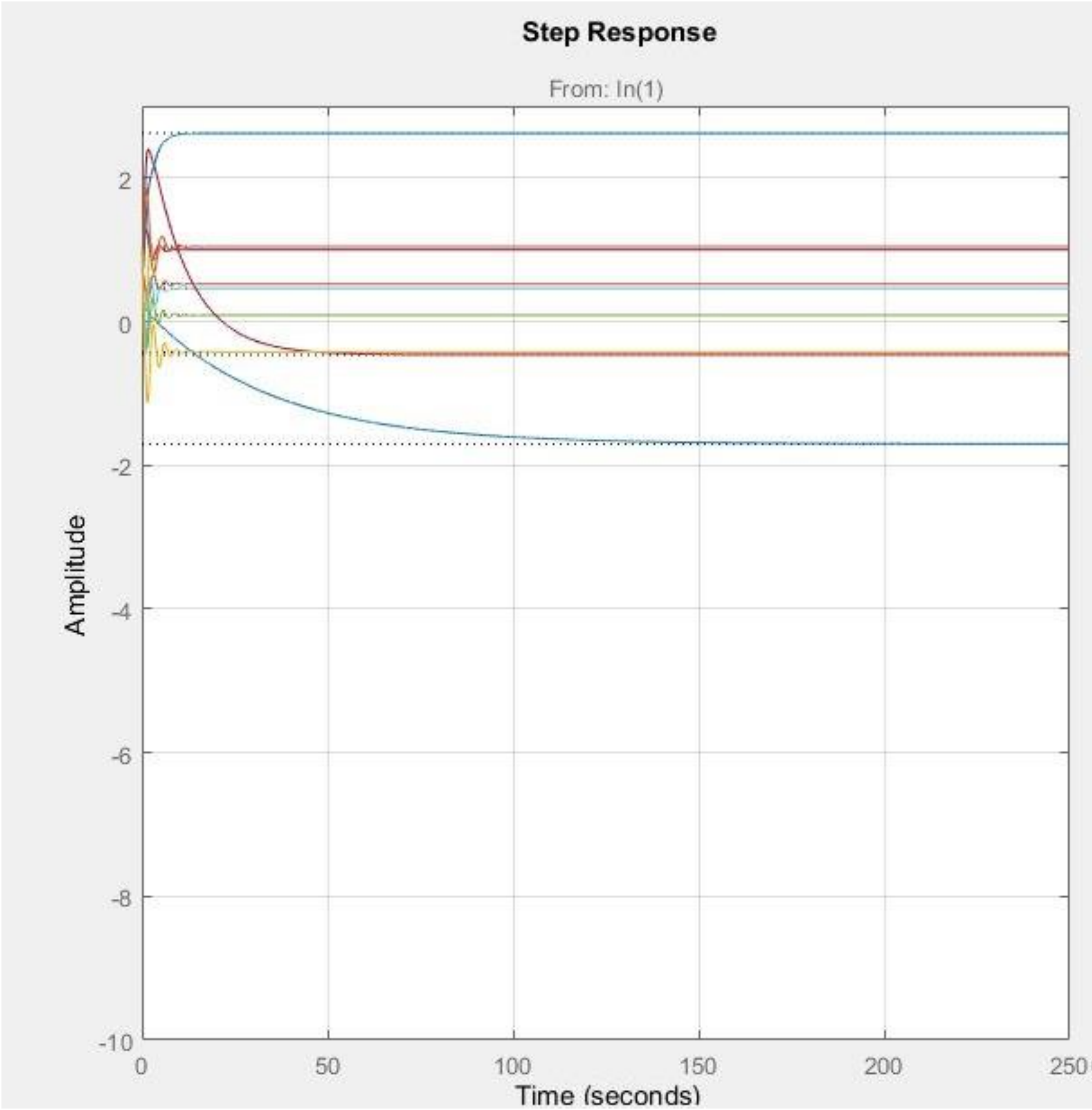

Figure 5-7 the response due to the first input -b1

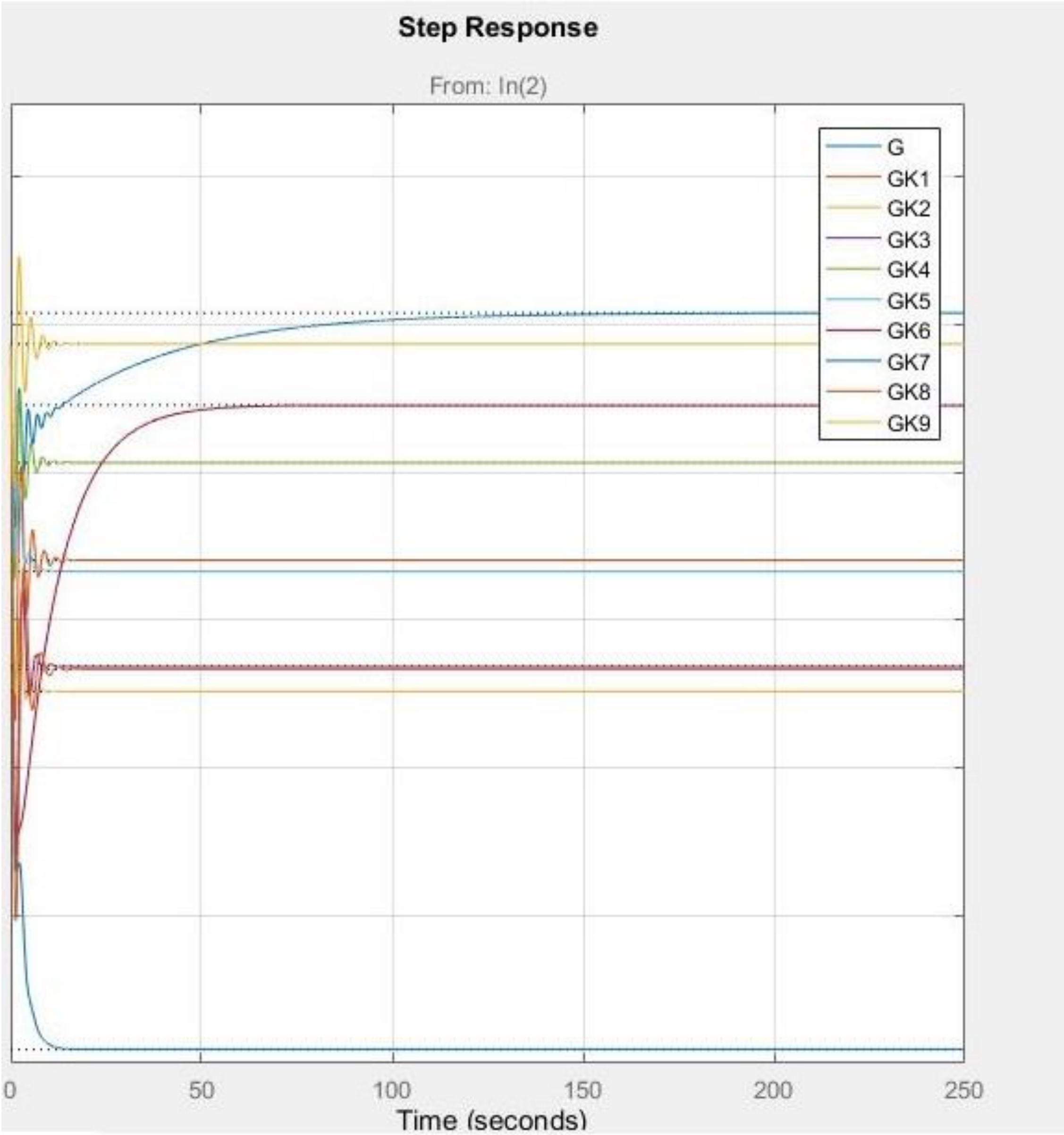

Figure 5-8 the response due to the second input -b2

5.3.5 Discussion

The discussion will be based on the following criteria:

- For feedback gains, small gains are desirable because they minimize noise amplification.

- For time specifications, the smaller the settling time, smaller rise time, smaller overshoot/undershoot the better in time response.
- For the sensitivities of the eigenvalues, we choose the one that has the lowest sensitivity.
- For the robust stability the greater the value of its measure the more robustly stable the system.
- For robust performance, the smaller the value of relative change the better the performance.

In order to do a comparison study between left/ right/ left and right solvents placement we gather the preceding results in tables as shown below. To make the analysis easier and be clear, tables 5-16, 5.-17 ,5-18, 5.-19 can be summarized into table: 5-20 & 5-21

Table 5-20 Comparative study between solvents in terms of gain, sensitivity and robustness

		Diagonal form	**Controller form**	**Observer form**
$\|\|K\|\|$		1 Left 1 Right Solvent	1 Left 1 Right Solvent	1 Left 1 Right Solvent
$S(V)$		1 Left 1 Right Solvent	1 Left 1 Right Solvent	Right Solvents
Robust stability	$s(\lambda_i)$	1 Left 1 Right Solvent	1 Left 1 Right Solvent	Right Solvents
	M_2	1 Left 1 Right Solvent	1 Left 1 Right Solvent	Right Solvents
	M_3	1 Left 1 Right Solvent	1 Left 1 Right Solvent	Left Solvents
Robust performance	$r_i(\lambda_i)$	Right Solvents	Right Solvents	Right Solvents

Table 5-21 Comparative study between solvents in terms of time specifications

		Diagonal form	**Controller form**	**Observer form**
Time specifications	T_S	Right Solvents	1 Left 1 Right Solvents	Right Solvents
	T_r	1 Left 1 Right Solvents	Right Solvents	Left Solvents
	Overshoot/ undershoot	Left Solvents	Left Solvents	Right Solvents

From table 5-20 we conclude that the best control design in terms of feedback gain, sensitivity and robustness:

- For the diagonal form the design using 1 right and 1 left solvent is recommended and more suitable.
- For the controller form the design using 1 right and 1 left solvent is recommended and more suitable.
- For the observer form the design using rights solvents is recommended and suitable.

From table 5-21 we conclude that the best design in terms of time response:

- For a best settling time, using right solvents is suitable.
- For a best rise time, using either right or left is recommended.
- For a small overshoot/ undershoot, using left solvents is suitable.

5.3.6 Conclusion

The choice of feedback gain matrix is done by the comparison between the three forms of the block poles in terms of best response characteristics and system robustness. In our case we can say that the choice between left/ right/ left & right solvents is made according to the form of these solvents. We can conclude that the choice of the feedback gain matrix is made according to the application itself from a desirable system's response hence we are providing the designer a flexibility to choose the best depending on the specified need.

5.3.7 Effect of moving the dominant pole from/to the $j\omega$-axis

When a pole (or pole pair) is further to the left into the negative s-plane, the real component will be a large negative number and so the decay will be rapid. Conversely if a pole or pole pair is close to the imaginary axis, sigma is negative but not very large so the response decays much less rapidly. Hence the response of the system is dominated by poles or pole pairs close to the imaginary axis. Now the eigenvalue λ_2 is the closest to the $j\omega$-axis. It's being pushed closer to and away from the imaginary axis to investigate the effect of this pole's placement on the previous sited criteria.

The following tables summarizes the obtained results in terms of state feedback gain, sensitivity and robustness for the new placement of the eigenvalue $\lambda_2 = -0.01$ (the same previous design procedure).

5.3.7.1 Case 1: $\lambda_{2=}-0.1$

- **Diagonal Form**

Table 5-22 Diagonal Form Results Summary case $\lambda_{2=}-0.1$

			Diagonal form		
			2 Rights Solvents	**2 Lefts Solvents**	**Right & Left Solvents**
			K_1	K_2	K_3
$\|\|K\|\|$			4.4254	4.4041	3.8689
$S(V)$			172.5939	535.8998	144.0884
Robust stability	$s(\lambda_i)$	-6.0	28.2343	33.6084	38.5332
		-0.1	21.2986	87.0032	12.6699
		$-1.5+j0.75$	65.0177	203.6481	52.3102
		$-1.5-j0.75$	65.0177	203.6481	52.3102
	M_2		5.7939e-04	1.8660e-04	6.9402e-04
	M_3		0.0047	0.0011	0.0079
Robust performance	$r_i(\lambda_i)$	-6.0	1.4516e-04	1.3874e-04	2.2945e-04
		-0.1	0.0021	0.0032	0.0052
		-1.5+0.75i	2.3400e-04	4.1437e-04	3.1432e-04
		-1.5-0.75i	2.3400e-04	4.1437e-04	3.1432e-04

- **Controller Form**

Table 5-23 Controller Form Results Summary case λ_2 = -0.1

			Controller form		
			2 Rights Solvents	**2 Lefts Solvents**	**Right & Left Solvents**
			K_4	K_5	K_6
$\|\|K\|\|$			3.2513	3.6186	2.3077
$S(V)$			227.2524	496.1741	112.5588
Robust stability	$s(\lambda_i)$	-6.0	5.7381	58.2382	30.7830
		-0.1	40.5504	73.2688	12.3308
		$-1.5 + j0.75$	86.6896	188.9880	42.0090
		$-1.5 - j0.75$	86.6896	188.9880	42.0090
	M_2		4.4004e-04	2.0154e-04	8.8842e-04
	M_3		0.0025	0.0014	0.0081
Robust performance	$r_i(\lambda_i)$	-6.0	6.5738e-05	4.0859e-04	3.3940e-04
		-0.1	0.0051	0.0016	6.9126e-04
		-1.5+0.75i	2.2111e-04	7.4984e-04	5.3799e-04
		-1.5-0.75i	2.2111e-04	7.4984e-04	5.3799e-04

- **Observer Form**

Table 5-24 Observer Form Results Summary case λ_2 = -0.1

			Observer form		
			2 Rights Solvents	**2 Lefts Solvents**	**Right & Left Solvents**
			K_7	K_8	K_9
$\lVert K \rVert$			3.8815	3.7867	2.1302
$S(V)$			139.3179	206.1877	327.2328
Robust stability	$s(\lambda_i)$	-6.0	28.7905	53.5639	12.2558
		-0.1	30.8743	19.1941	60.8189
		$-1.5 + j0.75$	51.5823	75.4486	123.5061
		$-1.5 - j0.75$	51.5823	75.4486	123.5061
	M_2		7.1778e-04	4.8499e-04	3.0559e-04
	M_3		0.0032	0.0052	0.0016
Robust performance	$r_i(\lambda_i)$	-6.0	2.9302e-04	3.6359e-04	1.4343e-04
		-0.1	0.0069	0.0069	0.0088
		-1.5+0.75i	2.7383e-04	4.8452e-04	4.8305e-04
		-1.5-0.75i	2.7383e-04	4.8452e-04	4.8305e-04

- **Time specifications**

Table 5-25 Time Response results case λ_2 = -0.1

System	Step info for S (1,1)	Step info for S (1,2)
G_{K1}	RiseTime: 0.5956 SettlingTime: 9.4844 SettlingMin: 0.3107 SettlingMax: 0.6611 Overshoot: 90.5164 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 0.3674 SettlingTime: 10.4849 SettlingMin: -5.3218 SettlingMax: -1.9885 Overshoot: 66.1920 Undershoot: 0 Peak: 5.3218 PeakTime: 1.1473
G_{K2}	RiseTime: 0.0337 SettlingTime: 6.9668 SettlingMin: 0.0917 SettlingMax: 1.0279 Overshoot: 13.0334 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 0.7257 SettlingTime: 5.8014 SettlingMin: -4.7781 SettlingMax: -3.7055 Overshoot: 3.2605 Undershoot: 0 Peak: 4.7781 PeakTime: 1.2720
G_{K3}	RiseTime: 0.0033 SettlingTime: 8.6565 SettlingMin: 0.9013 SettlingMax: 1.2986 Overshoot: 25.6327 Undershoot: 0 Peak: 1.2986 PeakTime: 1.1915	RiseTime: 0.4436 SettlingTime: 7.2589 SettlingMin: -7.0503 SettlingMax: -3.6942 Overshoot: 52.2190 Undershoot: 0 Peak: 7.0503 PeakTime: 1.3050
G_{K4}	RiseTime: 0.5302 SettlingTime: 8.9066 SettlingMin: -0.3512 SettlingMax: 0.3613 Overshoot: 785.3872 Undershoot: 301.9102 Peak: 1.0300 PeakTime: 0	RiseTime: 0.2738 SettlingTime: 10.0461 SettlingMin: -3.4664 SettlingMax: -0.9948 Overshoot: 81.2611 Undershoot: 0 Peak: 3.4664 PeakTime: 0.9693

G_{K5}	RiseTime: 0.2168 SettlingTime: 6.9178 SettlingMin: -0.4953 SettlingMax: 0.6062 Overshoot: 161.0935 Undershoot: 125.5464 Peak: 1.0300 PeakTime: 0	RiseTime: 0.7010 SettlingTime: 7.7594 SettlingMin: -3.0528 SettlingMax: -2.0864 Overshoot: 0.0044 Undershoot: 0 Peak: 3.0528 PeakTime: 11.6256
G_{K6}	RiseTime: 0.0588 SettlingTime: 8.2782 SettlingMin: 0.6580 SettlingMax: 2.2295 Overshoot: 162.5117 Undershoot: 0 Peak: 2.2295 PeakTime: 1.4259	RiseTime: 0.3703 SettlingTime: 8.5491 SettlingMin: -6.9592 SettlingMax: -3.3077 Overshoot: 85.9140 Undershoot: 0 Peak: 6.9592 PeakTime: 1.5097
G_{K7}	RiseTime: 4.7658 SettlingTime: 8.9462 SettlingMin: 3.3709 SettlingMax: 3.6288 Overshoot: 0 Undershoot: 0 Peak: 3.6288 PeakTime: 20.5736	RiseTime: 4.3901 SettlingTime: 8.4175 SettlingMin: -12.5564 SettlingMax: -11.3315 Overshoot: 0 Undershoot: 0 Peak: 12.5564 PeakTime: 20.5736
G_{K8}	RiseTime: 0.2176 SettlingTime: 9.6531 SettlingMin: 1.3442 SettlingMax: 2.6918 Overshoot: 58.5759 Undershoot: 0 Peak: 2.6918 PeakTime: 1.4345	RiseTime: 0.5008 SettlingTime: 9.4767 SettlingMin: -10.0521 SettlingMax: -5.2168 Overshoot: 55.4214 Undershoot: 0 Peak: 10.0521 PeakTime: 1.5780
G_{K9}	RiseTime: 0.7826 SettlingTime: 8.5450 SettlingMin: -0.8153 SettlingMax: 0.0158 Overshoot: 211.0513 Undershoot: 392.9517 Peak: 1.0300 PeakTime: 0	RiseTime: 0.1077 SettlingTime: 9.3711 SettlingMin: -2.6580 SettlingMax: 0.4164 Overshoot: 277.1355 Undershoot: 59.0858 Peak: 2.6580 PeakTime: 0.8087

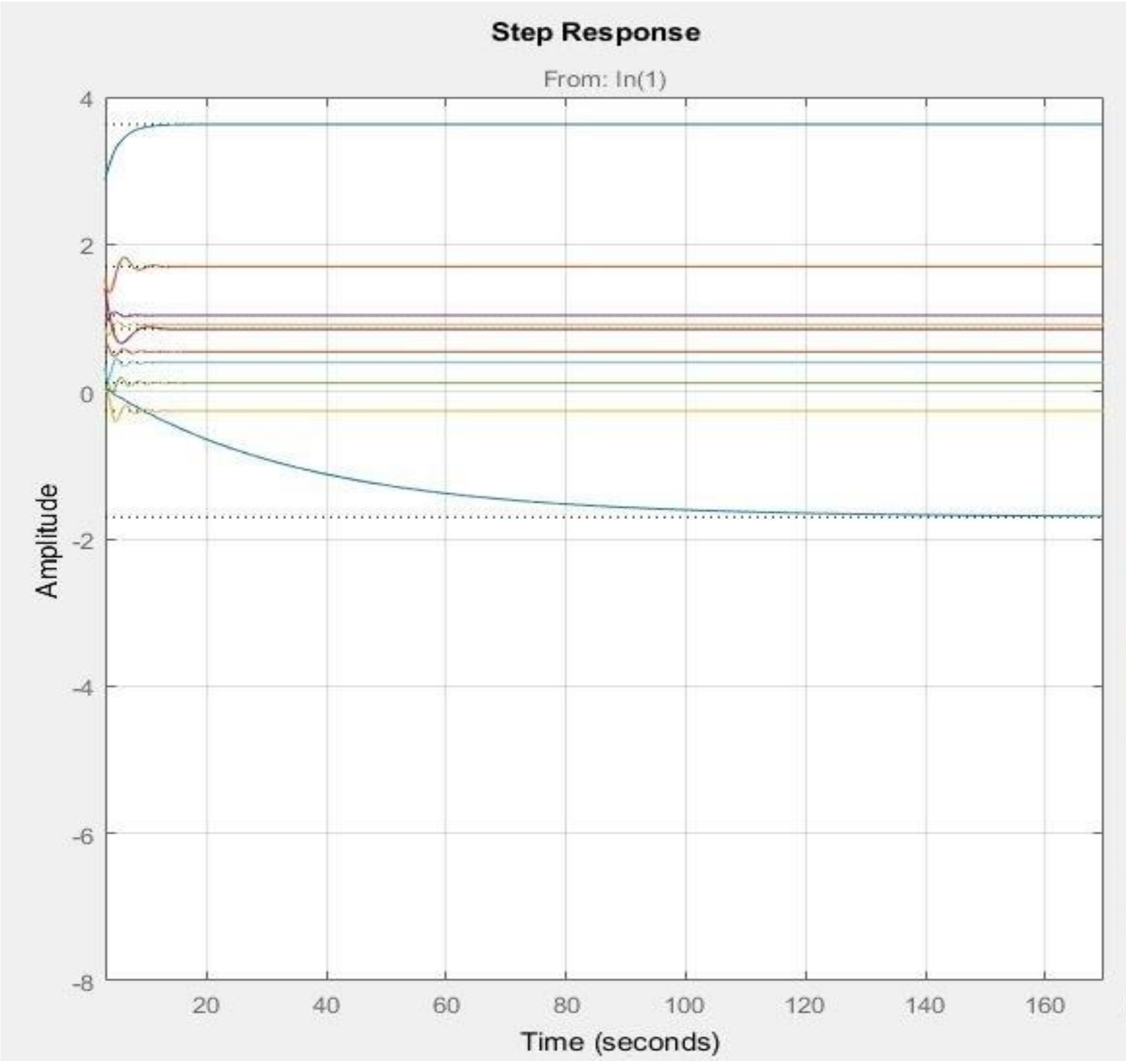

Figure 5-9 The response due to the first input b-1 case λ_2= -0.1

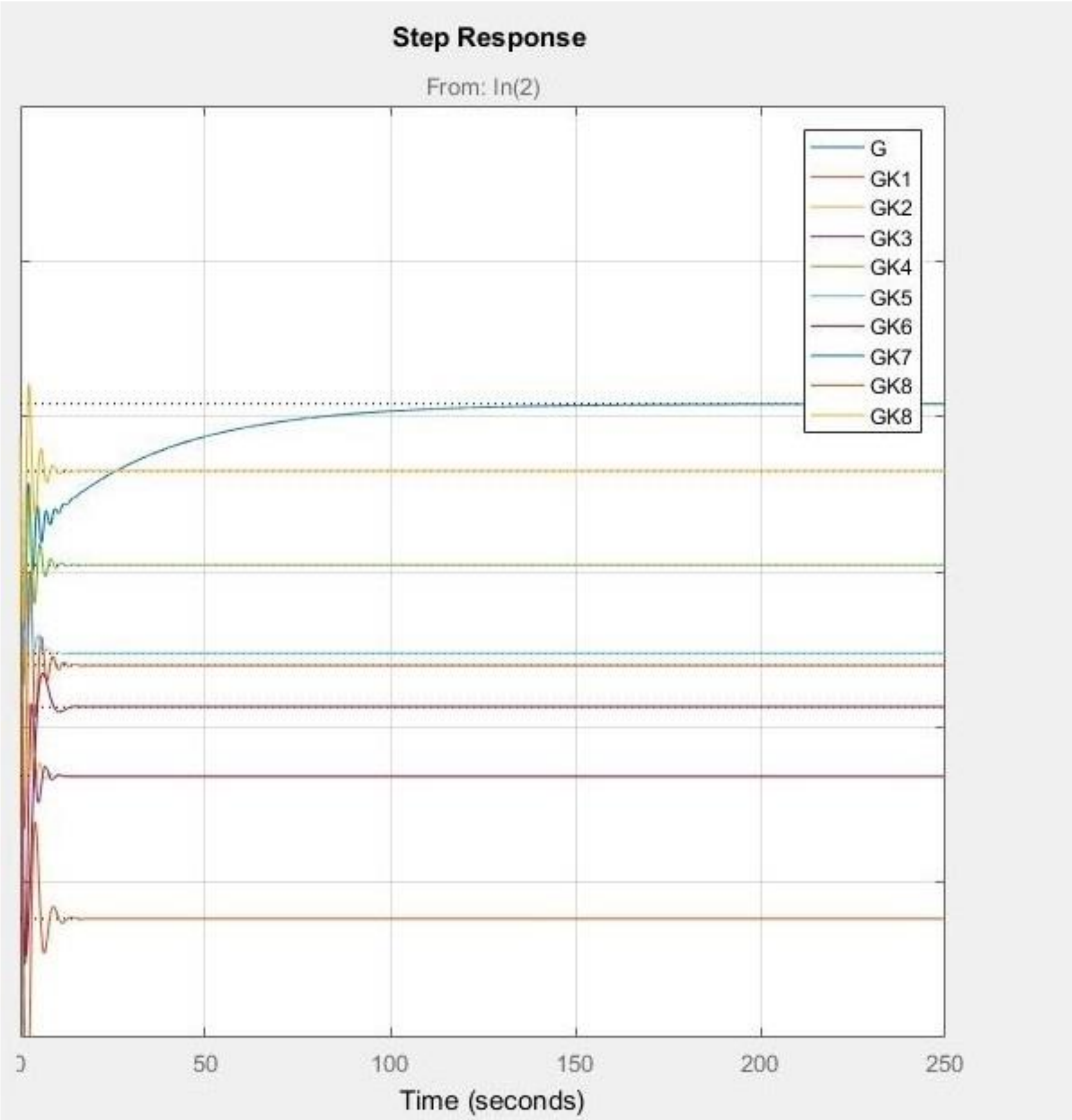

Figure 5-10 The response due to the second input b-2 case λ_2= -0.1

5.3.7.2 Case 2: $\lambda_2 = -0.001$

- **Diagonal form**

Table 5-26 Diagonal Form results case λ_2 = -0.001

			Diagonal form		
			2 Rights Solvents	**2 Lefts Solvents**	**Right & Left Solvents**
			K_1	K_2	K_3
$\|\|K\|\|$			4.3884	4.3328	3.8682
$S(V)$			167.1422	478.2137	142.5549
Robust stability	$s(\lambda_i)$	-6.0	27.9122	28.7963	38.5332
		-0.001	19.6623	75.1458	12.0215
		$-1.5 + j0.75$	64.0742	185.3810	52.5660
		$-1.5 - j0.75$	64.0742	185.3810	52.5660
	M_2		5.9829e-06	2.0911e-06	7.0148e-06
	M_3		5.0859e-05	1.2960e-05	8.2808e-05
Robust performance	$r_i(\lambda_i)$	-6.0	1.4594e-04	1.1845e-04	2.2945e-04
		-0.001	0.2180	0.3124	0.5473
		-1.5+0.75i	2.2849e-04	3.7025e-04	3.0683e-04
		-1.5-0.75i	2.2849e-04	3.7025e-04	3.0683e-04

- **Controller Form**

Table 5-27 Controller Form results case λ_2 = -0.001

<table>
<tr><td colspan="3" rowspan="3"></td><td colspan="3">Controller form</td></tr>
<tr><td>2 Rights Solvents</td><td>2 Lefts Solvents</td><td>Right & Left Solvents</td></tr>
<tr><td>K_4</td><td>K_5</td><td>K_6</td></tr>
<tr><td colspan="3">$\|K\|$</td><td>3.0386</td><td>3.2892</td><td>2.4206</td></tr>
<tr><td colspan="3">$S(V)$</td><td>203.8608</td><td>402.5157</td><td>94.0467</td></tr>
<tr><td rowspan="6">Robust stability</td><td rowspan="4">$s(\lambda_i)$</td><td>-6.0</td><td>5.3239</td><td>47.8571</td><td>30.2996</td></tr>
<tr><td>-0.001</td><td>35.0448</td><td>57.5645</td><td>7.6608</td></tr>
<tr><td>$-1.5 + j0.75$</td><td>79.4014</td><td>156.3518</td><td>33.6818</td></tr>
<tr><td>$-1.5 - j0.75$</td><td>79.4014</td><td>156.3518</td><td>33.6818</td></tr>
<tr><td colspan="2">M_2</td><td>4.9053e-06</td><td>2.4844e-06</td><td>1.0633e-05</td></tr>
<tr><td colspan="2">M_3</td><td>2.8535e-05</td><td>1.7071e-05</td><td>1.3245e-04</td></tr>
<tr><td rowspan="4">Robust performance</td><td rowspan="4">$r_i(\lambda_i)$</td><td>-6.0</td><td>6.4760e-05</td><td>3.5e-04</td><td>3.4644e-04</td></tr>
<tr><td>-0.001</td><td>0.4680</td><td>0.2</td><td>0.0608</td></tr>
<tr><td>-1.5+0.75i</td><td>2.0717e-04</td><td>6.6667e-04</td><td>5.5345e-04</td></tr>
<tr><td>-1.5-0.75i</td><td>2.0717e-04</td><td>6.6667e-04</td><td>5.5345e-04</td></tr>
</table>

- **Observer Form**

Table 5-28 Observer Form results case λ_2 = -0.001

			Observer form		
			2 Rights Solvents	**2 Lefts Solvents**	**Right & Left Solvents**
			K_7	K_8	K_9
$\|\|K\|\|$			3.4309	3.3979	2.4355
$S(V)$			138.8385	201.8651	437.0383
Robust stability	$s(\lambda_i)$	-6.0	17.8224	44.7262	10.9721
		-0.001	27.9449	21.5160	79.1015
		$-1.5 + j0.75$	52.8491	76.8252	168.0810
		$-1.5 - j0.75$	52.8491	76.8252	168.0810
	M_2		7.2026e-06	4.9538e-06	2.2881e-06
	M_3		3.5785e-05	4.6369e-05	1.2511e-05
Robust performance	$r_i(\lambda_i)$	-6.0	1.9275e-04	3.3430e-04	1.1667e-04
		-0.001	0.5465	0.4793	0.8
		-1.5+0.75i	1.3870e-04	4.5492e-04	3.0683e-04
		-1.5-0.75i	1.3870e-04	4.5492e-04	3.0683e-04

- **Time Specifications**

Table 5-29 Time response results case λ_2 = -0.001

System	Step info for S (1,1)	Step info for S (1,2)
G_{K1}	RiseTime: 0.6108 SettlingTime: 9.6215 SettlingMin: 0.2730 SettlingMax: 0.6531 Overshoot: 98.1895 Undershoot: 0 Peak: 1.0300 PeakTime: 0	RiseTime: 0.3636 SettlingTime: 10.6752 SettlingMin: -5.3686 SettlingMax: -1.9027 Overshoot: 68.4700 Undershoot: 0 Peak: 5.3686 PeakTime: 1.1475
G_{K2}	RiseTime: 0.0063 SettlingTime: 7.0114 SettlingMin: 0.1431 SettlingMax: 1.0919 Overshoot: 8.6167 Undershoot: 0 Peak: 1.0919 PeakTime: 2.4513	RiseTime: 0.7805 SettlingTime: 5.8719 SettlingMin: -5.1147 SettlingMax: -3.9891 Overshoot: 2.2715 Undershoot: 0 Peak: 5.1147 PeakTime: 4.0379
G_{K3}	RiseTime: 0.0088 SettlingTime: 8.9039 SettlingMin: 0.7529 SettlingMax: 1.2814 Overshoot: 28.3925 Undershoot: 0 Peak: 1.2814 PeakTime: 1.1574	RiseTime: 0.4375 SettlingTime: 7.4310 SettlingMin: -7.1998 SettlingMax: -3.5557 Overshoot: 55.8297 Undershoot: 0 Peak: 7.1998 PeakTime: 1.2985
G_{K4}	RiseTime: 0.5465 SettlingTime: 8.9053 SettlingMin: -0.3796 SettlingMax: 0.3224 Overshoot: 1.1231e+03 Undershoot: 450.7490 Peak: 1.0300 PeakTime: 0	RiseTime: 0.2660 SettlingTime: 10.1311 SettlingMin: -3.4584 SettlingMax: -0.9249 Overshoot: 86.0564 Undershoot: 0 Peak: 3.4584 PeakTime: 0.9730

G_{K5}	RiseTime: 0.1907 SettlingTime: 7.0709 SettlingMin: -0.4401 SettlingMax: 0.6280 Overshoot: 119.0418 Undershoot: 93.5922 Peak: 1.0300 PeakTime: 0	RiseTime: 0.8685 SettlingTime: 7.9727 SettlingMin: -3.3682 SettlingMax: -2.2683 Overshoot: 0 Undershoot: 0 Peak: 3.3682 PeakTime: 11.8969
G_{K6}	RiseTime: 90.2064 SettlingTime: 163.3662 SettlingMin: -4.4126 SettlingMax: -3.8752 Overshoot: 0 Undershoot: 54.8352 Peak: 4.4126 PeakTime: 283.1093	RiseTime: 90.2064 SettlingTime: 163.7090 SettlingMin: 6.2020 SettlingMax: 6.9047 Overshoot: 0 Undershoot: 100.6842 Peak: 6.9672 PeakTime: 1.6180
G_{K7}	RiseTime: 4.4927 SettlingTime: 8.5057 SettlingMin: 2.4028 SettlingMax: 2.5527 Overshoot: 0 Undershoot: 0 Peak: 2.5527 PeakTime: 14.9596	RiseTime: 3.9779 SettlingTime: 7.7570 SettlingMin: -9.6265 SettlingMax: -8.7042 Overshoot: 0 Undershoot: 0 Peak: 9.6265 PeakTime: 14.9596
G_{K8}	RiseTime: 0.0104 SettlingTime: 10.2973 SettlingMin: 0.6288 SettlingMax: 1.7875 Overshoot: 79.9439 Undershoot: 0 Peak: 1.7875 PeakTime: 1.1567	RiseTime: 0.3943 SettlingTime: 10.3051 SettlingMin: -7.9232 SettlingMax: -3.1309 Overshoot: 74.8578 Undershoot: 0 Peak: 7.9232 PeakTime: 1.3550
G_{K9}	RiseTime: 0.6632 SettlingTime: 9.5510 SettlingMin: -1.1598 SettlingMax: -0.0328 Overshoot: 166.8664 Undershoot: 237.0099 Peak: 1.1598 PeakTime: 1.4913	RiseTime: 0.0113 SettlingTime: 10.3473 SettlingMin: -0.8882 SettlingMax: 0.9696 Overshoot: 853.7874 Undershoot: 437.8686 Peak: 2.1121 PeakTime: 0.7000

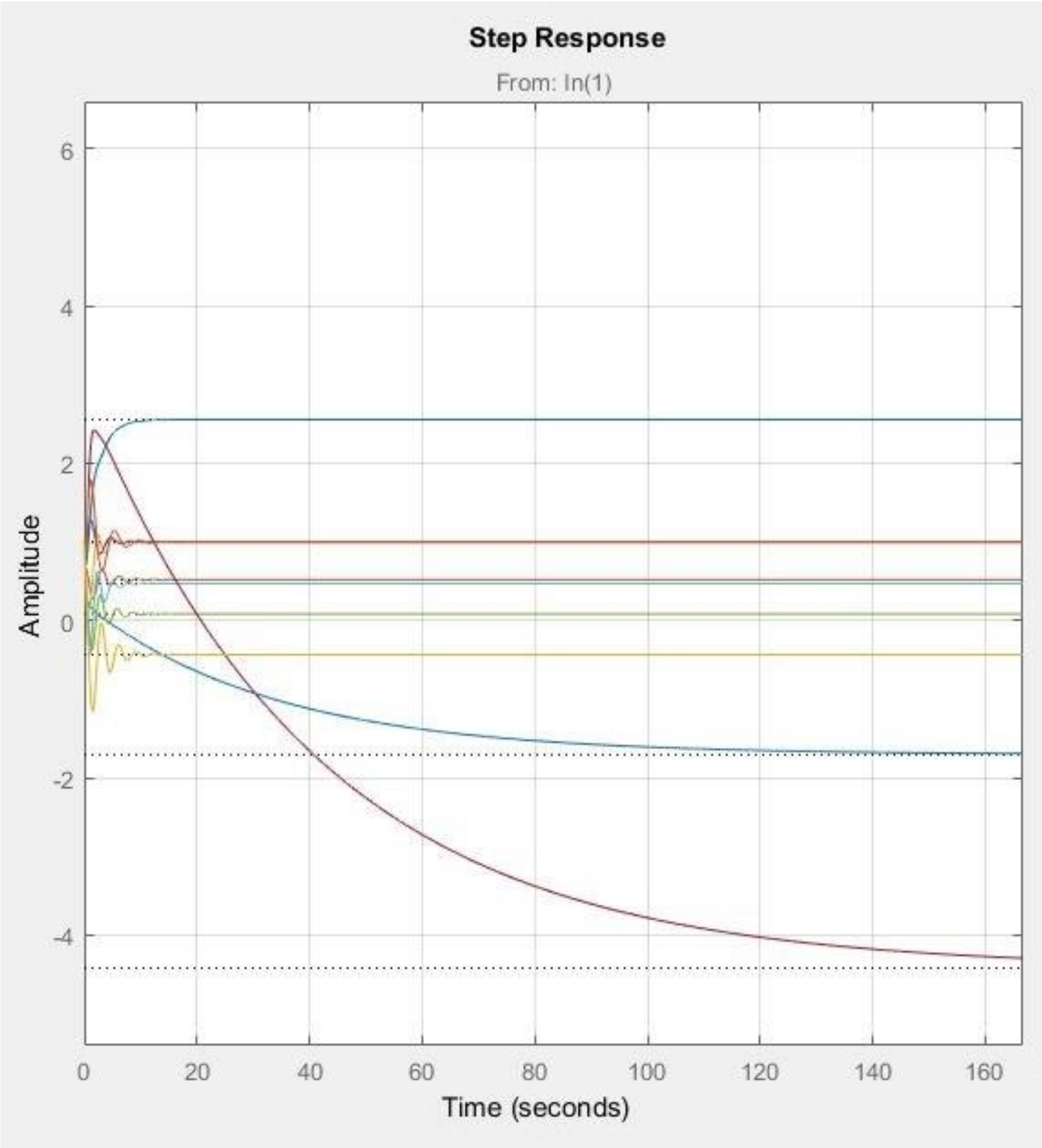

Figure 5-11 The response due to the first input b-1 case λ_2= -0.001

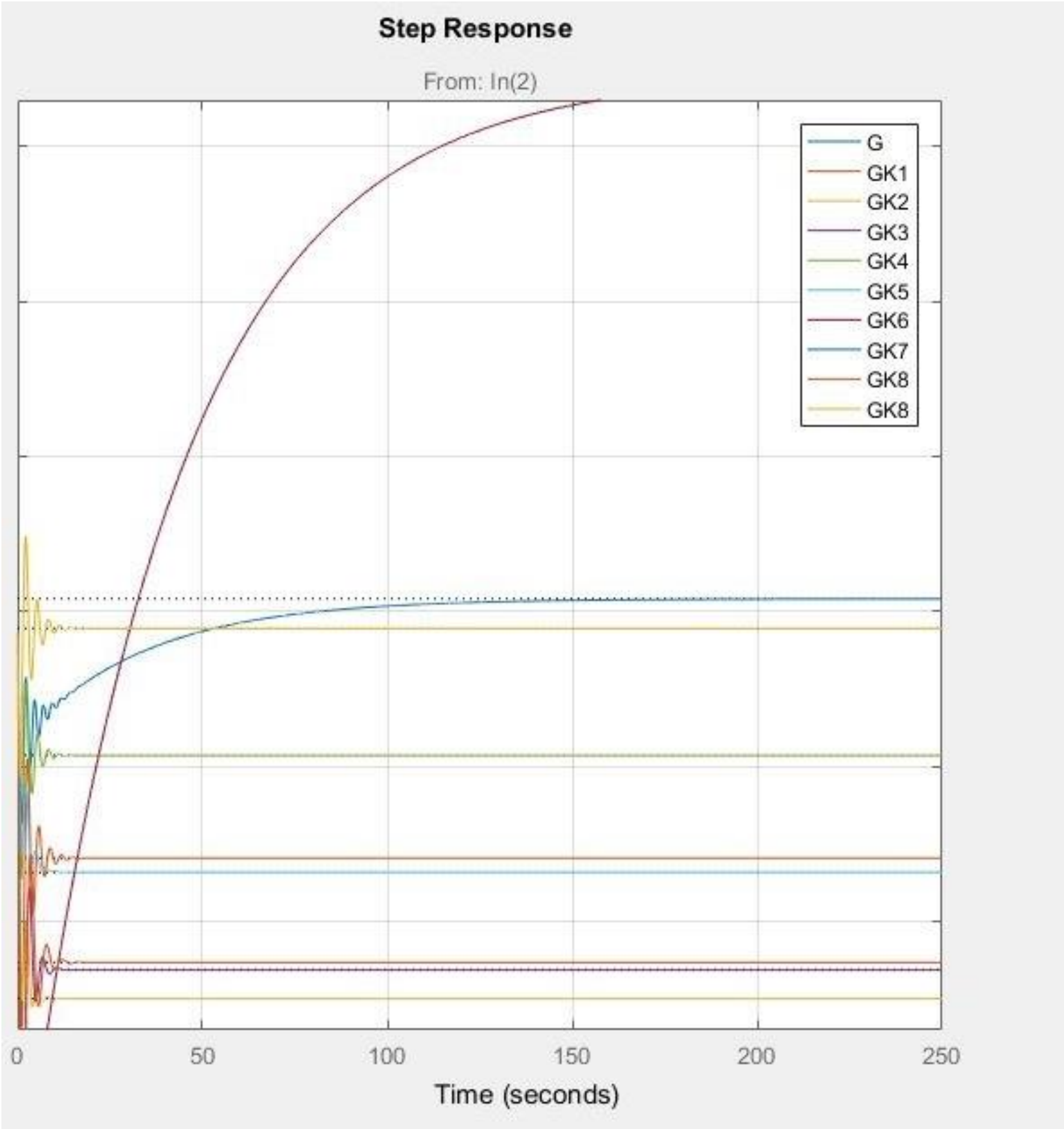

Figure 5-12 The response due to the second input b-2 case λ_2= -0.001

- **Discussion**

Based on the data collected in tables (5-16, 5-17, 5-18, 5-19, 5-22, 5-23, 5-24, 5-25, 5-26, 5-27, 5-28, 5-29) and figures (5-7, 5-8, 5-9, 5-10, 5-11, 5-12) we can conclude that:

- For the state feedback gains, moving away from *jω-axis* would increase the state feedback gains which will maximize to noise amplification.
- For time specifications, moving away from *jω*-axis would reduce the rising time and settling time but increases the overshoot/ undershoot. The reverse happens when moving it closer instead.
- For the sensitivities, since the sensitivity has proportional relationship with the state feedback gain, we can see clearly that moving away from *jω*-axis would increase the sensitivities (overall sensitivity and individual sensitivity) which would maximize modelling inaccuracies and parameter variations.
- For the robustness, moving away from *jω*-axis means moving to stable region, we can see clearly an increasing in the values of stability measures M_2 and M_3 as well as the robust performance measures (relative changes $r_i(\lambda_i)$), meaning that having a more robust system.

General conclusion

State feedback design, in multivariable control systems, may be achieved using block pole assignment. Not only, given a set of desired eigenvalues, the state feedback gain in a MIMO system is not unique but the construction of the block poles isn't either. Thus, different resulting state feedback gains means different design characteristics and performances.

In this thesis the two different canonical forms that have been used to investigate state feedback multivariable control design based on similarity transformation gave clear results on how one MIMO system may perform differently in terms of feedback gain magnitude, robust stability, robust performance, EV sensitivity, time response and the placement of the dominant pole from the way the state feedback gain have designed.

In the nine different forms obtained using solvent assignments and the six different forms obtained from the general controller form, the latter lead mostly to the best results in terms of gain magnitude and robust stability, with the smallest gain magnitudes, individual and overall eigenvalue sensitivities, and biggest stability measures M_2 and M_3. However, the fact that there is infinity of ways of assigning the solvents in the block companion form, doesn't mean that the general controller form will always be the one leading to the best results. The best robust performance was obtained from the solvent assignments with the smallest relative change values.

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