Null Steering of Dolph-Chebychev Arrays using Taguchi Method

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Abstract: *Dolph-Chebychev arrays are known to exhibit the best compromise between sidelobe level and directivity. However, they place a constraint on the null locations. Any attempt to impose nulls or get them deeper will impact the directivity/sidelobe level trade-off. In this work, null placement in Dolph-Chebychev arrays through element position perturbation is carried out based on Taguchi method while preserving the array aperture. Several examples are considered for single, double, multiple and broad null placement to demonstrate the ability of the Taguchi method to explore the search space and reach the global optimum.*

Keywords: *Pattern nulling, dolph-chebychev arrays, taguchi method, element position perturbation.*

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1. Introduction

In modern wireless communication systems, reducing and/or canceling the interference is a vital issue. This is because capacity of the system is directly proportional to the amount of interference that it can tolerate. The control of the radiation pattern has its importance also. To improve the radiation efficiency, this pattern must be oriented to the desired directions, while nulls (zero radiation energy) directed towards the interferers. The null steering in antenna radiation pattern of a linear array aims at rejecting unwanted interference while receiving the desired signal from a chosen direction has received considerable attention in the past and is still of great interest [2].

The null steering techniques are based on the variations of the array parameters such as the element excitations (amplitude and/or phase) and positions of array elements. The element position control with the use of a mechanical driving system, such as servomotors, is an alternative way to create nulls in the radiation pattern. These techniques, however, turn out to be expensive considering the cost of the controllers used for phase shifters and variable attenuators. Moreover, when the number of elements in the array increases, the computational time to find these parameters will also increase [2].

Dolph-Chebychev arrays have the important property that all side lobes in their radiation pattern are of equal magnitude. Furthermore, the relationship between the directivity and sidelobe level for these arrays is optimum in that for a specified sidelobe level the beam width is the smallest, and, alternatively, for a given beam width the sidelobe level is the lowest [1, 5]. These fine radiation characteristics, however, put a restriction on the flexibility of placing nulls in the sense that once the sidelobe level or directivity is fixed, the nulls have directions dictated by the Dolph-Chebychev excitations. Ideally, one would require an array with the best directivity/sidelobe level compromise such as the one of Dolph-Chebychev along with flexibility in null placement. This turns out to be unachievable as any attempt to impose nulls in directions other than the ones constrained by the Dolph-Chebychev coefficients or force the nulls to be deeper will alter the trade-off and introduce deterioration in sidelobe level or directivity.

Thanks to the rapid development of computer technology, many optimization techniques such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Simulated Annealing (SA), Artificial Neural Network (ANN), and gradient-based techniques have been implemented in the form of computer codes [11]. These global optimizers while more familiar, traditional techniques such as conjugate gradient and the quasi-Newtonian methods are classified as local optimizers. The distinction between local and global search of optimization techniques is that the local techniques produce results that are highly dependent on the starting point or initial guess, while the global methods are highly independent of the initial conditions [1]. Though they possess the characteristic of being fast in convergence, local techniques, in particular the quasi-Newtonian techniques have a direct dependence on the existence of at least the first derivative. In addition, they place constraints on the solution space such as differentiability and continuity, conditions that are hard or even impossible to deal with in practice [1]. Compared with traditional optimization techniques, Taguchi's optimization method is easy to implement and very efficient in reaching optimum solutions. Taguchi's optimization method is developed based on the Orthogonal Array (OA) concept, which offers a systematic and efficient way to select design parameters. In addition, it reduces the number of tests required in the optimization process compared to GA or PSO [11, 12].

In this work, the problem of imposing nulls in arrays fed by Dolph-Chebychev excitations through element position perturbation is carried out based on the Taguchi method. The idea is to keep the trade-off directivity/sidelobe level within an allowable rate with the nulls constrained to be as deep as possible in the desired directions. Another constraint is imposed which is that the array size must stay unchanged which is beneficial as the perturbed array occupies the same size of the original Dolph-Chebychev one.

The rest of the paper is organized as follows: section 2 presents the problem with its mathematical formulation to cast it as an optimization task. Section 3 describes briefly the idea behind the taguchi method along with the algorithm this technique follows towards the optimum solution. The results are given in section 4 and finally concluding remarks are drawn in section 5.

2. Problem Formulation

For a linear array of isotropic elements placed and excited symmetrically along the x-axis, the array factor is given as [9]:

$$AF(\theta) = \sum_{k=1}^{N} a_n \cos[2\pi x_k(\cos\theta - \cos\theta_0)] \tag{1}$$

where:

$2N$ is the number of elements.

a_n is the element excitation.

θ is the scanning angle range and varies from 0° to 180°.

θ_0 is the main beam direction (90° for broadside).

The pattern produced is symmetrical with respect to the broadside angle which suggests placing the nulls just on half the angle range (0-90°) and the other half will be automatically symmetrical. Particularly, the Dolph-Chebychev coefficients are known to be symmetrical with respect to the center which justifies the use of the above formula.

The position symmetry dictates that the optimization on this dimension should be done on half the array with the other half symmetrically constructed. The outmost element is fixed to have the same length of the original dolph-chebychev array while the other elements are varying which reduces the problem of optimizing a $2N$ element array to an N-1 dimensions. Starting from an equally spaced dolph-chebychev array with an a priori set sidelobe level and directivity, the optimization process tries to alter the positions of the elements so that the null(s) in the desired direction(s) is (are) placed with the directivity/sidelobe level ration kept within a tolerable change from the original one. The desired pattern is a modified version of the initial Dolph-Chebychev absolute array factor with the nulls imposed in the desired directions as follows:

$$Desired\ pattern = \begin{cases} 0 \text{ for the desired null directions} \\ \text{Initial array factor elsewhere} \end{cases} \tag{2}$$

This process is summarized by the fitness function:

$$f = \sum_{q=0°}^{180°} W_q \left|(AF_d(q) - AF_p(q)\right| + SLL \tag{3}$$

where, $AF_d(\theta)$ and $AF_p(\theta)$ are the desired and produced patterns at the angle θ, respectively. W_θ is a weighting coefficient to force the pattern to exhibit nulls at the desired angles and preserving the initial pattern elsewhere. It is defined as:

$$W_\theta = \begin{cases} 100 & \text{if } \theta = \text{desired directions} \\ 1 & \text{elswhere} \end{cases} \tag{4}$$

The term SLL is introduced to force the sidelobe level to stay within an allowable value set at the starting of the optimization procedure. The initial Dolph-Chebychev array is designed to have a sidelobe level of -40dB which is a value largely satisfactory for modern communication systems. Throughout the optimization procedure, we allow a 2dB increase in sidelobe level due to null imposing without any change in directivity. If we denote SLL_p to be the produced sidelobe level, the last sentence is interpreted by introducing a penalty of 10 if the sidelobe level increases above -38dB i.e., SLL is then:

$$SLL = \begin{cases} 10 & \text{if } SLL_p > -38\text{ dB} \\ 0 & \text{Otherwise} \end{cases} \tag{5}$$

3. The Taguchi Method

Taguchi's method was developed based on the concept of the OA, which can effectively reduce the number of tests required in a design process [11, 12]. It provides an efficient way to choose the design parameters in an optimization procedure.

Before presenting the Taguchi procedure, it is worth understanding what OAs are and how are they generated [11, 12]. Let S be a set of s symbols or levels (the simplest symbols are integers 1, 2, 3…). A matrix A of N rows and k columns with entries from S is said to be an OA with s levels and strength t ($0< t< k$) if in every $N\times t$ subarray of A, each t-tuple based on S appears exactly the same times as a row. The notation OA (N, k, s, t) is used to represent an OA.

3.1. Initialization Procedure

The optimization procedure starts with the problem initialization, which includes the selection of a proper OA and the design of a suitable fitness function. The selection of an OA (N, k, s, t) mainly depends on the number of optimization parameters. In general, to characterize the nonlinear effect, three levels (s=3) are found sufficient for each input parameter. Usually, an OA with a strength of 2 (t=2) is efficient for most problems because it results in a small number of rows in the array [11, 12].

3.2. Design of input Parameters

The input parameters need to be selected to conduct the experiments. When the OA is used, the corresponding numerical values for the three levels of each input parameter should be determined. In the first iteration, the value for level 2 is selected at the center of the optimization range. Values of levels 1 and 3 are calculated by subtracting/adding the value of level 2 with a variable called Level Difference (LD). The Level Difference in the first iteration (LD1) is determined by the following equation:

$$LD_1 = \frac{Max - Min}{Number\ of\ levels + 1} \tag{6}$$

where *Max* is the upper bound of the optimization range and *Min* is the lower bound of the optimization range.

3.3. Conduct Experiments and Build a Response Table

After determining the input parameters, the fitness function for each experiment can be calculated. These results are then used to build a response table for the first iteration by averaging the fitness values for each parameter n and each level m using the following equation:

$$F_{av} = \frac{s}{N} \sum_{i,OA(i,n)=m} f_i \tag{7}$$

as an example, consider that parameter x in an N dimensional problem has levels 1, 2 and 3 as described earlier. With s=2; the fitness values are evaluated based on equation 7 for each level and hence a response table is constructed for each parameter that can be used to choose which level produces the best fitness value (minimum value).

3.4. Identify Optimal Level Values and Conduct Confirmation Experiment

Finding the largest fitness value ratio in each column can identify the optimal level for that parameter. When the optimal levels are identified, a confirmation experiment is performed using the combination of the optimal levels identified in the response table. This confirmation test is not repetitious because the OA based experiment is a fractional factorial experiment, and the optimal combination may not be included in the experiment table. The fitness value obtained from the optimal combination is regarded as the fitness value of the current iteration.

3.5. Reduce the Optimization Range

If the results of the current iteration do not meet the termination criteria, the process is repeated in the next iteration. The optimal level values of the current iteration are used as central values (values of level 2) for the next iteration. To reduce the optimization range for a converged result, the *LDi* is multiplied with a reduced rate (rr) to obtain LD_{i+1} for the $(i+1)^{th}$ iteration:

$$LD_{i+1} = rr \times LD_i = RR(i) \times LD_i \tag{8}$$

where $RR(i)$ is called reduced function. When a constant rr is used, $RR(i)=rr^i$. The value of rr can be set between 0.5 and 1 depending on the problem. The larger rr is, the slower the convergence rate.

If LD_i is a large value, and the central level value is located near the upper bound or lower bound of the optimization range, the corresponding value of level 1 or 3 may reside outside the optimization range. Therefore, a process of checking the level values is necessary to guarantee that all level values are located within the optimization range. A simple way is to use the boundary values directly.

3.6. Check the Termination Criteria

When the number of iterations is large, the level difference of each element becomes small from equation 8. Hence, the level values are close to each other and the fitness value of the next iteration is close to the fitness value of the current iteration. The following equation may be used as a termination criterion for the optimization procedure:

$$\frac{LD_i}{LD_1} < converged\quad value \tag{9}$$

usually, the converged value can be set between 0.001 and 0.01 depending on the problem. The iterative optimization process will be terminated if the design goal is achieved or if equation 9 is satisfied.

4. Results and Discussions

The procedure described in section 2 and the algorithm of section 3 have been applied to a set of null placement tasks with satisfactory results obtained. The dolph-chebychev array factor for a broadside main beam and -40dB sidelobe level is shown in Figure 1. This array factor exhibits a constant sidelobe level with the nulls placed in fixed angles.

In the first example it is attempted to place a single null in the pattern at the angle 76°. The placement has been successfully achieved where the null depth reached a value of -115.6dB with the sidelobe level of -38.46dB and the same initial directivity. Figure 2 shows the resulting pattern.

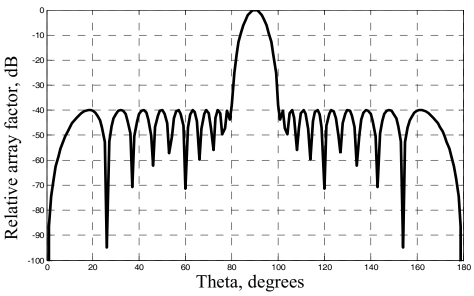

Figure 1. The radiation pattern of a broadside uniformly spaced Dolph-Chebychev fed linear array.

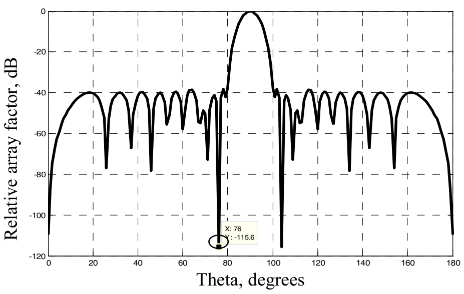

Figure 2. The produced radiation pattern with one null imposed at 76°.

This result is satisfactory as the null has been placed exactly at the desired direction while the sidelobe level kept within the tolerable value. The second example concerns placing two distinct nulls at 55° and 65°. Figure 3 shows the resulting pattern where it is clearly shown that the placement is successful with the null depth is at least at -115.8dB down the main beam and the sidelobe level value of -38.58dB with the same dolph-chebychev directivity. This demonstrates the versatility of the Taguchi method to explore the search space and find the optimal solution for the null placement.

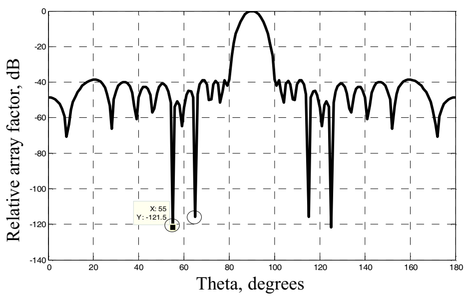

Figure 3. The produced radiation pattern with two nulls imposed at 55° and 65°.

In the third example, it is attempted to go further with the null placement task by placing three distinct nulls at 55°, 65° and 76°. Figure 4 shows the resulting pattern. Surprisingly, the taguchi method again reached our desired objectives by placing the three nulls at exactly their corresponding angles with null depths reaching even -114.3dB at a sidelobe level of -38.5dB and the same initial directivity.

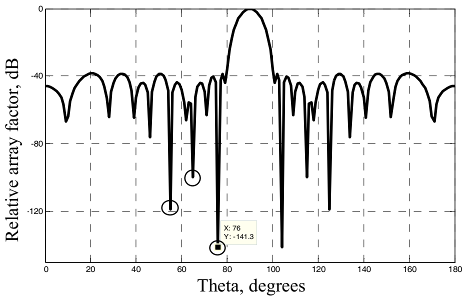

Figure 4. The produced radiation pattern with three nulls imposed at 55°, 65° and 76°.

This is again a proof of the capability of the Taguchi method to explore the search space rigorously to find the optimum solution of the problem. As an even further null placement case, in the next example, it is attempted to place six distinct nulls at 15°, 39°, 48°, 55°,65° and 72°. The optimization procedure has been terminated successfully with the resulting radiation pattern shown in Figure 5. It is clearly seen that the nulls are placed as it was desired with some nulls reaching -120dB with the trade-off sidelobe level/directivity within the tolerable range. Indeed, the sidelobe level is at -38dB. Here again we demonstrate the usefulness of the Taguchi method in placing even a large number of nulls with preservation of the best characteristics of the initial Dolph-Chebychev array. The previous examples treated the case of placing distinct nulls and next it is desired to place a broad null that centered at 57° and extends from 55° to 59°.

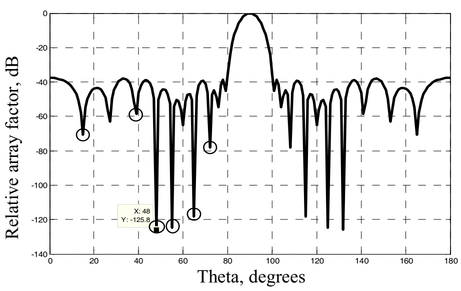

Figure 5. The produced pattern with six nulls imposed at 15°, 39°, 48°, 55°, 65° and 72°.

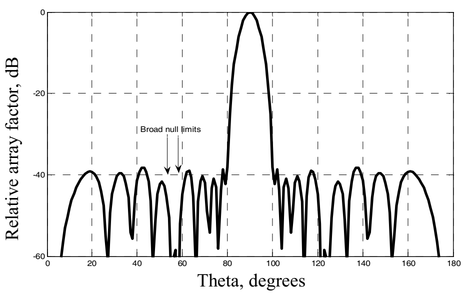

Figure 6. The produced pattern with a broad null imposed extending from 55° to 59°.

Figure 6 shows the resulting pattern that reveals the fact that this broad null has been placed despite the fact that the depths vary along its width but on overall this depth is less than -60dB. The achieved sidelobe level is again within the tolerable range and is at -38.28dB.

The cases treated previously have been devoted to the broadside. In the next examples it is attempted to explore the main beam angle steering case. As a first example, assume the main beam is directed towards 60°. The initial radiation pattern of a uniformly spaced dolph-chebychev fed linear array is shown in Figure 7. The original pattern has already a very deep null at 120° with a null depth of -305.9dB. It is desired now to place a null at 79° in this pattern. The null is successfully placed as it is depicted by Figure 8 with a null depth reaching -105.6dB while the original null depth at 120° reduced to -54.14dB. The sidelobe level again stayed within the limit and is at -38.15dB.

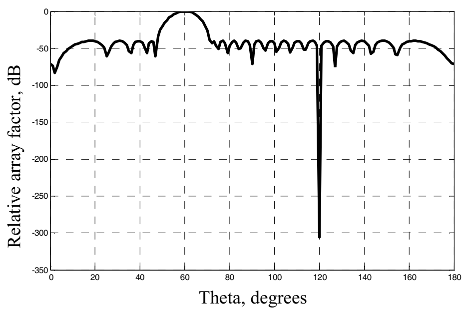

Figure 7. The radiation pattern of a uniformly spaced dolph-chebychev fed linear array steered towards 60°.

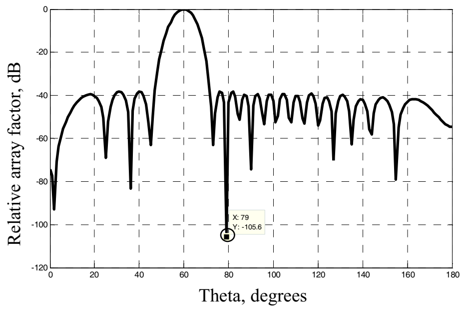

Figure 8. The produced pattern in the 60° steered case with one null at 79°.

In the next example, consider that the main beam is directed towards 130°. It is desired to impose a null at 87°. This null is place successfully as it is shown in Figure 9 with a null depth of -98.21dB. The sidelobe level is very good as it stayed close to the original value and is at -39.21dB.

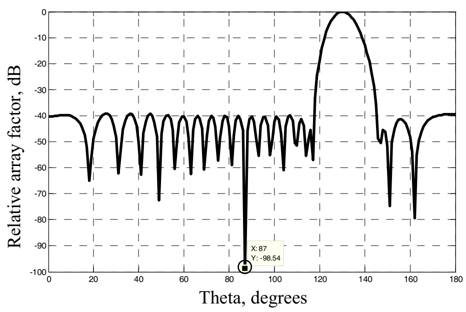

Figure 9. The produced pattern in the130° steered case with one null at 87°.

5. Conclusions

The problem of null placement in dolph-chebychev arrays using the taguchi optimization method has been addressed. The idea is based on element position perturbation with the extreme elements of the array fixed. The null placement has been successfully achieved with the characteristics of the initial array kept within tolerable values. The Taguchi method has proved to be powerful at reaching the global optimum solutions. The produced arrays possess the characteristics similar to the ones of the dolph-chebychev arrays along with the imposed nulls having significant depths and the overall array length kept the same. Overall, the optimization procedure involving the Taguchi method achieved the design objectives with appropriate characteristics for modern communication systems.

As a future work, the problem of using piratical antenna elements in the array design is to be addressed. This gives rise also to the problem of mutual coupling between the elements that can dramatically affect the performance of the array antenna in terms of interference rejection.

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