People's Democratic Republic of Algeria

Ministry of Higher Education and Scientific Research

**University M'Hamed BOUGARA – Boumerdes**

Institute of Electrical and Electronic Engineering

Department of Power and Control

Final Year Project Report Presented in Partial Fulfillment of the Requirements for the Degree of

# MASTER

In **Control**

Option: **Control**

Title:

# COMPARATIVE STUDY OF THREE NONLINEAR OBSERVERS

Presented by:

- **TERKMANE MUSTAPHA**
- **LOUAHCHI ZAKARYA**

Supervisor:

**Dr.AKROUM MOHAMED.**

Registration Number:………/2021

# DEDICATION

"What we know is a drop, what we don't know is an ocean."

ISAAC NEWTON

*First of all we thank god the almighty for his protection through our life and guidance to follow the right way and pray for him to show us the path to success.*

**We** *dedicate this work*

*To our parents, brothers and sisters. May this humble work make you proud and hopefully reassure your hearts that the son and sibling you have, lives out to be the image you have of him and hoped he grows into. May God preserve you and keep you as the beacon and light of our life.*

*To all our friends, in particular those who will recognize themselves here, for all the support and laughs during the long years leading to this moment.*

ACKNOWLEDGEMENT

All praise be to Allah, the Most Merciful and Beneficent. There is no God but Allah and Prophet Muhammad (Peace be upon him) is the Messenger of Allah. With His wills and gracious, we have successfully completed our thesis. First of all, we wish to express our gratitude to our supervisor **Dr. AKROUM MOHAMED** for giving us such a challenging opportunity, for his inspiring direction for the research and for his great support in all aspects of our study

We would also like to acknowledge honorary all the teachers of the Institute of Electrical and Electronics Engineering of the University of M'Hamed Bougara- Boumerdes for their support and help during the last five years. I have learned a lot from their knowledge and moralities.

We also thank our parents for the support and encouragement they offered us and all the resources and time they put so that we reach this point in life.

# ABSTRACT

This report presents a comparison study of performances and characteristics of three advanced state observers, including the high-gain observers, the sliding-mode observers and the extended state observers. These observers were originally proposed to address the dependence of the classical observers, such as the Kalman Filter and the Luenberger Observer, on the accurate mathematical representation of the plant. The results show that, over all, the nonlinear extended state observer is much superior in dealing with dynamic uncertainties, disturbances and sensor noise. Several novel nonlinear gain functions are proposed to address the difficulty in dealing with unknown initial conditions. Simulation results are provided.

**Keywords:** High Gain Observer, Sliding Mode Observer, Nonlinear Extended State Observer.

# TABLE OF CONTENTS

**DEDICATIONS** .......... **I**
**ACKNOWLEGEMENT** .......... **II**
**ABSTRACT** .......... **III**
**TABLE OF CONTENTS** .......... **IV**
**NOMENCLATURE** .......... **VI**
**LIST OF TABLES** .......... **VII**
**LIST OF FIGURES** .......... **VIII**

**GENERAL INTRODUCTION** .......... **1**

Problem Formulation .......... 4
Motivation of the Project .......... 5
Work Methodology .......... 5

**I. CHAPTER I .NONLINEAR CONTROL SYSTEMS** .......... **6**

**1.1.** Introduction .......... 7
**1.2.** Feedback Linearization .......... 8
**1.3.** Variable Structure, Sliding-Mode Control .......... 9
**1.4.** Backstepping control .......... 11
**1.5.** Conclusion .......... 13

**II. CHAPTER II. NONLINEAR OBSERVERS** .......... **14**

**2.1.** Introduction .......... 15
**2.2.** Basic Linear Observer Design .......... 18
**2.2.1.** Duality with state feedback control .......... 22
**2.3.** High Gain Observer .......... 23
**2.3.1.** Peaking Phenomenon .......... 25
**2.4.** Sliding Mode Observer .......... 27
**2.5.** Nonlinear Extended State Observer .......... 28

**2.5.1.** Selection of Nonlinear Gains for NESO ........ 32

**2.6.** Conclusion ........ 33

**III. CHAPTER III. COMPARISON OF ADVANCED STATE OBSERVER DESIGN TECHNIQUES ........ 34**

**3.1.** Servo Motor Plant Model ........ 35

**3.2.** Simulated Results........ 36

**3.2.1.** Observers Designs ........ 37

**3.2.2.**High gain observer (HGO) ........ 39

**3.2.3.**Sliding Mode observer(SMO)........ 43

**3.2.4.** Nonlinear Extended State Observers (NESO) ........ 47

**3.3.** Comparison study of the characteristics and performances of the HGO, SMO, and NESO ........ 52

**3.3.1.** NOMINAL SYSTEM ERRORS ........ 53

**3.3.2.** SYSTEM UNDER DISTURBANCE ERRORS ........ 54

**3.3.3.** SYSTEM WITH UNCERTAINTY ERRORS ........ 54

**3.4.** Conclusion ........ 55

**IV. CHAPTER IV.GENERAL CONCLUSIONS ........ 56**

**V. REFRENCES ........ 59**

# NOMENCLATURE

$A$ System matrix of the state space control

$B$ Input matrix of the state space control

$C$ Output matrix of the state space control

$L$ Observer gain matrix of the state space control

$x$ State vector of the state space control

$y$ Output vector of the state space control

$u$ Control command

$\hat{x}$ Estimate state vector of the state space control

$hi$ Observer gains of the high gain observer

$ki$ Observer gains of the sliding mode observer

$f(.)$ Dynamics of the plant and the disturbance

$zi$ Estimate states of the ESO

$w(t)$ External disturbance of the plant

$J$ Inertia of the load

$B_i$ Gains of the ESO

$g_i$ Nonlinear functions of ESO

$\mathbf{T_{TR}}$ Transient time

$F_S$ Static friction

$F_c$ coulomb friction

# LIST OF TABLES

**Table 3.1**: Parameters of high gain observer........................................................................39

**Table 3.2**: Parameters of sliding mode observer...................................................................43

**Table 3.3**: Parameters of nonlinear extended state observer...............................................47

**Table 3.4**: comparison factors of observers .........................................................................51

# LIST OF FUGURES

**Figure1.** General Classification of Observers ........ 4

**Figure 1.1**.Feedback Linearization control block diagram ........ 8

**Figure1.2**.Sliding mode demonstration ........ 9

**Figure1. 3**The Sliding condition ........ 11

**Figure1. 4** Back stepping control system ........ 12

**Figure 2.1**.luenberger observer ........ 15

**Figure 2.2**.Kalman filter demonstration ........ 15

**Figure 2.3.**Non-linear observer design hierarchy ........ 16

**Figure 2.4**.State-Feedback Design using an Observer ........ 18

**Figure 2.5.**Comparison of Linear and Nonlinear Gains ........ 30

**Figure 2.6**.Nonlinear Gain Functions ........ 32

**Figure 3.1.**Stribeck friction ........ 37

**Figure 3.2.**HGO Matlab Simulink Model ........ 39

**Figure 3.3.**HGO Matlab simulation of Nominal system ........ 40

**Figure 3.4.**HGO Matlab simulation of Nominal system with DISTURBANCE ........ 41

**Figure 3.5.**HGO Matlab simulation of Nominal system with UNCERTAINTY ........ 42

**Figure 3.6.**SMO Matlab Simulink Model ........ 43

**Figure 3.7.**SMO Matlab simulation of Nominal system ........ 44

**Figure 3.8.**SMO Matlab simulation of Nominal system with DISTURBANCE ........ 45

**Figure 3.9.**SMO Matlab simulation of Nominal system with UNCERTAINTY ........ 46

**Figure 3.10** NESO Matlab Simulink Model ........ 47

**Figure 3.11.**NESO Matlab simulation of Nominal system ........ 48

**Figure 3.12.**NESO Matlab simulation of Nominal system with DISTURBANCE ............................49

**Figure 3.13.**NESO Matlab simulation of Nominal system with UNCERTAINTY............................51

**Figure 3.14.**Over all Simulation Model ........................................................................................52

**Figure 3.15.**Estimation Error of the Nominal Plant ..........................................................................53

**Figure 3.16.**Estimation Error of the Plant with stribeck Friction........................................................54

**Figure 3.17.**Estimated Error of the Plant with 100% Change of Inertia .............................................54

# GENERAL INTRODUCTION

In many applications, estimating the current state of a dynamical system is crucial either to build a controller or simply to obtain real time information on the system for decision-making or surveillance. A common way of addressing this problem is to place some sensors on/in the physical system and design an algorithm, called *observer*, whose role is to process the incomplete and imperfect information provided by the sensors and thereby construct a reliable estimate of the whole system state. Of course, such an algorithm can exist only if the measurements from the sensor somehow contain enough information to determine uniquely the state of the system, namely the system is *observable*.

The number and quality of the sensors being often limited in practice due to cost and physical constraints, the observer plays a decisive role in a lot of applications. Many efforts have thus been made in the scientific community to develop universal methods for the construction of observers. Several conceptions of this object exist, but in this work, we mean by observer a finite-dimensional dynamical system fed with the measurements, and for which a function of the state must converge in time to the true system state. Although very satisfactory solutionsare known for linear systems, nonlinear observer designs still suffer from a significant lack of generality. The very vast literature available on the subject consists of scattered results, each making specific assumptions on the structure and observability of the system. In other words, no unified and systematic method exists for the design of observers for nonlinear systems [1].

On the contrary of linear observer theory, which has approximately reached to a saturation point, researches on nonlinear systems observers are still premature and far away from complete. Actually, design methodologies, stability analysis and formulation of nonlinear observer for nonlinear systems still encounter hard difficulties. In this context, the enlargement of stability region of attraction for nonlinear observer is the challenging problem which attracted many researchers who proposed many approaches to solve this problem. One solution is based on expansion or linearization irrespective to system complexity such as Leunberger observer and Kalman filters for nonlinear systems [2].

The Luenberger observer (1971), which is a linear observer, has been the essential approach in designing the state estimators in control theory. The works proposed by Arthur J. Krener and Alberto Isidori (1983), Arthur Krener andWiltold Respondek (1985), and Xiaohua Xia and Wei Gao (1989) had firstly addressed the theory of observers in nonlinear system by approximating the nonlinear dynamic of observation error to linear structure by imposing a set of conditions. However, the necessary and sufficient conditions of such observation approaches, like the feedback linearization problem, are somewhat restrictive [1, 2].

Another contribution to the linearization technique is made by Zeitz (1987), which proposed an algorithm that extends the Luenberger observer for nonlinear systems. This algorithm used input time derivatives and it was easy to implement. However, the critical issue with this technique is that the convergence of the Luenberger observer cannot be guaranteed. Later in 1989, Tornambe presented an approach to cancel the nonlinearity based on high gain approximation. The main drawback with this algorithm is that it cannot guarantee asymptotic convergence of estimation error to zero with arbitrarily finite high gain in spite that the error might be bounded and the initial conditions of both system and observer states have to be set synchronously [1, 2].

In 1990, an adaptive observer was proposed by Marino for Single Input-Single Output (SISO) nonlinear systems. The difficulty with this observer is that the nonlinear system is either in (or transformed to) an observable canonical form. The work presented by Bastin and Gevers (1988) could establish the necessary and sufficient conditions that transform the nonlinear system into observable canonical form. However, such conditions are restrictive since transforming the observer to canonical form may be difficult to be found. Although this adaptive observer does not require the full information of dynamic systems model, it can guarantee asymptotic stability to only finite error [3].

In 1990, Tsinias proposed an observer which is able to guarantee the convergence of estimated states of observer to the actual states. In 1992, Gauthier et al. presented a contribution to the nonlinear observation theory by introducing an observer which can asymptotically track the states of nonlinear system in such a way that the Lyapunov equation can be determined by observer gain. However, the existence of globally defined and globally Lipschitzian change of coordinates is a prerequisite of this observation method. It was shown that for any nonlinear system which is observable to any input, an observer with global convergence can be found. Gauthier et al. could present an alternative proof to show this hypothesis [2].

In 1992, Khalil and Esfandiari presented a new observer for output feedback control design called High-Gain Observer (HGO). HGO shows robust characteristics in estimating the unmeasured states and asymptotic attenuation of disturbances. Later in 1999, At tassi etal proved that the separation principle can be achieved with HGO for a wide class of systems and this was the basis in solving many nonlinear system problems [4, 5]. In 2008, a modified version HGO named as an Extended High Gain Observer (EHGO) has been proposed by Freidovich. The observer was used to reduce the effect of model errors and unknown disturbances in fully actuated mechanical systems [6,7].

Other efficient tool for observation is the sliding mode observers. The development of this type is contributed by pioneers of researchers such as Slotine, Utkin and Walcott [8]. These observers are basically based on sliding theory and can solve the problem of peaking phenomenon seen in HGO. They are able to offer finite-time convergence, and robustness with respect to uncertainties and the possibility of uncertainty estimation. Second sliding mode observer, super twisting sliding mode observer and adaptive sliding mode observer are other advanced versions of sliding mode observer, which recently used in many applications [8, 9, 36].

In 1995, J. Han introduced a unique observer design by class of Nonlinear Extended State Observers (NESO). The main feature with this observer is that it does not depend on plant mathematical model. Thus, enhanced robustness has been achieved and it was verified and applied in different industrial observer-based control applications [10 - 12].

Generally, the observers can be divided into three groups: linear, non-linear and disturbance observers. The linear and nonlinear observers mainly rely on the mathematical model of considered systems including the knowledge of existing noises and disturbances. More exact model information will give better estimation accuracy of such observers. On the other hand, the disturbance observer is concerned with input-output data. This type of observer can tackle systems of high nonlinearities and uncertainties and has the capability to disturbance rejection effectively. Fig .1 illustrates the details of observer classification [1, 2].

The present work focuses on a comparison of performances and characteristics of threse observers. The criterion for comparison is based on the observer tracking errors, both at steady state and during transient, and the robustness of the performance with respect to the uncertainties of plant. To further enhance the performance of NESO in the presence of unknown initial conditions, several nonlinear gain functions are introduced. The simulations conducted assisted in gaining insight of observer behavior in a nonlinear servo motor system. It results are provided to give realism.

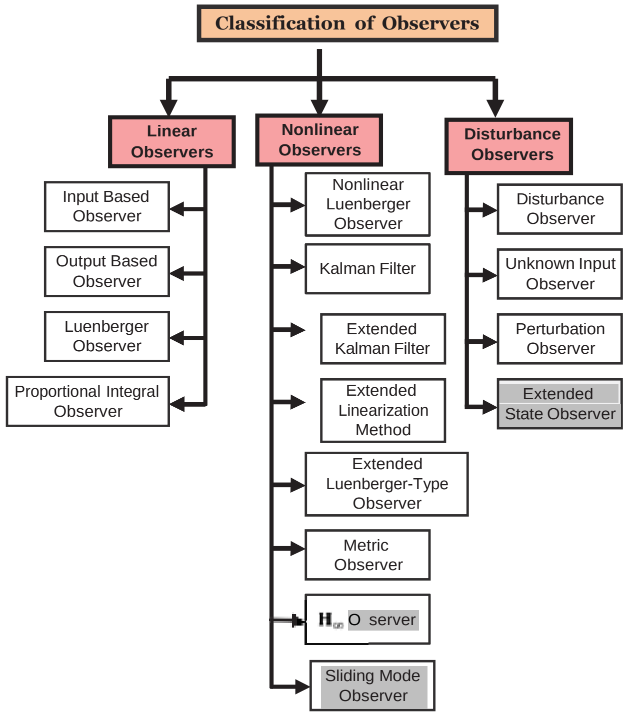



**Figure 1.**General Classification of Observers

## Problem Formulation

The design of observers have faced critical challenges in practical applications due to the presence of nonlinearities, disturbances and dynamic uncertainties. Thus, obtaining high-performance robust observer design was the target of many researchers. In the last two decades, several advanced observer design techniques have been proposed like high gain observers (HGOs), sliding mode observers (SMO), extended state observer (ESO), nonlinear extended state observer (NESO).

## Motivation of the project

From the literature review, it appears that several unresolved issues exist in the current nonlinear control methods.

Most nonlinear controllers are state feedback controllers. When the state of the system is unavailable or the sensor is expensive, observers must be used in estimation. The motivation of this work is to find a set of observer based the control systems that are relatively independent of the mathematical model, perform better, and are simple to implement.

The objectives of this thesis can be divided into several items as follows:

1. To design and develop non-linear high-gain observer HGO to estimate the system states to be used in the comparative study of the observers.
2. To design and develop non-linear sliding mode observer SMO to estimate the system states to be used in the comparative study of the observers.
3. To design and develop non-linear extended state observer NESO to estimate the system states to be used in the comparative study of the observers.
4. To presents a comparison study of the characteristics and performances of the HGO, SMO, and NESO.

## Work methodology

The Nonlinear Extended State Observer (NESO), proposed by Professor Jingqing Han, can estimate the state without a mathematical model of the system. It is a novel concept for observer design, estimating not only the state, but also the internal and external disturbances, thus making disturbance rejection control possible. The invention of NESO isa revolutionary concept for control theory and application. It has several properties including model-independence, active estimation, compensation for disturbances, simple design, and strong robustness. This method has evolved as an important technique for the state feedback control of nonlinear systems.

In this dissertation, the concepts of the NESO, the High Gain Observer (HGO), and the Sliding Mode Observer (SMO) are introduced in Chapter II. Chapter III presents a comparative study of these three observers, which is based on the robustness of the performance with respect to the uncertainties of the plant and the observer trackingerrors, both at steady-state and transient.

# CHAPTER I

# NONLINEAR CONTROL SYSTEMS

This chapter is divided to into four sections. The first section discusses nonlinear control systems in general; linear and non-linear controllers are briefly discussed. The feedback linearization control is reviewed in the second section. The third section highlights the use of variable structure with sliding mode control, the last section explained another nonlinear control theory under the name back stepping control, and all of the above have been conclude in the end of this chapter.

# 1.1. Introduction:

Nonlinear control theory is the area of control theory which deals with systems that are nonlinear, time-variant, or both. Control theory is an interdisciplinary branch of engineering and mathematics that is concerned with the behavior of dynamical systems with inputs, and how to modify the output by changes in the input using feedback, feed forward, or signals filtering. The system to be controlled is called the "plant". One way to make the output of a system follow a desired reference signal is to compare the output of the plant to the desired output, and provide feedback to the plant to modify the output to bring it closer to the desired output.

Control theory is divided into two branches. Linear control theory applies to systems made of devices which obey the superposition principle. They are governed by linear differential equations. A major subclass is systems which in addition have parameters which do not change with time, called *linear time invariant* (LTI) systems. These systems can be solved by powerful frequency domain mathematical techniques of great generality, such as the Laplace transform, Fourier transform, Z transform, Bode plot, root locus, and Nyquist stability criterion.

Nonlinear control theory covers a wider class of systems that do not obey the superposition principle. It applies to more real-world systems, because all real control systems are nonlinear. These systems are often governed by nonlinear differential equations. The mathematical techniques which have been developed to handle them are more rigorous and much less general, often applying only to narrow categories of systems. These include limit cycle theory, Poincaré maps, Lyapunov stability theory, and describing functions. If only solutions near a stable point are of interest, nonlinear systems can often be linearized by approximating them by a linear system obtained by expanding the nonlinear solution in a series, and then linear techniques can be used. Nonlinear systems are often analyzed using numerical methods on computers, for example by simulating their operation using a simulation language. Even if the plant is linear, a nonlinear controller can often have attractive features such as simpler implementation, faster speed, more accuracy, or reduced control energy, which justify the more difficult design procedure [18].

## 1.2. Feedback Linearization Control:

Feedback linearization is considered as a non-linear control methodin which non-linear system is indirectly linearised, where linearization is achieved by ex- act feedback and exact state transformations rather than linearising the system dynamics directly using linear approximation, Tylor series or Jacobian transformation.

Feedback linearization is to employ a non-linear control law so that the controlled sys- tem combined with non-linear control behaves linear and controllable in the closed-loop. As shown in Figure 1.1, feedback linearization is implemented using two control loops, the inner control loop is referred to as non-linear linearising control law, while the outer control loop is to control the resultant linear system achieved by the inner loop control [19].

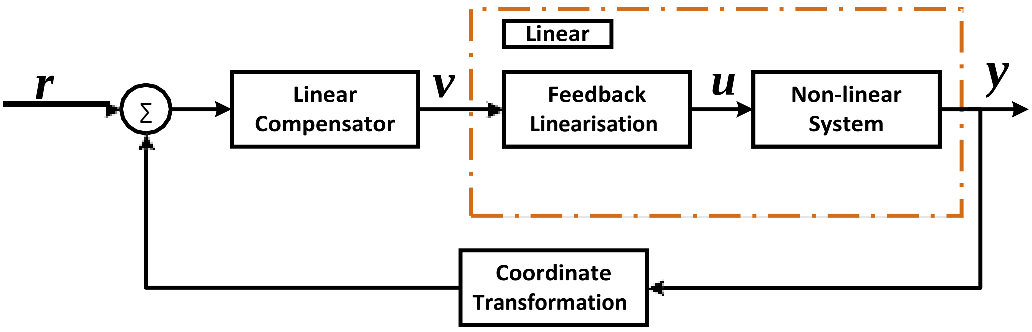



**Figure 1.1.**Feedback linearisation control block diagram

There are two types of feedback linearization, input-output linearization in which input- output map is completely linearised, yet the state equations might be partially linearised. Another type of feedback linearization is input-state linearization or referred to by some asfull-state linearization where the state equation is completely linearised.

In input-output linearization, the method of linearising input-output map can be briefly explained by writing the state and output equations in the form of

$$\dot{x}(t)=f(x,u)$$
$$\dot{x}(t)=g(v,y) \quad (1.1)$$

Where $\dot{x}$ is the dynamic change of the system, *x*, *u* and *y* represent the states, input and output of the system receptively and *v* is the control synthesis or the input to the linearisedsystem. State equation is said to be related to output equation if there is a diffeomorphism as

$$T: \rightarrow R^{n+1} \rightarrow\rightarrow R^{n+1} \tag{1.2}$$

Such that $T_1(x, u) = y_1$, $T_2(x, u) = y_2$, ... and $T_{n+1}(x, u) = v$. Thus the feedback equivalent to the system can be represented in terms of state equation as:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ . \\ . \\ . \\ \dot{y}_n \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 1 \end{pmatrix} v$$

This is considered as linear, time-invariant and controllable single input system.

## 1.3. Variable Structure, Sliding-Mode Control:

Sliding mode control is a subset of variable structure control [20, 21], in which the states of the system are guided into a switching surface and then the states slide to the origin, as shown in Figure 1.2. Variable structure system and control were developed by Utkin and sliding mode control was introduced by Utkin as well (Utkin, 1978).

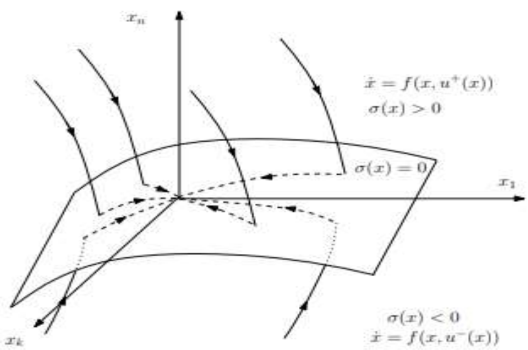



**Figure1.2.**Sliding mode demonstration

Variable structure control [20, 21] has been widely used over the last decade for the control of uncertain systems because of its robustness to modeling uncertainties and disturbances, consider the dynamic system:

$$x^{(n)}(t) = f(X;t) + b(X;t)u(t) + d(t) \quad (1.1)$$

Where *u(t)* is scalar control input, *x* is the scalar output of interest, and $X = \left[x, \dot{x}, \dots, x^{(n-1)}\right]^T$ is the state. The function $f(X;t)$ is not exactly known, but the extent of imprecision $|\Delta f|$ on $f(X;t)$ is upper bounded by a known continuous function of *X* and *t*; similar to the control gain $b(X;t)$. The disturbance *d (t)* is unknown but bounded in absolute value by a known continuous function of time. The control problem is to get the state *X* to track a specific state $X_d = \left[x_d, \dot{x}_d, \dots, x_d^{(n-1)}\right]^T$ in the presence of model imprecision on *f(X;t), b(X;t*) and disturbance of *d(t)*. To guarantee that this is achievable using a finite control *u*, it must be assumed:

$$\tilde{X}\big|_{t=0} = 0 \quad (1.2)$$

Where $\tilde{X} = X - X_d = \left[\tilde{x}, \dot{\tilde{x}}, \dots, \tilde{x}^{(n-1)T}\right]$. A time-varying sliding surface *s(t)* is defined in the state-space $R^n$as $S(\tilde{x};t) = 0$ with:

$$S(\tilde{x};t) = (\frac{d}{dt} + \lambda)^{n-1}\tilde{x}, \lambda > 0 \quad (1.3)$$

Where λ is a positive constant

Given initial condition (1.2), the problem of tracking $X = X_d$ is equivalent to that of remaining on the surface $S(t)$ for all $t > 0$; indeed $S \equiv 0$ represents a linear differential equation whose unique solution is $\tilde{X} \equiv 0$ thus, the problem of tracking the n-dimensional vector $x_d$ can be reduced to that of keeping the scalar quantity *s* at zero. A sufficient condition for such positive invariance of *s (t)* is to choose the control law *u(t)* of (1.1) to satisfy

$$\frac{1}{2}\frac{d}{dt}s(x;t) \leq -\eta|s| \quad (1.4)$$

Where η is a positive constant. Inequality (1.4) constrains trajectories to point towards the surface *s (t*) (Fig. 1.3), and is referred to as the sliding condition.

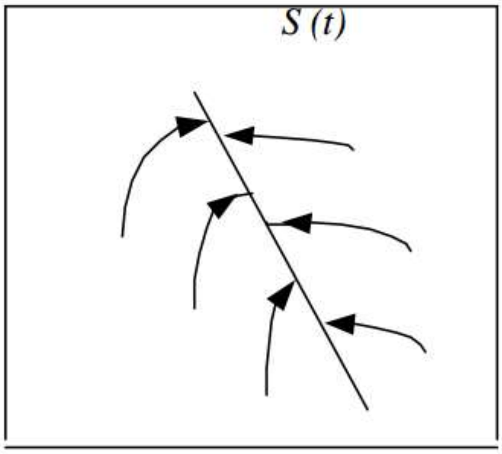



**Figure 1.3**.The Sliding condition

## 1.4. Back stepping control:

In the last few decades, several control techniques are deployed for the stabilization, regulation, and control of linear and nonlinear dynamical systems. For linear autonomous systems, it is easy to find a control Lyapunov function for stability and optimization problems. However, finding a suitable control Lyapunov function is a challenging problem for nonlinear control systems.

The back stepping control method is a recursive design procedure that links the choice of a control Lyapunov function with the design of a feedback controller and guarantees global asymptotic stability of strict feedback systems. The active back stepping control method is a practical tool to overcome the limitations of the feedback linearization approach in the control literature. The block back stepping control method is a general back stepping control method with more applicability in the control literature. The adaptive back stepping control method is a modified form of back stepping control method that uses estimates for unknown parameters in the systems. The robust back stepping control method is an effective back stepping technique for control systems with uncertainties.

The basic idea of this method is to leave some states of the system act as virtual inputs. The back stepping uses a form of the system chain of integrators, after a coordinate transformation of a triangular system and based on the direct method of Lyapunov. The method is to split the system into a set of subsystem nested descending order. From there, it is possible to design systematically and recursively controllers and corresponding Lyapunov functions [22].

**Back stepping theory:** The control objective is to design a robust controller for the output y(t) of the system to track the output $\mathbf{y_{ref}}$ of the reference model asymptotically .assume that not only $y_{ref,}$ but also it is first two derivatives $\dot{y}_{ref}$ and $\ddot{y}_{ref}$ are all bounded functions of time [23]. The diagram block of this back stepping control is shown in Fig.1.4

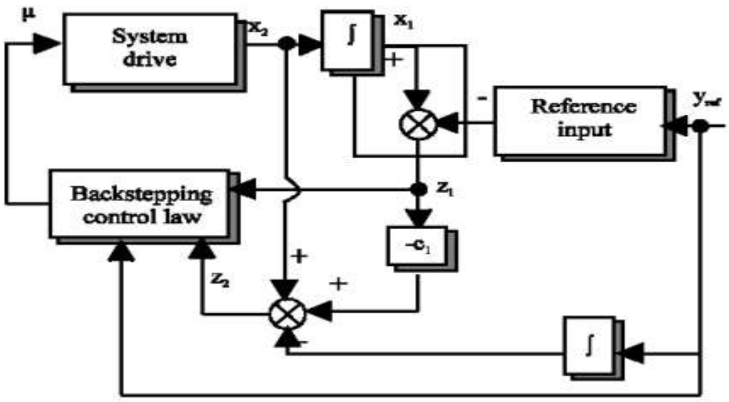



**Fig.1.4.** Back stepping control system

The back stepping design to achieve the position tracking objective is described step by step as follows.

Consider a drive system:

$$\dot{x_1} = x_2$$
$$\dot{x_2} = u + \varphi(x_1, x_2)\theta \qquad (1.5)$$
$$y = x_1$$

Where u: control input

$\theta$: System parameter

y: output state

x: variable state

**Step 1:** for the position tracking objective, define the tracking error as

$$z_1 = y - y_{ref} \qquad (1.6)$$

And its derivative as:

$$\dot{z}_1 = \dot{y} - \dot{y}_{ref} \qquad (1.7)$$

The $x_2$ can be viewed as a virtual control in above equation. Define the following stabilizing function

$$\alpha_1 = -c_1 z_1 \qquad (1.8)$$

Where $c_1$ is a positive constant. So, the second regulated variable is chosen as:

$$z_2 = x_2 - \alpha_1 - \dot{y}_{ref} \qquad (1.9)$$

The first Lyapunov function is chosen as:

$$v_1 = \frac{1}{2} z_1^2 \qquad (1.10)$$

Then the derivative of $v_1$ is

$$\dot{v}_1 = z_1 z_2 - c_1 z_1^2 \tag{1.11}$$

**Step2:** hence, the derivative of the second regulated variable is calculated as

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 - \ddot{y}_{ref} \tag{1.12}$$

To design the controller, add terms concerning $z_2$ to $v_1$ to form the following Lyapunov function

$$v_2 = v_1 + \frac{1}{2} z_2^2 \tag{1.13}$$

Using eq.1.11 and 1.13 the derivative of $v_2$ can be derived as follows:

$$\begin{aligned} \dot{v}_2 &= \dot{v}_1 + z_2 \dot{z}_2 \\ &= -c_1 z_1^2 + z_2 (z_1 + \dot{z}_2) \end{aligned} \tag{1.14}$$

According to eq.1.14, the control law u is designed as follows:

$$u = -c_2 z_2 - z_1 - \varphi(x_1, x_2)\theta - c_1 \dot{z}_1 + \ddot{y}_{ref} \tag{1.15}$$

Where $c_2$ is a positive constant. Substituting eq.1.15 into eq.1.14, the following equation can be obtained

$$\dot{v}_2 = -c_1 z_1^2 - c_2 z_2^2 \leq 0 \tag{1.16}$$

So, the back stepping control is asymptotically stabilizing.

## 1.5. Conclusion

Nonlinear control systems are those control systems where nonlinearity plays a significant role, either in the controlled process (plant) or in the controller itself. Nonlinear plants arise naturally in numerous engineering and natural systems, including mechanical and biological systems, aerospace and automotive control, industrial process control, and many others. Nonlinear control theory is concerned with the analysis and design of nonlinear control systems which provides its basic analysis tools such as feedback linearization control, sliding mode control and back stepping control.

# CHAPTER II

# NONLINEAR OBSERVERS

Observers provide state estimates of the plant to closed-loop control algorithms. The control algorithm is designed in two parts: a "full-state feedback" part based on the assumption that all of the state variables can be measured and an observer to estimate the state of the process based on the observer output. In this chapter, the concepts of a nonlinear linear observer are introduced first, a linear observer have been explained as a basic design. The high gain observers, sliding mode observers and extended state observers are reviewed.

# 2.1. Introduction

Estimating a state of a dynamical system, whether it is estimated from system input or output, is known as "observing the state", that is where the name of "observer" in the theory of systems comes from. In the early works, the principle of observer has been widely studied and proven in many linear systems, Linearised systems, in the so-called "observer-based control". Figure2.1.(Luenberger, 1979).

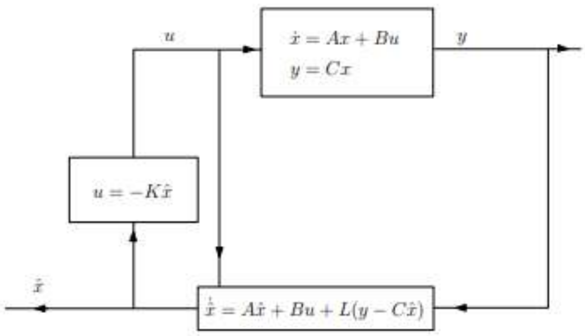



**Figure 2.1**.luenberger observer

However, applying such type of linear observer to non-linear system theory has been successfully implemented by using the extended Kalman filter (Primbs, 1996).

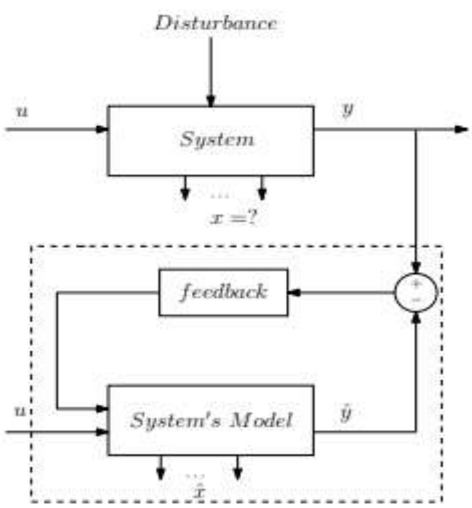



**Figure 2.2**.Kalman filter demonstration.

Attempts have continued to construct a non-linear observer using tools developed from pure non-linear systems theories. One of the most highlighted results, which used Lyapunov stability theory, was presented by Thau (1973) and Kou et al. (1975). Primbs (1996) has considerably simplified both these results and presented with examples [39, 41, 43].

Techniques relying on Lie-algebraic approach have been introduced in non-linear observer design by converting non-linear states of the system to linear states where any applicable linear theory can be utilized. Non-linear state transformations method in non-linear observer design was primarily developed and introduced by Zeitz (1987) who has designed non-linear observer by transformation into a generalized canonical from and Keller (1987) has extended the Luenberger observer for non-linear control systems [40, 44].

Baumann and Rugh (1986) introduced the method of injecting non-linear output [42], of single-input multi-output (SIMO) non-linear system, based on system linearization in order to place the eigenvalues of the family of linearised closed-loop systems at specific values, so that the linearised error dynamics would have locally constant eigenvalue with respect to the closed-loop operating points.

Figure2.3 summarises the methodology that can be used to design a non-linear observer.

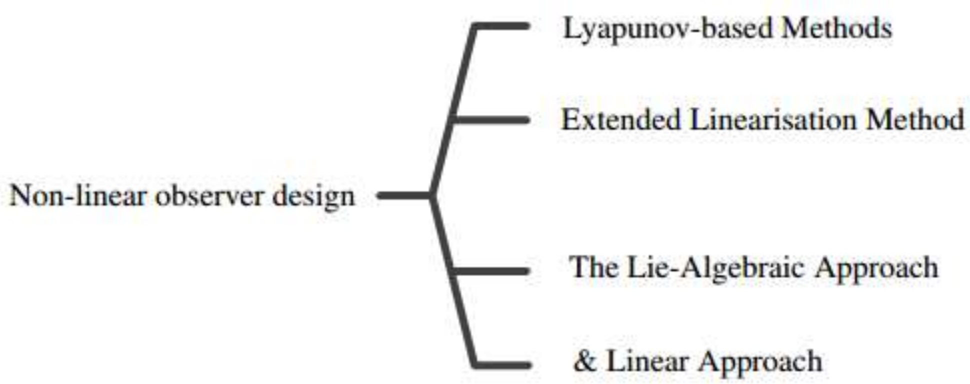



**Figure 2.3.**Non-linear observer design hierarchy

In the late of 1980's, high-gain observer was used in controlling non-linear systems two groups of researches simultaneously developed techniques in high-gain observer for non-linear terms. These were a French group lead by Hammouri, Gauthier and a US group lead by Khalil. The French group focused on stabilizing the non-linear system globally under global Lipschitz and the structure of non-linear zeros in this work was more general. However, the work done by Khalil's group was in lack of global Lipschitz conditions, i.e. the characteristic of this work doesn't require that non-linearity of the system to satisfy Lipschitz conditions, as when the observer gain is sufficiently increased that could destabilize the system, which means only semi-global results were achieved in this work with a compact set which can be made arbitrary large. The first demonstration of the presence of peaking phenomena in non-linear feedback control was in Esfandiari and Khalil's work, where undesirable peak would make the transient response worse and worse as the observer gain increased. It was found that the interaction of peaking phenomena with non-linearity of the system drives the system to unstable region and causes the system to have finite escape time [13].

In the case of designing an observer for linear/ linearised system, a high value of the linear Luenberger observer gain "$L$" will make the estimated states converge very quickly to the system states. Nevertheless the initial estimator error can be prohibitively large leading to the so-called peaking phenomenon. The aforementioned problem justifies utilizing sliding mode observer. The sliding mode observer drives the estimated states to hyper surface (sliding surface) around zero estimated error utilizing instead non-linear high gain. This non-linear gain can be implemented with any scaled function such as *signum*, saturation, or *tanh* function. This kind of non-linear gain will maintain the observer trajectories slide along a surface once the estimated states hit this sliding surface where the observed outputs exactly match the measured outputs. Hence, this attractive feature would reduce the sensitivity of the estimated states to many types of noise [36].

The extended state observer first proposed by Jingqing Han in can efficiently estimate disturbances, uncertainties and sensor noise. ESO is used in the control system to estimate and compensate disturbances via a feed-forward cancellation technique. Also, it can be extended to estimate uncertainties and disturbances for multi-input–multi-output (MIMO) systems as well [16].

## 2.2. Basic Linear Observer Design

The concept of separating the state feedback control design into the full-state feedback part and observer is known as the separation principle, which has rigorous validity in linear systems, as well as a limited class of nonlinear systems. In the linear control community, the well-known separation principle states that, for a controller designed using an observer and a constant-gain state-feedback gain can be designed separately since the overall closed-loop system eigenvalues are the union of those due to the observer alone and those due to the state-feedback controller alone [11].

Figure2.4. illustrates the state-feedback design using an observer.

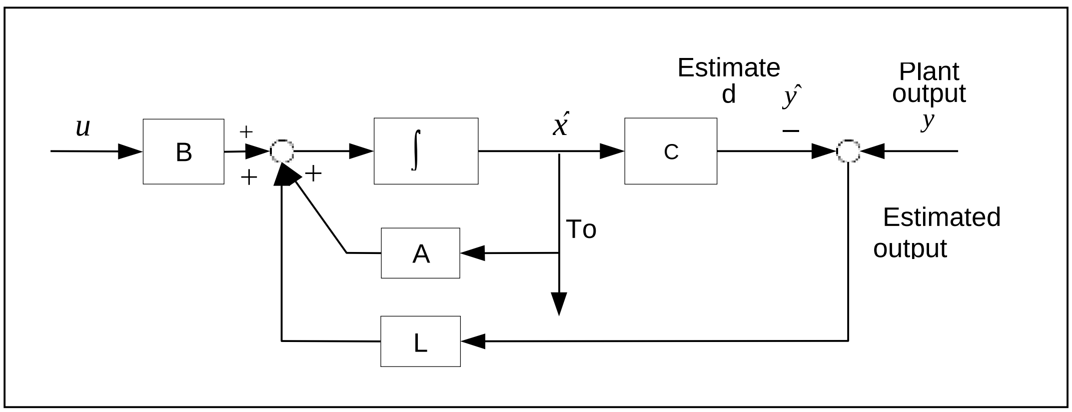



**Figure 2.4**.State-Feedback Design using an Observer

Consider a linear, continuous-time dynamic system:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (2.1)$$

$$y(t) = Cx(t)$$

Given the control input u(t) and the initial condition we can predict the evolution of x(t) and y(t)

The state evolution is given by:

$$x(t)=e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(t)d\tau \qquad (2.2)$$

Where x(0) is the initial condition, the second term is the convolution contains u(t) which is well known because we assume that A, B and C are exactly known.

If we simulate a model with state $\hat{x}$ in parallel with the process:

The process output $y(t) = Ce^{At}x(0) + C\int_0^t e^{A(t-\tau)}Bu(t)d\tau$ (2.3)

The model output $y_m(t) = Ce^{At}\hat{x}(0) + C\int_0^t e^{A(t-\tau)}Bu(t)d\tau$ (2.4)

The model parameters and input u(t) are known exactly, then :

$t > t_1$ and $t_1$ is too large so $e^{At} \approx 0$ and A must be Hurwitz.

For that it will be:

$$y(t) \approx y_m(t) \quad (2.5)$$
$$x(t) \approx \hat{x}$$

A simple observer is an open loop model simulation in parallel with the actual process. But in practice this will fail because the model parameters are not known exactly, there are also external signals affecting a real system such disturbance.

Open loop simulation is inadequate because it makes no use of system information and system measurements to recalibrate.

Let have:

$$y(t) = Ce^{At}\hat{x}(0) + C\int_0^t e^{A(t-\tau)}Bu(t)d\tau + \hat{C}\int_0^t e^{A(t-\tau)}\hat{B}v(t)d\tau \quad (2.6)$$

Where v(t) is the disturbance signal and the third term in y(t) is the disturbance effects.

So our model output will be:

$$y_m(t) = C\,e^{At}\hat{x}(0) + C\int_0^t e^{A(t-\tau)}Bu(t)d\tau \quad (2.7)$$

The model parameters are slightly different from the real ones.

So even with stable dynamics the difference $y-y_m \neq 0$ at $t \longrightarrow \infty$ due to to parameters error and unknown signal v(t).

For that the parallel model simulation need to be recalibrate to ensure it converges to match the true model states, irrespective of v(t) and that by forcing the error between the model output system and the real output to zero.

$$e(t) = y(t)-y_m(t) \; ; \quad \lim_{t\longrightarrow\infty} e(t) = 0$$

$$e_x(t) = x(t)-\hat{x} \quad ; \quad \lim_{t\longrightarrow\infty} ex(t) = 0$$

A full-order observer for the linear process is defined by

$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x}) + Bu \tag{2.8}$$

The estimation error is

$$e = x - \hat{x} \tag{2.9}$$

From (2.8) and (2.9) it is given that:

$$\dot{e} = (A-LC)*e = \hat{A}e \tag{2.10}$$

The estimation error will converge to zero if $\hat{A}$ is a stability matrix. When $\hat{A}$ is constant, its eigenvalues must be in the open left half-plane. This asymptotic state estimator is known as the Luenberger observer [2]. Since the matrices *A*, *B* and *C* are defined by the plant, the only freedom in the design of the observer is in the selection of the gain matrix *L*. Optimization and pole placement are two standard design methods.

Since the observer given by (2.8) has the structure of a Kalman filter, its gain matrix can be chosen as a Kalman filter gain matrix [25], i.e,

$$L = PC'R^{-1} \tag{2.11}$$

Where *P* is the covariance matrix of the estimation error and satisfies the matrix Riccati equation:

$$\dot{P} = AP + PA' - PC'R^{-1}CP + Q \tag{2.12}$$

*R* is a positive definite matrix and *Q* is a positive semi-definite matrix. In most applications the steady-state covariance matrix is used in (2.11). This matrix is given by setting $\dot{P}$ in (2.12) to zero. The resulting equation is known as the algebraic Riccati equation. Algorithms to solve the algebraic Riccati equation are included in popular control system software packages such as MATLAB and CONTROL-C. In order to make the gain matrix given by (2.11) and (2.12) to be genuinely optimum, the process noise and the observation noise must be white, with the matrices *Q* and *R* as their spectral densities.It is nearly impossible to determine these spectral density matrices in practical applications. Hence, the matrices Q and R are best treated as design parameters that can be varied to achieve overall system design objectives.

An alternative to solving the algebraic Riccati equation in order to obtain the observer gain matrix is to select *L* to place the poles of the observer [26, 27, 28, 29], i.e. the eigenvalues of $\hat{A}$ in (2.10). From (2.10), the characteristic equation of the error is now given by:

$$\mathbf{det[sI - (A - LC)] = 0} \tag{2.13}$$

If *L* is chosen so that *A-LC* has stable and reasonably fast eigenvalues, i.e. we make the transient matrix *A-LC* has a stable dynamics then *e* will decay to zero and remain there, independent of the known forcing function *u (t)*, its effect on the state *x (t)*, and irrespective of the initial condition *e (0)*. Therefore, $\hat{x}(t)$ will converge to x(t) regardless of the value $\hat{x}(0)$ . Furthermore, the dynamics of the error can be chosen for stability, as well as for speed, as opposed to the open-loop dynamics determined by *A*.If we do not have an accurate model of the plant *(A, B, C)*, the dynamics of the error are no longer governed by (2.10).

However, we can typically choose *L* so that the error system is at least stable and the error remains acceptably small, even with (small) modeling errors and disturbing inputs. It is important to emphasize that the nature of the plant and that of the estimator are quite difference.

## 2.2.1. Duality with state feedback control

The selection of $L$ can be approached in exactly the same fashion that $K$ is selected in the control-law design. If $S_i$ is the desired location of the estimator error poles than it is specified as:

$$S_i = \beta_1, \beta_2 \ldots\ldots \beta_n$$

Where $\beta_i$ are the desired poles

Then the desired estimator characteristic equation $\alpha_e$ is

$$\alpha_e\ (s) = (s - \beta_1)\ (s - \beta_2) \ldots (s - \beta_n) \tag{2.14}$$

We can solve for $L$ by comparing coefficients in (2.7) and (2.8).

The observer is used mainly to estimate the state for the purpose of feedback control.

$$u = -K\hat{x} \tag{2.15}$$

Where:

$$\hat{x} = \mathrm{x} - \mathrm{e} \tag{2.16}$$

The closed-loop dynamics is given in part by:

$$\dot{x} = Ax - BK(x - e) \tag{2.17}$$

When a full-order observer is used

$$\dot{e} = \hat{A}e = (A - LC)\ e \tag{2.18}$$

Thus, the complete closed-loop dynamics is

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \tag{2.19}$$

Suppose:

$$\bar{\mathrm{A}} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix}$$

$$|SI - \bar{\mathrm{A}}| = |SI - \mathrm{A} + \mathrm{BK}||SI - \mathrm{A} + \mathrm{LC}| = 0 \tag{2.20}$$

The closed-loop eigenvalues are the eigenvalues of *A-BK*, the full-state feedback system; and the eigenvalues of *A-LC*, the dynamics matrix of the observer. This is a statement of the well-known separation principle, which permits one to design the observer and the full-state feedback control independently, with the assurance that the poles of the closed- loop dynamic system will be the poles selected for the full-state feedback system and those selected for the observer.

## 2.3. High Gain Observer

High gain observer is used to estimate unknown state for a nonlinear system with the assumption that the system is observable. The design process for high-gain observer is very simple; the observer gain is determined based on a positive constant that should be selected as small as possible to have a fast state estimation. The first problem of high-gain observer is the so-called peaking phenomenon (the state estimation exhibits a large output during transients), but such an issue can easily be solved by saturating the control input during transients. Note that the control saturation does not affect the transient performance of the closed-loop system, as the state observer can compensate for the effect of the saturation blocks. Global asymptotic stability under high-gain observer is guaranteed for nonlinear systems [13, 30, 31].

More importantly, nominal transient performance achieved under feedback linearization is retained with high-gain observer provided that the observer gain is high enough. Such a feature cannot be guaranteed under the conventional composite controllers. Here, separation principle can be adopted to prove the stability of the closed-loop system under the composite controller consisting of high-gain observer and feedback linearization. Furthermore, high-gain observer can be employed to estimate the unknown disturbance representing model uncertainty and external disturbance to ensure asymptotic stability of the close-loop system.

Last important point is that fast state estimation requires high-observer gain, which raises concerns about measurement noises sensitivity. Therefore, measurement noises put limit on how fast could the state observer. Prof. H. Khalil has proposed different strategies to overcome such an issue.
As an example, one can use high observer gain during transients to ensure fast state estimation, and once the difference between the measured output and its estimate becomes small enough (within specified limits), the observer gain can be reduced to reduce the effect of the measurement noise during steady-state regime.

Consider the second-order nonlinear system:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \Phi(x, u, w)$$

Where Φ(x,u,w) is some nonlinear function and w is a disturbance.

Supposing that u = γ(x, w) is a state feedback control that stabilizes the origin *x = 0* of the closed-loop system, so the system is given as:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \Phi(x, \gamma(x, w), w) \qquad (2.21)$$

The following observer is used:

$$\dot{\hat{x}}_1 = \hat{x}_2 + h_1(y-\hat{x}_1) \qquad (2.22)$$

$$\dot{\hat{x}}_2 = \Phi_0(\hat{x}, u) + h_2(y-\hat{x}_1)$$

Where, $\Phi_0(\hat{x}, u)$ is a nominal model of the nonlinear function Φ(x ,u, w). The estimation error equations are:

$$\tilde{x}_1 = x_1 - \hat{x}_1 \qquad (2.23)$$

$$\tilde{x}_2 = x_2 - \hat{x}_2$$

So it derivatives are:

$$\dot{\tilde{x}}_1 = \dot{x}_1 - \dot{\hat{x}}_1 = x_2 - \hat{x}_2 - h_1(y-\hat{x}_1)$$

$$\dot{\tilde{x}}_2 = \dot{x}_2 - \dot{\hat{x}}_2 = \Phi(x, \gamma(x, w), w) - \Phi_0(\hat{x}, u) - h_2(y-\hat{x}_1)$$

After that:

$$\dot{\tilde{x}}_1 = -h_1 \tilde{x}_1 + \tilde{x}_2 \qquad (2.24)$$

$$\dot{\tilde{x}}_2 = -h_2 \tilde{x}_1 + \delta(x, \tilde{x}, w)$$

Where $\delta(x, \check{x}, w) = \Phi(x, \gamma(\tilde{x}, ,w), w) - \Phi_0(\hat{x}, \gamma(\tilde{x}, ,w),)$ is the disturbance term.

As in any asymptotic observer, the observer gain H = $\begin{bmatrix} h1 \\ h2 \end{bmatrix}$ is designed to achieve asymptotic error convergence; that is, $\lim_{t \to \infty} \tilde{x}(t) = 0$. In the absence of the disturbance term δ(x, $\tilde{x}$, w) , asymptotic error convergence is achieved by designing the observer gain such that the matrix $A_0 = \begin{bmatrix} -h1 & 1 \\ -h2 & 0 \end{bmatrix}$ is Hurwitz; that is, its eigenvalues have negative real parts. For this second-order system, $A_0$ is Hurwitz for any positive constants $h_1$ and $h_2$. In the presence of δ , the observer gain must be designed with the additional goal of rejecting the effect of the disturbance term δ on the estimation error $\breve{x}$. This is ideally achieved, for any disturbance term δ if the transfer function from δ to $\tilde{x}$ is identically zero. The observer gain can then be designed $h_1 >> h_2 >> 1$, such that the transfer function $H_0$ from the input to states is arbitrarily close to zero.

$$H_0(s) = \frac{1}{s^2 + h_1 s + h_2} \begin{bmatrix} 1 \\ s + h_1 \end{bmatrix} \tag{2.25}$$

In particular, taking:

$$h_1 = \frac{\alpha_1}{\varepsilon}, \; h_2 = \frac{\alpha_2}{\varepsilon^2} \tag{2.26}$$

For some positive constant $\alpha_1$, $\alpha_2$ and $\varepsilon$, with $\varepsilon \ll 1$, $\lim_{\varepsilon \to 0} H0(s) = 0$.

The scaled estimation errors are defined as:

$$\eta_1 = \frac{\tilde{x}_1}{\varepsilon}\; ; \; \eta_2 = \tilde{x}_2 \tag{2.27}$$

Then the newly defined variables satisfy the following equations:

$$\varepsilon \dot{\eta}_1 = -\alpha_1 \eta_1 + \eta_2 \tag{2.28}$$

$$\varepsilon \dot{\eta}_2 = -\alpha_2 \eta_1 + \varepsilon\, \delta(x, \tilde{x})$$

This equation shows that reducing ε diminishes the effect of the disturbance term δ.

It also shows that, for small ε , the dynamics of the estimation error will be much faster than the dynamics of *x*.

## 2.3.1. Peaking Phenomenon

The change of variable (2.27) may cause the initial condition $\eta_1(0)$ to be divided by ε, even when $\tilde{x}_1(0)$ is of order *O*(1).

$\eta_1$ is given by:

$$\eta_1 = \frac{x_1 - \tilde{x}_1}{\varepsilon} = \frac{\tilde{x}_1}{\varepsilon}$$

If $x_1(0) \neq \hat{x}_1(0)$

$\eta_1(0)$ could be divided by ε.

With this initial condition the solution of (2.27) will contain a term of the form $\frac{1}{\varepsilon} e^{-at/\varepsilon}$ for some *a>0.* While this exponential mode decays rapidly, it exhibits an impulse-like behavior where the transient peaks to high values according to ε before decaying toward zero.

This behavior is known as the peaking phenomenon, which is an intrinsic feature of any HGO design that rejects the effect of the disturbance term δ in (2.24) ; that is, any design with $h_2 >> h_1 >> 1$. The peak phenomenon could destabilize the closed-loop system, as the impulse-like behavior is transmitted from the observer to the plant.

The HGO is basically an approximate differentiator, which can be easily seen in a special case when the nominal function $\Phi_0$ is chosen to be zero; for which the observer is linear. For the full-order observer (2.22) the transfer function from *y* to *x* is given by:

$$\frac{\alpha_2}{(\varepsilon s)^2 + \alpha_1 \varepsilon s + \alpha_2} \begin{bmatrix} 1 + (\varepsilon \alpha_1/\alpha_2)s \\ s \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ s \end{bmatrix} \text{ as } \varepsilon \rightarrow 0$$

Realizing that the HGO is basically an approximate differentiator, the measurement noise and unmodeled high frequency sensor dynamics will put a practical limit on how small ε could be.

The combination of globally-bounded state feedback control with an HGO allows for a separation approach where the state feedback control is designed to meet the design objectives first. The HGO follows quickly enough to recover the performance achieved under state feedback. Most papers that use an HGO incorporate this separation approach.

The peaking phenomenon can be solve by saturating u or $\hat{x}$ outside a compact set of intrest, we can either saturate the components of the estimate themselves or saturate the control signal by finding the maximum value that the control can take over that compact set and saturate it at a value higher than that. The HGO follows quickly enough to recover the performance achieved under state feedback.

## 2.4. Sliding Mode Observer

Sliding mode observers have unique properties [24], in that the ability to generate a sliding motion on the error between the measured plant output and the output of the observer ensures that a sliding mode observer produces a set of state estimates that are precisely commensurate with the actual output of the plant. It is also the case that analysis of the average value of the applied observer injection signal, the so-called equivalent injection signal, contains useful information about the mismatch between the model used to define the observer and the actual plant. These unique properties, coupled with the fact that the discontinuous injection signals which were perceived as problematic for many control applications have no disadvantages for software-based observer frameworks, have generated a ground swell of interest in sliding mode observer methods in recent years.

Slotine, Hedrick, and Misawa [33, 35] proposed the design of state observers using sliding surfaces. Consider the function f to be:

$$x^{(n)} = f(x,t) \tag{2.29}$$

Where $f(x,t)$ is a nonlinear, uncertain function and $x_1$ is the measurement

The observer used is on the form:

$$\begin{aligned} \dot{\hat{x}}_1 &= -\alpha_1 e_1 + \hat{x}_2 - k_1 sgn(e_1) \\ \dot{\hat{x}}_2 &= -\alpha_2 e_1 + \hat{x}_3 - k_2 sgn(e_1) \\ &\ldots\ldots. \\ \dot{\hat{x}}_n &= -\alpha_n e_1 + \hat{f} - k_n sgn(e_1) \end{aligned} \tag{2.30}$$

$\alpha_i$ Chosen as for a Luenberger observer to ensure asymptotic error decay when $k_i = 0$

$e_1 = \hat{x}_1 - x_1, \hat{f}$ is an estimate of $f(x,t)$

Errors dynamics are:

$$\begin{aligned} \dot{e}_1 &= -\alpha_1 e_1 + e_2 - k_1 sgn(e_1) \\ \dot{e}_2 &= -\alpha_2 e_1 + e_3 - k_1 sgn(e_1) \\ &\ldots\ldots\ldots\ldots.. \\ \dot{e}_n &= -\alpha_n e_1 + \Delta f - k_n sgn(e_1) \end{aligned} \tag{2.31}$$

$\Delta f = \hat{f} - f$ is assumed bounded and $k_n \geq |\Delta f|$

The sliding condition $\frac{d}{dt}(e_1)^2 < 0$ is satisfied in the region

$$e_2 \leq k_1 + \alpha_1 e_1 \ if \ e_1 > 0$$
$$e_2 \geq -k_1 + \alpha_1 e_1 \ if \ e_1 < 0$$

Now revisit the error dynamics imposing the sliding condition.

**Sliding mode dynamics when $e_1 = 0$:**

It follows from $\dot{e}_1$ dynamic equation that

$$e_2 - k_1 sgn(e_1) = 0$$

And therefore :

$$\begin{aligned} \dot{e}_2 &= e_3 - \frac{k_2}{k_1} e_2 \\ &\ldots\ldots\ldots.. \\ \dot{e}_n &= \Delta f - \frac{k_n}{k_1} e_2 \end{aligned} \tag{2.32}$$

By making the first error converges to zero $e_1 \to 0$ using the sliding condition ie. consider $e_1$ as a sliding surface and using the sliding condition $\frac{d}{dt}(e_1)^2 < 0$ it will be asymptotically stable and that causes $e_1 = 0$, Sliding mode observer allow all the other states errors to slide to $e_1$ results in an estimated states exactly match the real ones.

# 2.5. Nonlinear Extended State Observer

The Nonlinear Extended State Observer (NESO), proposed by Professor Jingqing Han [16, 34], can estimate the state without a mathematical model of the system. It is a novel concept for observer design, estimating not only the state, but also the internal and external disturbances, thus making disturbance rejection control possible. The invention of NESO isa revolutionary concept for control theory and application. It has several properties including model-independence, active estimation, compensation for disturbances, simple design, and strong robustness. This method has evolved as an important technique for the state feedback control of nonlinear systems.

Han proposed the following extended state observer (NESO):

$$\dot{\hat{x}}_1 = \hat{x}_2 - \beta_1\, g_1(\hat{x}_1 - y) \qquad (2.33)$$

$$\dot{\hat{x}}_2 = \hat{x}_3 - \beta_2\, g_2(\hat{x}_1 - y)$$

$$\vdots$$

$$\dot{\hat{x}}_n = \hat{x}_{n+1} - \beta_n\, g_n(\hat{x}_1 - y) + b\,u(t)$$

$$\dot{\hat{x}}_{n+1} = -\beta_{n+1}\, g_{1n+1}(\hat{x}_1 - y)$$

With $e = \hat{x}_1 - y$

For an n-dimensional SISO nonlinear system

$$x^{(n)}(t) = f\,(x(t), \dot{x}(t), \ldots\ldots, x(n-1)(t), w(t)) + bu$$

$$y(t) = x(t) \qquad (2.34)$$

This can be written as:

$$\dot{x}_1(t) = x_2(t) \quad ;\ x_1(0) = x_{10}$$

$$\dot{x}_2(t) = x_3(t) \quad ;\ x_2(0) = x_{20} \qquad (2.35)$$

$$\vdots$$

$$\dot{x}_n(t) = f\,(t, x_1, x_2, \ldots\ldots, x_n, w(t)) + b\,u(t) \quad ;\ x_n(0) = x_{n0}$$

$$\dot{x}_{n+1} = a(t)$$

$$y(t) = x_1(t)$$

Where $a(t) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}\dot{x}_i + \frac{\partial f}{\partial w}\dot{w}$

where *f (.)* is an uncertain function, *w( t )* is the unknown external disturbance, *u (t)* is the known control input, and *y (t)* is the measured output**,** $\beta_i$ , $\alpha_i$ and i = 1,2,…,n are constants.

$g_I(.)$ is defined as a modified exponential gain function :

$$g_i(e, \alpha_i, \delta) = \begin{cases} |e|^{\alpha} sign(e) & ; \ |e| > \delta \\ \frac{e}{\delta^{1-\alpha}} & ; \ |e| < \delta \end{cases} \quad (2.36)$$

The nonlinear function in (2.36) which is used to make the observer more efficient, was selected heuristically based on experimental results. Intuitively, it is a nonlinear gain function where small errors correspond to higher gains, and large errors correspond to smaller gains. When the error is small, it prevents excessive gain, which causes high frequency chattering in some simulation studies.

Figure 2.5 illustrates the difference between the linear and nonlinear gain.

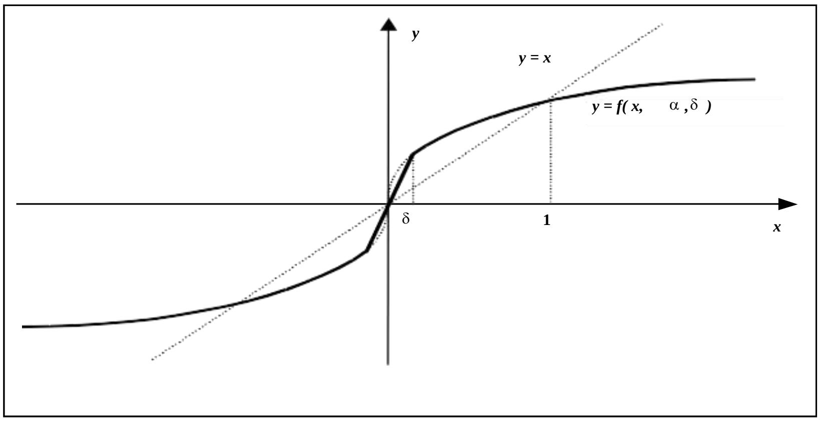



**Figure 2.5.**Comparison of Linear and Nonlinear Gains

If $\alpha_i$ are chosen as unity, then the observer is equal to the well-known Luenberger observer.

The main idea of the extended state observer is that for the appropriately chosen functions $g_i$ , the state of the observer $\dot{\hat{x}}_i$ , i = 1,2,…,n and $\dot{\hat{x}}_{n+1}$ can be, through regulating αi, considered as the approximations of the corresponding state xi for i =1, 2, . . . , n, and the total disturbance f , respectively.

The NESO does not include a model, yet can reconstruct states reliably for nonlinear plants. Because it does not use a model, it is simpler and easier to construct, easier to implement due to its efficiency in many cases, and freer from model uncertainties such as parameter variations and external disturbances. Therefore, in this case, it is robust.

If part of f (t, $x_1$, $x_2$, w) says $f_1$ (t, $x_1$,$x_2$, w) is known, and then it should be incorporated into the observer as:

$$\dot{\hat{x}}_1 = \hat{x}_2 - \beta_1\, g_1(\hat{x}_1 - y, \beta_1, \delta_1) \tag{2.37}$$
$$\dot{\hat{x}}_2 = \hat{x}_3 - \beta_2\, g_2(\hat{x}_1 - y, \beta_2, \delta_2)$$
$$\vdots$$
$$\dot{\hat{x}}_n = \hat{x}_{n+1} - \beta_n\, g_n(\hat{x}_1 - y, \beta_n, \delta_n) + b\,u(t)$$
$$\dot{\hat{x}}_{n+1} = h_1(t, x_1, x_2, w) - \beta_{n+1}\, g_{1n+1}(\hat{x}_1 - y, \beta_{n+1}, \delta_{n+1})$$

Where $h_1(t,x_1,x_2,w) = \dot{f}(t, x_1,x_2,w)$ . This will make the observer more efficient.

For example if we have:

$$\ddot{x} = f(x(t), \dot{x}(t), w) + b_0 u$$

Which is equivalent to

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = x_3(t) + b_0 u \tag{2.38}$$
$$\dot{x}_3(t) = h$$

So f $(x(t), \dot{x}(t), w)$ is treated as an extended state, **x,** . Here both f $(x(t), \dot{x}(t), w)$and $h(t,x_1,x_2,w) = \dot{f}(t, x_1,x_2,w)$ are unknown however, it is **now** possible *to* estimate f $(x(t), \dot{x}(t), w)$ by using a state estimator.

$$\dot{\hat{x}}_1 = \hat{x}_2 - \beta_1\, g_1(e) \tag{2.39}$$
$$\dot{\hat{x}}_2 = \hat{x}_3 - \beta_2\, g_2(e) + b_0 u$$
$$\dot{\hat{x}}_3 = -\beta_3\, g_3(e)$$

The NESO is not only a state observer, but also a disturbance observer. Since Han's observer uses nonlinear functions, it was named Nonlinear Extended State Observer (NESO) and can be applied for MIMO systems also.

## 2.5.1. Selection of Nonlinear Gains for NESO

Research revealed that, for the plant with unknown initial conditions, a new nonlinear function $f_i$ *(.)*, as shown in equation (2.40), could be used in NESO to avoid significant transient estimation error:

$$f_i(e, K_{1i}, K_{2i}) = \begin{cases} K_{2i}e + sign(e) * (K_{1i} - K_{2i}) * \delta & ; \ |e| > \delta \\ K_{1i} * e & ; \ |e| \leq \delta \end{cases} \tag{2.40}$$

With $K_{1i}$, $K_{2i} > 0$. Furthermore, by choosing $\alpha_i < 0$ in (2.36), the transient error was significantly reduced. Three curves from (2.36) and (2.40) are shown in Figure2.6 to illustrate the differences. As in (2.36), $\delta$ defines the range of a high gain section where the observer is very aggressive. This range is usually small.

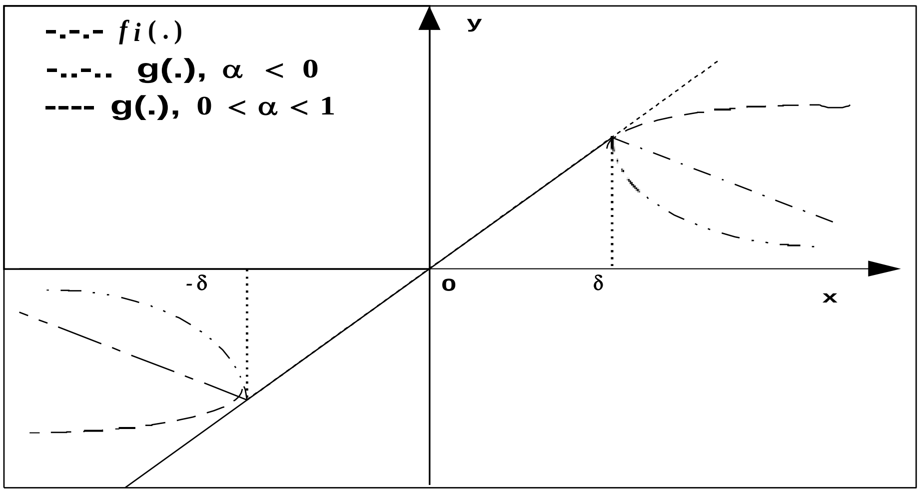



**Figure 2.6.**Nonlinear Gain Functions

# 2.6. Conclusion

Observer structures usually need to include a plant model in their equations which inevitably accompanies some practical burdens. Without a model, observers cannot be constructed: even if it is available, unless it is accurate enough, a reliable state reconstruction could not be expected. Even when a model is accurate enough, the observer could often become too complicated (because of model complexity) to have any practical use, especially on a real-time basis. The advantages and benefits of an accurate and efficient model cannot be emphasized too much, when it is available, unless such a model is difficult to obtain. HGOs, SMOs, and NESOs are three kinds of observers designed for the plant in the presence of disturbances, dynamic uncertainties, and nonlinearities in practical applications.

# CHAPTER III

# COMPARISON OF ADVANCED STATE OBSERVER DESIGN TECHNIQUES

This chapter presents a comparison study of the characteristics and performances of the HGO, SMO, and NESO. These observers were originally proposed to address the dependence of the classical observers, such as the Kalman Filter and the Luenberger observer, on the accurate mathematical representation of the plant. The simulations conducted give insight into observer behavior in servo system in three cases, nominal system, system with disturbance and system with uncertainty, it results are provided to give realism.

# 3.1. The Servo Motor Plant Model

A typical servo motor plant is made up of a motor, a servo drive amplifier, a system of gears, and a load. This plant is used for software in the loop simulations.

A current drive is typically used in high performance servo motor systems to reduce the order of the mathematic model of the servomotor by eliminating the effect of the inductance. In motion control literature, **a servo motor can be approximated as a linear**, **time-invariant** system.

The expression describes the model of a servo fed by a current amplifier is on the following form:

$$J\ddot{y} + c\dot{y} + f\,\mathrm{sign}(\dot{y}) = \tau + \tau_c \tag{3.1}$$

Where y, $\dot{y}$, and $\ddot{y}$ are the angular position, velocity, and acceleration, respectively. J is the sum, in that order, of servomotor $J_m$ and load inertia $J_l$, ***c*** and f are, respectively, the viscous and Coulomb friction coefficients; $\tau_C$ is a constant disturbance; and τ is the driving torque. The term τ is equal to Ku, where u is the input voltage of the servo amplifier, and K = KT KE/Kc, KT is the servomotor torque constant, KE is the amplifier gain, and Kc is the gain in the amplifier current loop. The gain K is assumed known.

The parameters associated with the Coulomb and with the constant disturbance are not available. Defining the following relationship:

$$\ddot{y} = \frac{c}{J}\dot{y} + \frac{b}{J}u \tag{3.2}$$

Where *b* is the total hardware gain which typically includes the gear ratio

The transfer function is:

$$G(s) = \frac{b}{s(Js+c)} \tag{3.3}$$

The criteria for comparison is based on the robustness of the performance with respect to the uncertainties of plant and the observer tracking errors, both at steady state and during transients. Simulations are conducted to give insight into observer behavior.

The linear model for this study purpose is derived as:

$$\ddot{y} = -1.41\,\dot{y} + 23.2\,u \qquad (3.4)$$

Where $y$ is the output position and $u$ is the control voltage sent to the power amplifier that drives the motor. The servo motor system is already stable [11].

Initially, no friction, disturbance or backlash is intentionally added.

## 3.2. Simulated Results:

The quality of observers is measured by the speed and accuracy of the states of the observer converging to those of the plant. To make the comparison fair, the parameters of the observers are adjusted so that their sensitivities to measurement noise are roughly the same. The exact outputs of $\boldsymbol{y}$ and $\dot{\boldsymbol{y}}$ are **obtained directly from the simulation model of the plant to calculate the state estimation error.**

For the tests, the input to the plant is a sine wave function, and the observers are evaluated according to their capability in tracking the input response. The tests were run in three conditions:

**I.** Nominal plant;

**II.** Nominal plant plus **stribeck friction** as **disturbance.**

A stribeck effect is a kind of friction which occurs when a liquid or solid oils is used for the contact surfaces of moving mechanical parts. At low velocity the friction will decrease with the increase velocity. Stribeck friction is usually expressed by following equation [15]:

$$\mathbf{F(x_2) = F_C + (F_S - F_c)\, e^{-\beta\, x2}} \qquad (3.10)$$

Where $F_c$, $F_s$, $1/\beta$ are Coulomb friction, static friction and stribeck velocity, respectively.

The stribeck friction is in the Figure 3.1 shown.

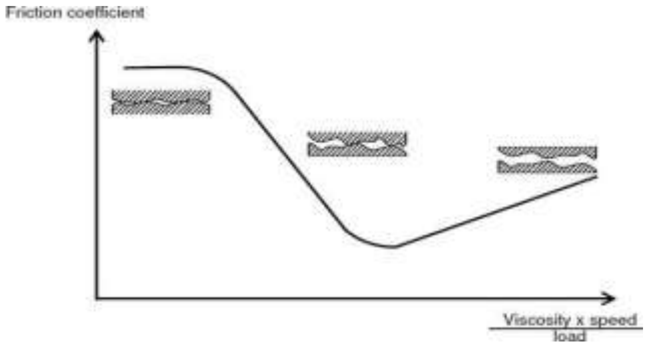



**Figure 3.1.**Stribeck friction

For our simulation the same disturbance **F($x_2$)** is used, where **$F_C$ = 0.1, $F_S$= 0.5 and β=0.07.**
**There for:**

$$\text{Dis}= \mathbf{F(x_2) = 0.1+ 0.4}\, e^{-0.07\, x2} \qquad (3.11)$$

And the function is bounded **by $0 \le e^{-\beta\, x2} \le 1.5$**

**III.** Nominal plant with **100% increase in inertia**

The system has uncertainty in load moment of inertia such that its value will increase 100% over the nominal value.

Profile generator has been used for the system input as a sine wave with low-frequency which provides the desired state trajectory in both y and $\dot{y}$.
The input u(t) = 0.05sin(t) is used in all simulations.
The same set of observer parameters are used in all simulations.

## 3.2.1.Observers Designs:

Define the states $x_1$=$y$ (position), $x_2$=$\dot{y}$ (velocity) and the system input $u$ = $v$ (applied voltage). The state space model of the servo motor system can be expressed as:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = 23.2\, u - 1.41\, x_2 \qquad (3.5)$$

**The HGO for the model (3.5) is:**

$$\dot{\hat{x}}_1 = \hat{x}_2 + h_1 (y-\hat{x}_1) \tag{3.6}$$
$$\dot{\hat{x}}_2 = 23.2\, u - 1.41\, \hat{x}_2 + h_2 (y-\hat{x}_1)$$

The observer gains are adjusted as:

$$h_1 = \frac{\gamma_1}{\varepsilon} \qquad h_2 = \frac{\gamma_2}{\varepsilon^2} \tag{3.7}$$

Where $\mathbf{0 < \varepsilon << 1}$

**The SMO is designed as:**

$$\dot{\hat{x}}_1 = -\gamma_1 e_1 + \hat{x}_2 - k_1 sgn(e_1) \tag{3.8}$$
$$\dot{\hat{x}}_2 = -\gamma_2 e_1 + 23.2\, u - 1.41\, \hat{x}_2 - k_2 sgn(e_1)$$

**The extended state-space model for (3.4) is:**

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3 + 23.2\, u$$
$$\dot{x}_3 = 0$$
$$y = x_1$$

Where $-1.41x_2$ is treated as $f(t,x_1,x_2,w)$, as well as an extended state, $x_3$.

**The NESO is given as:**

$$\dot{\hat{x}}_1 = \hat{x}_2 - \beta_1\, g_1(e) \tag{3.9}$$
$$\dot{\hat{x}}_2 = \hat{x}_3 - \beta_2\, g_2(e) + 23.2\, u$$
$$\dot{\hat{x}}_3 = -\beta_3\, g_3(e)$$

Where $e = y - \hat{x}_1$

$g_i(.)$ for i= 1,2,3 are the same as (2.30)

## 3.2.2. High gain observer (HGO):

Pole placement is used to determine the position of the poles at –4.2, $\{\gamma_1, \gamma_2\}$ in HGO.

Initial conditions of the non-linear Servo model were different from the ones for the non-linear observer as they were set for non-linear model to $x_{10} = 0.05$ for the position, the velocity was set to $x_{20} = 0$ and the initial conditions for the observer are **[0, 0].**

**Table 3.1.**Parameters of high gain observer

| Parameter | Symbol | Value |
|---|---|---|
| positive constant_1 | $\gamma_1$ | 1.4 |
| positive constant_2 | $\gamma_2$ | 0.311 |
| positive constant | $\varepsilon$ | 0.2 |
| First observer gain | $h_1$ | 70 |
| Second observer gain | $h_2$ | 777.5 |

**Simulation Model with Matlab Simulink:**

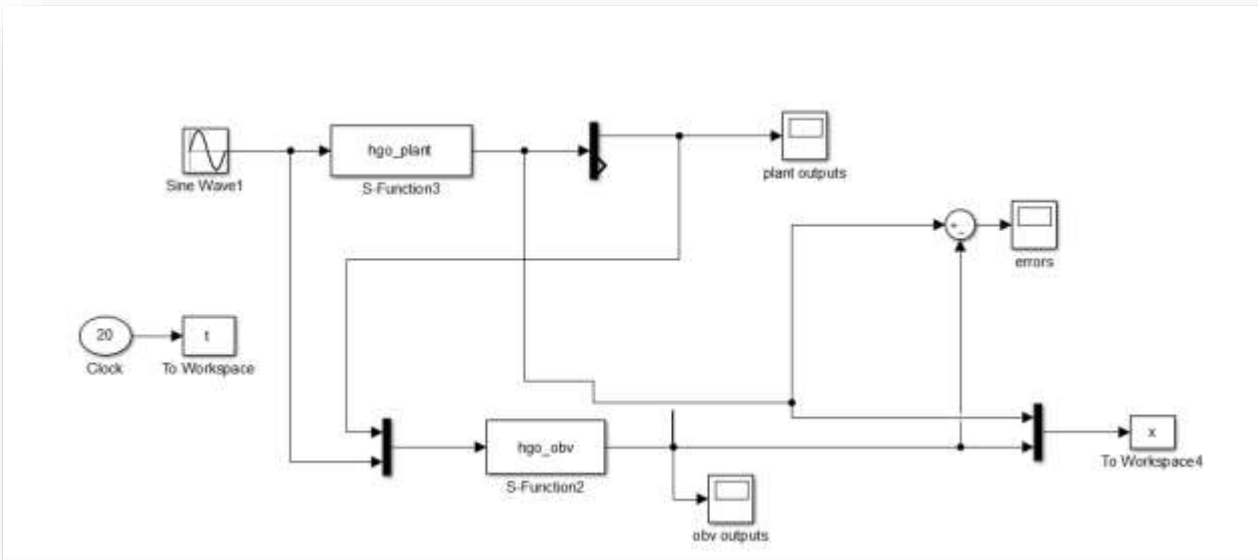
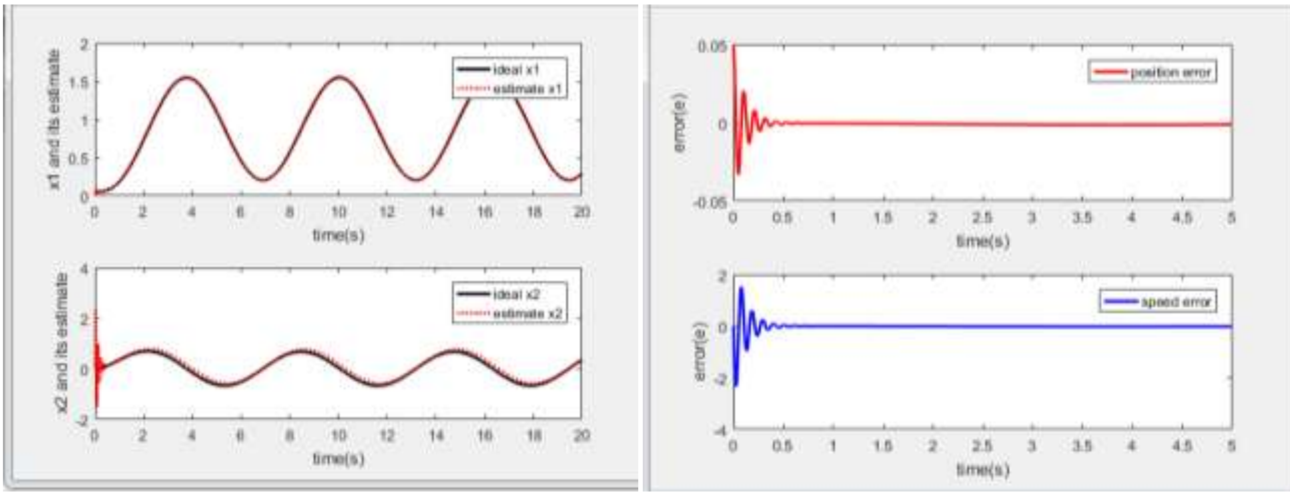
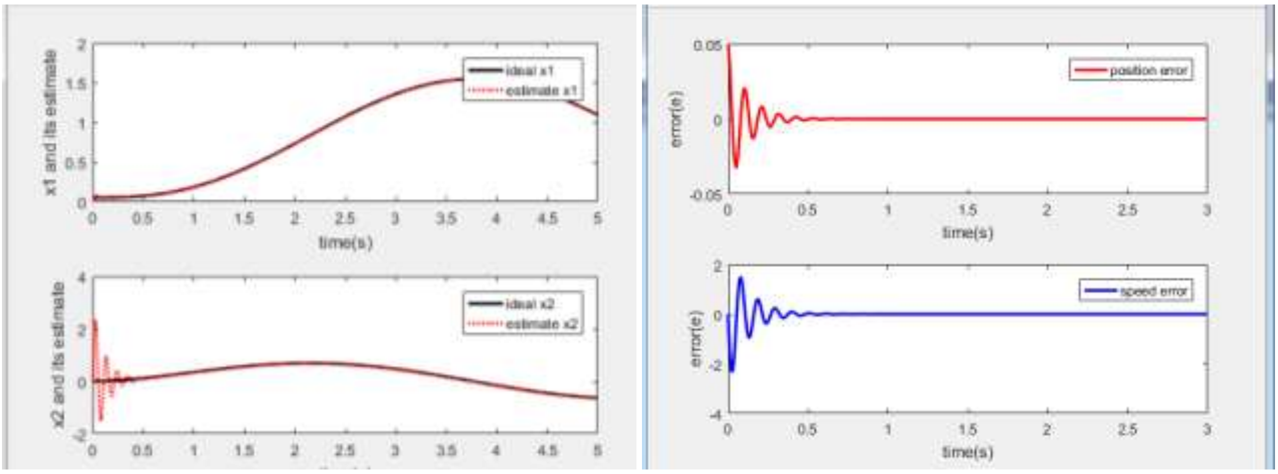



**Figure 3.2.**HGO Matlab Simulink Model

### 3.2.2.1. NOMINAL SYSTEM:




(a) Actual and estimate states of the Nominal plant (b) Estimation Error of the Nominal Plant



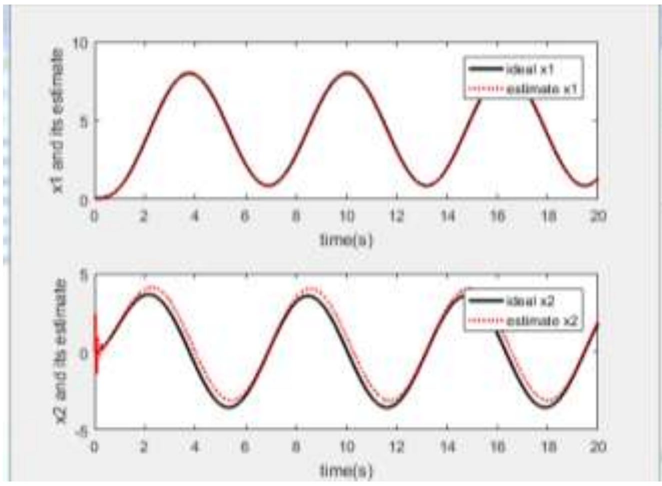

Zoom in on (a) Zoom in on (b)

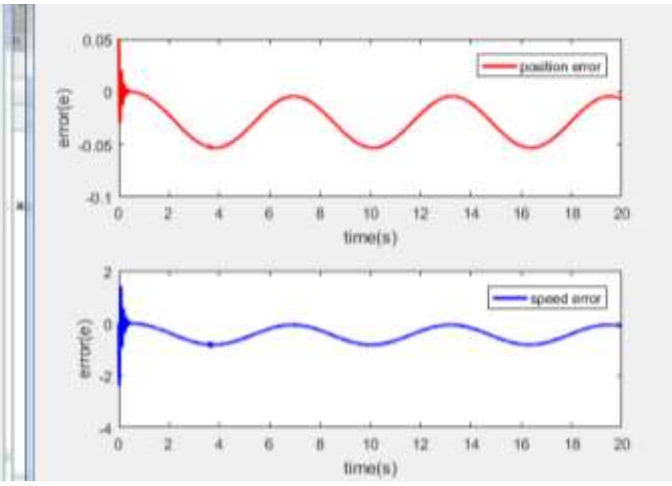
**Figure 3.3.**HGO Matlab simulation of Nominal system

Figure 3.3 gives the actual and estimate responses of the angular position ($x_1$, $\hat{x}_1$) and angular speed ($x_2$, $\hat{x}_2$) for the servo nominal plant. Also, the error for angular position and its estimate ($e_1$) and angular velocity and its estimate ($e_2$) are also shown in the figure.

From figure 3.3 (b) the HGO has an estimation position residuals of **$e_1$= -0.025** to **0.05**
And **$e_2$≈ -02** to **1.6** for the speed estimation error
The time of transient response for the HGO in the nominal case is **0.5s** for both position and speed.

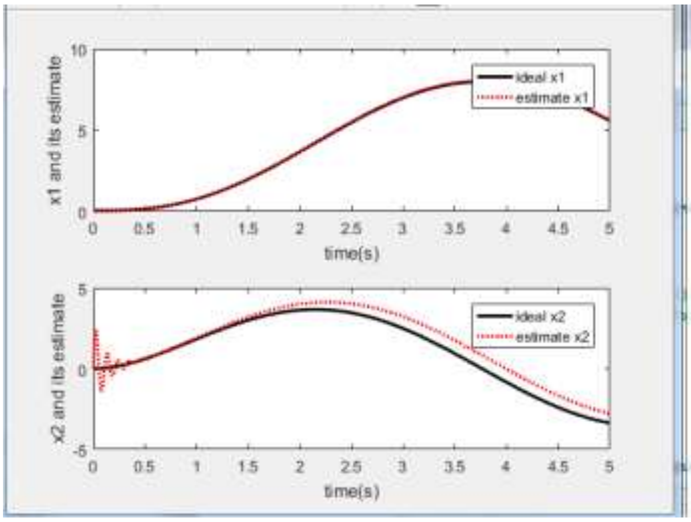
### 3.2.2.2. SERVO MOTOR SYSTEM UNDER DISTURBANCE :

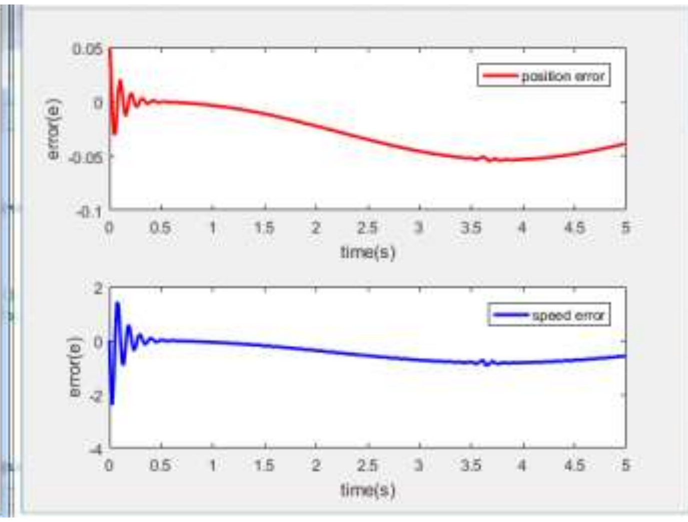



(a) Actual and estimate states of the Nominal plant With disturbance

(b) Estimation Error of the Nominal Plant With disturbance




Zoom in on (a)

Zoom in on (b)

**Figure 3.4.**HGO Matlab simulation of Nominal system with DISTURBANCE

In figure 3.4 the same set of behaviors shown in previous scenarios are repeated with the system is subjected to disturbance and the performance of the observer will be assessed accordingly. The applied disturbance is a stribeck friction in the form:

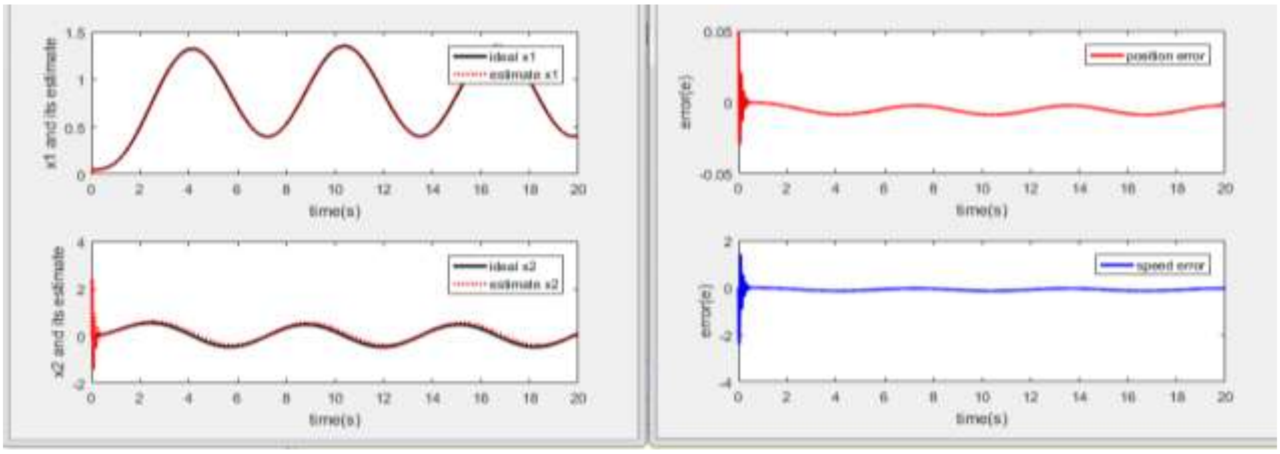
$\mathbf{F(x_2) = 0.1 + 0.4}\, e^{-0.07\, x2}$.

The time of transient response for the HGO in the nominal case with disturbance **is $T_{TR}$= 0.5s**.

The same states and estimation errors as before are shown in Figure 3.4 (b). It has seen that the HGO has a residuals of **$e_1$= -0.025** to **0.05** for the position estimation. After **$T_{TR}$** the error will be bounded by **$-0.05 \leq e_1 < 0$.**

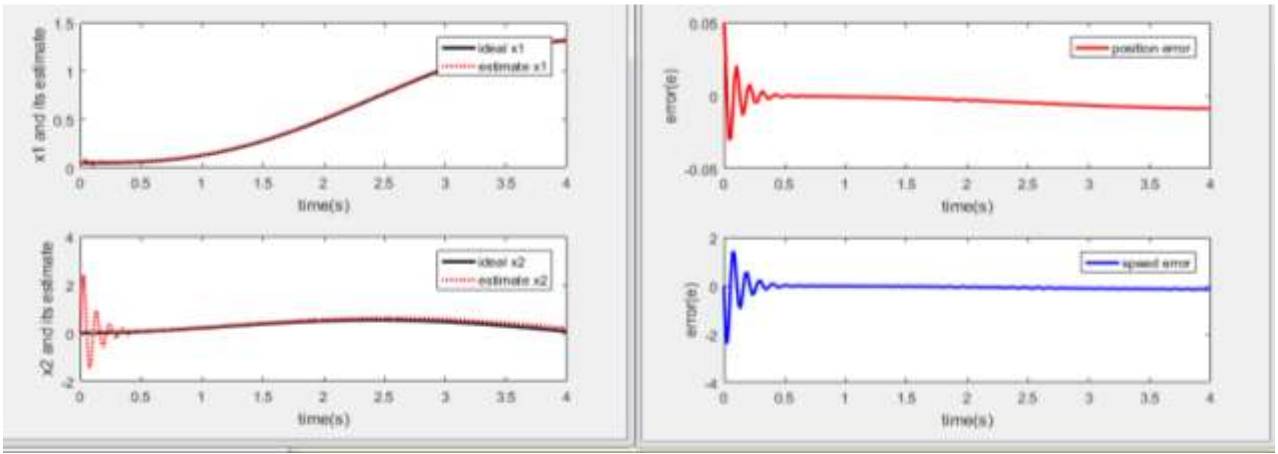
And **$e_2$= -02 to 1.8** for the speed estimation. After **$T_{TR}$** the error will be bounded by **$-1 \leq e_2 < 0$.**

### 3.2.2.3. SERVO MOTOR SYSTEM WITH UNCERTAINTY:




(a) Actual and estimate states of the Nominal plant With uncertainty

(b) Estimation Error of the Nominal Plant With uncertainty




Zoom in on (a)

Zoom in on (b)

**Figure 3.5.**HGO Matlab simulation of Nominal system with UNCERTAINTY

It is assumed that the system has uncertainty in load moment of inertia such that its value will increase 100% over the nominal value. Figure 3.5 shows the responses of actual and estimated states and also the estimation error for both angular position and speed for the servo system. It is evident from the figure that, the HGO has an estimation position residuals of $e_1$= **-0.025** to **0.05.** And $e_2$= **-02** to **1.8** for the speed estimation error.

The time of transient response for the HGO in the nominal case with uncertainty is **$T_{TR}$= 0.5s**.

## 3.2.2. Sliding Mode Observer (SMO):

Pole placement is used to determine the position of the poles at –4.2, **{** $\boldsymbol{\gamma_1, \gamma_2}$ **}** in SMO).

Initial conditions of the model were **[0.05, 0]** whereas the initial conditions for the observer were **[0, 0].**

**Table 3.2**.Parameters of sliding mode observer

| Parameter | Symbol | Value |
|---|---|---|
| positive constant_1 | $\boldsymbol{\gamma_1}$ | 1.4 |
| positive constant_2 | $\boldsymbol{\gamma_2}$ | 0.311 |
| First observer gain | $k_1$ | 0.5 |
| Second observer gain | $k_2$ | 15 |

**Simulation Model with Matlab Simulink:**

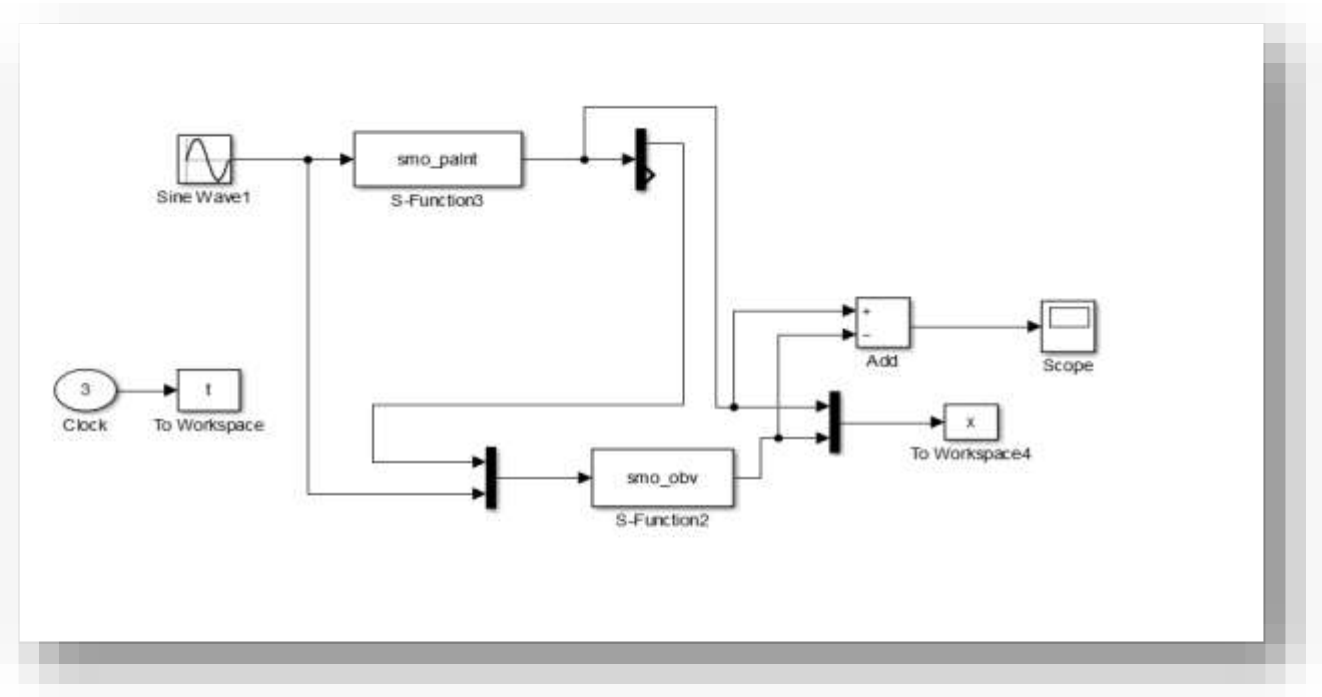
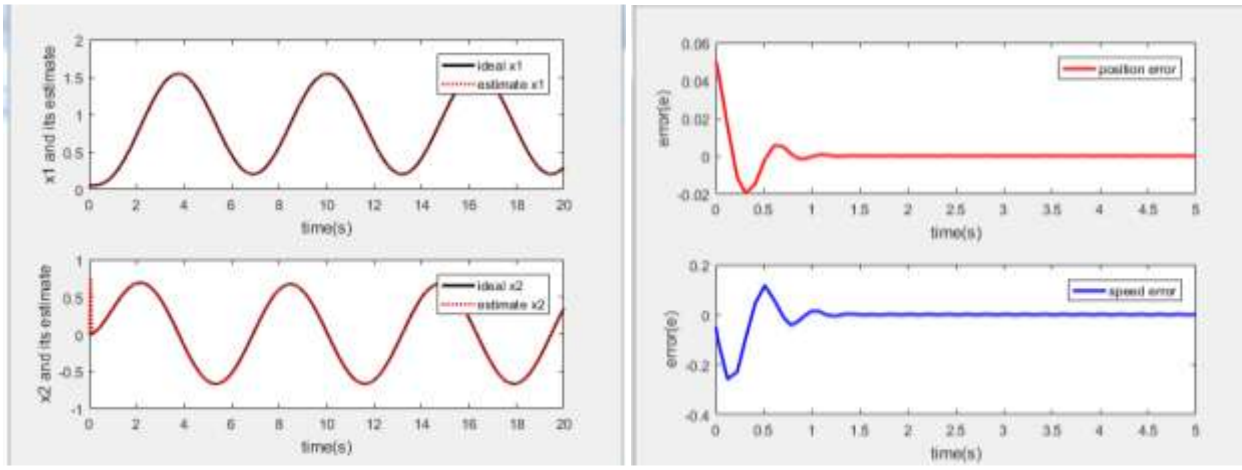
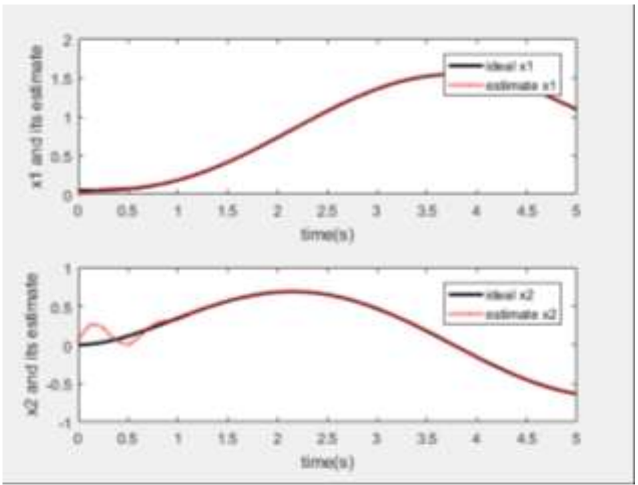



**Figure 3.6.**SMO Matlab Simulink Model

### 3.2.3.1. NOMINAL SYSTEM:




(a) Actual and estimate states of the Nominal plant (b) Estimation Error of the Nominal Plant



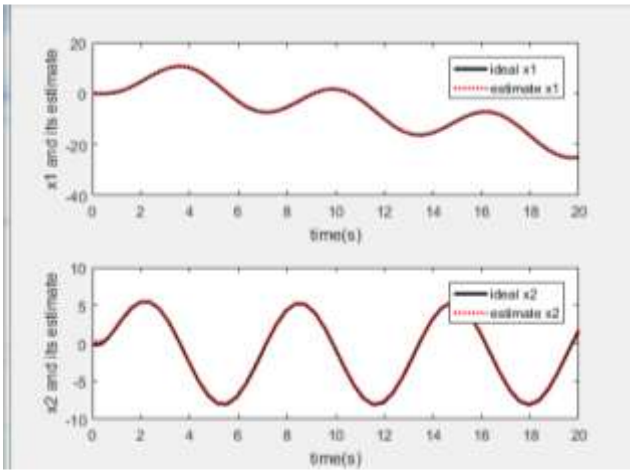

Zoom in on (a)

**Figure 3.7.**SMO Matlab simulation of Nominal system

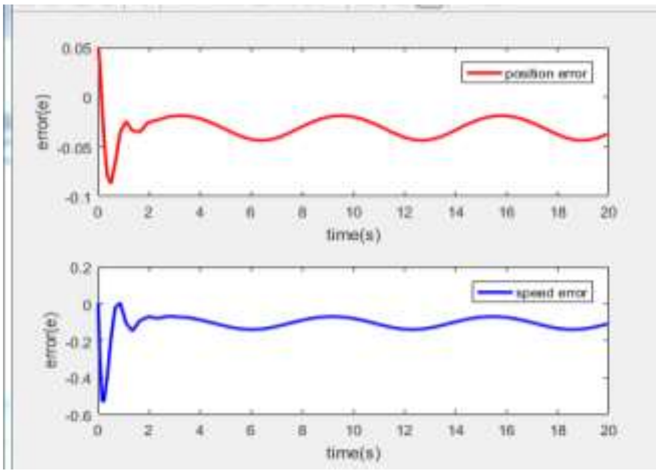
Figure 3.7 gives the actual and estimate responses of the angular position ($x_1$, $\hat{x}_1$) and angular speed ($x_2$, $\hat{x}_2$) for the servo nominal plant. Also, the error for angular position and its estimate ($e_1$) and angular velocity and its estimate ($e_2$) are also shown in the figure.

From figure 3.7 (b) the SMO has an estimation position residuals of **$e_1$≈ -0.02** to **0.05**
And **$e_2$≈ -0.25** to **0.1** for the speed estimation error
The time of transient response for the SMO in the nominal case is **$T_{TR}$= 0.8s** for position and **0.6s** for the speed.

### 3.2.3.2. SERVO MOTOR SYSTEM UNDER DISTURBANCE:




(a) Actual and estimate states of the Nominal plant With disturbance




(b) Estimation Error of the Nominal Plant With disturbance

**Figure 3.8.**SMO Matlab simulation of Nominal system with DISTURBANCE

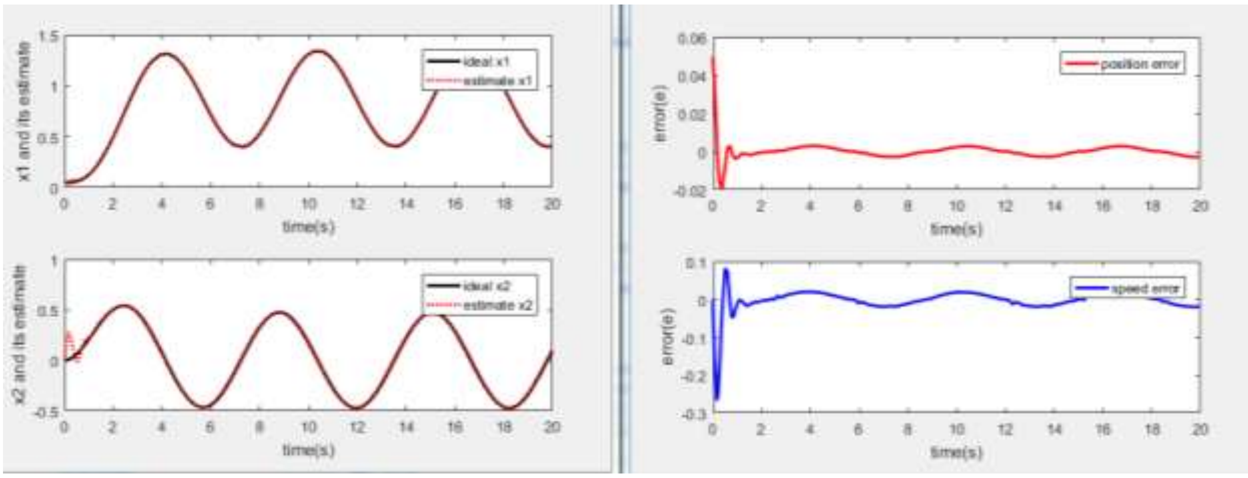
In figure 3.8 the same set of behaviors shown in previous scenarios are repeated with the system is subjected to disturbance and the performance of the observer will be assessed accordingly. The applied disturbance is a stribeck friction in the form:

**$F(x_2) = 0.1 + 0.4\, e^{-0.07\, x2}$.**

The same states and estimation errors as before are shown in Figure 3.8 (b). It has seen that the SMO has a residuals of **$e_1$= -0.075** to **0.05** for the position estimation. After **$T_{TR}$** the error will be bounded by **$-0.05 \leq e_1 < 0$** and **$e_2$= -0.5 to 00** for the speed estimation. After **$T_{TR}$** the error will be bounded by **$-0.2 \leq e_2 < 0$.**

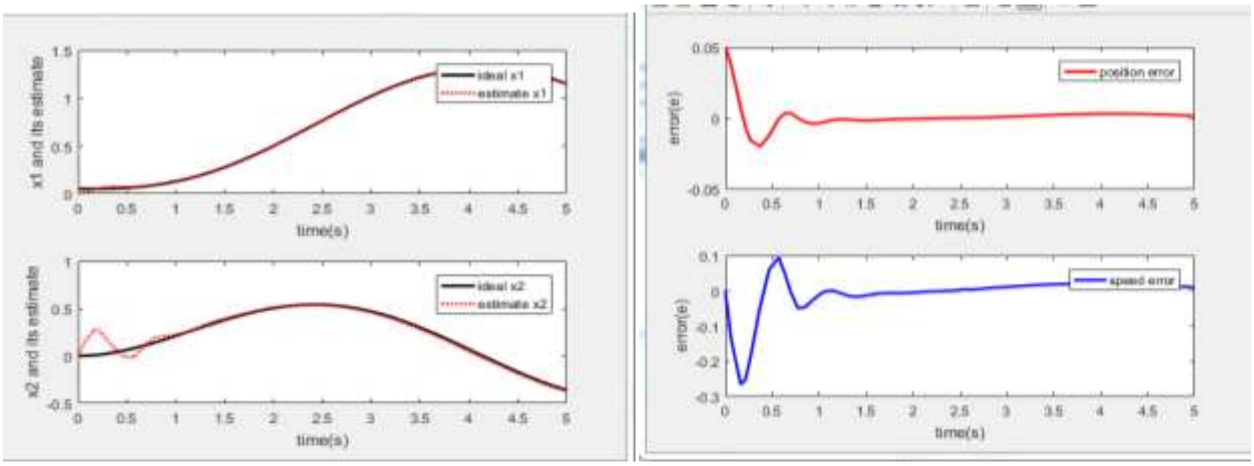
The time of transient response for the SMO in the nominal case with disturbance **is $T_{TR}$ = 02s**.

### 3.2.3.3. SERVO MOTOR SYSTEM WITH UNCERTAINTY:




(a) Actual and estimate states of the Nominal plant With uncertainty

(b) Estimation Error of the Nominal Plant With uncertainty




Zoom in on (a)

Zoom in on (b)

**Figure 3.9.**SMO Matlab simulation of Nominal system with UNCERTAINTY

It is assumed that the system has uncertainty in load moment of inertia such that its value will increase 100% over the nominal value. Figure 3.9 shows the responses of actual and estimated states and also the estimation error for both angular position and speed for the servo system. It is evident from the figure that, the SMO has an estimation position residuals of $e_1$= **-0.02** to **0.05.** And $e_2$= **-0.25** to **0.1** for the speed estimation error.

The time of transient response for the SMO in the nominal case with uncertainty is $\mathbf{T_{TR} \approx 1.5s}$.

## 3.2.4. Nonlinear Extended State Observer (NESO):

Pole placement is used to determine the position of the poles at –4.2, $\beta i$ (i= 1, 2, 3) for NESO.

Initial conditions of the model were **[0.05, -2,0]** whereas the initial conditions for the observer were **[0, 0,0].**

**Table 3.3.**Parameters of nonlinear extended state observer

| **Parameter** | **Symbol** | **Value** |
|---|---|---|
| First observer gain | $\beta_1$ | 12.6 |
| Second observer gain | $\beta_2$ | 52.92 |
| Third observer gain | $\beta_3$ | 74.088 |
| Regulable constant_1 | $\alpha_1$ | 01 |
| Regulable constant_2 | $\alpha_2$ | 0.5 |
| Regulable constant_3 | $\alpha_3$ | 0.25 |
| Gain function constant | $\delta$ | $10^{-3}$ |

**Simulation Model with Matlab Simulink:**

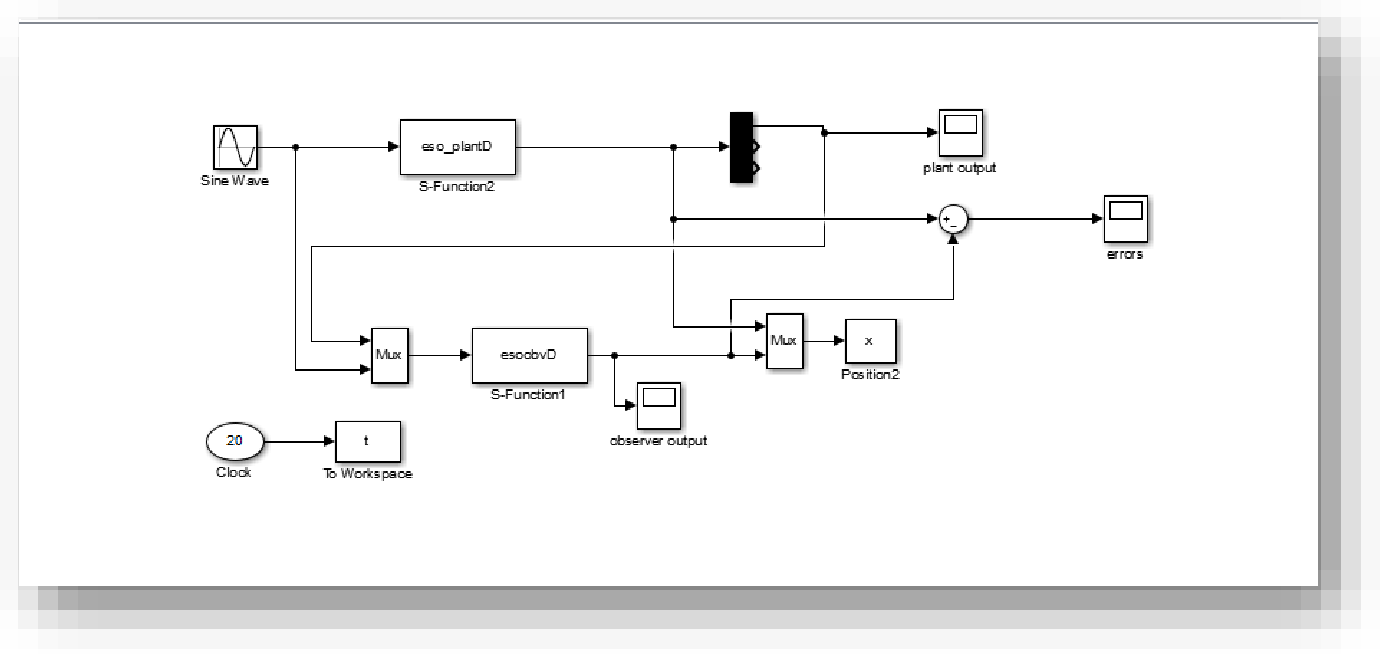
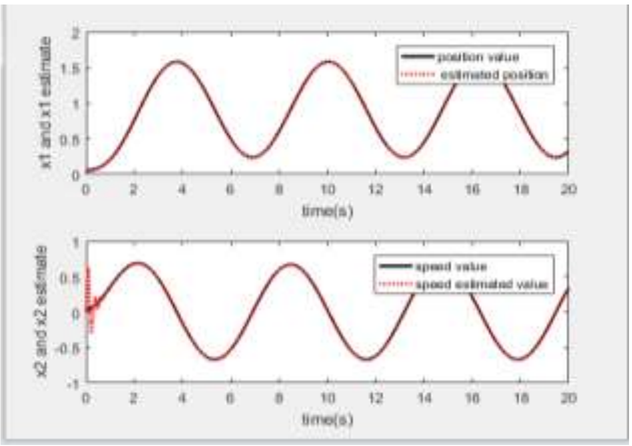
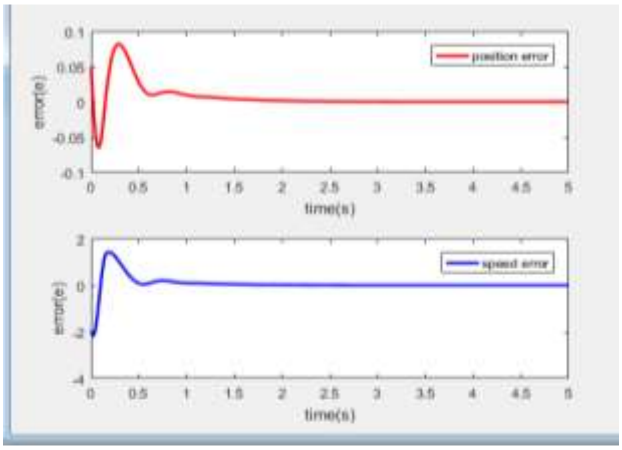


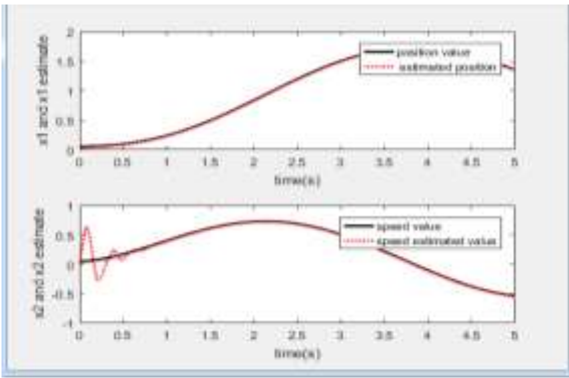

**Figure3.10.** NESO Matlab Simulink Model

## 3.2.4.1. NOMINAL SYSTEM:

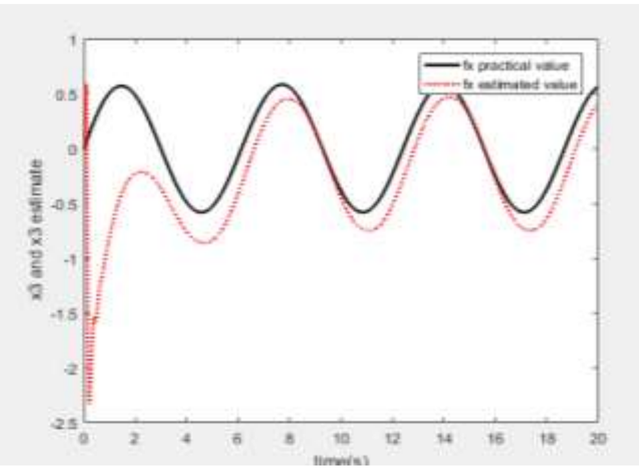


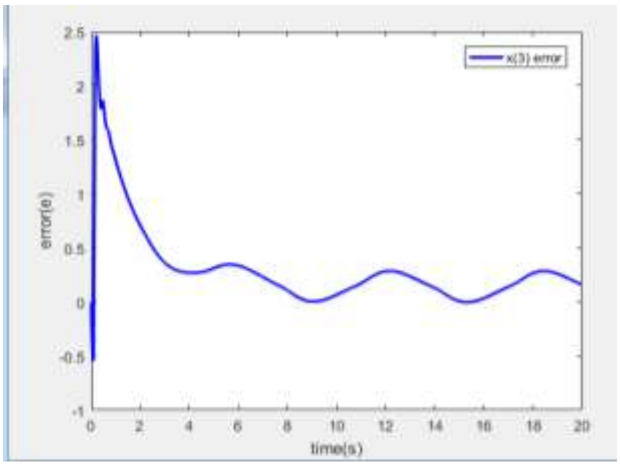




(a) Actual and estimate states of the Nominal plant (b) Estimation Error of the Nominal Plan




Zoom in on (a)




(c) Actual and estimate states of f( x) (d) Estimation Error of f( x) and x(3).

**Figure 3.11.NESO** Matlab simulation of Nominal system

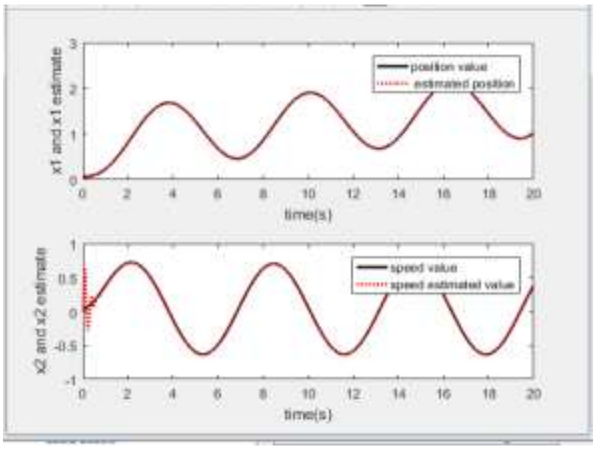
Figure 3.11 gives the actual and estimate responses of the angular position ($x_1$, $\hat{x}_1$) and angular speed ($x_2$, $\hat{x}_2$) for the servo nominal plant. Also, the error for angular position and its estimate ($e_1$) and angular velocity and its estimate ($e_2$) are also shown in the figure.

From figure3.11 (b) the NESO has an estimation position residuals of **$e_1$= -0.06** to **0.08** And **$e_2$≈ -02** to **1.5** for the speed estimation error.

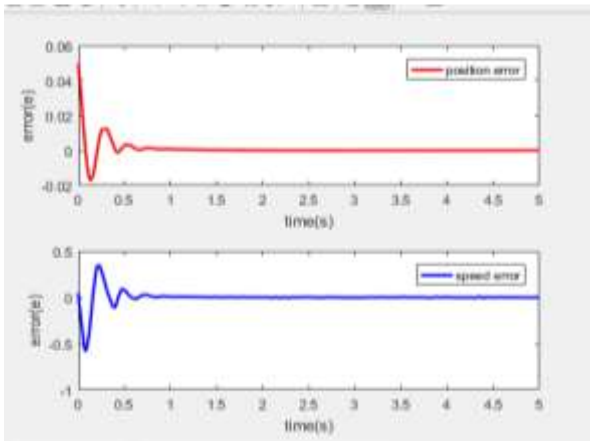
The time of transient response for the NESO in the nominal case is **01s** for both position and speed

For the disturbance NESO has an estimation residuals of **$e_3$ = -0.5** to **2.5, with $T_{TR}$= 5s**. After **$T_{TR}$** the error will be bounded by **$0 \leq e_3 \leq 0.25$.**

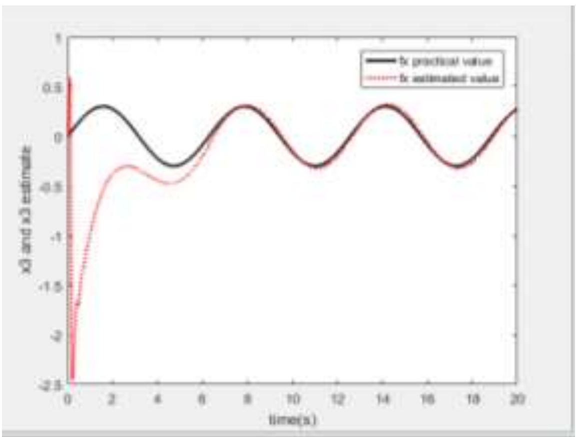
### 3.2.4.2. SERVO SYSTEM UNDER DISTURBANCE:



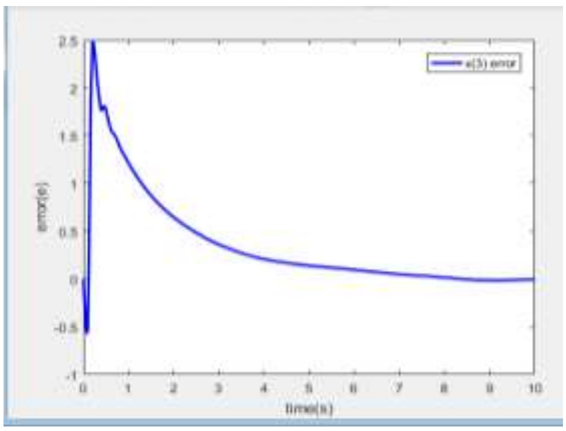

(a) Actual and estimate states of the Nominal Plant with disturbance




(b) Estimation Error of the Nominal Plant with disturbance




(c) Actual and estimate states of f(x)




(d) Estimation Error of f(x) and x(3).

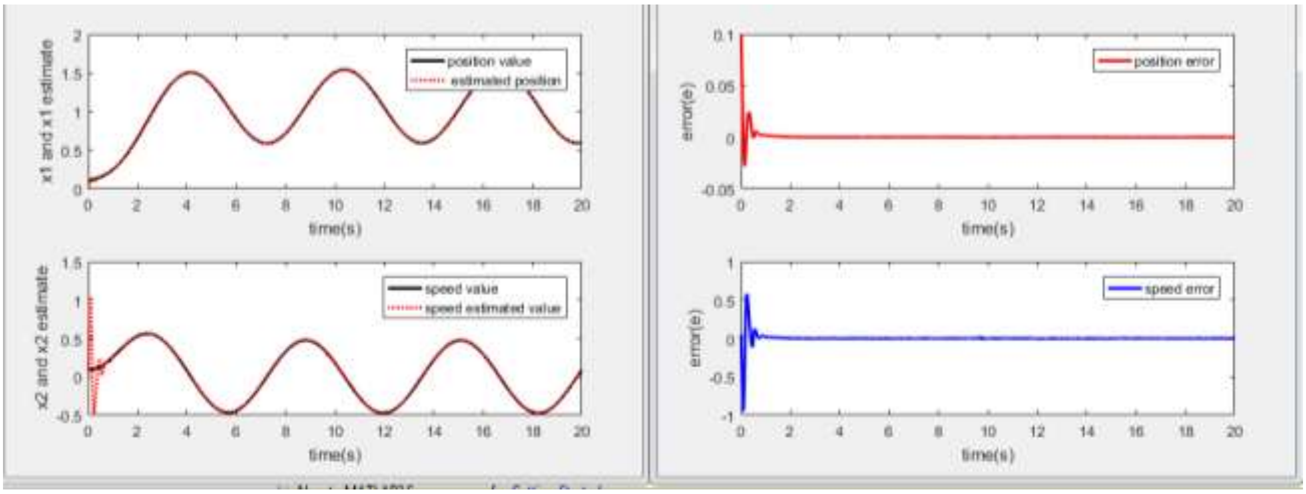
**Figure 3.12.**NESO Matlab simulation of Nominal system with DISTURBANCE

In figure 3.12 the same set of behaviors shown in previous scenarios are repeated with the system is subjected to disturbance and the performance of the observer will be assessed accordingly. The applied disturbance is a stribeck friction in the form: **F(x2) = 0.1+ 0.4** $e^{-0.07\,x2}$.

The same states and estimation errors as before are shown in Figure 3.12 (b). It has seen that the NESO has residuals of **$e_1$= -0.018** to **0.05** for the position estimation and **$e_2$= -0.5 to 0.3** for the speed estimation. For the disturbance NESO has an estimation residuals of **$e_3$ = -0.5** to **2.5.**

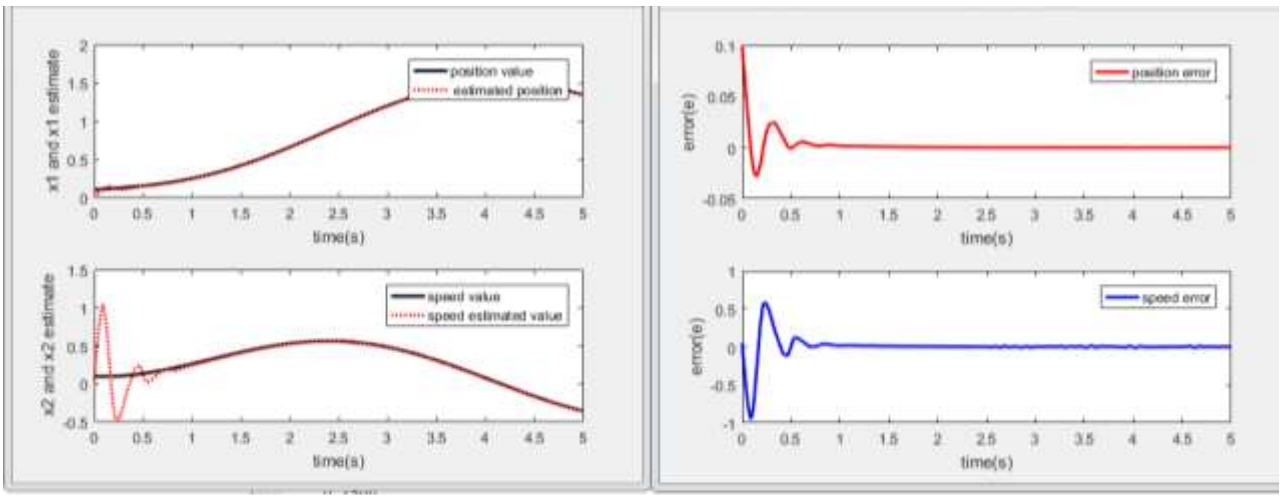
The time of transient response for the NESO in the nominal case is **0.75s** for both position and speed. For the disturbance **$T_{TR}$ = 5s**.

### 3.2.4.3. SERVO MOTOR SYSTEM WITH UNCERTAINTY:



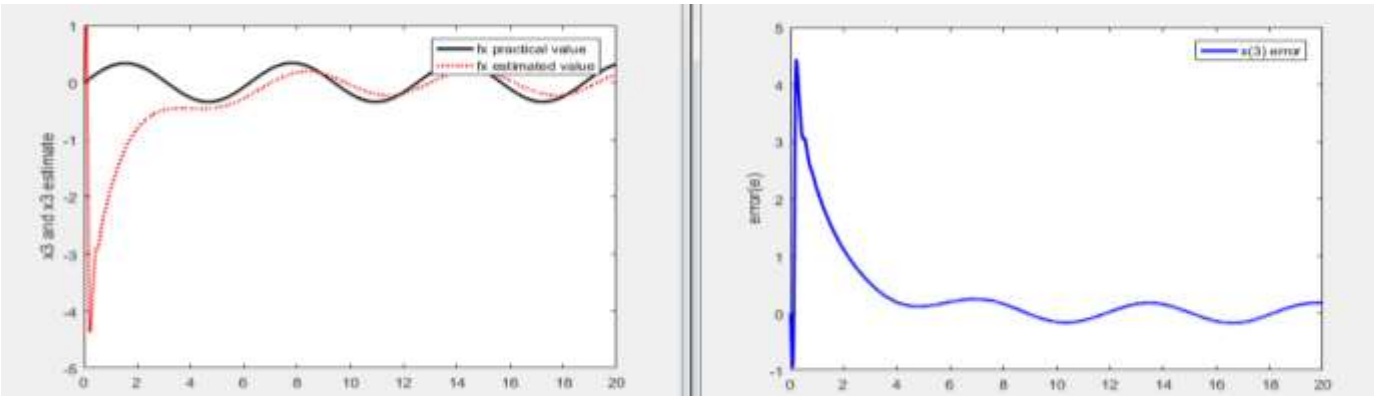

(a) Actual and estimate states of the Nominal plant With uncertainty

(b) Estimation Error of the Nominal Plant With uncertainty




Zoom in on (a)

Zoom in on (b)




(c) Actual and estimate states of f x) (d) Estimation Error of f(x) and x(3)

**Figure 3.13.**NESO Matlab simulation of Nominal system with UNCERTAINTY

It is assumed that the system has uncertainty in load moment of inertia such that its value will increase 100% over the nominal value. Figure 3.13 shows the responses of actual and estimated states and also the estimation error for both angular position and speed for the servo system. It is evident from the figure that, the NESO has an estimation position residuals of $e_1$= **-0.025** to **0.1**And $e_2$= **-01** to **0.5** for the speed estimation error. For the disturbance NESO has an estimation residuals of $\mathbf{e_3}$ **= -01** to **4.4** and $\mathbf{T_{TR}}$ **= 4s**. After $\mathbf{T_{TR}}$ the error will be bounded by **-0.25**≤ $\mathbf{e_3}$ ≤ **0.25.** The time of transient response for the NESO in the nominal case is **01s** for both position and speed.

**Table 3.4.**comparison factors of observers

| | **HGO** | | **SMO** | | **NESO** | |
|---|---|---|---|---|---|---|
| | **Residuals** | $\mathbf{T_{TR}}$ | **Residuals** | $\mathbf{T_{TR}}$ | **Residuals** | $\mathbf{T_{TR}}$ |
| | $\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$ | $\begin{bmatrix} TR_{TR1} \\ TR_{TR2} \end{bmatrix}$ | $\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$ | $\begin{bmatrix} TR_{TR1} \\ TR_{TR2} \end{bmatrix}$ | $\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$ | $\begin{bmatrix} TR_{TR1} \\ TR_{TR2} \\ TR_{TR3} \end{bmatrix}$ |
| Nominal plant | -0.025 to 0.05<br>-2 to 1.6 | 0.5s<br>0.5s | -0.02 to 0.05<br>-0.25 to 0.1 | 0.8s<br>0.6s | -0.06 to 0.08<br>-2 to 1.5<br>-0.5 to 2.5 | 01s<br>01s<br>5s |
| Plant plus disturbance | -0.025 to 0.05<br>-2 to 1.8 | 0.5s<br>0.5s | -0.075 to0.05<br>-0.5 to 0 | 02s<br>02s | -0.018 to 0.05<br>-0.5 to 0.3<br>-0.5 to 2.5 | 0.75s<br>0.75s<br>5s |
| Plant with 100% increase in inertia | -0.025 to 0.05<br>-2 to 1.8 | 0.5s<br>0,5s | -0.02 to 0.05<br>-0.25 to 0.1 | 1.5s<br>1,5s | -0.025 to 0.1<br>-1 to 0.5<br>-01 to 4.4 | 0.75s<br>0,75s<br>4s |

## 3.3. Comparison study of the characteristics and performances of the HGO, SMO, and NESO:

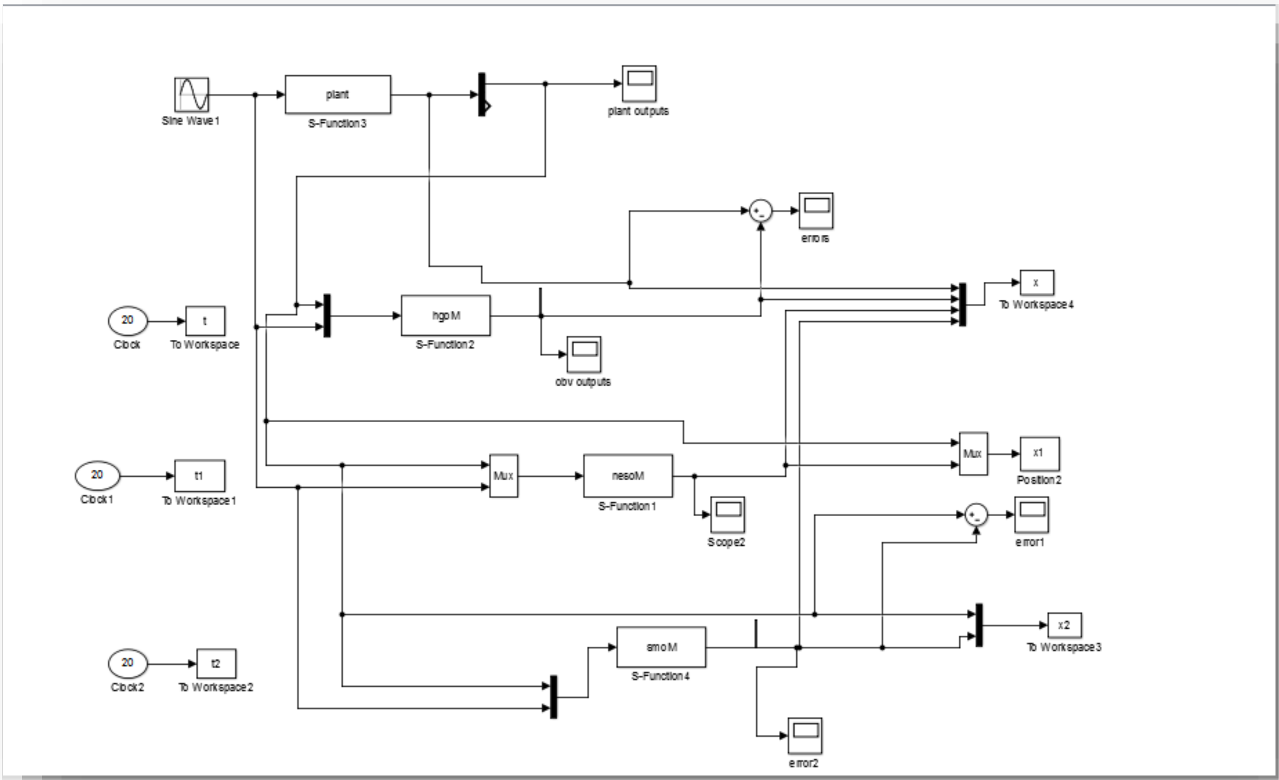



**Figure 3.14.**Over all Simulation Model

The performance comparison of observers is assessed in terms of to their capability in tracking and residuals of estimation error. The simulation with three cases (nominal, disturbance and uncertainty)
are shown below:

Table 3.4 shows the comparison factors depending on the results of simulations of the designed observers. For nominal case all three observers perform well in steady-state and have roughly the same accuracy and sensitivity to the noise. As expected in the transient state NESO takes longer to reach steady state $\mathbf{T_{TR}=1s}$, because it does not assume the knowledge of the plant dynamics and it has the highest range of residuals $\mathbf{-0.06< e_1 <0.08, -2< e_2 <1.6}$ where the residuals of SMO are the least ones $\mathbf{-0.02< e_1 < 0.05, -0.25< e_2 < 0.1}$ but the quickest to reach steady state is the HGO with $\mathbf{T_{TR}(e_1) = 0.5s}$ for both position and speed**.**

Comparing according to additive disturbance, figure 3.16 illustrates the tracking errors for the plant with added stribeck friction. As we can see only the estimated errors of NESO are zero at steady state while we have a sine wave tracking errors for SMO and HGO caused by the imperfect state tracking of these two observers, where for HGO in steady state **$-0.05<e_1<0$** and**$-01< e_2<0$,** for the SMO in steady state **$-0.05<e_1<0$** and**$-0.2< e_2<0$**. NESO has the least range of residuals in transient state **$-0.018<e_1<0.05$, $-0.5< e_2<0.3$** clearly demonstrating that NESO is much more robust than HGO and SMO in the presence of disturbance. As shown in Figure 3.12, $\dot{\hat{x}}_3$ converges quickly **$T_{TR}$=5s** with **$-0.5<e_3<2.5$**, and accurately to the combination of unknown dynamics and disturbance .NESO is a good estimator for both states and disturbances.

Figure 3.17 illustrates the simulation results for the plant with a 100% increase of inertia. As we can see even the sliding mode observer behaves well in the presence of uncertainty due to its small range of residuals during the transient state **$-0.02<e_1<0.05$**, **$-0.25<e_2<0.1$** with **$T_{TR}$=1.5s** almost like those for NESO **$-0.025<e_1<0.1$, $-1<e_2<0.5$** and **$T_{TR}$=1s**, but only the estimated errors of NESO are zero again at steady state which make it the most accurate in following the states. Its performance is the best overall, followed by SMO.

### 3.3.1. NOMINAL SYSTEM ERRORS:

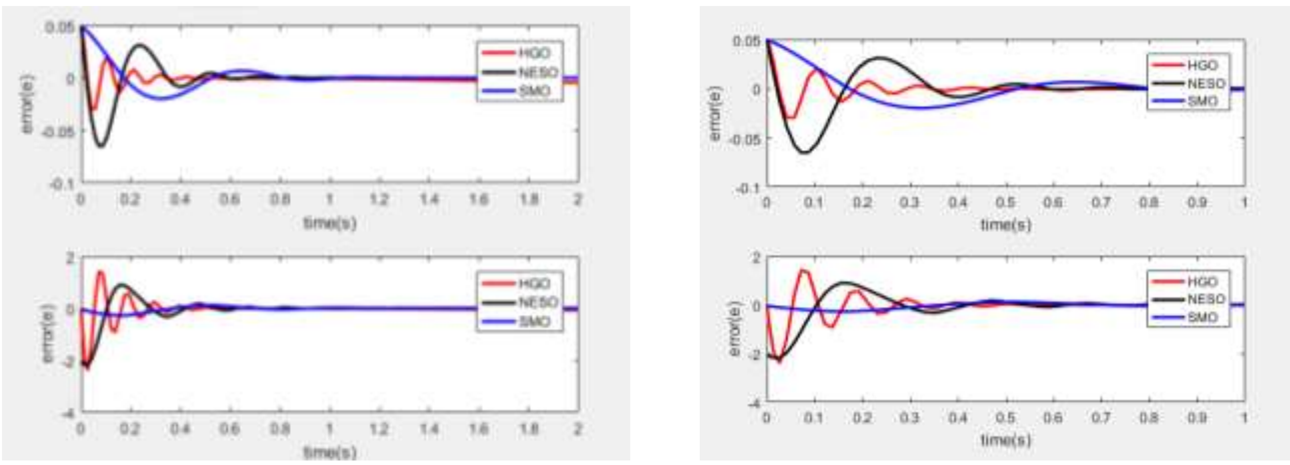



**Figure 3.15.**Estimation Error of the Nominal Plant

### 3.3.2. SYSTEM UNDER DISTURBANCE ERRORS:

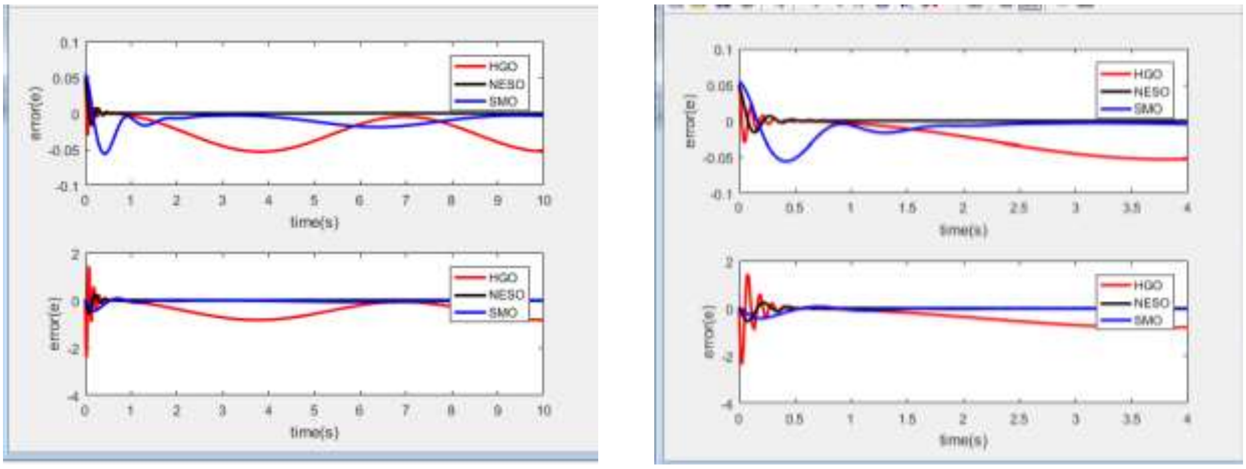



**Figure 3.16.**Estimation Error of the Plant with stribeck Friction

### 3.3.3. SYSTEM WITH UNCERTAINTY ERRORS:

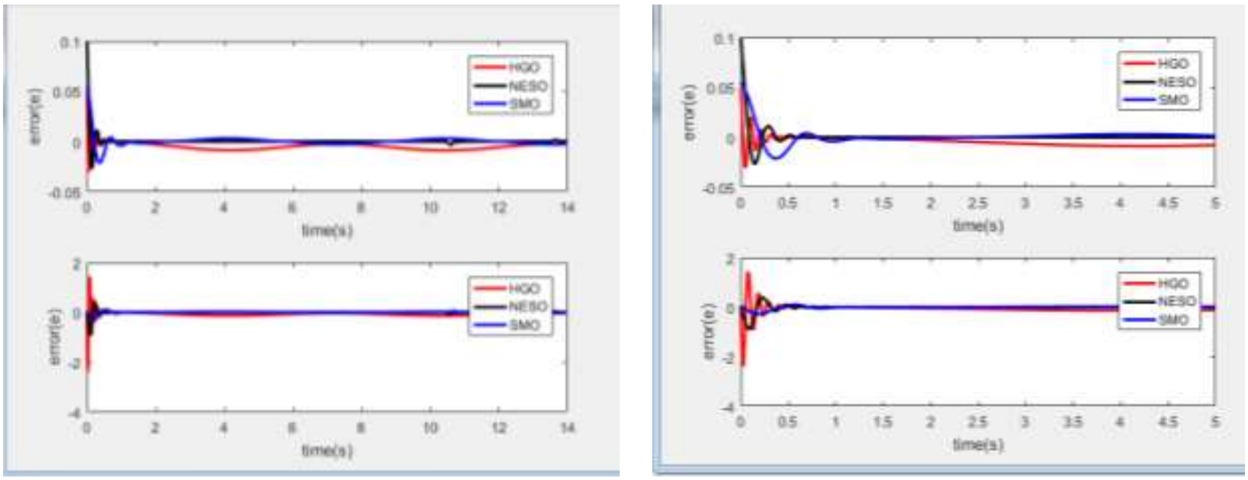



**Figure 3.17**.Estimated Error of the Plant with 100% Change of Inertia

## 3.4. Conclusion

In chapter III a software simulation with MATLAB of advanced observer designs, including NESO, HGO, and SMO, was performed for a nonlinear servo motor system. A gain modification method is proposed for the NESO to deal with the unknown initial conditions.

The computational criterion chosen is the amplitude of residuals and how fast is the system to reach the actual states. As it has been reported using simulation, for every case and for every link, there is salient observer outperforms the others in terms of minimum residuals range and high dynamic performance. The simulated results show that the performance of the extended state observer is superior in terms of the disturbance rejection and uncertainties compared with other observers for this class of nonlinear systems.

# CHAPTER IV

# GENERAL CONCLUSIONS

Most of controlled systems, in one aspect or another, are non-linear, with non-linear dynamics due to saturation of actuators, non-linearity of sensors or systems governed by non-linear differential equations. Although linear control tools may work well in many non-linear systems, but in some cases non-linear effects need to be taken in consideration to stabilize the system. Furthermore, considering the non-linear aspect may significantly enhance the performance and improve the overall robustness.

In the literature many researches are based on linearised models around a specific nominal operating point then linear controller is utilized to control the system. The associated problem with the linear control technique is that the system can be only be adequately controlled in a small region around the equilibrium point but the variation of operating regions in such non-linear system are wide following major disturbance .Furthermore, linear controllers provide large actuation and zero tracking error cannot be guaranteed in the in presence of disturbance. Thus, non-linear control is considered as a better choice to ensure the stability of non-linear systems in the presence of large disturbance and over larger operating regions. In the design of any optimal controller, whether it is linear or non-linear, it is essential that all state variables of the system are available, whether measured all (which in many cases not applicable) or estimated. State estimation can resolve the difficulties associated with unmeasured states.

As we have seen through the past chapters, a comparison study of performances and characteristics of three advanced state observers, including the NESO,HGO, and SMO, was performed. These observers were originally proposed to address the dependence of the classical observers, such as the Kalman Filter and the Luenberger Observer, A gain modification method is proposed for the NESO to deal with the unknown initial conditions. As a general overview of the comparison study which is based on the robustness of the performance with respect to the uncertainties of the servo motor as a model plant and the observer trackingerrors, both at steady-state and during transients. The tests were run in three conditions:

- Nominal plant.
- Nominal plant plus stribeck friction as disturbance.
- Nominal plant with **100% increase in inertia**

These simulations were run in Matlab simulink and the following observations were concluded:

- For nominal case, in terms of accuracy and sensitivity to the noise all three observers perform well in steady-state. NESO has the longest transient response, because it does not assume the knowledge of the plant parameters. HGO takes the least time to reach steady-state with least range of residuals which make it superior in nominal condition.

- For disturbance and uncertainty cases simulation results shows that NESO is the best in terms of uncertainties and disturbance rejection and that is because NESO provides a smaller steady state errors, i.e. closer adherence to the desired states .
- The state estimates errors for the Sliding Mode Observer appear somewhat comparable to those of the Extended state observer specially for the speed estimates errors although the response of SMO is far more oscillatory. This is because of the switching function (saturation function) associated with the Switching Gain Matrix K.
- Although the three observers are successful in providing estimates to such a degree of accuracy as to meet design requirements, the Extended State Observer formulated in this research is typically more accurate in terms of steady state relative speed and position estimation errors, and usually has less standard deviation in such estimates as well, indicating a less oscillatory response. This oscillation indicates that there will be a non-zero relative velocity and position, and the Follower plant will require control effort of increasing frequency for higher standard deviations in $|\hat{x}|$. The underlying reason for recommending this formulation of the Extended State Observer lies in the desire to minimize relative velocity and speed errors, in order to save on control effort.

# REFERENCES

[1]. Pauline Bernard. Observer design for nonlinear systems. Automatic Control Engineering. Université Paris sciences et lettres, 2017. Englis NNT :2017PSLEM010. tel-02094225.

*Automatic Control*, vol. 11, pp. 190-197, 1966.

[2]. Radke, and Z. Gao, "A survey of state and disturbance observers for practitioners", *Proce Ame Cont Conf,* 2006 Minneapolis, Minnesota, USA.

[http://dx.doi.org/10.1109/ACC.2006.1657545]

[3]. H.K. Khalil, and L. Praly, "High-gain observers in nonlinear feedback control", *Int. J. Robust Nonlinear Control,* vol. 24, pp. 993-1015, 2014.

[http://dx.doi.org/10.1002/rnc.3051]

[4].H.K. Khalil, *Nonlinear Control.,* Pearson Educational Limited: England, 2015.

[5].Amjad J. Humaidi, and A.A. Mohammed, "Performance comparison of different observers for pendubot", *j engin technol, sci,* vol. 34, no. 1, pp. 34-53, 2017.

[6]. E. Bou Rachel, and K. Hassan, "Application of the extended high gain observer to underactuated mechanical systems', IEEE", *Ame Cont Conf (ACC),* 2012

[7]. L.B. Freidovich, and H.K. Khalil, "Performance recovery of feedback-linearization based designs' Automatic Control", *IEEE Trans,* vol. 53, no. 10, pp. 2324-2334, 2008.

[8]. S.K. Spurgeon, *Sliding Mode Observers-toward a constructive design framework.,* Spring School: Aussois, 2015.

[9]. H. Rios, D. Efimov, and W. Perruquetti, An adaptive sliding mode observer for a class of uncertain nonlinear systems, International J Signal Processing.

[http://dx.doi.org/10.1002/acs.2857]

[10]. Amjad J. Humaidi, and H. M. Badir, Linear and nonlinear active rejection controllers for single-link flexible joint robot manipulator based on PSO tuner, APRN, J. Eng Applied Sciences, Vol. 13, No. 6.

[11]. W. Wang, *"A comparative study of nonlinear observers and control design strategies "*, Ph.D. Thesis, Guangxi University, P.R. China, 2003.

[12]. G. Wang, *"Observer-Based on Feedback Control Methods for Underactuated Robot System"*, M.Sc. Thesis, Simon Fraser University, Canada, 2003.

[13]. F. Esfandiari and H.K. Khalil, "Output feedback stabilization of fully linearizable systems," *Int. Journal of Control*, vol. 56, pp. 1007-1037, 1992.

[14].Henrik Rehbinder and Xiaoming Hu, "Nonlinear Pitch and Roll Estimation for Walking Robots," *Proc. of the IEEE International Conference on Robotics & Automation,* San Francisco, CA, April 2000.

[15]. E.S.Shin and K.W.Lee, "Robust output feedback control of robot manipulators using high-gain observer," *Proc. of the 1999 IEEE Int. Conference on Control Application*, HI, August 22-27, 1999.

[16]. J. Han. "A class of extended state observers for uncertain systems", Control and Decision, Vol.10, No.1, pp: 85-88, 1995. (In Chinese)

[17]. F.L.Lewis, Applied Optimal Control and Estimation, Prentice Hall, 1992.

[18]. en.wikipedia.org/wiki/Nonlinear_control.

[19] .Erbatur, K., Vinter, R. B., and Kaynak, O. (1994). Feedback linearization control for a3-dof flexible joint elbow manipulator. In *Proceedings of the 1994 IEEE International Conference on Robotics and Automation*, pages 2979–2984 vol.4.

[20]. William S. Levine, Ed, *The Control Handbook,* CRC Press, 1996.

[21]. Jean-Jacques E. Slotune and W. Li, *Applied Nonlinear Control,* Prentice-Hall, 1991.

[22].Souad chaouch and mouhamed-Said Nait-Said.Backstepping control design for position and speed tracking o DC motors, pp. 2-4.

[23]. S Vaidyanathan, AT Azar. Backstepping Control of Nonlinear Dynamical systems,2021,pp.1

[24]. Niloofar Nasiri Kamran , Sliding Mode Observers for Distributed Parameter Systems: Theory

and Applications.

[25]. Jongguk Yim and Jong Hyeon Park, "Nonlinear H/sub infinity/ control of robotic manipulator," *1999 IEEE International Conference on Systems, Man, and Cybernetics*, vol. 2, pp. 866-871, 1999.

[26]. Gene F. Franklin, J. David Powell, and Abbas Emami-Naeini, *Feedback Control of Dynamic Systems*, Addison-Wesley Publishing Company, 1994.

[27].Guangren Duan and Steve Thompson, "Separation principle for robust pole assignment - An advantage of full-order state observer," *Proc. of the 38th Conference on Decision & Control,* pp. 76-78, December 1999.

[28].Carla Seatzu and Alessandro Giua, "Observer-controller design for gains via pole placement and gain-scheduling," *Proc. of the 6th IEEE Mediterranean Conference*, Alghero, Sardinia, Italy, June 1998.

[29]. Gene F. Franklin, J. David Powell, and Michael Workman, *Digital Control of Dynamic Systems*, Third Edition, Addison Wesley Longman, Inc., 1997.

[30]. H.K. Khalil, "High-gain observers in nonlinear feedback control," *New Directions in Nonlinear Observer Design (Lecture Notes in Control and Information Sciences)* vol. 24, no. 4, pp. 249-268, 1999.

[31]. Gildas Besancon, "Further results on high-gain observers for nonlinear systems," *Proc. of the 38th Conference on Decision & Control,* AZ, pp. 2904-2902, Decemenber1999.

[32]. Seungrohk OH and Hassan K. Khalil, "Nonlinear Output-Feedback Tracking Using High-gain Observer and Variable Structure Control," *Automatic*. vol. 33, no. 10. pp. 1845-1856, 1997 .

[33]. Sarah K.Spurgeon, Sliding mode observers – historical background and basic introduction.

[34]**.** J. Han, "Nonlinear Design Methods for Control Systems," *Proc. of the 14th IFAC World Congress*, Beijing, 1999.

[35] J.J. E. Slotune, J.K. Hedrick, and E.A. Misawa, "On sliding observers for nonlinear systems,"

*Journal of Dynamic Systems, Measurement, and Control*, vol. 109, pp. 245-251, 1987.

[36] V.I. Utkin. *Sliding-modes in control optimization*. Springer-Verlag, 1992.

[37].Primbs, J. (1996). Survey of nonlinear observer design techniques. *Penn State Notes*,1(1):1–18.

[38]. Keller, H. (1987). Non-linear observer design by transformation into a generalized observer canonical form. *International Journal of Control*, 46(6):1915–1930.

[39]. Kou, S. R., Elliott, D. L., and Tarn, T. J. (1975). Exponential observers for nonlinear dynamic systems. *Information and control*, 29(3):204–216.

[40]. Baumann, W. and Rugh, W. (1986). Feedback control of nonlinear systems by extended linearization. *IEEE Transactions on Automatic Control*, 31(1):40–46.

[41]. Thau, F. (1973). Observing the state of non-linear dynamic systems. *International journal of control*, 17(3):471–479.

[42]. Zeitz, M. (1987). The extended luenberger observer for nonlinear systems. *Systems &Control Letters*, 9(2):149–156.