Modeling of Radiative Heat Transfer of a Real Gas in 2D Enclosure

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Abstract

A two-dimensional radiation code based on the discrete ordinates method (DOM) coupled with spectral line-based weighted sum of grey gases (SLW) model for radiative heat transfer in non-grey absorbing–emitting media for use in conjunction with a computational fluid dynamics (CFD) code was developed. The code was applied to four different kinds of participating media: two homogeneous and isothermal and two non-homogeneous and non-isothermal cases with a single participating gas (CO_2 or H_2O). Predictive accuracy of the code is compared with the available results in the literature. Comparisons reveal that the discrete ordinates method (DOM) coupled with SLW model provides accurate solutions for radiative heat fluxes and source terms and can be used with confidence in conjunction with CFD codes.

Keywords
Natural convection, Double diffusion, Gas radiation, Discrete ordinates method, SLW model, 2D cavity.

1. Introduction

Radiative heat transfer plays an important role in many engineering problems, including atmospheric processes and combustion applications, namely, in furnaces, boilers, engines, rocket nozzles and fires **(**Kihwan Kimet al. 2008). The modeling of thermal radiation in these systems necessitates reliable evaluation of the medium radiative properties and accurate solution of the radiative transfer equation (RTE) in conjunction with the time-dependent conservation equations for mass, momentum, energy and chemical species. The computational effort associated with coupled solution of these governing equations can be minimized by using efficient and compatible solution techniques together with computationally feasible gas spectral radiative property models. In this paper, the radiative transfer equation (RTE) is solved by the discrete ordinates method (DOM) which is an attractive method as it provides efficient and flexible computation by enabling use of various higher-order approximations for temporal and spatial discretization. This method is associated with the spectral line weighted-sum-of-gray-gases (SLW) model which is an efficient model for prediction of radiative properties in high temperature gaseous media; the SLW model approaches the exact solution with an increase of the number of gray gases in the model **(**Vincent Goutière et al. 2000).

2. Modelling of radiation and gas radiative property

2.1 The radiative transfer equation

The radiative transfer equation RTE in an emitting-absorbing non-scattering medium may be written as

$$\mu_m \frac{dI_{\nu,m}}{dx} + \eta_m \frac{dI_{\nu,m}}{dy} = -k_\lambda I_{\nu,m} + k_\lambda I_{b,m} \quad m = 1,\dots,M \tag{1}$$

Where I_ν is the spectral radiation intensity, $k\nu$ is the spectral absorption coefficient of the medium, s is the direction of propagation of radiation, and the subscripts ν and b denote wave number and blackbody, respectively.

The boundary condition for a gray surface that emits and reflects diffusely is given by:

$$I_{w\nu}(\vec{s}) = \varepsilon_w I_{b\nu} + \frac{1-\varepsilon_w}{\pi} \int_{\vec{n}.\vec{s}'<0} |\vec{n}.\vec{s}'| I_{w\nu}(\vec{s}') d\Omega' \tag{2}$$

Where ε_w is the emissivity of the surface, $\vec{n}$ is the unit normal vector, $\vec{s}$ is the direction of the outgoing radiation intensity, and $\vec{s}'$ is the incoming direction associated with the elementary solid angle $d\Omega'$.

2.2 Gas radiative property model (SLW)
The motivation for using the SLW model is due to the work carried out by Goutière et al. (2000), who had compared several non gray methods in simple cases, where radiation is the dominant mode of heat transfer. Homogeneous as well as inhomogeneous media are considered. They judge the SLW method as the best, taking into account the computational cost, in terms of calculation time and memory requirements, and the quality of the results. However, the best results are obtained with the SNB and SNBCK models. The same conclusion, referring to the SLW, is stated by Coelho (2002). Furthermore, Bedir et al. (1997) compare both the SNB and SLW models for a one dimensional diffusion flame, and stated that the two models are in good agreement.

Denison and Webb (1993) introduced the Spectral Line Weighted sum of gray gases model, SLW, which can be considered as a refinement to the WSGG model, since it can be applied to inhomogeneous media.

In the SLW model, the non grey gas is replaced by a number of grey gases, each of which is represented with a constant absorption cross-section, $\tilde{C}abs, j$ and associated weight, aj. In this study, 15 absorption cross-sections logarithmically spaced between $3\text{x}10^{-5}$ and 60 m^2/mol for water vapor and $3\text{x}10^{-5}$ and 600 m^2/mol for carbon dioxide were utilized. These logarithmically spaced absorption cross-sections are called supplemental absorption cross-sections, $\tilde{C}abs, j$ as they are used to determine the grey gas weights, but do not appear directly in the corresponding grey gas absorption coefficient calculation. Each of the spaces between two consecutive supplemental absorption cross-sections, $\tilde{C}abs,j$ and $\tilde{C}abs,j+1$, is considered as a separate grey gas associated with a constant absorption cross-section calculated as follows:

$$\tilde{C}abs\,,j\; = exp\left[\frac{ln\left(\tilde{C}abs\,,j\;\right)+ ln\left(\tilde{C}abs\,,j\;+1\right)}{2}\right] \tag{3}$$

Once the grey gas absorption cross-sections are obtained, local absorption coefficients are calculated from:

$$k_{\,j} = N.\tilde{C}abs, j \tag{4}$$

The grey gas weights, a_j, are calculated through absorption-line black body distribution functions derived from high-resolution HITRAN database. Correlations based on this function exist in the literature for the computation of the function F for CO_2 (Denison and Webb, 1995), **(**Modest and Mehta, 2004), (Hongmei and Modest, 2003) and for H_2O (Denison and Webb, 1993)**.**

3. Results and Discussion
To show the validity and the accuracy of the current code, different test problems are considered and compared with the available results of the literature.

3.1 Validation of the RTE solver
Before studying the real-gas cases, a test is made with a grey gas to ensure that the RTE solvers will not influence the results obtained with the real-gas models. The value of the grey gas absorption coefficient is taken as 0.5 m^{-1}. The temperature of the medium is kept uniform at 1000 K and the walls are black and cold. Figure 1 shows that the results yielded by the three RTE solvers are in good agreement.

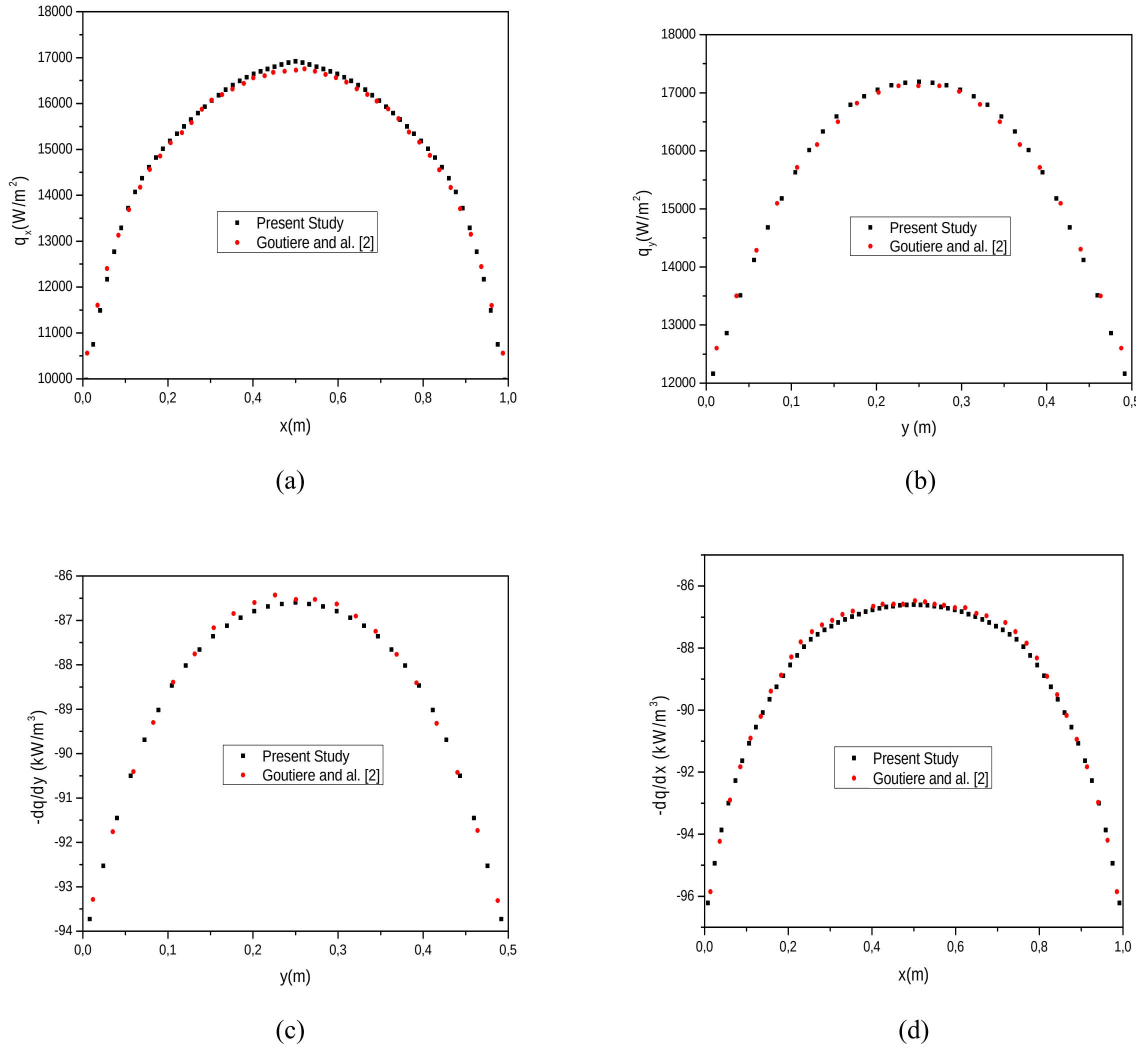


Figure 1: (a) Evolution of the heat flux q_x for the grey case;
(b) Evolution of the heat flux q_y for the grey case;
(c) Evolution of the source term-(divq)x for the grey case;
(d) Evolution of the source term-$(divq)_y$ for the grey case.

As can be observed in Fig.2, the results yielded by the RTE solver is in good agreement.

3.2 Validation of the RTE solver associated with SLW

In this study, we follow the same steps as in the reference Vincent Goutière et al. (2000). The enclosure used for all the tests is rectangular (1x0.5 m) and its walls are black and kept at 0 K (Figure 2). The grid is uniform (61x31) and the S_{16} quadrature with 178 directions is used (T_7 quadrature with 392 directions was used in the reference of Vincent Goutière et al. (2000). In order to make meaningful comparisons, we have chosen four different kinds of participating media: two homogeneous and isothermal cases with a single participating gas (CO2 and H2O), two non-homogeneous and non-isothermal cases with a single participating gas (CO2 and H2O). For all the four cases, the non-participating gas is N2 and the pressure of the gas medium is 1 atm.

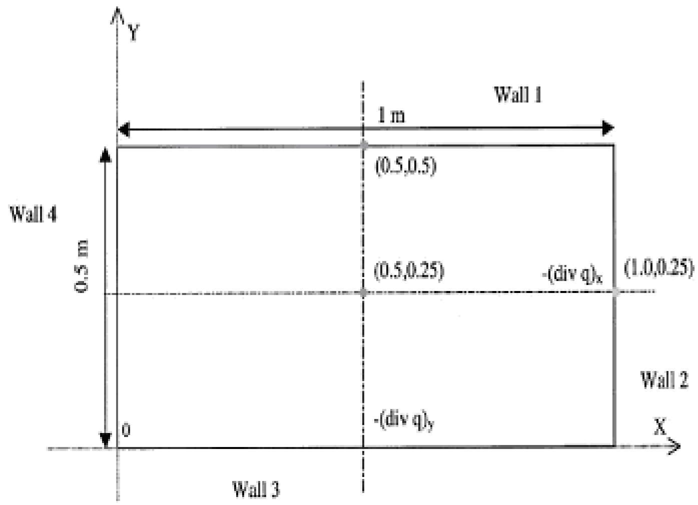
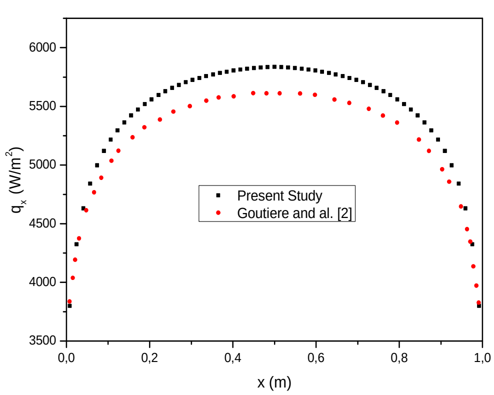


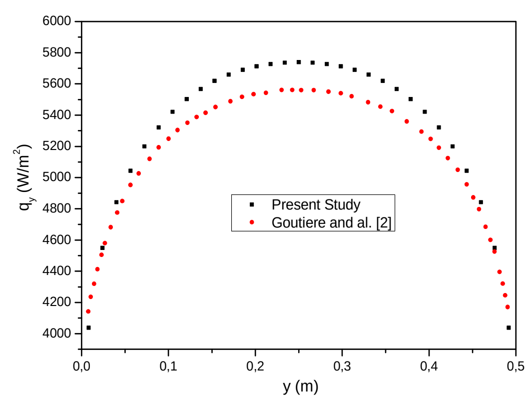
Figure 2: The enclosure

3.2.1 Homogeneous and isothermal medium

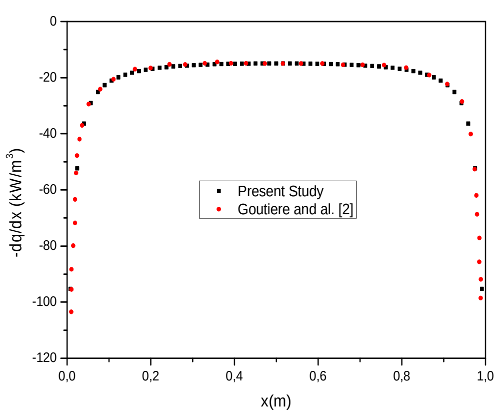
The cavity is filled with either CO_2 (10%) (Case 1) or H_2O (20%) (Case 2), and maintained at 1000K. The wall heat flux and the divergence of heat flux are calculated.



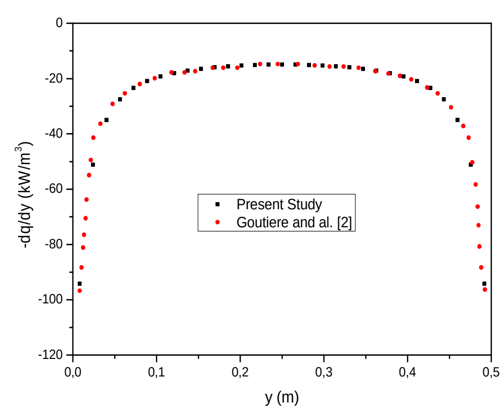
(a)



(b)



(c)



(d)

Figure 3: (a) Evolution of the heat flux q_x ;
(b) Evolution of the heat flux q_y;

(c) Evolution of the source term -(div q)x;
(d) Evolution of the source term-(div q)y
for the case 1(Homogeneous, isothermal medium, CO_2).

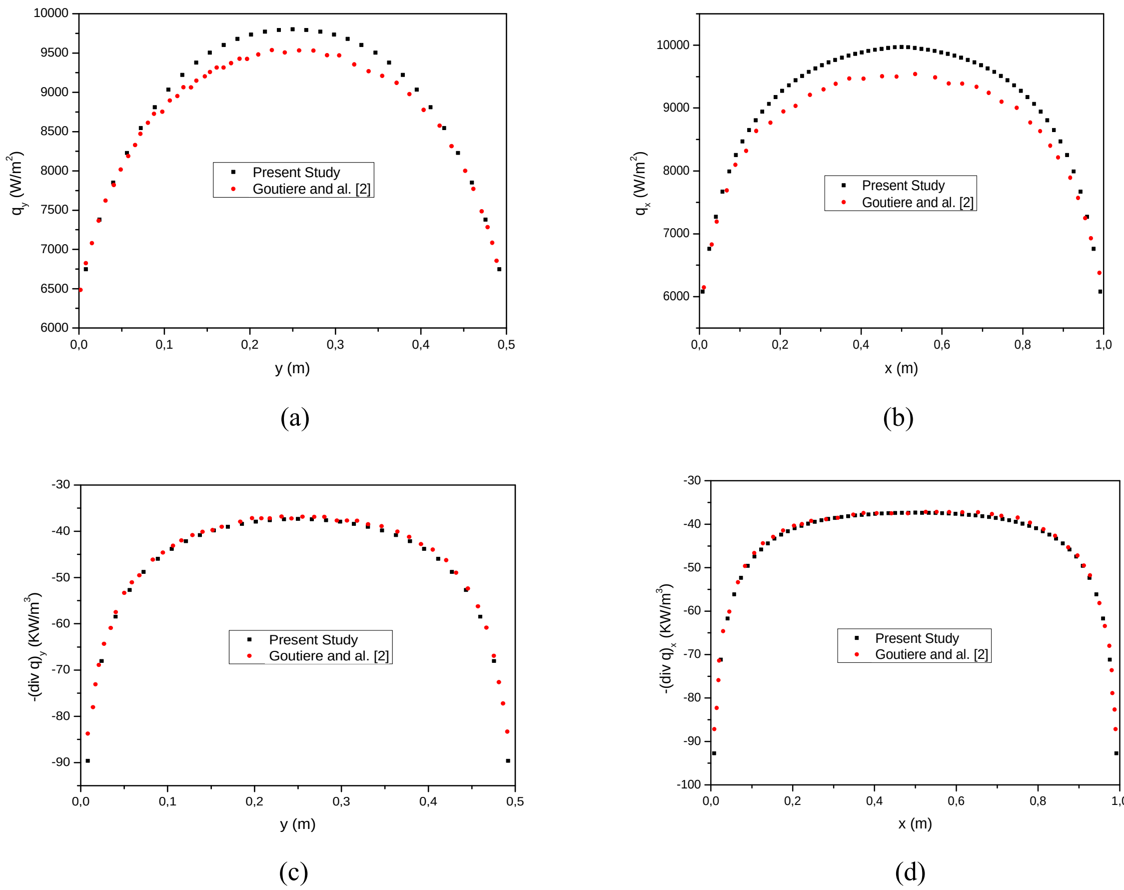


Figure 4: (a) Evolution of the heat flux q_x
(b) Evolution of the heat flux q_y
(c) Evolution of the source term-(div q)x
(d) Evolution of the source term-(div q)y
for the case 1(Homogeneous, isothermal medium, H_2O).

3.2.2 Non homogeneous and non isothermal medium

The cavity is the same as in the cases above, this time with variable temperature and concentration of the radiating gases, CO_2 or H_2O, according to the following functions. The temperature and the concentration are:

$$T(x, y) = T_0 \left[0.3333 \left(1 - 2 \left| x - 0.5 \right| \right) \left(1 - 4 \left| y - 0.25 \right| \right) + 1 \right] \tag{5}$$

$$c(x, y) = c_0 \left[4 \left(1 - 2 \left| x - 0.5 \right| \right) \left(1 - 4 \left| y - 0.25 \right| \right) + 1 \right] \tag{6}$$

Where T_0 is 1200 K and c_0 is 0.02 for CO_2 (case 3) and 0.04 for H_2O (case 4).
The mesh used is the same of that one used in the previous cases. The reference temperature from which the effective absorption coefficient is implicitly determined has some impact on the final result. For this particular case, the reference temperature has been taken to be the volumetric mean and a similar criterion has been used to define the reference concentration for the radiating gas case.

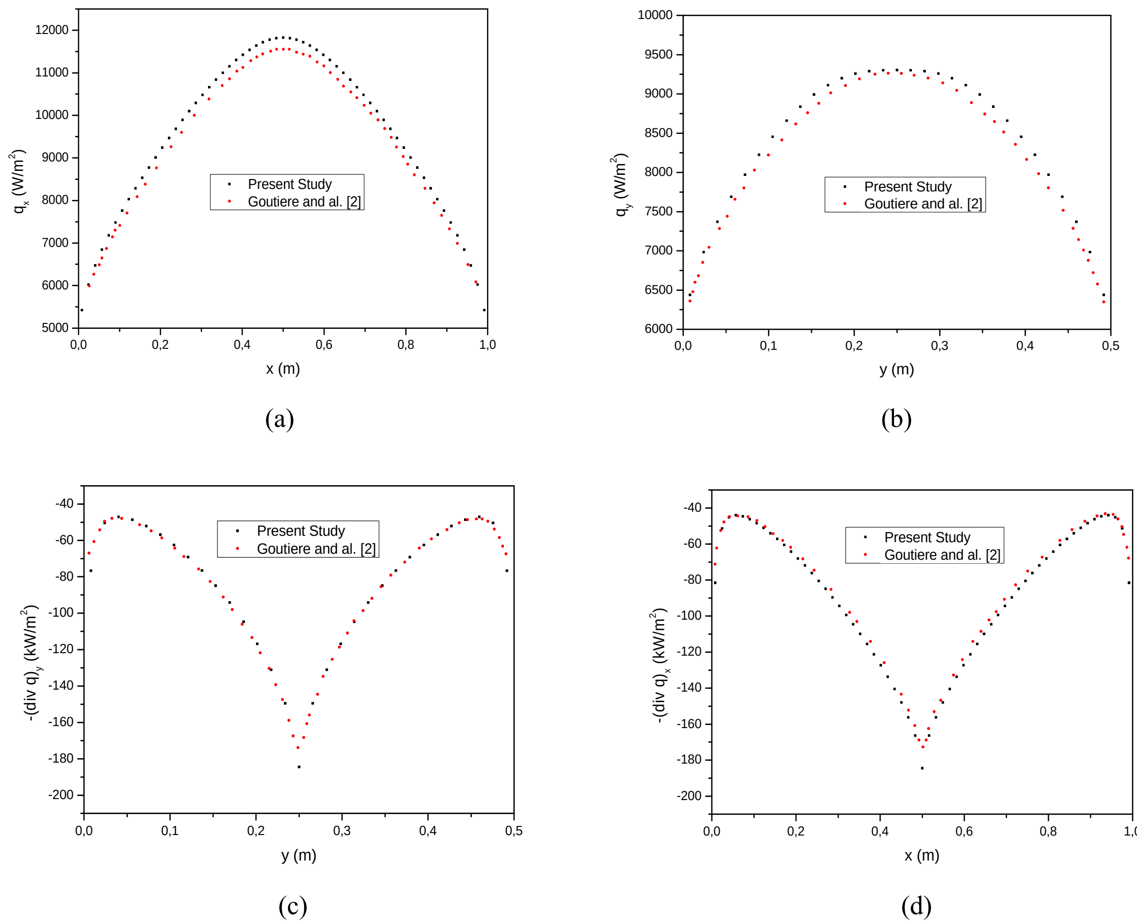
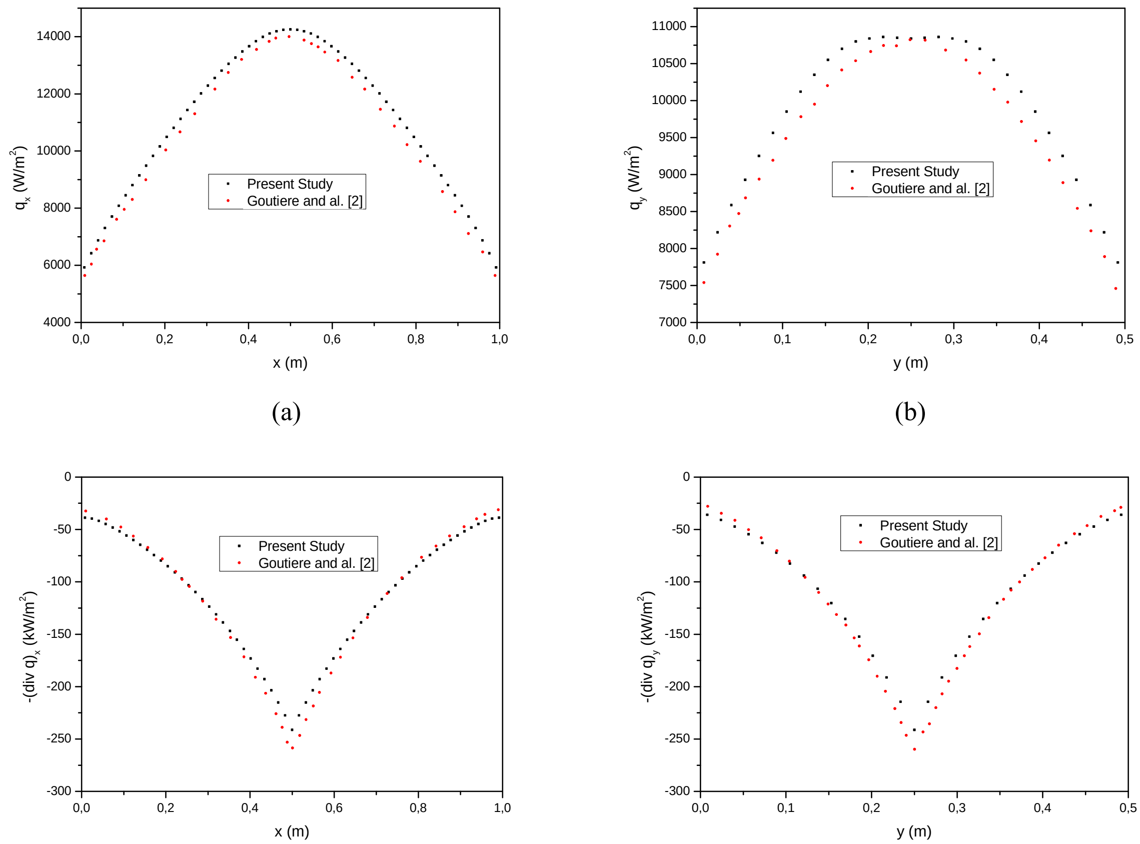


Figure 5: (a) Evolution of the heat flux q_x
(b) Evolution of the heat flux q_y
(c) Evolution of the source term-(div q)x
(d) Evolution of the source term-(div q)y
for the case 3(non Homogeneous, non isothermal medium, CO_2).



(a) (b)

(c) (d)

Figure 6: (a) Evolution of the heat flux q_x ;
(b) Evolution of the heat flux q_y;
(c) Evolution of the source term -(div q)x;
(d) Evolution of the source term-(div q)y
for the case 3 (non Homogeneous, non isothermal medium, H_2O).

As it can be observed from figures 3-5, a good agreement between the calculated values and those reported in the reference of Vincent Goutière et al. (2000) with a small difference for the heat flux divergence. The greatest difference is in the calculated wall heat flux which is probably due to the different reference concentration and temperature used in both calculations and to the ray effects due to the quadrature type (S_{16} quadrature with 178 directions is used in this study and T_7 quadrature with 392 directions was used in the reference of Vincent Goutière et al. (2000)).

5. Conclusions

A two-dimensional radiation code based on the discrete ordinates method (DOM) coupled with spectral line-based weighted sum of grey gases (SLW) model for radiative heat transfer in non-grey absorbing–emitting media for use in conjunction with a computational fluid dynamics (CFD) code was developed. Comparisons reveal that the discrete ordinates method (DOM) coupled with SLW model provides accurate solutions for radiative heat fluxes and source terms and can be used with confidence in conjunction with CFD codes.

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Biography

Mohammed Cherifi is Ph.D. student. He earned his Master degree in 2009. The subject of his thesis is devoted to the Combined Effect of Radiation and Natural Convection in a Square Cavity.

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