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MASTER

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Title:

Active Surge Control Of The Compressor Recycle System Using Feedback Linearization

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DEDICATIONS

I dedicate this work to:

My beloved mother and father who have always been and still the support in my life;

To my sisters and brothers;

To my Family and all my friends.

| | *Oussama*

I dedicate this modest work to:

My beloved parents;

My dear brothers and sisters;

All dear friends:

My dear university mates,

All igee teachers, students and administrative staff.

>>>> Issam

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First and foremost, Praise and glorification be only to **Allah**, the Almighty, the most beneficent and the most merciful, whose blessing and guidance have helped us to finish this work.

A special thanks to our supervisor Dr. BOUSHAKI who managed to find a very helpful internship for us, and also for her great support during all the period of this project.

We would like also to thank all the IGEE members, the guards, the teachers, the head of the department and everyone who helped us somehow during these 5 years.

ABSTRACT

Surge control in the centrifugal compressor recycle system is our main focus in this project.

Surge is a term that is used for instability or oscillation through a compressor and is highly unwanted. The recycle system feeds compressed gas back to the intake via the recycle valve to ensure the safety of the system.

A mathematical model of the recycle system which contains the compressor characteristic is extended and simulated in SIMULINK. The recycle system is proven to be stable as long as the slope of the compressor characteristic is negative.

Two control techniques were used: The first is Surge Avoidance in which a PID controller keeps the operating point far from the unstable region and ensure the total safety of the system. The second method is Active Surge Control in which feedback linearization method is used to linearize the system then linear controllers were used to stabilize the system near the unstable region where efficiency is high.

TABLE OF CONTENTS

Chapter **Page**

TABLE OF CONTENTS

LIST OF TABLES

Table **Page**

LIST OF FIGURES

Figure **Page**

NOMENCLATURE

Symbol	Meaning
w	Mass Flow (kg/s)
ω	Angular Speed (rad/s)
w_t	Throttle Flow (kg/s)
w_f	Feed Flow (kg/s)
w_r	Recycle Flow (kg/s)
τ_d	Driving Torque (N.m)
τ_c	Compressor Torque (N.m)
ψ_c	Compressor Characteristics
a_p or a	Speed Of Sound(m/s)
J	The Moment Of Inertia (kg.m^2)
p_{01}	Ambient Pressure(Pa)
p_p	Pressure In The Plenum(Pa)
p_1	Pressure In Plenum 1(Pa)
p_2	Pressure In Plenum 2(Pa)
$p_{upstream}$	Up Stream Pressure(Pa)
μ	Flow Coefficient
ρ	Density Of The Fluid(kg/m^3)
V_1	Volume Of Plenum 1(m^3)
V_2	Volume Of Plenum 2(m^3)
A	Cross Sectional Area Of Duct(m^2)
L	Length Of The Duct(m)

General Introduction

Centrifugal compressor is a mechanical device that increases the pressure of a gas by reducing its volume. Compressors are used in a wide variety of applications. These includes turbojet engines used in aerospace propulsion, power generation using industrial gas turbines, turbocharging of internal combustion engines, pressurization of gas and fluids in the process industry, transport of fluids in pipelines and many other applications.

The safe and efficient operation of centrifugal compressor is very important in every process. Unfortunately centrifugal compressor are subjected to a very dangerous instability called: surge. This phenomenon is characterized by oscillations in system states such as pressure and mass flow, and is undesirable since it introduces the possibility of severe damage to the machine due to vibrations and high thermal loading resulting from lowered efficiency.

In this project our major goal is to deal with surge instability in centrifugal compressors and try to control and eliminate its negative effects while increasing the efficiency of its output. There are many methods to cope with such problem, the two common and most used are **Surge avoidance** and **Active surge control**.

Traditionally, surge has been avoided using surge avoidance schemes. Such schemes use various means in order to keep the operating point of the compressor away from the region where surge occurs. This method restricts the operating range of the machine to the region in which the system is open loop stable, and efficiently limited. Usually a recycle valve around the compressor is used for actuation which will be the main controlled actuator in this project. Active surge control is fundamentally different from surge avoidance. In an active surge control scheme, the open loop unstable region of the compressor map is sought stabilized with feedback rather than avoided. Thus, the operating regime of the machine is enlarged and the efficiency is increased.

In order to introduce more about the centrifugal compressor, the surge phenomenon, and the different schemes to deal with this instability; this thesis is divided into four main chapters:

In the first chapter: a background about centrifugal compressor will be presented , this will include : Definitions , The main components , The working principle and a deep explanation of the main instabilities that occurs in centrifugal compressor : surge and rotating stall .

In the second chapter: a mathematical model for centrifugal compressor will be developed. At the beginning a broad literature of modelling will be discussed, then a model for both open loop and recycle compressor system will be constructed, tested and validated using MATLAB and SIMULINK simulation results.

In the third chapter: The surge avoidance method will be discussed in order to deal with surge phenomenon and a PID control technique will be used to implement the controller.

In the fourth chapter: Active surge control is used to increase the efficiency of the compressor by allowing the operating point to work near the unstable region of the compressor performance map. And in order to ease that, a mathematical technique called feedback linearization will be employed to linearize the system, then common linear control techniques will be applied and tested.

CHAPTER I: Generalities on compressor and instability phenomenon

1.1 Introduction:

Compressors are used in a wide variety of applications. They are considered as vital tool which insures the efficient production and work of every process.

Commonly we find two types of compressors in use today:

- The dynamic compressors
- Positive displacement compressors

The most common positive displacement compressor is the reciprocating compressor, while the most common dynamic compressor is the centrifugal compressor which will be our major interest in this project.

In this chapter we will be covering centrifugal compressor principles and its major parts, then we will introduce the two common instability phenomena in compressors operation: surge and rotating stall.

1.2 Centrifugal Compressor:

A centrifugal compressor is a type of dynamic compressor, or called also turbo compressor, with a radial design. Unlike displacement compressors that work at a constant flow, dynamic compressors work at a constant pressure and the performance is affected by external conditions such as changes in inlet temperatures. [1]

The physical size (diameter) of a centrifugal compressor is determined by the volumetric flow rate at the inlet. The compression ratio (or head) determines the number of stages (length). The rotating speed of a centrifugal compressor is an inverse function of diameter to maintain a desired peripheral speed at the outer diameters of the impellers regardless of the physical size of the compressor. Very large flow compressors may operate at speeds as low as 3,000 rpm. Conversely, low-volume flow compressors may operate at speeds up to 30,000 rpm. Power requirement is related to mass flow, head, and efficiency.

Depending on the particular application, centrifugal compressor powers can range from as low as 500 hp (400 kW) to more than 50,000 hp (40 MW) [2].

Remark: The centrifugal compressor that we will be working on in this project is called (Vortech S-Trim turbocharger .figure1-1). The data are found in Appendix A

Figure 1-1: Centrifugal Compressor (VORTESH s trim)

1.3 Main components:

The main components of a simple centrifugal compressor are [3]:

1- **Inlet or suction port :** which is a simple pipeline , it may include additional features such as : valve , stationary vanes and both pressure and temperature sensors .these additional devices have an important uses in the control of the centrifugal compressor

2- **Impeller:** the centrifugal compressor's name came from the centrifugal impeller which contain a set of rotating blades that gradually rises the velocity hence the energy of the working gas. In many modern high-efficiency centrifugal compressors the gas exiting the impeller is travelling near the speed of sound.

   There are many types of impellers including: open (visible blades), "covered shrouded", "with splitters" and "w/o splitters ".

3- **Diffuser:** the key role of the diffuser is to convert the kinetic energy of the gas into pressure by gradually slowing the gas velocity.

4- **Collector or Discharge:** as the name implies, the collector purpose is to gather the flow from the diffuser discharge annulus and deliver this flow to a downstream pipe. The collector of the centrifugal compressor can take many shapes and forms. When the diffuser discharges into a large empty chamber; the collector in this case called the plenum. The collector may also contain valves such as the throttle valve and instrumentation to control the compressor.

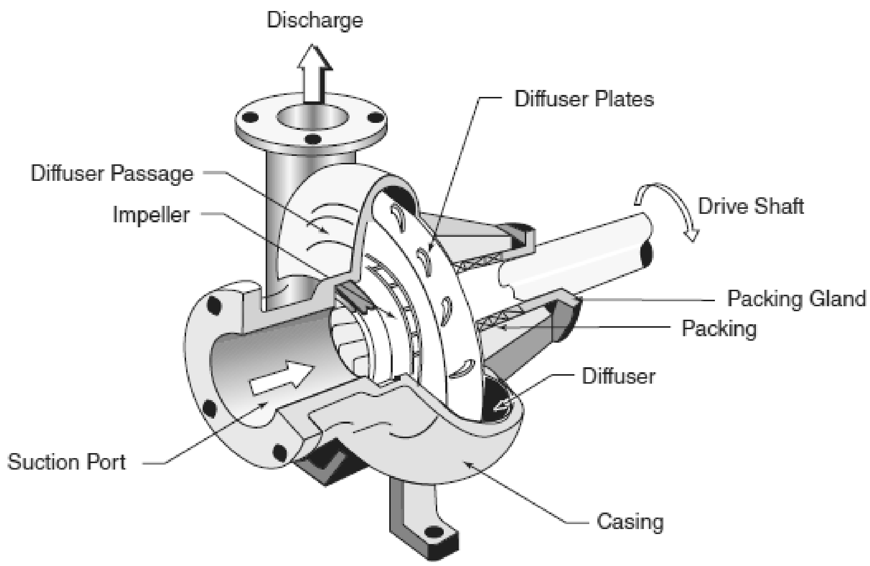


Figure 1-2 The main components of a simple Centrifugal compressor

1.4 Working principle:

When the gas enters the compressor, several forces are acting on the moving gas: The *Aerodynamic force* which helps moving gas from low pressure at the inlet of impellers to higher pressure at the outer edge. Another force is the *centrifugal force* which lifts and moves the gas away from the center of the impeller as well as creating a suction at its center. Another thing is as the gas leaves the outside edge of the impeller it enters the diffuser which is designed so that the flow area increases gradually which allows the gas to decrease in velocity, as the gas slows down the motion energy is converted into increase pressure. Finally when the gas leaves the compressor, the combination of these forces: centrifugal force, aerodynamic force and change in velocity will help produce the gas discharge pressure that is several times higher than the suction pressure.

1.5 Compressor instabilities:

In compressors as the mass flow is reduced the pressure rise increases up to a given point at which further reduction of mass flow leads to more or less severe instabilities. The conventional terminology for this point is the 'surge point' even if different types of instabilities may occur, depending on the compressor and the system in which it is operating. The instabilities are typically classified into two main classes: rotating stall and surge.

1.5.1 Compressor Surge:

Surge is defined as the operating point at which centrifugal compressor peak head capability and minimum flow limits are reached [4].

When the plenum pressure behind the compressor is higher than the compressor inlet pressure, the fluid tends to reverse or even flow back in the compressor. As a consequence, the plenum pressure will decrease, inlet pressure will increase and the flow reverses again. This phenomenon, called surge, repeats and occurs in cycles with frequencies varying from 1 to 6 Hz. So, the compressor loses the ability to maintain the peak head when surge occurs and the entire system becomes unstable. A collection of surge points during varying compressor speed is fitted as surge line [4]. To understand more about the development of surge cycle let us consider the *figure 1.3*

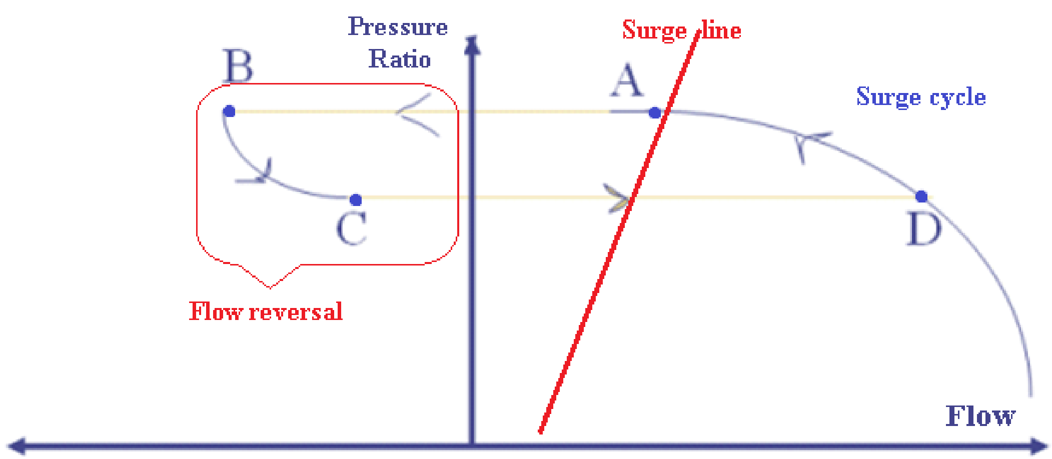


Figure 1-3 Suge cycle in the compressor map

Assume that the system is operating at steady state at Point D. If the demand for gas is reduced, the operating point will move toward Point A, the surge point. If the load is reduced enough, the compressor operating point will cross Point A. Beyond Point A, the compressor loses the ability to increase the discharge pressure which will become less than Plenum pressure (or receiver). The flow will be reversed and the operating point will then jump to Point B.

Point B is not a stable operating point. When the flow reversal occurs, the discharge pressure drops. This forces the operating point to move from Point B to Point C. At Point C, the flow rate is insufficient to build the necessary pressure to return to Point A. Thus, the operating point moves to Point D where the flow rate is in excess the load demanded and the pressure builds until Point A is finally reached. This completes a single surge cycle. The next cycle begins again with another flow reversal and the process repeats until an external force breaks the surge cycle.

1.5.2 Consequences of surge:

Surging can cause the compressor to overheat to the point at which the maximum allowable temperature of the unit is exceeded. Also, surging can cause damage to the thrust bearing due to the rotor shifting back and forth from the active to the inactive side.

1.5.3 Rotating stall:

Stall is another instability that occurs in turbo-compressors. This should not be confused with surge. In stall, the flow oscillates in localized regions around the rotor [5].

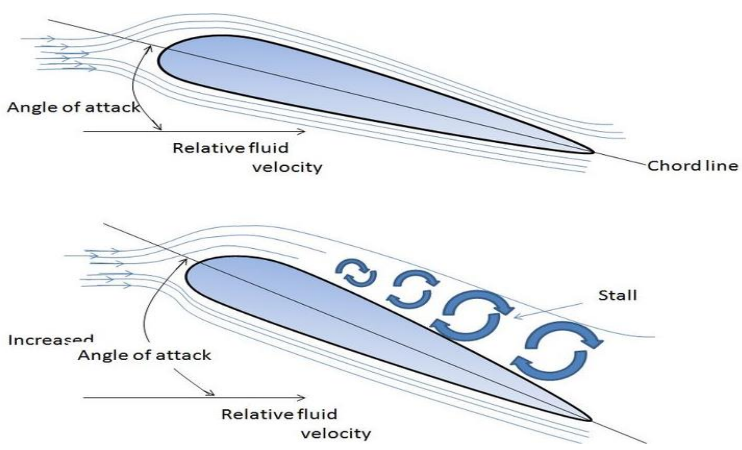


Figure 1-4: Rotating stall phenomenon

Stall is characterized by a distortion of circumferential flow patterns. One or more regions of stagnant flow (“stall cells”) travel around the circumference of the compressor at 10% to 90% of the shaft speed. The stall cells can reduce or completely block the flow, resulting in large vibratory stresses and thermal loads on the compressor rotating components. Stall introduces a gradual or abrupt drop of the pressure rise. Moreover, stall can introduce hysteresis into the system, implying that the flow-rate has to be increased beyond the stall initiation point in order to bring the compression system out of its stalled operating mode [5].

Although rotating stall can bring serious damage to the all types of dynamic compressors, its effect is less common in centrifugal compressors .For this reason, this project will be more concerned about surge phenomena and its effects.

1.6 Conclusion:

In this chapter we have seen a small introduction to compressors, then we specifically talked about centrifugal compressor, how it works, its main components and the major instabilities that occur during its operation. In the next chapter we will try to develop a mathematical model for centrifugal compressor then test and check the validity from simulation results.

CHAPTER II: Modelling of centrifugal compressor

2.1 Introduction:

A mathematical model that is able to predict the onset of limit cycle and the dynamics of the compression system while the compressor is operating, is necessary to design an effective and robust controller.

Modeling of compression systems can be divided into two categories : those that capture the phenomenon of surge which are usually one dimensional, and those that captures the phenomena of both surge and rotating stall which are two dimensional since rotating stall is a two dimensional phenomena . As discussed in Chapter 1, surge is an instability resulting in all system states entering a limit cycle. So a model for this phenomenon can in general be described by a set of ordinary differential equations of various system states, where it is required to reproduce both steady and transient behavior for states, such as pressure and flow, in accordance with that of the experimentally observed.

2.2 Previous work on Modelling:

The first extensive work in the area of modeling compressor instabilities was done by *Greitzer* [6] whose model was originally developed for low speed axial compressors. *Hansen et al [7]*. Then successfully adapted *Greitzer's* model to a small centrifugal compressor. After that *Fink et al* [8] showed that *Greitzer's* model can be applied to predict surge in a turbocharger and also included the effects of rotor speed variations in the *Greitzer's* model. Others have done similar work to produce more complicated models that would also include the compressibility effects. *Table 2.1* [9] provides an overview of the previous work on modelling compressors.

Table 1: Previous work on Modelling of compressor

Model	Year	Flow description	Speed variation	Machine	Instability
Greitzer	1976	1D incompressible	Not Included	Axial	Surge
Hansen et al	1981	1D incompressible	Not Included	Centrifugal	Surge
Maedougal and Elder	1983	1D incompressible	Not Included	Both	Surge
Elder and Gill	1985	1D incompressible	Not Included	Centrifugal	Surge
Fink et al	1992	1D incompressible	Included	Centrifugal	Surge
Botros	1994	1D incompressible	Included	Both	Surge
Badmus et al	1995	1D incompressible	Not Included	Both	Surge
Gravdahl and Egeland	1997	1D incompressible	Included	Centrifugal	Surge
Moore and Greitzer	1986	2D incompressible	Not Included	Axial	Surge & R.S
Feulner et al	1996	1D 2D compressible	Not Included	Axial	Surge &R.S
Islur and Kashwabara	1996	2D compressible	Not Included	Axial	Surge & R.S
Gravdahl and Egeland	1997	2D incompressible	Included	Centrifugal	Surge & R.S

R.S = rotating stall   1D = 1 dimension .

2.3 Centrifugal compressor modelling :

2.3.1 Open loop model :

Axial and centrifugal compressors have similar flow instabilities according to *Gravdahl and Egeland (1999) [10]*. In this section a multispeed model for centrifugal compressors is presented. If the speed is assumed constant, the model reduces to the dimensional model of *(Greitzer, 1976)* .The compressor system modeled by Hansen et al. (1981) in Figure 2.2 was developed firstly for axial compressors than it was shown that it is also valid for centrifugal compressor, with a duct of length L, a plenum of volume Vp, a throttle, and a drive unit imparting a torque on the compressor.

The expression for the pressure is found from the mass balance applied on the plenum. The expression for the mass flow is found from the momentum balance applied on the duct, and the expression for the shaft dynamics is found from the angular momentum relation. The model is [6]:

$$\begin{cases} \dot{p}_p = \dfrac{a_p^2}{V_p}\,(w - w_t) \\ \dot{w} = \dfrac{A}{L}\,\left(\psi_c(w, \omega)\, p_{01} - p_p\right) \\ \dot{\omega} = \dfrac{1}{J}\,(\tau_d - \tau_c) \end{cases} \tag{2.1}$$

Where:

p_p Is the pressure in the plenum

w: is the mass flow

ω : Is the angular speed of the impeller

a_p: is the speed of sound of the gas in the plenum

V_p: Is the volume of the plenum

w_t: is the mass flow through the throttle

A: cross sectional area of the duct

L: the length of the duct

p_{01}:Is the ambient pressure

J: is the moment of inertia of the drive unit

τ_d : Is the driving torque

τ_c: the torque on the shaft from impeller blades

ψ_c: Is the compressor characteristics

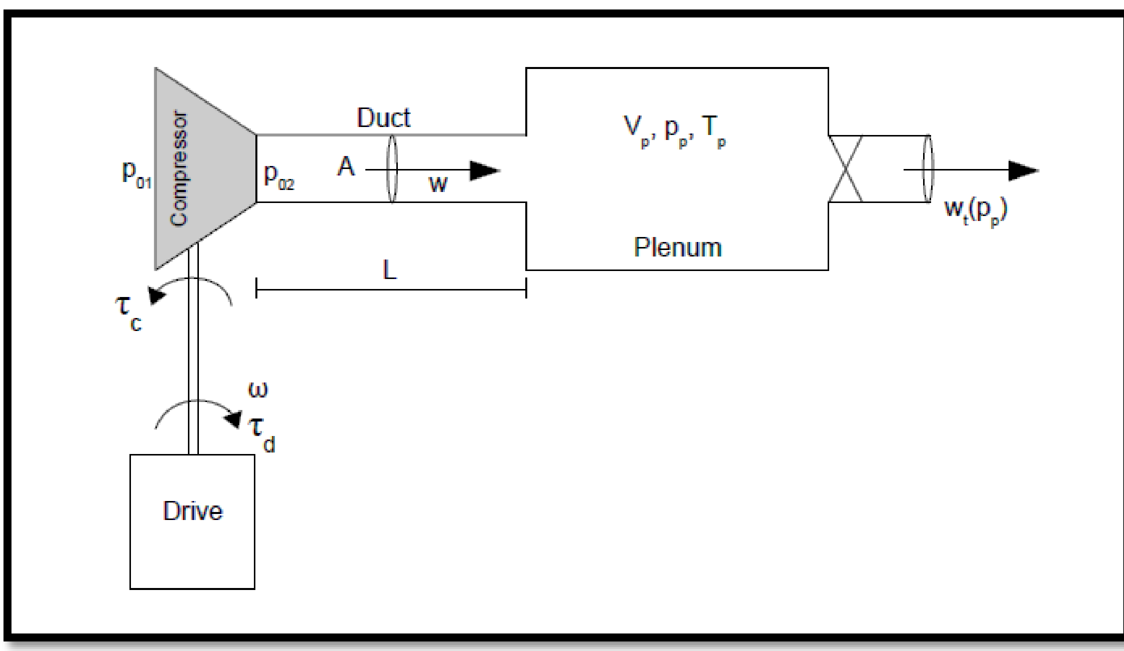


Figure 2-1 Compressor Model by Hansen and al

2.3.2 Torque Model:

The expression of the compressor torque τ_c , was shown in *Gravdahl and Egland* model as :

$$\tau_c = |w| r_2^2 \mu \omega \tag{2.2}$$

Where:

μ: is the flow coefficient r^2 : is the radius of the impeller exit

w: is the mass flow ω: is the angular velocity

2.3.3 Throttle Valve model:

The flow through the throttle $w_t\ (p_p)$ is modeled in [10] as a flow through a valve which can be modeled as flow through an orifice that is:

$$w_t = C\ A_t \sqrt{2\rho\ (p_1 - p_2)} \tag{2.3}$$

Where

w_t : is the mass flow through the orifice C: is the flow coefficient

A_t: Is the area of the orifice opening ρ : is the density of the fluid

$p_1 - p_2$ Is the pressure drop through the orifice

In general this equation is only valid for incompressible flow. Gas is indeed compressible, which is in our case, but since a comparison between an incompressible and compressible flow through a valve will show a small difference we can assume incompressible flow in a small area around the orifice.

For simplicity we will be using a flow coefficient of C =0.65 which is a good approximation to some loss of energy, and for air then we assume C$\sqrt{2\rho} \approx 1$ and equation 2.3 becomes:

$$w_t = A_t \sqrt{(p_1 - p_2)} \tag{2.4}$$

And in order to control the flow through the valve, A_t will be adjusted from 0% to 100% and equation 2.4 becomes:

$$w_t = A_{t\%} \sqrt{(p_1 - p_2)} \tag{2.5}$$

To be able to account for the special case where $p_1 < p_2$ the equation 2.4 can be modified as follow:

$$w_t = sgn(p_1 - p_2)\sqrt{|p_1 - p_2|} \tag{2.6}$$

The sign function is not continuous, and it can be approximated as :

$$sgn(x) = \lim_{\xi \to \infty} \tanh(\xi x) \tag{2.7}$$

And the absolute value can be written:

$$|x| = x \, . sgn(x) = x \lim_{\xi \to \infty} \tanh(\xi x) \tag{2.8}$$

The flow through the throttle is finally given as

$$w_t = \tanh\left(\xi\left(p_p - p_{01}\right)\right) A_{t\%} \sqrt{\left(p_p - p_{01}\right) \tanh\left(\xi\left(p_p - p_{01}\right)\right)} \tag{2.9}$$

2.3.4 The Compressor characteristics:

The compressor characteristics or also known as performance map of compressor shows how the pressure developed by the compressor varies with mass flow for a given compressor speed. It is very important to have a mathematical model of the performance map in order to get an accurate model of our compressor.

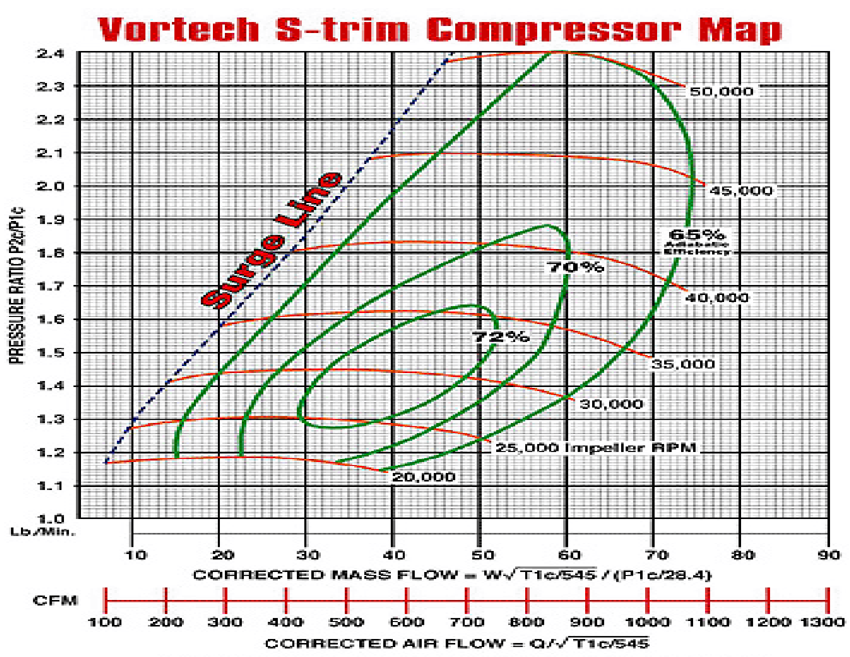


Figure 2-2 : Vortech S-trim Compressor Map

A simple cubic approximation of the characteristics curve is proposed by Moore and Greitzer (1986) , which has found a wide acceptance in numerous paper regarding surge and stall phenomena . This equation is only valid for a constant speed [11].

$$\psi_c(\phi) = \psi_{c0} + H \left[1 + \frac{3}{2}\left(\frac{\phi}{W} - 1\right) - \frac{1}{2}\left(\frac{\phi}{W} - 1\right)^3\right] \qquad (2.10)$$

Where :

ϕ : is the mass flow (ϕ *instead of w is used here to not be confused with w*)

ψ_{c0} , H and W : Are parameters derived from performance curve *figure 2.3*

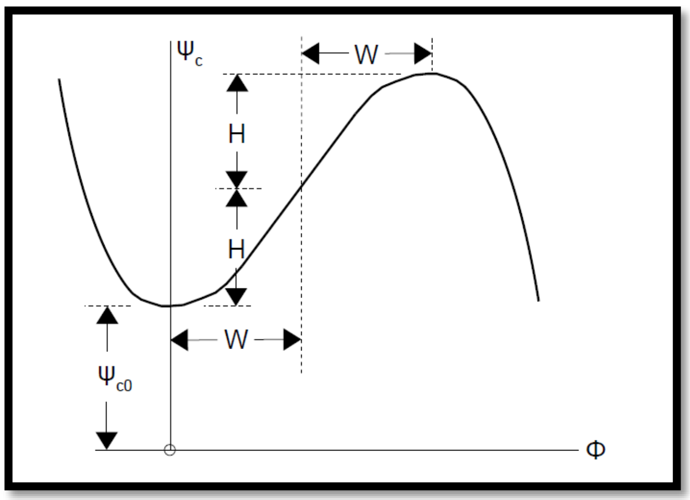


Figure 2-3: Notation used in definition of cubic compressor characteristic

The equation – could be viewed as a third order fit witch captures much of the general performance properties of the compressor.

Now, let us consider some measurements that was taken from the Vortech S-trim supercharger *figure 2.2*(www.vortechsupercharger.com),The measurements represent points of pressure ratio for different mass flows and speed. To make the characteristic continuous in both mass flow and speed, a MATLAB function “Polyfit“ which is based on polynomial curve fitting technique is used where each speed line is approximated by :

$$\psi_c(w , N) = C_0(N)w^3 + C_1(N)w^2 + C_2(N)w + C_3(N) \qquad (2.11)$$

Where:

N : is the rotational speed w : is the flow

$[C_0\ C_1 C_2 C_3\ ]$: Coefficients corresponding to a given speed.

The results of the approximation is shown in the *figure 2.4 :*

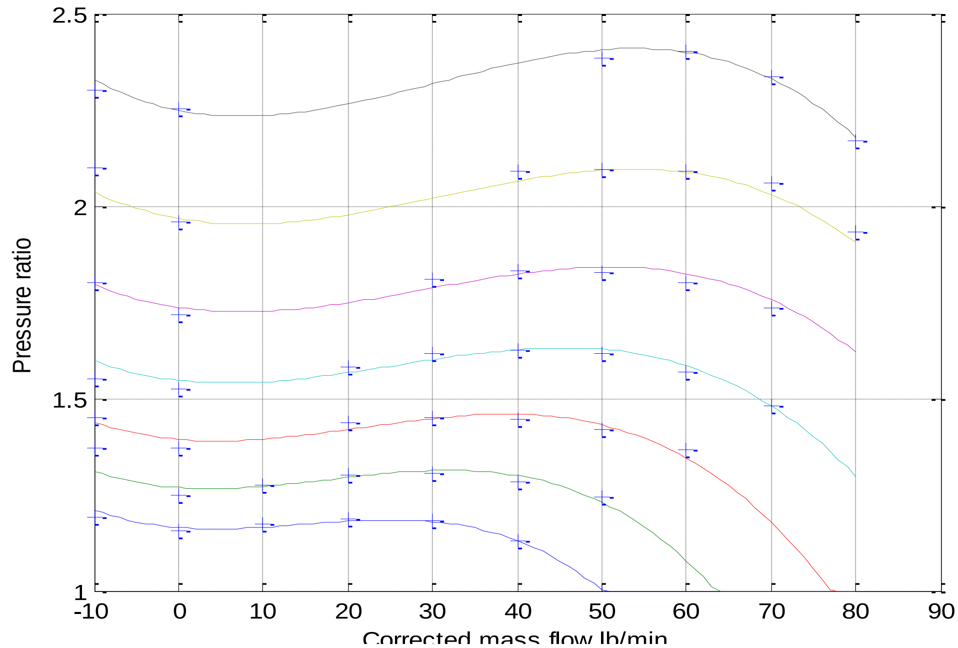


Figure 2-4:Polyfit approximation of compressor map (Vortech S-trim supercharger)

Remark : The *polyfit* function is found in the "approximation_polyfit.m" file in the simulation files

2.3.5 Stability of the model:

By assuming a constant speed the model of compressor system is reduced to :

$$\begin{cases} \dot{p}_p = \dfrac{a_p^2}{V_p}\,(\,w - \,w_t) \\ \dot{w} = \dfrac{A}{L}\,\left(\psi_c(w\,)\,p_{01} - p_p\,\right) \end{cases} \tag{2.12}$$

The equilibrium points can be clearly defined as:

$$w^* = \,w_t({p^*}_p)$$
$$p_p^* = \,\psi_c(w^*\,)\,p_{01}$$

That is: the mass flow is equal to the throttle flow and the plenum pressure is equal to the pressure developed by the compressor.

In order to study the stability, first let us linearize the system around the equilibrium point .Using the Jacobian evaluated at the equilibrium points we get:

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial p_p} & \frac{\partial f_1}{\partial w} \\ \frac{\partial f_2}{\partial p_p} & \frac{\partial f_2}{\partial w} \end{bmatrix} = \begin{bmatrix} -\frac{a_p^2}{V_p}\frac{\partial w_t}{\partial p_p}|p_p^* & \frac{a_p^2}{V_p} \\ -\frac{A}{L} & \frac{A}{L}p_{01}\frac{\partial \psi_c}{\partial w}|w^* \end{bmatrix} \tag{2.13}$$

By defining:

$$k_1 = \frac{a_p^2}{V_p} \qquad k_2 = \frac{a_p^2}{V_p} \qquad a = p_{01} \qquad \partial_1 = \frac{\partial w_t}{\partial p_p}|p_p^* \qquad \partial_2 = \frac{\partial \psi_c}{\partial w}|w^*$$

Then A becomes:

$$A = \begin{bmatrix} -k_1\partial_1 & k_1 \\ -k_2 & k_2 a\partial_2 \end{bmatrix} \tag{2.14}$$

The characteristic polynomial is given by $det(\lambda I - A) = 0$ hence we get:

$$\lambda^2 + (k_1\partial_1 - k_2 a\partial_2)\lambda + k_1k_2(1 - a\partial_2\partial_1) = 0 \tag{2.15}$$

The eigenvalues are the solution of the second order equations and given by:

$$\lambda = \frac{-(k_1\partial_1 - k_2 a\partial_2) \pm \sqrt{(k_1\partial_1 - k_2 a\partial_2)^2 - 4k_1k_2(1 - a\partial_2\partial_1)}}{2} \tag{2.16}$$

It can be seen from the equation that if either $(k_1\partial_1 - k_2 a\partial_2)$ or $(1 - a\partial_2\partial_1)$ is negative we will have an unstable system. So we have from the second term:

$$\frac{1}{p_{01}}\left(\frac{\partial w_t}{\partial p_p}|p_p^*\right)^{-1} < \left.\frac{\partial \psi_c}{\partial w}\right|w^* \Rightarrow \frac{\partial \psi_t}{\partial w_t}|p_p^* < \frac{\partial \psi_c}{\partial w}|w^* \tag{2.17}$$

That means that if the slope of the compressor characteristic $(\frac{\partial \psi_c}{\partial w})$ becomes bigger than the slope of the throttle characteristic $(\frac{\partial \psi_t}{\partial w_t})$, the system is in this case *statically unstable* (tends to happen a distance to the left of the compressor characteristic).

And from the first term we have:

$$\frac{\partial \psi_c}{\partial w}|w^* < \frac{k_1}{k_2 a}\frac{\partial w_t}{\partial p_p}|p_p^* \tag{2.18}$$

In this case the system is termed *dynamically unstable* and the instability tends to happen just to the left of the peak of the compressor characteristics.

Particularly if we write *equation 2.18* in the form:

$$\frac{\partial \psi_c}{\partial w}\Big|w^* \;\; \frac{\partial \psi_t}{\partial w_t}\Big|p_p^* \quad > \; \frac{k_1}{k_2 a} \qquad (2.19)$$

Where the expression to the right is very small, and we know that the flow through the throttle $\frac{\partial \psi_t}{\partial w_t}\Big|p_p^*$ is increasing ($\partial_1 > 0$) , so for higher mass flow and pressures, the point of instability moves towards the compressor characteristic maximum (the peak). **Conclusion:** if the slope of compressor characteristic is negative or zero. That is $k_1\partial_1 - k_2 a\partial_2 > 0$ and $(k_1\partial_1 - k_2 a\partial_2)^2 - 4k_1k_2(1 - a\partial_2\partial_1) < k_1\partial_1 - k_2 a\partial_2$ then the eigenvalues are strictly negative and the system is asymptotically stable.

2.3.6 Simulation of the open loop model:

SIMULINK program is used to simulate Vortech s-trim compressor model. Simulation files can be found in the folder under the name (Open_loop_model) .

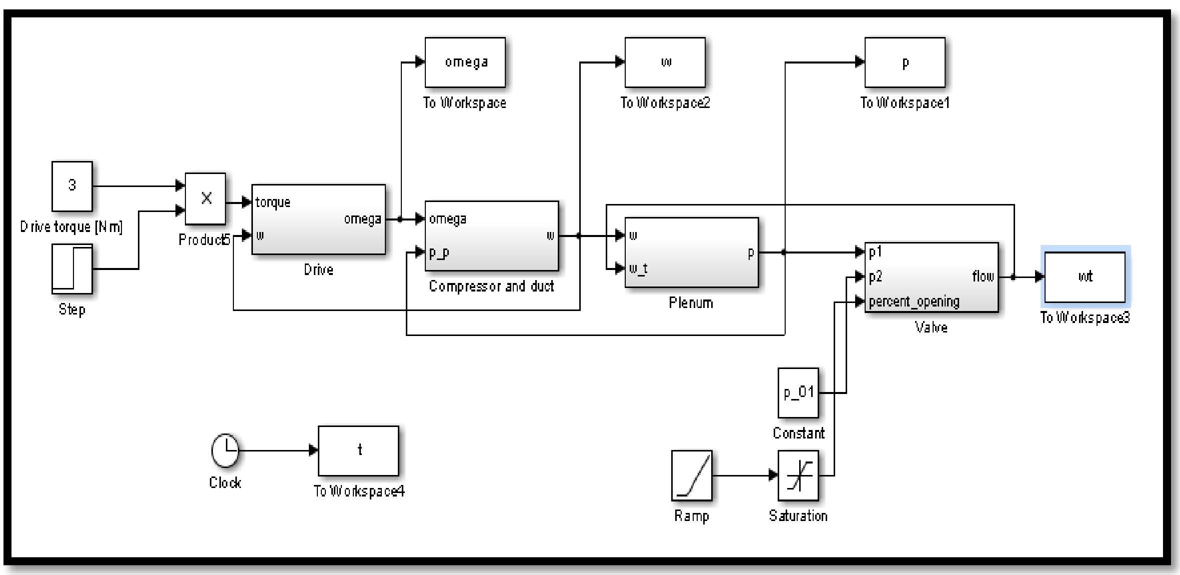


Figure 2-5:Open loop model using SIMULINK

2.3.7 Simulation results:

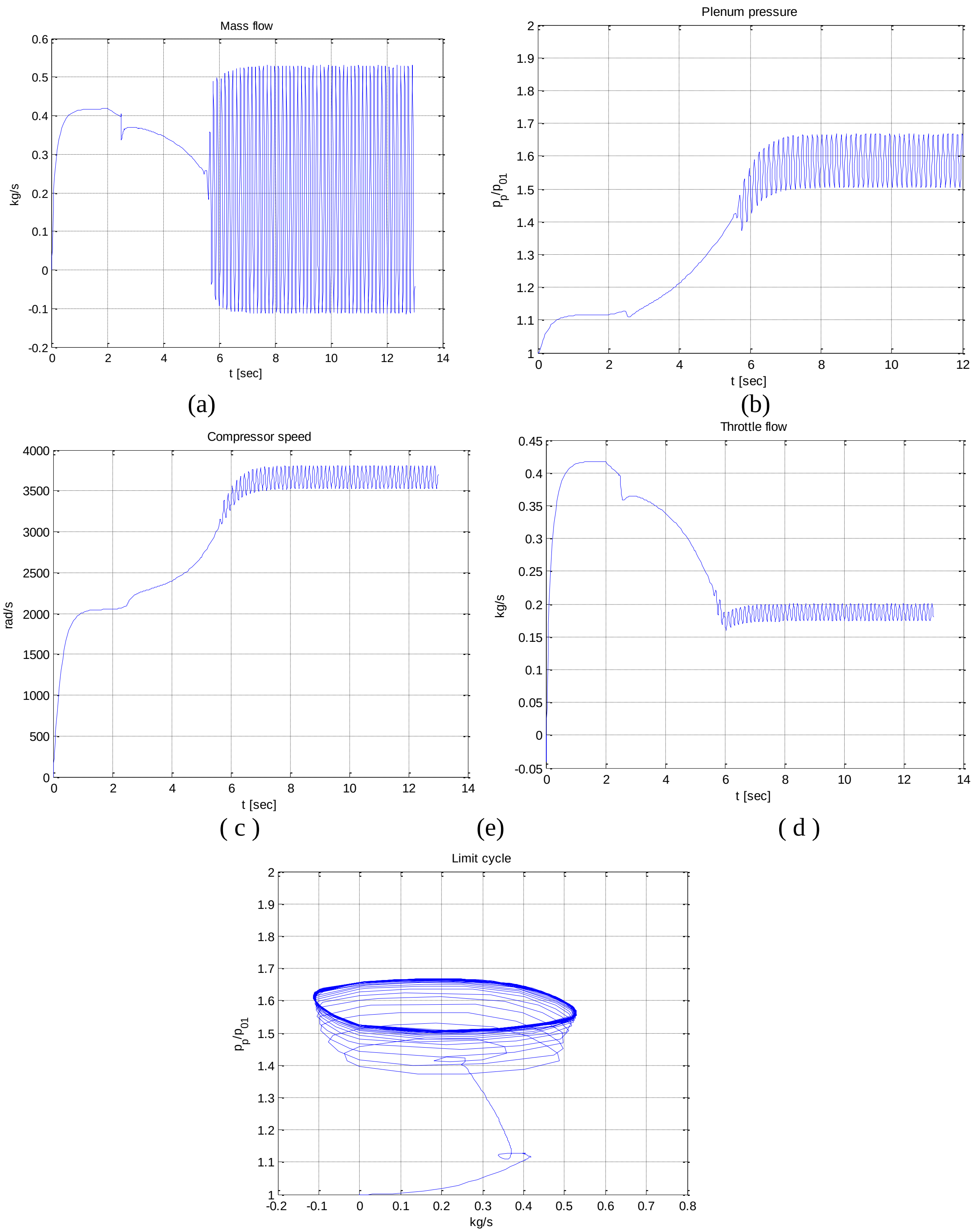


Figure 2-6:Simulation results of open loop model . (a) mass flow , (b)plenum pressure, (c) compressor speed, (d) throttle flow, (e) limit cycle

Discussion :

1- The throttle valve is initially 100 % open **(figure 2.6, d).** At time t= 2 seconds, it starts closing and when it reaches a point (not completely closed) the system enters the surge phenomenon (limit cycle).
2- At t=2s the throttle flow **(figure 2.6, d)** and mass flow **(figure 2.6, a)** starts decreasing while the plenum pressure **(figure 2.6, b)** and compressor speed **(figure 2.6, c)** increase gradually, at t=5.8 s the oscillations happen which prove the effects of surge phenomenon on the system.
3- The speed at which the system enters a limit cycle is approximately 3153 rad/s (about 30000 rpm) **(figure 2.6,b)**
4- The mass flow at which the system enters a limit cycle is approximately 2.25 kg/s**(figure 2.6,a)**
5- The surge cycle has a frequency of approximately 6 Hz. **(figure 2.6,e)**
6- Closing the throttle valve will obviously lead to the previous results hence our model is correct and acceptable.

2.4 Recycle Model:

As previously mentioned, surge is a highly unwanted phenomena in continuous flow compressor system.

.In this section a recycle valve will be added to the previous model and manual control will be done to prove its effect in stabilizing the system.

2.4.1 Recycle valve model:

Applying the mass balance equation (Equation 3.20 in White (2008)[22]) on plenum1.figure 2.- Yields:

$$\left(\frac{dm}{dt}\right)_{syst} = 0 = \frac{d}{dt}\left(\iiint_{CV} \rho dV\right) + \iint_{CS} \rho\left(\overrightarrow{V_r}.\overrightarrow{n}\right) dA \tag{2.20}$$

$$\frac{d}{dt}(\rho_1 V_1) = w_f + w_r - w \tag{2.21}$$

where w_r is the flow through the recycle valve and it is given by

$$w_r = A_r\sqrt{p_2 - p_1} \tag{2.22}$$

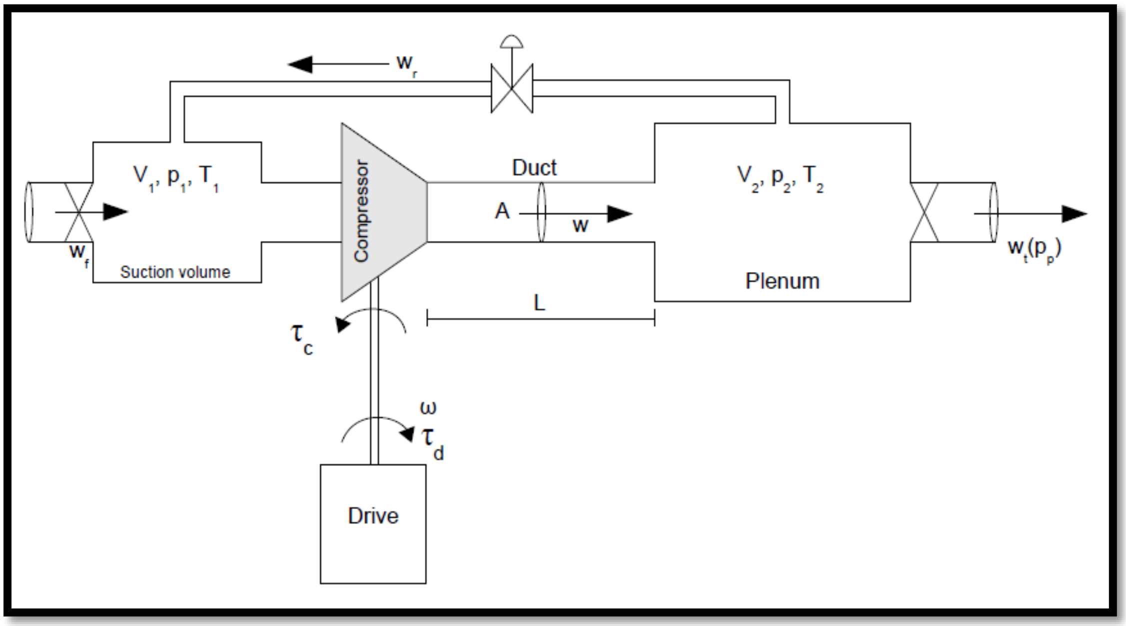


Figure 2-7: Recycle valve model

Now if we assume that the gas is ideal and isentropic. That means ignoring the molecular size of the gas along with intermolecular forces. So there is no energy losses due to viscosity or heat conduction. The following relation can be deduced from (Equation 9.15, White (2008) [22])

$$dp = a^2 \, d\rho \tag{2.23}$$

Where

a: is the speed of sound in gas .

from equation 2.21 and equation 2.23 We get the equation of pressure in plenum 1:

$$\frac{d}{dt}(p_1) = \frac{a^2}{V1} \, (w_f + w_r - w) \tag{2.24}$$

The same thing is done for plenum 2:

$$\frac{d}{dt}(p_2) = \frac{a^2}{V2} \, (w - w_t - w_r) \tag{2.25}$$

The final model is:

$$
\begin{cases}
\dot{p_1} = \dfrac{a^2}{V_1}\,(w_f + w_r - w) \\
\dot{p_2} = \dfrac{a^2}{V_2}\,(w - w_t - w_r) \\
\dot{w} = \dfrac{A}{L}\,(\psi_c(w\,,\omega)\,p_1 - p_2) \\
\dot{\omega} = \dfrac{1}{J}\,(\tau_d - \tau_c)
\end{cases}
\tag{2.26}
$$

But a problem has appeared when trying to simulate this model. The cause of this problem is a raise of pressure in volume 1 due to a constant supply of the feed flow. In reality the feed flow should not be constant, instead it should be dependent on the compressor flow somehow. This is why the feed flow should be modeled as a flow through an orifice hence a feed flow valve and it is modeled as before:

$$
w_f = A_f\sqrt{p_{upstream} - p_1} \tag{2.27}
$$

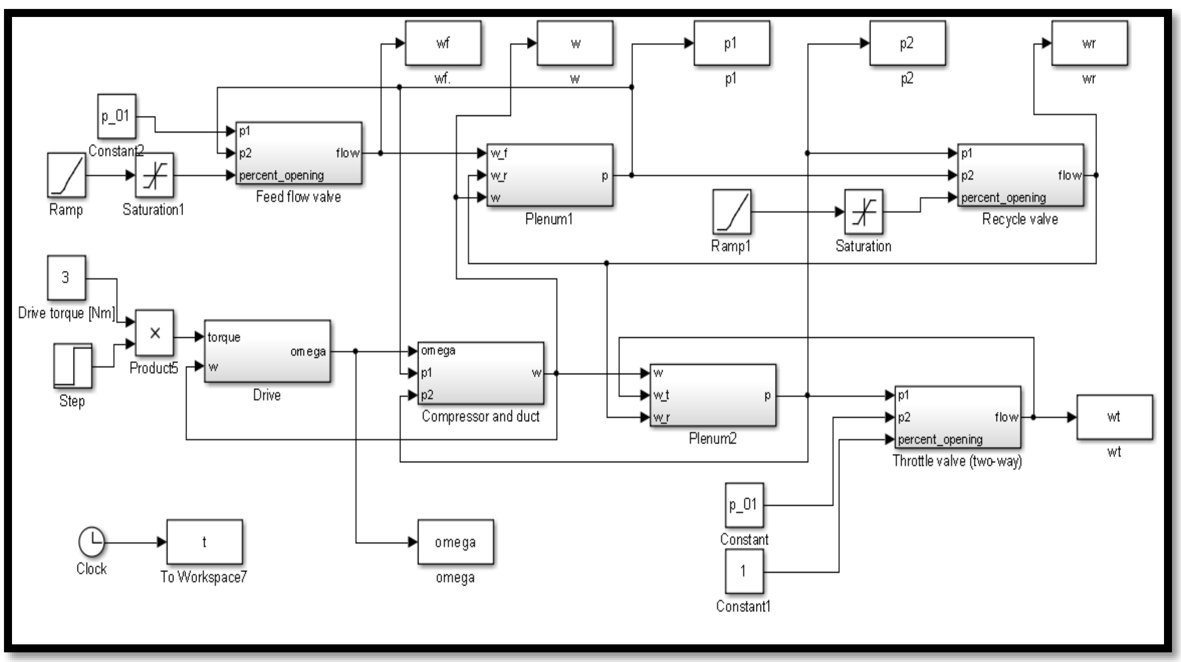


Figure 2-8: SIMULINK model of the recycle model

2.4.2 Simulation results:

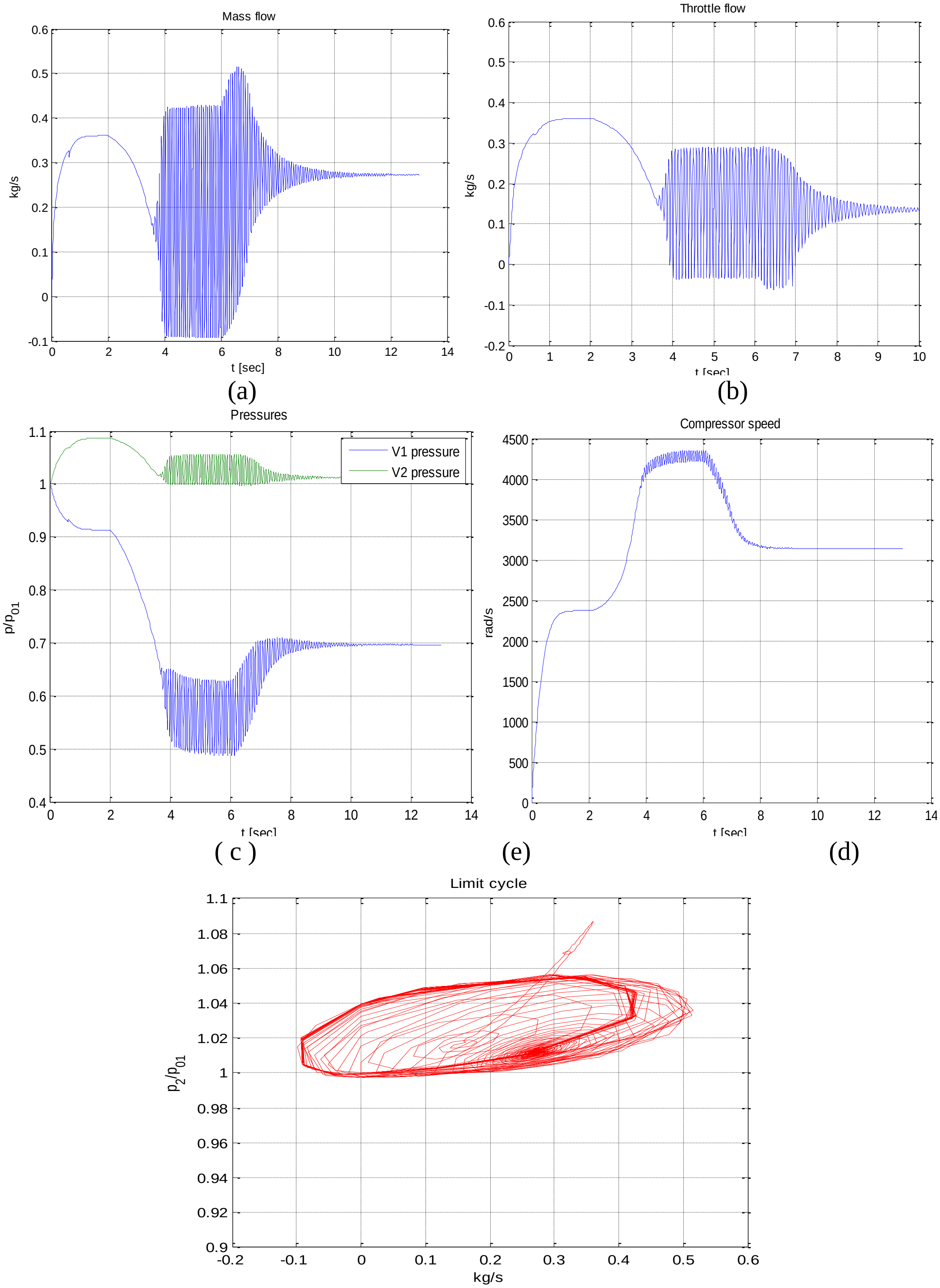


Figure 2-9: Simulation results of the recycle valve model. (a) mass flow , (b)plenum pressure, (c) compressor speed, (d) throttle flow, (e) limit cycle

2.4.3 Discussions:

1- Initially, the pressures in the two volumes are set to ambient pressure, the compressor speed and the mass flow are set to zero while the throttle and feed flows are taken at their initial states.

2- After starting simulation we see that the system will stabilize at t=1 s.(start-up stabilization)

3- The feed flow valve is initially open, at t= 2.0 second it start closing. It can be clearly seen from *figure 2.9* that :

  - discharge pressure decreases from 10.8 x 10^4Pa to 10.1 x 10^4 Pa and the suction pressure decreases from 9.15 x 10^4 Pa to 6.5 x 10^4 Pa **(Fig.2.9.c)**
  - the mass flow enters in damping (oscillation) at time t = 3.5 s (**Fig.2.9.a**)
  - rotating speed increases from 2200 rad/s to 4250 rad/s with an overshoot of 48% (**Fig.2.9.d**)
  - The feed flow here act as a disturbance for our system.

4- At t= 3.5 the system enters a surge phenomenon and starts oscillating (**Fig.2.9.e**).

5- during surge phenomena the operating point oscillates between positive and negative values as it is shown in (**Fig.2.9.a**)and (**Fig.2.9.b**)

6- In order to stabilize the system the recycle flow is used. At t = 6 second it start opening. And the oscillations start decreasing where it can be seen that :

  - The compressor speed decreases due to the higher mass flow load applied on the impellers and stabilizes at 3200 rad/s.
  - The mass flow stabilizes at 0.29 kg/s.
  - The suction and the discharge pressures stabilize at 7.05 x 10^4Pa and 10.2 x 10^4 Pa respectively.

As conclusion: the recycle valve has done a remarkable effect on stabilizing the system, hence it can be used as an actuator for an automatic control of compression system.

“The simulation files of the recycle model can be found on the folder under the name “Recycle_model”

2.5 Conclusion:

In this chapter, we presented a broad literature of modeling centrifugal compressors and various approaches. We started with the model presented by *Greitzer 1976* and we ended up with the model of *Gravdah and Egeland 1997* .

The model of the centrifugal compressor was implemented with Simulink and the results proved the validity of the model.

The simulation results have shown also that surge phenomena could happen whenever the flow is below certain value where the operating point oscillates between positive and negative values .One way to prevent this is by using some actuators like recycle valve and throttle valve, since we could stabilize the system when a perturbation occurs.

In the next chapter, Recycle valve will be used along with surge avoidance method and further work on controlling the surge will be discussed.

CHAPTER III: Surge Avoidance

3.1 Introduction:

One of the common methods used nowadays to prevent centrifugal compressor from surge phenomena is called surge avoidance (also called Anti-surge control) .it consists of a recycle loop that can be activated by a fast-acting valve (recycle valve or anti-surge valve) when the control system detects that the compressor approaches its surge limit .

In the previous chapter a model with recycle valve proved that surge phenomena can be prevented by manual opening of the recycle valve .In this chapter surge avoidance method will be discussed and a PID control will be applied to prevent compressor system from surge .

3.2 Surge avoidance method:

In the surge avoidance method, the compressor is prevented from operating beyond surge line or some area close to it .This is achieved by recirculation of flow back to the intake using recycle valve .The surge avoidance line or called also surge control line is often chosen with considerable margin which is called “surge margin” and it is defined as the distance between the surge line and the surge avoidance line. *(Flow control safety margin in figure 3.1)*

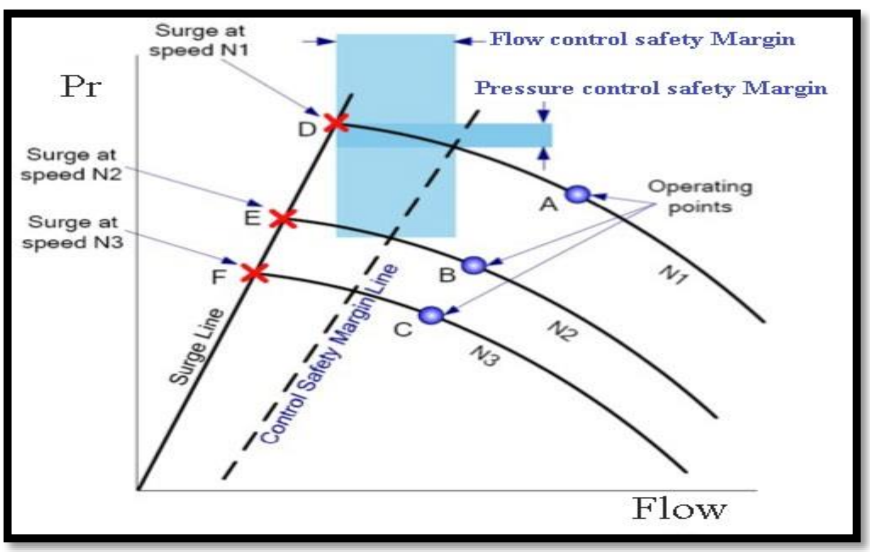


Figure 3-1: Surge avoidance using control safety margin

There are many ways to calculate the surge margin, one way is :

$$Surge\ Margin = \frac{F_{sl} - F_{sa}}{F_{sa}} \tag{3.1}$$

Where F_{sl} : is the mass flow corresponding to the surge line for a given speed.

F_{sa}: is the mass flow corresponding to surge avoidance line .

So the working principle of the surge avoidance scheme is as follow:

When the operating point of the compressor system is located right to the surge avoidance line, nothing is done. As the working point slides over the avoidance line the controller is activated and the valve will react in order to bring the operating point back to the right.

Remark: The selection of the surge margin will directly affect the efficiency of the compressor system .The shorter the surge margin the closer will be the operating point to the surge line where the efficiency is high, but the risks of entering surge will be high also. So there is a trade-off between security and efficiency. *(Figure 3-2*)

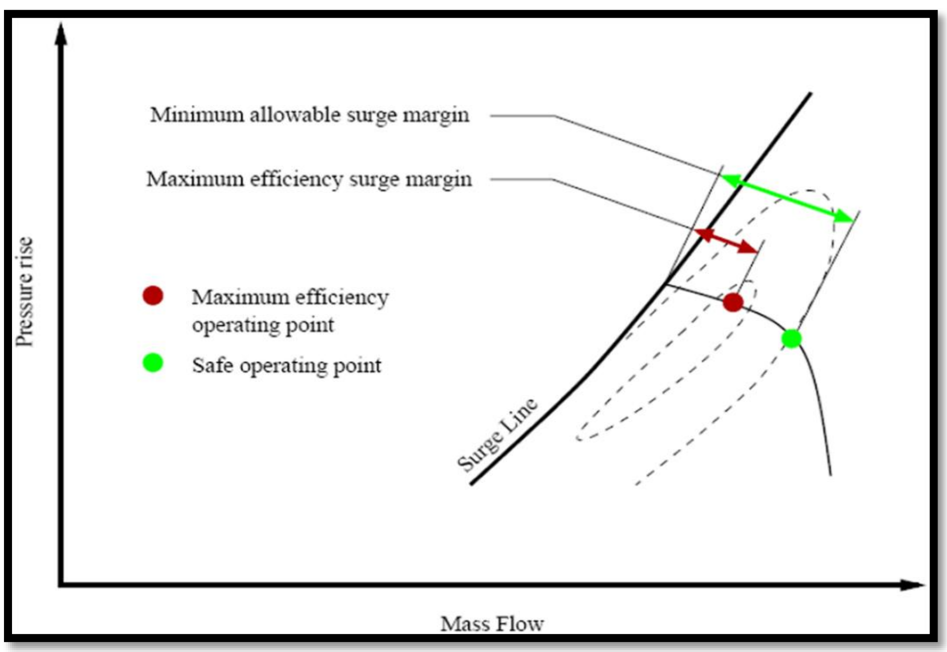


Figure 3-2 : relation between efficiency margin and safe region

3.3 Recycle valve PID controller:

A proportional – integral – derivative (PID controller) is a control loop feedback mechanism, commonly used in industrial control systems. A PID controller continuously calculates an error value as the difference between a desired set-point and a measured process variable (mass flow in our study). The controller attempts to minimize the error over time by adjustment of a control variable, such as the position of a control valve in our case, to a new value determined by a weighted sum [12]:

$$u(t) = k_p e(t) + \; k_i \int_0^t e(\tau)\, d\tau + k_d \frac{de(t)}{t} \tag{3.2}$$

Where **:** $\boldsymbol{k_p}$ $\boldsymbol{k_i}$ $\boldsymbol{k_d}$ are coefficient to be chosen according to the user specifications .

So in order to define the error, the controller should know about the current operating point at a specific speed line than compare it with the surge line. To do that, the surge line must be well defined on the compressor characteristic map. this can be easily achieved by choosing the maximum value on each speed line as a surge point using maximum algorithm , then fit all the points into a single linear line using “Polyfit” MATLAB function .

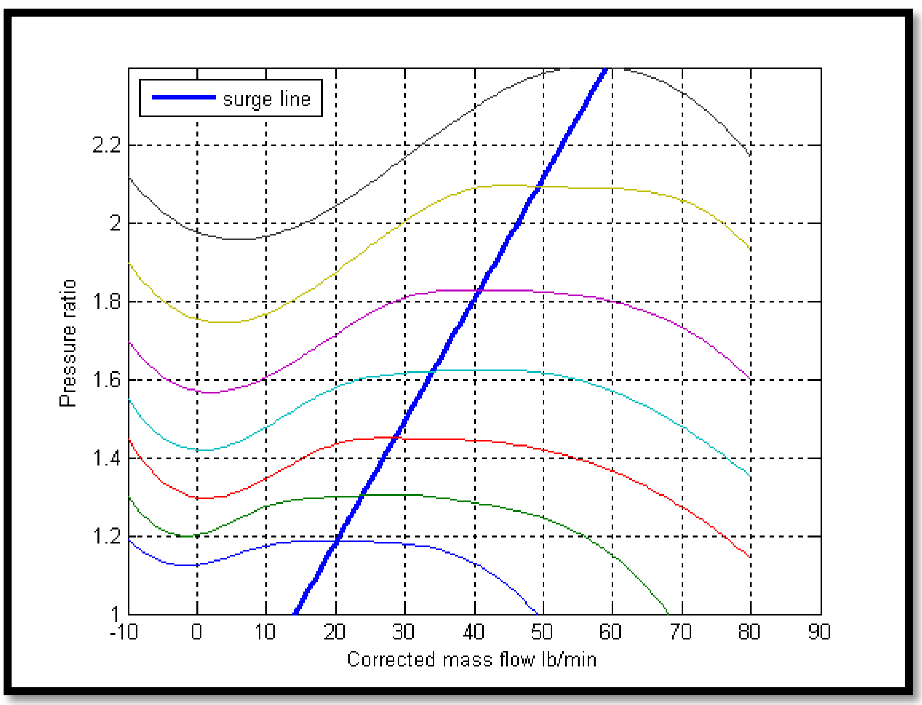


Figure 3-3: Surge Line using Polyfit equation in MATLAB

Next, let us define a surge avoidance line which is also linear:

$$SA\ (w) = a.w + b \tag{3.3}$$

Where a and b are the coefficient of the fitted 1st order polynomial(linear line) after adding the surge margin and they are given in the MATLAB function as **coef_avoid(1)**and **coef_avoid(2)** respectively .

Remark: we have added an option of manual selection of the surge margin , hence the user has the choice to work either with maximum or minimum efficiency line .

Now defining the horizontal distance between the operating point and the surge avoidance as:

$$d = W_{op} - W_{SA} \tag{3.4}$$

Where : W_{op} : the operating point flow .

W_{SA} : the surge line flow .

Remark : The surge avoidance flow can be calculated from the equation 3.3 and given by

$$W_{SA} = \frac{SA\ (w) - b}{a} \tag{3.5}$$

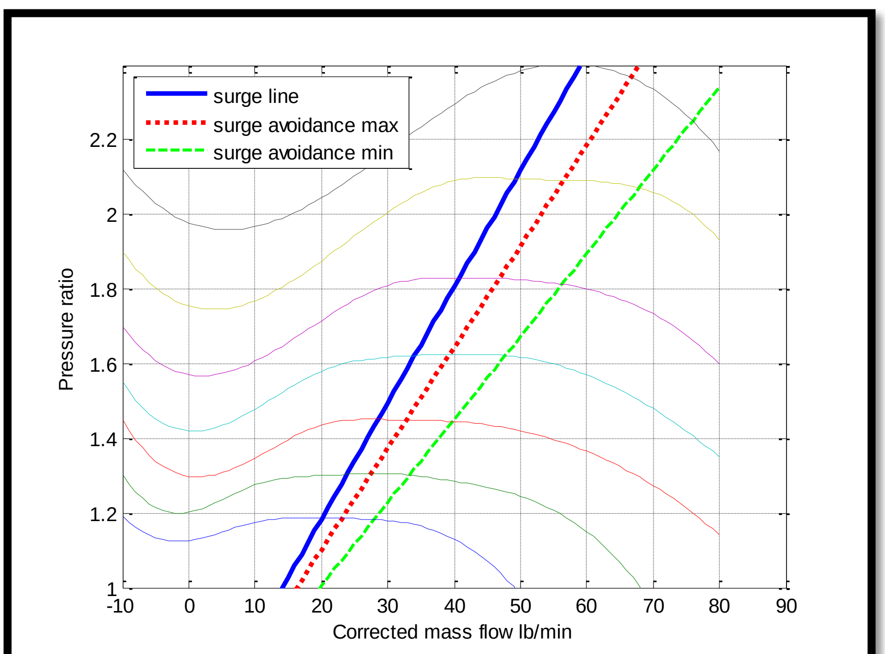


Figure 3-4: Defining surge avoidance lines with maximum and minimum efficiency

So when the operating point is to the right of surge avoidance line, the distance **d** is positive and nothing should be done. When the distance **d** is negative the operating point has passed the surge avoidance line and its positive value will be used as the error for the controller.

Hence the error is given as:

$$error = \begin{cases} 0 & when \;\; d > 0 \\ -d & when \; d < 0 \end{cases} \tag{3.6}$$

The PID controller is given as:

$$u = k_p e + \; k_i \int_0^t e(\tau)\, d\tau + k_d \frac{de}{t} \tag{3.7}$$

The control signal u will control the percentage opening of the recycle valve and its value varies between 0 and 1 (0% to 100 % opening):

$$w_r = \;\; u \,.\, A_r \sqrt{p_{2-} \, p_1} \tag{3.8}$$

Remark: The PID controller is implemented as MATLAB code function and used with the previous recycle system.

3.4 Tuning the PID parameters:

The gains of a PID controller can be obtained by trial and error method [12]. In this method, the I and D terms are set to zero first and the proportional gain is increased until the output of the loop oscillates. As we increase the proportional gain, the system becomes faster, but care must be taken not make the system unstable. Once P has been set to obtain a desired fast response, the integral term is increased to stop the oscillations. The integral term reduces the steady state error, but increases overshoot. Some amount of overshoot is always necessary for a fast system so that it could respond to changes immediately. The integral term is tweaked to achieve a minimal steady state error. Once the P and I have been set to get the desired fast control system with minimal steady state error, the derivative term is increased until the loop is acceptably quick to its set point. Increasing derivative term decreases overshoot and yields higher gain with stability but would cause the system to be highly sensitive to noise. So in our case, we did not need the derivative term since we got acceptable results from setting the P and I parameters and the final values are:

Kp = 35.5 Ki = 85 Kd = 0

Remark : The PID controller was implemented using a MATLAB code , it can be found in the folder under the name "Surge_avoidance".

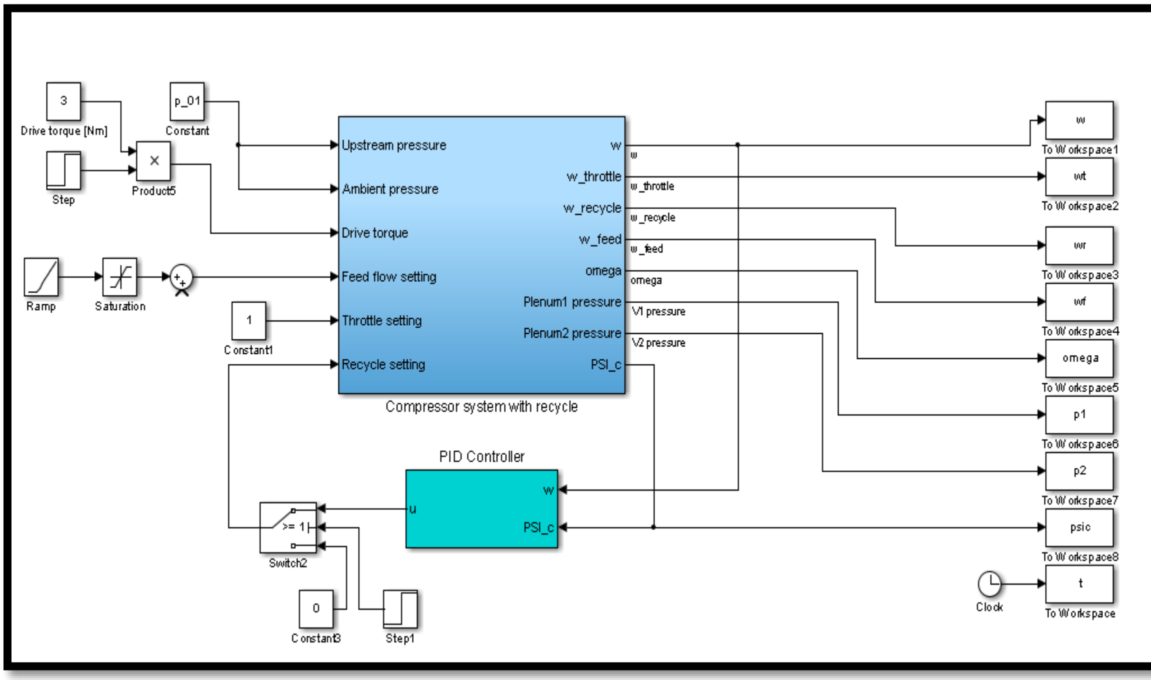


Figure 3-5: Simulink Block diagram of the PID controller

3.5 Simulation results:

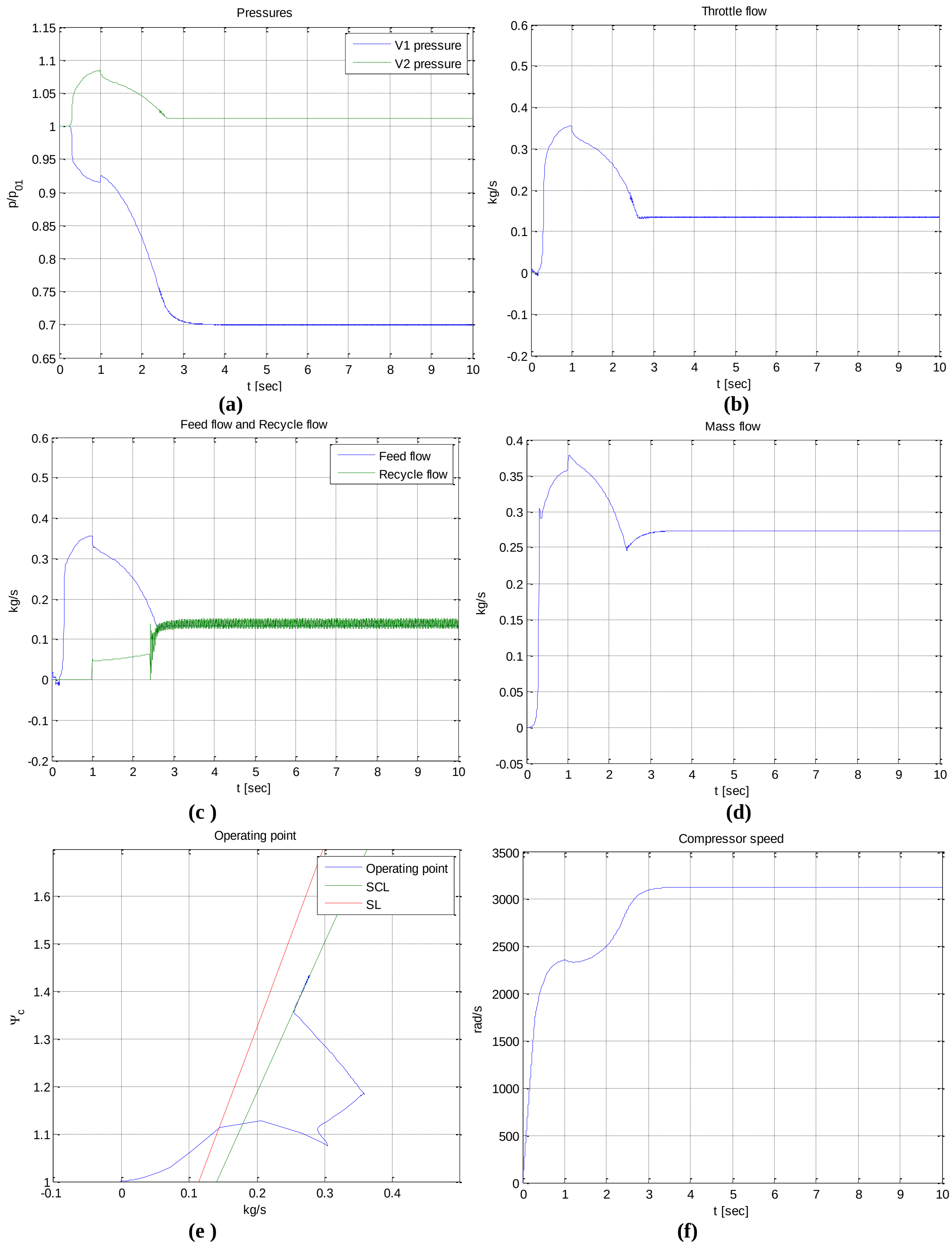


Figure 3-6 Simulation results : (a) pressure in V1 and V2 (b) Throttle flow (c) Feed flow and recycle flow (d) Mass flow (e) operating point (f) compressor speed.

Remarks :

1- Initially , the pressure in the two volumes are set to ambient , the compressor speed is set to zero , and the flows are set to zero .
2- At time t=1 second the disturbance is applied to the feed-flow , we remark drop in V1 and V2 pressure **(figure3-6 .a)** , and drop also in the mass flow and throttle flow **(figure3-6 .d,b)**
3- At t= 2.5 second the operating point is located to the left of the avoidance line , so the controller detects the drop in the mass flow and start opening the recycle valve **(figure3-6 .c)**
4- At t = 3 second and After stabilization , The mass flow is set to 0.28 kg/s , the pressure in V2 is 10.3×10^4 Pa and the pressure in V1 is : $70.9 \times 10^3 \, Pa$, while the compressor speed is set to 3100 rad/s .
5- We clearly notice that when the controller detected that the operating point is going beyond surge avoidance line, it behaved rapidly to keep the error as small as possible. **(figure3-6 .e)**

3.6 Discussion:

1- The controller has successfully detected the drop in the mass flow and start acting to stabilize the compressor and prevent it from reaching the surge line.
2- The well-tuned PID parameters made the controller set the position of the operating point exactly at the surge avoidance line which means that the error is zero.
3- The response of the compressor was remarkable .so after detecting drop in mass flow beyond the avoidance line at t = 2.5 second, it could successfully stabilize it after just 0.5 s. **(figure3-6 .d)**
4- The surge avoidance methods showed that the compressor can be prevented from entering the surge phenomena by proper choice of both surge line & PID controller parameters.
5- Although the controller prevented the compressor from reaching the surge line, the recycle flow showed some oscillations. **(figure3-6 .c)** which may damage the valve by quick opening and closing and negatively affect the whole system.

3.7 Conclusion:

We have seen in this chapter one of the traditional approaches in dealing with surge phenomenon: Surge avoidance, which successfully proved that surge can be prevented by using surge avoidance line and setting the recycle valve as a controller actuator. However, this method has many disadvantages, the most common one is that the system will always work at low efficiency operating point which cannot always supply the desired output.

The goal of the next chapter is to eliminate this problem by using another common approach in controlling centrifugal compressors which is called active surge control that has the ability to stabilize the unstable regime near the surge line where the efficiency is high. To do that, the feedback linearization approach will be used and further work on controlling the system will be presented.

CHAPTER IV: Active surge Control Using Feedback Linearization.

4.1 Introduction

One of the challenging problems in dealing with nonlinear systems is that there is few control theories compared to that of linear systems .but the problem is that all the real systems around us are actually nonlinear which is the case in the compressor system .

One way to deal with this problem is to linearize the system. In fact there are many approaches to do that, the famous and the most acceptable one is called feedback linearization.

In this chapter we are going to use the feedback linearization method to linearize the compressor recycle system, then Active surge control will be applied using a linear controller to control the actuators and get the desired outputs.

4.2 Feedback linearization method

4.2.1 What is feedback linearization?

Feedback linearization is a common approach used in controlling nonlinear systems. It involves coming up with a transformation of the nonlinear system into an equivalent linear system through a change of variables and a suitable control input. This is achieved by exact state transformations and feedback, which differs entirely from the conventional Jacobi-linearization[13].

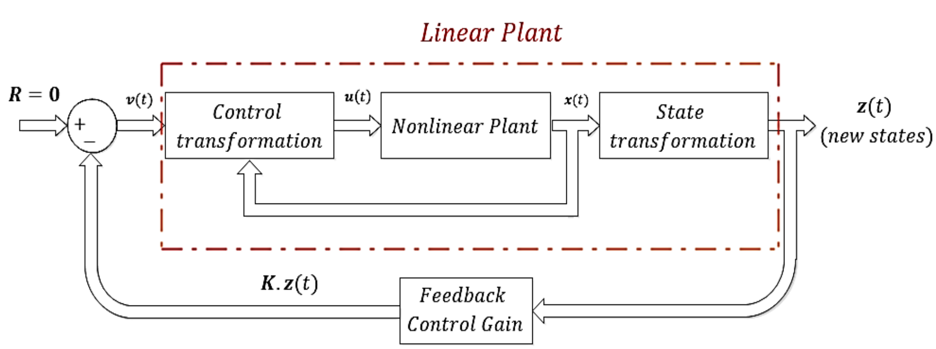


Figure 4-1:Feedback Linearization Overview

There are two types of feedback linearization which are: input-state linearization and input-output linearization .Depending on full or partial linearization of the system, input-state linearization aims at finding states transformation and linearizing input that linearizes the whole system, whereas input-output linearization aims at finding a relationship between the concerned output of the system and the input and then linearizes just this map.

4.2.2 Mathematical tools and basic theorems:

The basic mathematical tools that we should know in order to further work with feedback linearization approach are listed below [14], [15] ,[16] ,[17] ,[18], [19]:

- ✓ Lie derivative
- ✓ Lie brackets
- ✓ Diffeomorphism
- ✓ Distribution
- ✓ Brunovsky Form
- ✓ Controllability
- ✓ Relative degree

Definitions and theorems are well explained and provided in Appendix B .

4.2.3 Input-state feedback linearization:

Consider a system of the form:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u(t), u \in \mathbb{R} \tag{4.1}$$

With $\boldsymbol{f}$ and $\boldsymbol{g}$ being smooth vector fields.

The objective is to transform this system into one that is linear time-invariant by using a state feedback control law plus a coordinate transformation.

General case:

A system of the form (4.1) is said to be linearizable if there exist:

- A region D in $\mathbb{R}^n$.
- A diffeomorphism $T: D \to \mathbb{R}^n$.
- Non-linear feedback control law $u = \alpha(\boldsymbol{x}) + \beta(\boldsymbol{x})v$.

The new state variable $\boldsymbol{z} = \boldsymbol{T}(\boldsymbol{x})$ and the new input v satisfy a linear time invariant relation:

$$\dot{\boldsymbol{z}} = A\boldsymbol{z} + Bv$$

Where A and b are in Brunovsky form.

So, to perform input state linearization we follow the coming steps:

- ❖ Construct the victor fields $g, ad_f g, \dots, ad_f^{n-1} g$ for the given system.
- ❖ Check whether the controllability and involutivity conditions are satisfied.
- ❖ Find the first state z_1 that satisfy:

$$\nabla z_1 ad_{\boldsymbol{f}}^i \boldsymbol{g}(\boldsymbol{x}) = \frac{\delta z_1}{\delta x} ad_{\boldsymbol{f}}^i \boldsymbol{g}(\boldsymbol{x}) = 0 \qquad i = 0, \dots, n-2$$

$$\nabla z_1 ad_{\boldsymbol{f}}^{n-1} \boldsymbol{g}(\boldsymbol{x}) \neq 0$$

Where ∇ is the gradient vector:

- ❖ Compute the state transformation: $\boldsymbol{T}(\boldsymbol{x}) = \left[z_1 \; L_{\boldsymbol{f}} z_1 \dots \; L_{\boldsymbol{f}}^{n-1} z_1\right]^T$

And the input transformation:

$$\alpha(\boldsymbol{x}) = -\frac{L_{\boldsymbol{f}} z_1^n}{L_{\boldsymbol{g}} L_{\boldsymbol{f}}^{n-1} z_1}$$

$$\beta(\boldsymbol{x}) = \frac{1}{L_{\boldsymbol{g}} L_{\boldsymbol{f}}^{n-1} z_1}$$

4.2.4 Input-output feedback linearization:

Let's take the system of the form:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x}) u(t)$$
$$y = h(\boldsymbol{x})$$

The objective of I/O feedback linearization is to generate a linear differential relation between the output y and a new input v.

Depending on the relative degree of the system, there are two cases:

- Well defined relative degree.
- Undefined relative degree.

Case 1: Undefined relative degree:

The relative degree is undefined when $L_{\boldsymbol{g}} L_{\boldsymbol{f}}^{r-1} h(\boldsymbol{x_o}) = 0$, but nonzero at some points $\boldsymbol{x}$ close to $\boldsymbol{x_0}$. . In some particular cases, this problem may be solved by a simple change in the choice of the output. In general, however, I/O linearization at a point $\boldsymbol{x_0}$ cannot be straightforwardly achieved for undefined relative degree.

Case 2: Well defined relative degree

The output derivative which explicitly depends on the input control u is:

$$y^{(r)} = L_{\boldsymbol{f}}^{r}h(\boldsymbol{x}) + L_{\boldsymbol{g}}L_{\boldsymbol{f}}^{r-1}h(\boldsymbol{x})u(t) \quad \text{(a)}$$

Where r is the relative degree of the system. The relative degree is said to be well defined if $L_{\boldsymbol{g}}L_{\boldsymbol{f}}^{r-1}h(\boldsymbol{x_o}) \neq 0$, where $\boldsymbol{x_o}$ is the operating point. If we take:

$$u = \frac{1}{L_{\boldsymbol{g}}L_{\boldsymbol{f}}^{r-1}h(\boldsymbol{x})}\left(v - L_{\boldsymbol{f}}^{r}h(\boldsymbol{x})\right) \qquad \text{(b)}$$

Plugging (b) in (a) yields the desired linear differential relation between the output y and the new input v.

$$y^{(r)} = v$$

Remark: when $r = n$ the input output linearization and the input/state linearization coincides.

When $r < n$ the transformed description of the system is divided to two parts:

- A subsystem describing the external dynamics of the system.
- A subsystem describing the internal dynamics of the system.

The states of the external dynamics subsystem are given by:

$$\boldsymbol{z} = [z_1\ z_2\ \dots\ z_r]^T = \left[y\ \ \dot{y}\ \dots\ \ y^{(r-1)}\right]^T$$

Then the normal form of the system can be written as:

$$\dot{\boldsymbol{z}} = \begin{bmatrix} z_2 \\ \vdots \\ z_r \\ a(\boldsymbol{z},\boldsymbol{\Psi}) + b(\boldsymbol{z},\boldsymbol{\Psi})u(t) \end{bmatrix}$$
$$\dot{\boldsymbol{\Psi}} = \boldsymbol{w}(\boldsymbol{z},\boldsymbol{\Psi})$$

With the output defined as:

$$y(t) = z_1$$

Where: - $\dot{\boldsymbol{z}}$ gives the description of the external dynamics.

- $\dot{\boldsymbol{\Psi}}$ gives the description of the internal dynamics.

$\boldsymbol{\Psi}$ can be found by solving the following $n - r$ partial differential equations:

$$\nabla\Psi_j\boldsymbol{g}(\boldsymbol{x}) = 0 \qquad for\ 1 \leq j \leq n - r$$

The system can be transformed to this normal form if the transformation

$\boldsymbol{\varphi}(\boldsymbol{x}) = [z_1 \ldots z_r\ \Psi_1 \ldots \Psi_{n-r}]^T$ is a diffeomorphism.

The expressions of $a(\boldsymbol{z}, \boldsymbol{\Psi})$ and $b(\boldsymbol{z}, \boldsymbol{\Psi})$ are given by:

$$a(\boldsymbol{z}, \boldsymbol{\Psi}) = L_{\boldsymbol{f}}^{r} h(\boldsymbol{x}) = L_{\boldsymbol{f}}^{r} h\left(\boldsymbol{\varphi}^{-1}(\boldsymbol{z}, \boldsymbol{\Psi})\right)$$
$$b(\boldsymbol{z}, \boldsymbol{\Psi}) = L_{\boldsymbol{g}} L_{\boldsymbol{f}}^{r-1} h(\boldsymbol{x}) = L_{\boldsymbol{g}} L_{\boldsymbol{f}}^{r-1} h\left(\boldsymbol{\varphi}^{-1}(\boldsymbol{z}, \boldsymbol{\Psi})\right)$$

4.2.5 Zero dynamics:

The I/O feedback linearized system is now decomposed to an external (input-output) part and an internal (unobservable) part. Since the external part consists of a linear relation between y and$v$, it is easy to design an input v so that the output behaves as desired [20] . Since the input v has no effect on the internal dynamics, the behavior of these last must be taken into consideration. To study the behavior of the internal dynamics, we take the input u that maintains the output y at zero (hence the name zero dynamics). Doing so will make the internal dynamics the only dynamics acting on the system.

The input that satisfies this condition is:

$$u_0(\boldsymbol{x}) = -\frac{L_{\boldsymbol{f}}^{r} h(\boldsymbol{x})}{L_{\boldsymbol{g}} L_{\boldsymbol{f}}^{(r-1)} h(\boldsymbol{x})} = -\frac{a(\boldsymbol{0}, \boldsymbol{\Psi})}{b(\boldsymbol{0}, \boldsymbol{\Psi})}$$

Applying $u_0(\boldsymbol{x})$ the normal form becomes:

$$\dot{\boldsymbol{z}} = \boldsymbol{0}$$
$$\dot{\boldsymbol{\Psi}} = \boldsymbol{w}(\boldsymbol{0}, \boldsymbol{\Psi})$$

$\dot{\boldsymbol{\Psi}} = \boldsymbol{w}(\boldsymbol{0}, \boldsymbol{\Psi})$ Is the zero dynamics of the nonlinear system. For the linearization to be practically acceptable the zero dynamics should be stable.

4.3 Application of feedback linearization to recycle system:

As mentioned before there are two ways to linearize the system, hence it is always preferable to choose the simplest which leads to the desired characteristics easily and rapidly. The choice of the approach to be used would also depend on the relative degree r of the system:

- Case1: $(r = n)$

When the relative degree of the system is equal to the order of the system, we can use either I/O or I/S feedback linearization.

- Case 2: $(r < n)$

When the relative degree of the system is less than the order of the system, I/O linearization is used.

In our case, we have chosen to work with input –output for two reasons:

1- The relative degree of the system is $(r < n)$
2- It does not incorporates a lot of difficulties like solving the partial differential equations and checking the rank conditions.

To further simplify the linearization of the compressor system , we have chosen to work with a constant speed , that means working just on one line among the speed lines in the performance map and which led to eliminating the state equation of the angular velocity , hence the first thing to do next is to control the speed of the compressor .

4.3.1 Speed control of the compressor:

One easy way to control the speed of the system is using the PID control method, since this approach has proved its efficiency in the previous chapter. In order to do that we have to control the torque of the drive motor τ_d :

$$\tau_d = u \tag{4.2}$$

Where u is the control signal and it is equal to:

$$u = k_p e + k_i \int_0^t e(\tau)\, d\tau + k_d \frac{de}{t} \tag{4.3}$$

Here e is the error and it is defined as:

$$e = desired_{speed} - Omega \tag{4.4}$$

Omega is the angular velocity.

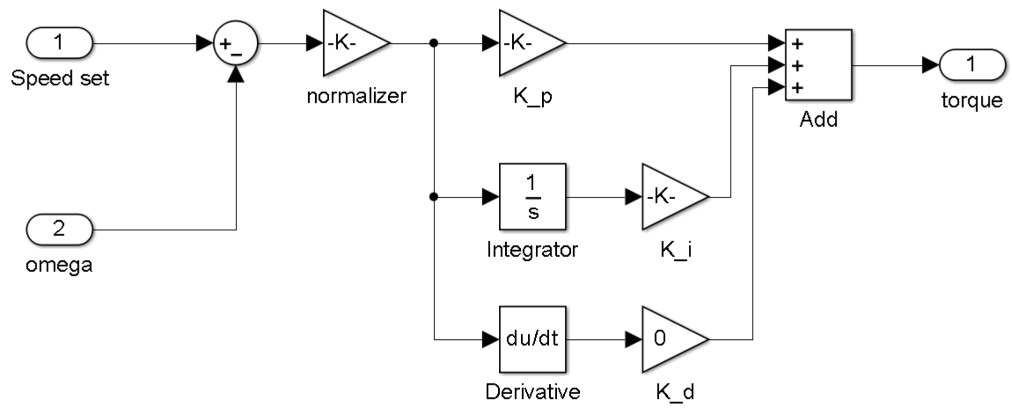


Figure 4-2: PID Control diagram of the speed of the compressor using SIMULINK

As said in the previous chapter there are many method to tune the PID parameters .In our case we found that the Trial and error do the work and give an excellent results although it take some time to get the desired output . The PID parameter that has been used are:

Kp = 2 Ki = 10 Kd = 0

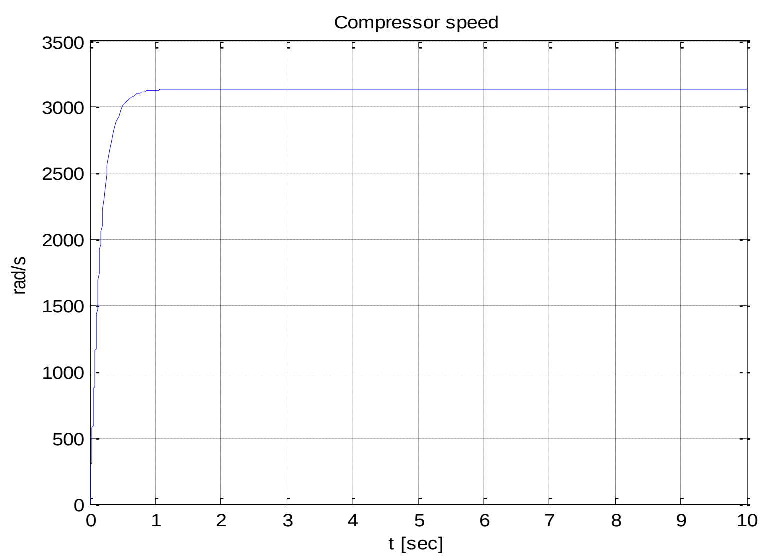


Figure 4-3: Compressor speed output after control

Remarks : The desired speed is set to 30000 rpm which corresponds to 3140 rad/s , we notice from figure 4.3 that both transient and steady state performance are acceptable .(0% overshoot , 1s settling time) .

4.3.2 Feedback linearization of the recycle system:

Since our goal is to control the flow by controlling the percentage opening of the recycle flow, the control signal $\boldsymbol{u}$ will be applied to the recycle valve. So the recycle system is now written as:

$$\begin{cases} \dot{p_1} = \dfrac{a^2}{V_1}\,(w_f + \ u * w_r - w) \\ \dot{p_2} = \dfrac{a^2}{V_2}\,(w - \ w_t - u * w_r\ ) \\ \dot{w} = \dfrac{A}{L}\,(\psi_c(w\,,\omega\,)\,p_1 - p_2\ ) \end{cases} \tag{4.5}$$

The system then can be written in the form:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u(t) \\ \mathbf{y} = \boldsymbol{h}(\boldsymbol{x}) \end{cases} \tag{4.6}$$

Where:

$\boldsymbol{x} = [x_1\ \ x_2\ \ x_3\,]^T = [p_1\ \ p_2\ \ w]^T$ $\quad$ $\boldsymbol{h}(\boldsymbol{x})$= w $\quad$ **u**=$A_{r\%}$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} k_1(w_f - w) \\ k_2\,(w - w_t) \\ k3\,(\psi_c(w\,,)\,p_1 - p_2\ ) \end{bmatrix} \quad ; \quad \boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} k_1 w_r \\ -k_2 w_r \\ 0 \end{bmatrix}$$

Where: $k_1 = \frac{a^2}{V_1}$ $\quad$ $k_2 = \frac{a^2}{V_2}$ $\quad$ $k3 = \frac{A}{L}$

After working on the mathematical calculations we found:

- the rank of $[\boldsymbol{g}(\boldsymbol{x})\ \ \boldsymbol{ad_f g}\ \ \boldsymbol{ad_f^2 g}\ ]$ is 3 for all **x** in D where :

  D =$\mathbb{R}^n - \{p2 = p1, p1 = p01\}$
- $[\boldsymbol{g}(\boldsymbol{x})\ \ ad_f g\ \ ]$ is involutive in D.

Since these two necessary conditions are hold, the system is feedback linearizable. Then the relative degree of the system was calculated and we found:

$$L_{\boldsymbol{g}} h(\boldsymbol{x}) = 0 \tag{4.7}$$

$$L_{\boldsymbol{g}} L_{\boldsymbol{f}}^{r-1} h(\boldsymbol{x}) = L_{\boldsymbol{g}} L_{\boldsymbol{f}}^{2-1} h(\boldsymbol{x}) = k_2.k_3.w_t \neq \mathbf{0}\ \boldsymbol{for}\ w_t \neq\ \mathbf{0} \tag{4.8}$$

So we conclude that the relative degree **r=2.**

Remark : since we got the relative degree equal to 2 , that means (r< n) .another problem arises which is the stability of the internal dynamics .So let us check them.

Internal dynamics analyses:

In order to find the normal form, let us take:

$$\mu_1 = x_3 \tag{4.9}$$

$$\mu_2 = \dot{x_3} = k_3 \ (\psi_c(w\,)\, x_1 - x_2\,) \tag{4.10}$$

The third function $\boldsymbol{\Psi}$ **(x)** required to complete the transformation should satisfy:

$$L_g\,\Psi = k_1 \frac{\boldsymbol{\partial \Psi}}{\boldsymbol{\partial x_1}} - k_2 \frac{\boldsymbol{\partial \Psi}}{\boldsymbol{\partial x_2}} = \mathbf{0} \tag{4.11}$$

One solution of this equation is:

$$\Psi = \frac{k_2}{k_1} x_1 + x_2 \tag{4.12}$$

Consider now the associated state transformation $\boldsymbol{Z} = [\mu_1 \ \ \mu_2 \ \ \Psi\,]^T$, its Jacobian matrix should be non-singular:

$$\frac{\boldsymbol{\partial Z}}{\boldsymbol{\partial X}} = \begin{bmatrix} 0 & 0 & 1 \\ k_3\ (\psi_c(x_3\ ) & -k_3 & x_1\, k_3\, \frac{\partial \psi_c(x_3\ )}{\partial x_3} \\ \frac{k_2}{k_1} & 1 & 0 \end{bmatrix} \tag{4.13}$$

Which is non-singular for any x.

So from equation 4.10 and 4.12 we get:

$$\begin{cases} x_1 = \dfrac{\mathrm{k}_1}{k_3} \left(\dfrac{\mu_2 + k_3\,\Psi}{k_2 + k_1\,\psi_c(\mu_1)}\right) \\ x_2 = \left(\dfrac{k_3\,\Psi\,\psi_c(\mu_1)\mathrm{k}_1 - \mu_2 k_2}{k_3(k_2 + k_1\,\psi_c(\mu_1)\,)}\right) \end{cases} \tag{4.14}$$

Derivation of equation 4.12 gives:

$$\dot{\Psi} \ = \frac{k_2}{k_1} \dot{x_1} + \dot{x_2} \tag{4.15}$$

Using the state space description equation 4.6:

$$\dot{\Psi} = k_2(w_f(x_1) - w_t(x_2)) \tag{4.16}$$

Stability analysis of such a system can performed around the equilibrium point:

$$x^*{}_3 = w^*{}_f + u.w^*{}_r \tag{4.17}$$
$$x^*{}_3 = w^*{}_t + u.w^*{}_r \tag{4.18}$$

From Equation 4.17 and 4.18 we have throttle flow equals the feed flow .so equation (4.16) becomes:

$$\dot{\Psi} = 0 \tag{4.19}$$

Remarks:

- The internal dynamics of the system are found to be marginal stable.
- Marginal stability means that the system can go either stable or unstable. But since the output is the mass flow, So physically we can conclude that the system will go stable if we could stabilize the mass flow since all other states depend on it , hence the compressor system given in equation 4.6 is said to be stable.(later In this chapter we will prove this assumption)

Linearized model:

The linear model derived from nonlinear coordinate transformation is given as:

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} h(x) \\ L_f h(x) \end{bmatrix} = \begin{bmatrix} w \\ \dot{w} \end{bmatrix} \tag{4.20}$$

The system can be written as:

$$\begin{cases} \dot{z} = Az + Bv \\ y = Cz \end{cases} \tag{4.21}$$

Where:

$$A = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \qquad B = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \qquad C = \begin{vmatrix} 1 & 0 \end{vmatrix} \qquad v = \ddot{w}$$

Up to this point we have dealt with feedback linearization and nonlinear system. Now we have a linear model, so linear control approaches can be applied directly.

System analysis and Controllability:

It is clear that the system is unstable since we have two eigenvalues equals to zero .So before we move into design a controller to the linear system lets first check its controllability:

$$R_{matrix} = [B \quad AB] \tag{4.22}$$

$$R_{matrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{4.23}$$

from the reachability matrix the system is full rank, so the system is controllable .

4.3.3 Controller design:

In this section state feedback linearization will be presented in order to control the linearized model.

First let us define the control v as:

$$v = -k * z$$

So the new system will be written as:

$$\dot{z} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} z - \begin{vmatrix} 0 \\ 1 \end{vmatrix} |k_1 \quad k_2| \begin{vmatrix} z_1 \\ z_2 \end{vmatrix} + \begin{vmatrix} 0 \\ 1 \end{vmatrix} r \tag{4.24}$$

Where: k_1 & k_2 are the feedback gains

r: is the reference or the desired value .

In order to select k_1 & k_2 we should first design the desired transient response for our system.

Transient response:

Let us suppose that the desired transient response would be:

Po = 0% overshoot and Ts 1s settling time.

So from the control theory courses we have:

$$Po\% = e^{\frac{\xi * \Pi}{\sqrt{1-\xi^2}}} \tag{4.25}$$

$$Ts = \frac{4}{\xi * \omega_n} \tag{4.26}$$

So applying the desired performance to the equation 4.25 and 4.26 we got:

$$\xi = 1 \qquad \& \qquad \omega_n = 4$$

So the desired characteristic polynomial is :

$$s^2 + 8s + 16 \tag{4.27}$$

Hence the desired poles are the same and equal to **-0.5** and they will be used as the desired eigenvalues in equation 4.24. we got the feedback gains k1 & k2 as:

$$k_1 = 16 \qquad \& \qquad k_2 = 8$$

Next we derive the equation of the control signal **u** with respect to the linear control signal **v**. From system described in 4.6 we have

$$\dot{w} = \ k3 * \ (\psi_c(w\ )\ p_1 - p_2\ ) \tag{4.28}$$

$$\ddot{w} = v = \ k3 * \ \left(\dot{\psi}_c(w\ )\ p_1 + \psi_c(w\ )\ \dot{p_1} - \dot{p_2}\ \right) \tag{4.29}$$

From 4.29 we derive the equation of the control u:

$$u = \frac{v - \ k_3\ [\dot{\psi}_c(w\ )\ p_1 + k_1\psi_c(w\ )\left(w_f - w\right) - \ k_2(w - w_t)}{(k_1\psi_c(w\ ) + k_2)k_3 w_r} \tag{4.30}$$

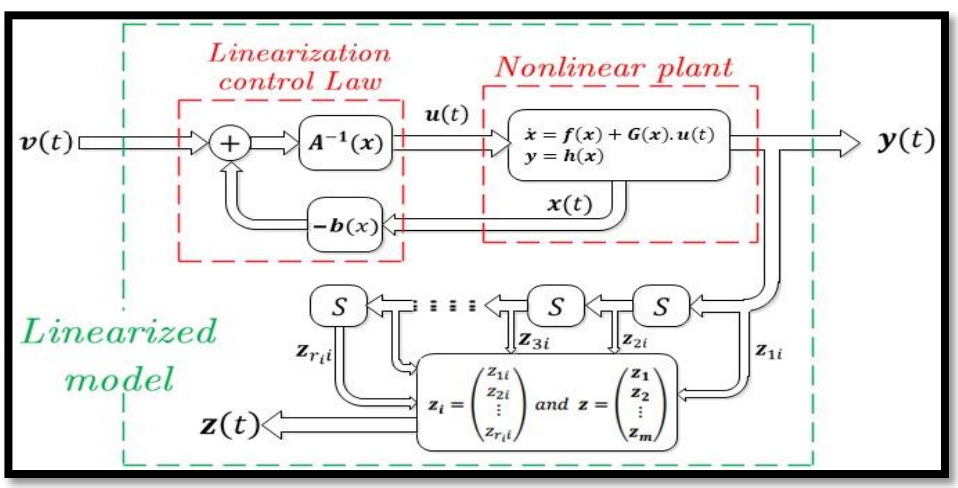


Figure 4-4: input-output feedback linearization

Steady state performance:

The steady state error is calculated using the flowing equation:

$$error \ = 1 + C(A - Bk)^{-1}B \tag{4.31}$$

$$error \ = 1 - \frac{1}{16} = 0.975$$

In order to ensure a zero steady state error, **steady state error design via integrator** [21] will be used .where a new state will be added to the system as:

$$\dot{z}_{new} = \ -Cz + \ \ r \tag{4.32}$$

And the new control v will be written as:

$$v = \ -kz - k_e z_{new} = \ [-k1 \quad -k2 \quad -ke] \begin{bmatrix} z1 \\ z2 \\ z_{nerw} \end{bmatrix} \tag{4.33}$$

This leads to an augmented state system .The new linear system will be written as

$$\begin{bmatrix} \dot{z} \\ \dot{z}_{new} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} z \\ z_{new} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \tag{4.34}$$

So substituting v into equation 4.34 we got:

$$\dot{z} = \begin{vmatrix} 0 & 1 & 0 \\ -k1 & -k2 & ke \\ -1 & 0 & 0 \end{vmatrix} z + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} r \tag{4.35}$$

Since the system has no zeros, we assume no zeros for the closed loop system and augment equation 4.27 by a third pole (s + 50) which has a real part much greater than five times than the dominant second order poles to ensure that the transient performance will not be affected . So the desired closed loop characteristic polynomial is given:

$$s^3 + 58s^2 + 416s \ + \ 800 \tag{4.36}$$

And the characteristic polynomial of system in equation:

$$s^3 + k_1 s^2 + \ k_2 s + ke \tag{4.37}$$

By identification between (4.28) and (4.29) we got:

$$k_1 = 58 \qquad\qquad k_2 = \ \ 416 \qquad\qquad k_3 = 800$$

Simulink block diagram:

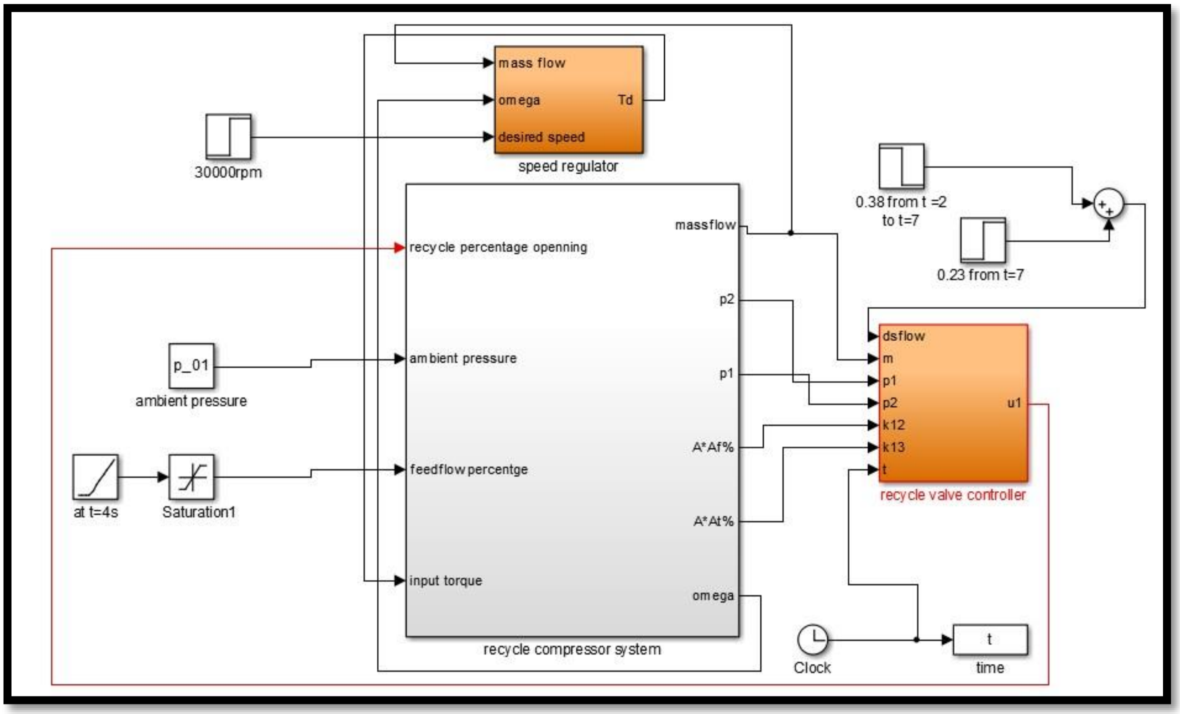
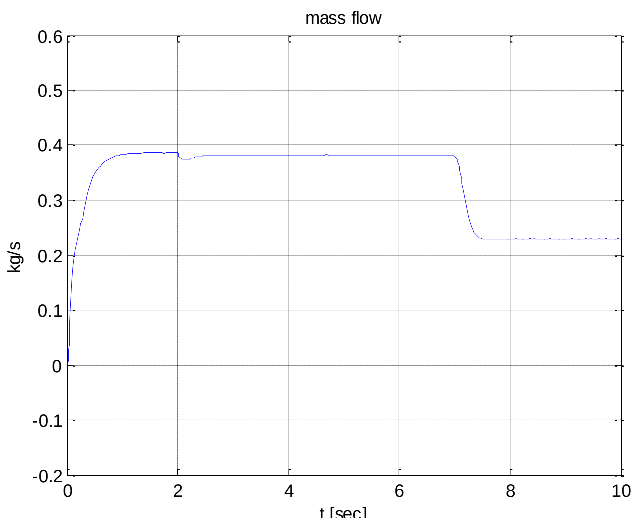
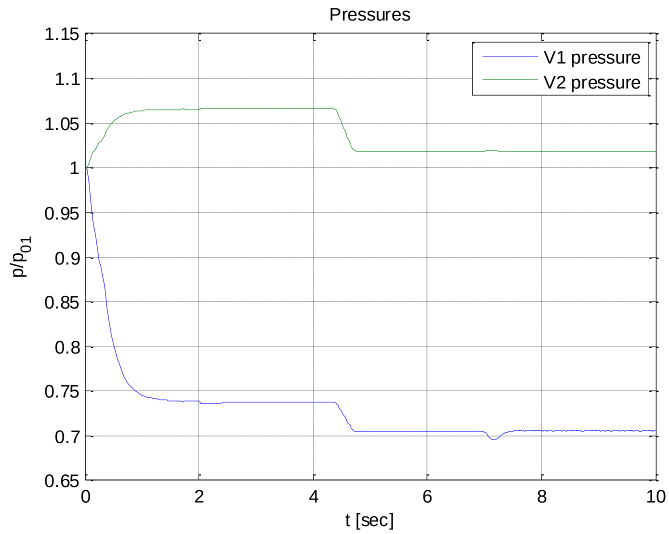


Figure 4-5: SIMULINK diagram of the controlled compressor system

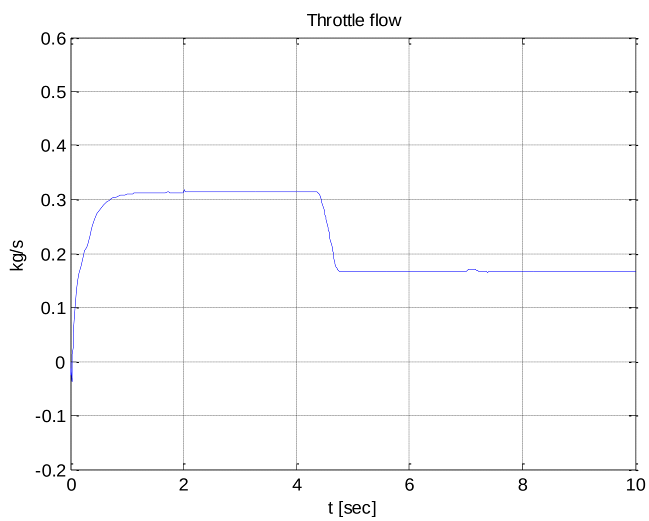
4.3.4 Simulation results:



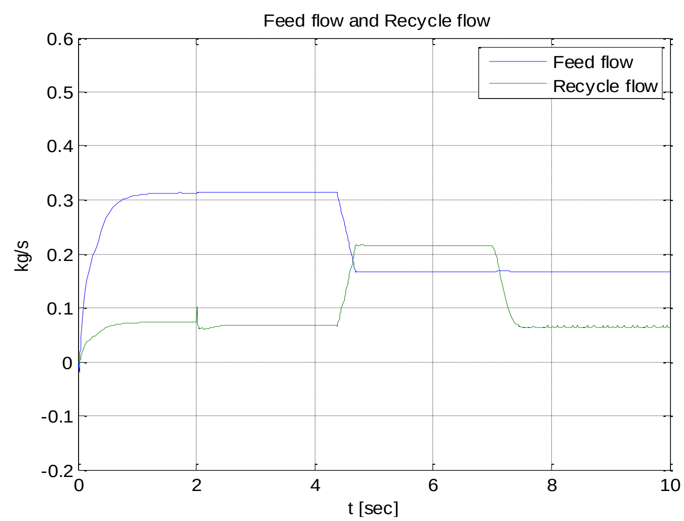
(a)



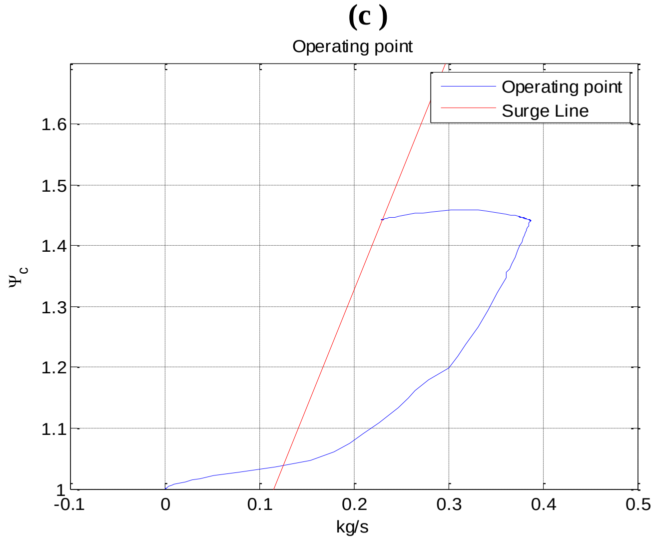
(b)



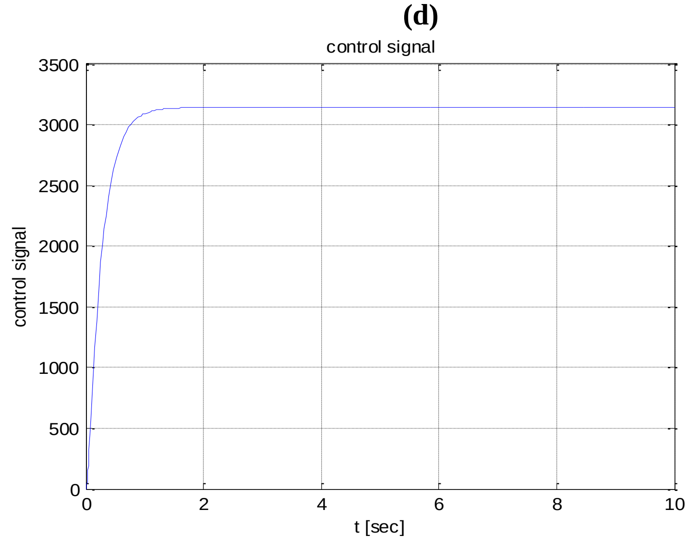
(c)



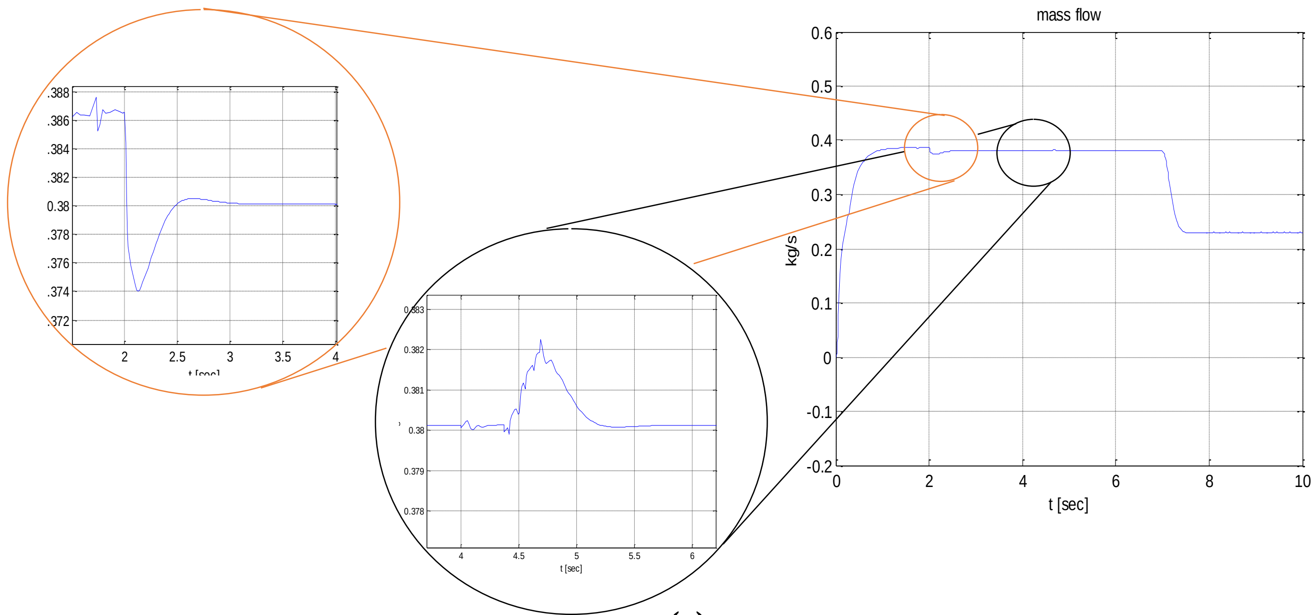
(d)



(e)



(f)



(g)

Figure 4-6:Simulation results : (a) mass flow (b) pressures in V1 & V2 (c) throttle flow(d) Feed flow and recycle flow (e) operating point (f) compressor speed (g) zoom of the mass flow

4.3.5 Remarks:

1- Initially , the pressure in the two volumes are set to ambient , the compressor speed is set to 3140 rad/s , and the throttle flow and feed-flow and mass flows are set to zero .

2- At t=2 second the controller is set ON. At t= 4 second the disturbance is applied

3- At t =2 the desired mass flow value is set to 0.38 kg/s, and at t= 7 it is set to 0.23kg/s.

4- The first remark is that the speed **(figure 4-6.f)** is constant during all the simulation and its value is 3140 rad/s (30000 rpm)

5- The mass flow **(figure 4-6.a)** has decreased to 0.38 kg/s at t=2s when the controller is activated, we notice a small overshoot of about 0.004 kg/s **(figure 4-6.g)** .

6- When the disturbance is applied at t=4s , we notice a small increase in mass flow from 0.38kg/s to 0.3823 kg/s but quickly comes back to 0.38kg/s **(figure 4-6.g)**

7- At t=7s the controller set the desired value to 0.23 kg/s, we see that the mass flow drop quickly and smoothly. **(figure 4-6.a)**

8- We notice that the operating point has moved near the surge line when we applied the desired value to 0.23kg/s **(figure 4-6.e)**.

4.3.6 Discussion

1- From the speed graph we concluded that the PID speed controller is keeping the speed at (30000rmp) **(figure 4-6.f)**

2- The state feedback with integrator controller proved also its high performance , where we notice at t=2 and at t=7s **(figure 4-6.a)** that the desired mass flows: 0.38kg/s and 0.23kg/s respectively, have been achieved with minimum rise and settling time and a small or negligible overshoot and zero steady state error .

3- As we said before, stabilizing the mass flow will automatically make the whole system stable as we see from pressures in V1 and V2. **(figure 4-6.b)**

4- Setting the desired mass-flow to 0.23kg/s has led the operating point **(figure 4-6.e)** to move near the surge line where the efficiency is high.

5- When the disturbance is applied at t=4s, the mass flow deviates by a small amount from the desired value (0.38kg/s), but the controller quickly stabilized it, and forced it to come back to the desired value, which proves that the controller has successfully rejected the disturbance.

4.3.7 Conclusion

In this chapter, Active surge control is applied using feedback linearization to linearize the recycle system and then state feedback control method is used to control it. First, a background on feedback linearization was given then input-output method was used to linearize the system. After that transient and steady state performance was calculated and applied along with the controller .finally the controller was tested using the Simulink software.

The obtained results were excellent were:

1- The response of the controller was good as we have expected , and the desired transient and steady state performance have been achieved .

2- The controller has successfully set the operating point near surge line without setting the limit cycle on.

3- Working near the surge line led to high efficiency output.

GENERAL CONCLUSION

Throughout this project, Surge control in centrifugal compressor was our main concern. First, a third order approximation of the compressor's performance map was constructed using a given data and the "Ployfit" MATLAB function. Then a mathematical model for the open loop compressor was developed and tested which proved that the compressor could enter the instable region with small disturbance, After that a recycle model was suggested to control the mass flow and stabilize the system by manual opening of the recycle valve .Hence the recycle valve was used as the main actuator in the entire project.

In the second part, two proposed schemes to control surge were presented:

- The first one is surge avoidance where a considerable margin from the surge line was created and set as the limit for the operating point. This method did really prevent the system from entering the surge but it restricted its efficiency to a minimum level .a PID controller was successfully implemented to detect and minimize the error when the operating point tend to go near the surge line. This method showed also another negative point where we have seen a considerable oscillations in the recycle flow.
- In the second part an attempt to increase the efficiency was done via Active surge scheme .starting with linearizing the system using feedback linearization and then applying state feedback control to set the operating point to the desired value. The results of this method were promising, where we could set the operating point to the near the surge line where efficiency is high without going into surge.

So the two main objectives of this project have been achieved:

- Prevent the centrifugal compressor from surge phenomenon and achieve high efficiency output.

As for further work we suggest this interesting topic:

- Active surge control using positive feedback control.

Appendix A

Vortech S-Trim turbocharger specifications

Table of performance:

Performance	Value
Max speed	50000 RPM
Max boost	20 PSI
Max flow	1000 CFM
Max power	680 HP
Peak efficiency	72 %

Table of Dimensions:

Dimension	Value
Discharge OD	2.75”
Inlet OD	3.5”
Discharge ID	2.38”
Inducer Diameter	3.1”

Appendix B

Mathematical tools and Theorems for feedback linearization approach

Lie Derivatives:

Definition 1: Consider a scalar function:

$h: D \subset \mathbb{R}^n \rightarrow \mathbb{R}$ and a vector field: $\boldsymbol{f}: D \subset \mathbb{R}^n \rightarrow \mathbb{R}^n$

The Lie derivative of h with respect to $\boldsymbol{f}$, denoted $\mathcal{L}_{\boldsymbol{f}} h$, is given by:

$$\mathcal{L}_{\boldsymbol{f}} h(\boldsymbol{x}) = \frac{\partial h}{\partial \boldsymbol{x}} . \boldsymbol{f}(\boldsymbol{x})$$

Lie Brackets:

Definition 2: Consider the vector fields

$$\boldsymbol{f}, \boldsymbol{g} : D \subset \mathbb{R}^n \rightarrow \mathbb{R}^n$$

The Lie bracket of $\boldsymbol{f}$ and $\boldsymbol{g}$, denoted by $[\boldsymbol{f}, \boldsymbol{g}]$, is the vector field defined by:

$$[\boldsymbol{f}, \boldsymbol{g}](\boldsymbol{x}) = \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}} . \boldsymbol{f}\,(\boldsymbol{x}) - \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} . \boldsymbol{g}(\boldsymbol{x})$$

When computing repeated Bracketing, the following notation is usually used:

$ad_f^i \boldsymbol{g}(\boldsymbol{x}) = \left[\boldsymbol{f}, ad_{\boldsymbol{f}}^{i-1} \boldsymbol{g}(\boldsymbol{x})\right]$ With "ad" stands for adjoint and:

$$ad_f^0 \boldsymbol{g}(\boldsymbol{x}) \triangleq \boldsymbol{g}(\boldsymbol{x})$$

$$ad_{\boldsymbol{f}} \boldsymbol{g}(\boldsymbol{x}) \triangleq ad_{\boldsymbol{f}}^1 \boldsymbol{g}(\boldsymbol{x}) = [\boldsymbol{f}, \mathbf{g}](\mathbf{x})$$

Diffeomorphism:

Definition 3: (Local diffeomorphism): A function $\boldsymbol{f} : D \subset \mathbb{R}^n \rightarrow \mathbb{R}^n$ is said to be a diffeomorphism on D if:

i. it is continuously differentiable on D.

ii. its inverse $\boldsymbol{f}^{-1}$ exists and is continuously differentiable.

Definition 4: (Global diffeomorphism): The function $\boldsymbol{f}$ is said to be a global diffeomorphism if in addition:

i. $D = \mathbb{R}^n$.

ii. $\lim_{\|\boldsymbol{X}\| \rightarrow \infty} \|\boldsymbol{f}(\boldsymbol{X})\| = \infty$.

Lemma 1: Let $\boldsymbol{f} : D \subset \mathbb{R}^n \rightarrow \mathbb{R}^n$ be continuously differentiable field on D. If the Jacobian matrix $\nabla \boldsymbol{f}(\boldsymbol{x}) = \frac{\delta f(x)}{\delta x}$ is nonsingular at a point $\boldsymbol{x_0} \in$ D, then $\boldsymbol{f}(\boldsymbol{x})$ is a diffeomorphism in a subset $\Omega \subset D$.

Distribution:

Let $\boldsymbol{f} : D \subset \mathbb{R}^n \rightarrow \mathbb{R}^n$. This vector field assigns the n-dimensional vector $\mathbf{f}\,(\boldsymbol{x})$ to each point $\boldsymbol{x} \in D$. Now consider "p" vector fields $\boldsymbol{f_1}, \boldsymbol{f_2}, \ldots., \boldsymbol{f_p}$ on $D \subset \mathbb{R}^n$. At each $\boldsymbol{x} \in D$;

$$\Delta(x) = span\{\boldsymbol{f_1}(\boldsymbol{x}), \boldsymbol{f_2}(\boldsymbol{x}), \ldots \ldots, \boldsymbol{f_p}(\boldsymbol{x})\}$$

is a subspace of $\mathbb{R}^n$.

<u>Definition 5:</u> Given the smooth functions $\boldsymbol{f_1}, \boldsymbol{f_2}, \ldots, \boldsymbol{f_p}: D \subset \mathbb{R}^n \rightarrow \mathbb{R}^n$, a smooth distribution is the process of assigning to each $\boldsymbol{x} \subset D$ a subspace:

$$\Delta(\boldsymbol{x}) = span\{\boldsymbol{f_1}(\boldsymbol{x}), \boldsymbol{f_2}(\boldsymbol{x}), \ldots, \boldsymbol{f_p}(\boldsymbol{x})\} \subset \mathbb{R}^n$$

The dimension of the distribution is defined as:

$$dim\big(\Delta(\boldsymbol{x})\big) = rank[\boldsymbol{f_1}(\boldsymbol{x}), \boldsymbol{f_2}(\boldsymbol{x}), \ldots, \boldsymbol{f_p}(\boldsymbol{x})]$$

- $\boldsymbol{x}$ is said to be "Regular Point" if $dim\big(\Delta(\boldsymbol{x})\big)$ is constant in a neighborhood of $\boldsymbol{x}$.
- Δ is said to be non-singular distribution in $D \subset \mathbb{R}^n$ if $\dim(\Delta(\boldsymbol{x}))$ is constant in an open set $D \subset \mathbb{R}^n$, otherwise Δ is said to be of variable dimension.

<u>Definition 6: (Involutive Distribution):</u> A distribution Δ is said to be Involutive if for $\boldsymbol{g_1}(\boldsymbol{x}) \in \Delta\ and\ \boldsymbol{g_2}(\boldsymbol{x}) \in \Delta$ then $[\boldsymbol{g_1}, \boldsymbol{g_2}](\boldsymbol{x}) \in \Delta$.

It then follows that $\Delta(x) = span\{\boldsymbol{f_1}(\boldsymbol{x}), \boldsymbol{f_2}(\boldsymbol{x}), \ldots \ldots, \boldsymbol{f_p}(\boldsymbol{x})\}$ is involutive if and only if $rank\big[\boldsymbol{f_1}(\boldsymbol{x}), \boldsymbol{f_2}(\boldsymbol{x}), \ldots, \boldsymbol{f_p}(\boldsymbol{x})\big] = rank[\boldsymbol{f_1}(\boldsymbol{x}), \boldsymbol{f_2}(\boldsymbol{x}), \ldots, \boldsymbol{f_p}(\boldsymbol{x}), [\boldsymbol{f_i}(\boldsymbol{x}), \boldsymbol{f_j}(\boldsymbol{x})]]$, $\forall \boldsymbol{x}$ and all i, j.

<u>Theorem (Forbenius):</u> A set of linearly independent vector fields $\{\boldsymbol{f}_i(\boldsymbol{x}), i = 1, \ldots p\}$ is completely integrable if and only if it is involutive.

It is alternative to say that: nonsingular distribution is completely integrable if and only if it is involutive. Forbenius theorem is an important tool in feedback linearization since it gives a necessary and sufficient condition for solvability of partial differential equations.

Brunovsky Form:

Consider the linear system with the general form:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t)$$

with $\boldsymbol{x}(t) \in \mathbb{R}^n$ *and* $\boldsymbol{u}(t) \in \mathbb{R}^m$ (n states and m inputs). Brunovsky form is the canonical form with:

$$\dot{\overline{\boldsymbol{x}}}(t) = A_c\overline{\boldsymbol{x}}(t) + B_c\overline{\boldsymbol{u}}(t)$$

Brunovsky theorem states that: Every linear system with n states and m inputs is equivalent by change of coordinates and feedback to the canonical form

$$A_c = diag\{A_{c1}, A_{c2}, \dots \dots, A_{cm}\} \;\; and \;\; B_c = diag\{B_{c1}, B_{c2}, \dots \dots, B_{cn}\}$$

where A_{ci} and B_{ci} are given by:

$$A_{ci} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \; and \;\; B_{ci} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Where A_{ci} is $n_i \times n_i$ and B_{ci} is $n_i \times 1$.

Single-Input Systems:

For single-input systems, m=1, the Brunovsky form is:

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \; and \;\; B_c = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Where A_c is $n \times n$ and B_c is $n \times 1$.

Controllability:

Definition 7**:** A nonlinear system is controllable if $\forall\, \boldsymbol{x_0}, \boldsymbol{x_f} \in \mathbb{R}^n$, there exist an admissible control $\boldsymbol{u}(t)$ on $[0, T]$, $T < \infty$, such that for $\boldsymbol{x(0)} = \boldsymbol{x_0}, \boldsymbol{x(T)} = \boldsymbol{x_f}$.

Consider the nonlinear system to have the form: $\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x}).\boldsymbol{u}(t)$ where $\boldsymbol{f}$ and $\boldsymbol{g}$ are two fields.

Single-Input Systems: A single input system is controllable if:

$$rank\left(\left[\boldsymbol{g}(\boldsymbol{x}), ad_f\boldsymbol{g}(\boldsymbol{x}), \dots, ad_f^{n-1}\boldsymbol{g}(\boldsymbol{x})\right]\right) = n$$

Relative Degree:

Consider the system described by the state space equation:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}(t)$$

$$y = h(\boldsymbol{x})$$

Where: $\boldsymbol{f}, \boldsymbol{g} : D \rightarrow \mathbb{R}^n$, and h are sufficiently smooth in a domain D.

$$\dot{y} = \frac{\partial h}{\partial \boldsymbol{x}}[\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}(t)] \stackrel{\text{def}}{=} \mathcal{L}_{\boldsymbol{f}}h(\boldsymbol{x}) + \mathcal{L}_{\boldsymbol{g}}h(\boldsymbol{x})\boldsymbol{u}(t)$$

$$\mathcal{L}_{\boldsymbol{g}}h(\boldsymbol{x}) = 0 \Rightarrow \dot{y} = \mathcal{L}_{\boldsymbol{f}}h(\boldsymbol{x})$$

$$y^{(2)} = \frac{\partial\left(\mathcal{L}_{\boldsymbol{f}}h\right)}{\partial \boldsymbol{x}}[\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}(t)] = \mathcal{L}^2{}_{\boldsymbol{f}}h(\boldsymbol{x}) + \mathcal{L}_{\boldsymbol{g}}\mathcal{L}_{\boldsymbol{f}}h(\boldsymbol{x})\boldsymbol{u}(t)$$

$$\mathcal{L}_{\boldsymbol{g}}\mathcal{L}_{\boldsymbol{f}}h(\boldsymbol{x}) = 0 \rightarrow y^{(2)} = \mathcal{L}^2{}_{\boldsymbol{f}}h(\boldsymbol{x})$$

$$\mathcal{L}_{\boldsymbol{g}}\mathcal{L}^{i-1}{}_{\boldsymbol{f}}h(\boldsymbol{x}) = 0, i = 1,2,\dots,r-1; \quad \mathcal{L}_{\boldsymbol{g}}\mathcal{L}^{r-1}{}_{\boldsymbol{f}}h(\boldsymbol{x}) \neq 0$$

$$y^{(r)} = \mathcal{L}^r{}_{\boldsymbol{f}}h(\boldsymbol{x}) + \mathcal{L}_{\boldsymbol{g}}\mathcal{L}^{r-1}{}_{\boldsymbol{f}}h(\boldsymbol{x})\boldsymbol{u}(t)$$

r is called the relative degree of the output .

<u>Definition 8:</u> Let the system described by the state space equation be:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}(t)$$

$$y = h(x)$$

This system has relative degree r, $1 \leq r \leq n$, in $D_0 \subset D$ if $\forall\ \boldsymbol{x} \in D_0$

$$\mathcal{L}_{\boldsymbol{g}}\mathcal{L}^{i}{}_{\boldsymbol{f}}h(\boldsymbol{x}) = 0, \quad i = 0,1,\dots,r-2;$$

$$\mathcal{L}_{\boldsymbol{g}}\mathcal{L}^{r-1}{}_{\boldsymbol{f}}h(\boldsymbol{x}) \neq 0$$

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