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Optimization of operators planning on a Petroleum site: Case of Sonatrach, Algeria

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I thank God for giving me the inspiration and strength to persevere through all circumstances over the course of this extremely challenging yet rewarding journey.
I dedicate this thesis to my loving family, whose unwavering support and encouragement have been my guiding light throughout this academic journey, your belief in me and constant encouragement have given me the strength to overcome challenges and pursue my dreams,i am forever grateful for your love, sacrifices, and understanding,this achievement is a testament to our collective effort, and i dedicate it to each of you with heartfelt appreciation and love.

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ملخص

تتطلب عملية الإنتاج في سوناطراك على مدار ٢٤ ساعة تخطيط فريق فعال لضمان استمرار العمليات وزيادة الربحية ، في هذه الدراسة ، ينصب تركيزنا على تخصيص المشغلين (المهندسين) ضمن فرق تعمل في نوبات متناوبة ، مما يسمح بفترات الراحة اللازمة مع الحفاظ على العمل المتسق الجودة ، يكمن التحدي الأساسي في إنشاء فرق متوازنة يمكنها استبدال بعضها البعض بسلاسة في موقع البترول ، وبالتالي ضمان نفس المستوى من الأداء والإنتاجية ، لمواجهة هذا التحدي ، نقترح نموذجًا رياضيًا موسعًا مستمدًا من مشكلة المهمة. الهدف هو تقليل الاختلافات في المنفعة بين الفرق وتحسين تكوينها ، لمعالجة هذه المشكلة ، قمنا بتطوير خوارزمية وراثية معدلة وخوارزمية هجينة معدلة. تم تصميم هذه الخوارزميات لحل مشكلة تخصيص الفريق بشكل فعال والمساهمة في الكفاءة الكلية لعملية إنتاج سوناطراك.

تشكيل فرق، الاستدلال الفوقي، مشكلة التعيين :**الكلمات المفتاحية**

Abstract

Sonatrach's 24-hour production process necessitates effective team planning to ensure uninterrupted operations and maximize profitability.In this study, our focus is on the allocation of operators (engineers) within teams that work in rotating shifts, allowing for necessary rest periods while maintaining consistent work quality, the primary challenge lies in creating balanced teams that can seamlessly replace each other on the petroleum site, thereby ensuring the same level of performance and productivity, to address this challenge, we propose an extended mathematical model derived from the assignment problem. The aim is to reduce the differences in utility between teams and optimize their composition, to tackle this problem, we have developed an adapted Genetic algorithm and an adapted Hybrid algorithm. These algorithms are designed to effectively solve the team allocation issue and contribute to the overall efficiency of Sonatrach's production process.

Keywords: Teams formation, Assignment problem, Metaheuristics

Résumé

Le processus de production 24 heures sur 24 de Sonatrach nécessite une planification d'équipe efficace pour assurer des opérations ininterrompues et maximiser la rentabilité. qualité,le premier enjeu est de créer des équipes équilibrées qui se substituent harmonieusement sur le site pétrolier, assurant ainsi le même niveau de performance et de productivité,pour relever ce défi, nous proposons un modèle mathématique étendu dérivé du problème d'affectation. L'objectif est de réduire les différences d'utilité entre les équipes et d'optimiser leur composition,pour résoudre ce problème, nous avons développé un algorithme génétique adapté et un algorithme hybride adapté. Ces algorithmes sont conçus pour résoudre efficacement le problème d'allocation des équipes et contribuer à l'efficacité globale du processus de production de Sonatrach.

Mots Clée : Formation des équipes, Problème d'affectation,Métaheuristiques

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General Introduction

In today's globalized world, work schedules vary significantly from one country to another, reflecting cultural, economic, and industrial differences,these variations encompass the number of working hours, breaks, and the organization of shifts, Algeria, as a country with a diverse industrial landscape, presents its own unique approach to work schedules.
In Algeria, work schedules are influenced by a combination of factors, including legal regulations, cultural norms, and industry-specific requirements,the standard workweek in Algeria typically consists of 40 to 48 hours, distributed across five to six days,however, it is important to note that specific industries may have different arrangements to accommodate their operational needs.
Shift schedules play a crucial role in industries that require continuous 24-hour coverage, such as production and oil industries,these sectors often operate around the clock to maximize efficiency and meet production demands,some industries opt for a three-shift system, where workers rotate through morning, afternoon, and night shifts, ensuring continuous operation,others employ a two-shift system, with longer working hours per shift, to maintain productivity while reducing transition periods.
In the case of Sonatrach, the state-owned oil and gas company in Algeria, work shifts in petroleum sites located in the southern regions have undergone a notable transformation,previously, a three-shift system was implemented, accommodating the demanding nature of the operations, however, in recent years, a shift to a two-shift system has been observed, aiming to optimize productivity while addressing workforce management challenges.
One of the key challenges faced by Sonatrach is the formation of balanced teams in terms of utility and experience,it is essential for teams to have a well-rounded combination of skills and expertise to ensure the consistent delivery of high-quality work throughout the shifts,achieving this balance becomes increasingly complex in industries with unique demands and dynamic work environments.
To address this challenge, operations research (OR) and optimization techniques prove to be valuable tools, OR methodologies can assist in optimizing workforce allocation, schedul-

ing, and team composition to maximize productivity and minimize disruptions,this thesis and case study focus on the application of OR and optimization techniques to enhance team formation and productivity within Sonatrach's petroleum sites.
By exploring the dynamics of work schedules, shift systems, and the importance of balanced teams in the context of Sonatrach and the petroleum industry, this thesis aims to contribute to the ongoing discussion on workforce management optimization,through the application of OR and optimization techniques, it seeks to provide practical insights and recommendations to improve the efficiency and effectiveness of work scheduling and team composition in the oil and gas sector.
The structure of this thesis is organized into four main chapters, each focusing on a specific aspect of the research :

- Chapter 1 serves as an introduction, presenting an overview of the host organization, SONATRACH, including its background, key figures, organizational structure, activities, and management,it also explores the Exploration-Production Activity and its significance within SONATRACH, along with an emphasis on human resources and training.

- Chapter 2 delves into the fundamentals and theoretical notions of Operations Research (OR), covering topics such as the origin of OR, its methodology and phases, tools and techniques employed, and the scope of OR,the chapter also highlights combinatorial optimization problems, focusing on the linear programming problem, solution methods (both exact and approximate), and the Assignment Problem.

- Chapter 3 addresses the modeling and development of a computer solution, providing a detailed problem formulation and its mathematical model,a case study is presented to demonstrate the application of the model, followed by an exploration of the model's complexity.

- Chapter 4 explores the practical implementation and analysis of two distinct algorithms, the Genetic Algorithm and the Hybrid Algorithm, for solving the team formation problem,the chapter elucidates the underlying principles and motivations behind the adoption of these algorithms, emphasizing the adaptations made to optimize their effectiveness in the team formation context,thorough examination of extensive results obtained from applying both algorithms allows for a comprehensive comparative analysis of their performance. Ultimately, the chapter aims to demonstrate the efficacy and efficiency of the Genetic Algorithm and the Hybrid Algorithm in effectively addressing the intricacies inherent in the team formation problem.

Chapter 1

Presentation of the host organization SONATRACH

1.1 Introduction

In this Chapter, we provide a comprehensive introduction to SONATRACH, the host organization of this study. We begin by presenting an overview of SONATRACH's background, key figures, organizational structure, and its diverse activities. Additionally, we shed light on SONATRACH's management practices and delve into the significance of the Exploration-Production Activity within the company. Furthermore, the chapter highlights the importance of human resources and training in supporting the organization's operations.

Note:
All information presented in this chapter is sourced from the official site of Sonatrach [Son23],By referencing the official site, we ensure the accuracy and reliability of the data and insights provided.

1.2 Background

SONATRACH is a state owned company formed to explore and develop the largest hydrocarbon resources of the country. It was established in the aftermath of Algeria's independence on December 31st1963 by the title of **So**ciété **Na**tionale de **Tra**nsport et de **C**ommercialisation des **H**ydrocarbures. This was the origin of the acronym **SONATRACH**, while in 1966 the name has been changed in Socièté Nationale pour la Recherche, la Production, le Transport, la Transformation et la Commercialisation des Hydrocarbures to take into account its whole activities. In fact, after the Arab-Israeli War (June 1967), Algeria decided to nationalize the refining and distribution activities, including all the French oil and gas holdings (February 1971), so as to control all Algerian petrochemical resources.

The Fundamental Law on Hydrocarbons, which was promulgated by the Algerian government on 12 April 1971, had two main purposes:

1. To formally abolish the system of concessions and establish that all mining titles, including the control of all petroleum reserves that might be discovered in the future in any part of Algeria, must be transferred to SONATRACH.

2. To make provision for foreign companies to enter into service contracts or joint ventures with SONATRACH, provided that 51% of the assets were held by the state company.

In the 1980s, the name changed again in Enterprise Nationale SONATRACH, which coincided with a reduction in the company's direct control over these assets. The company was divided into four enterprises and it became possible for foreign hydrocarbon companies to do business in Algeria being in a partnership with SONATRACH.

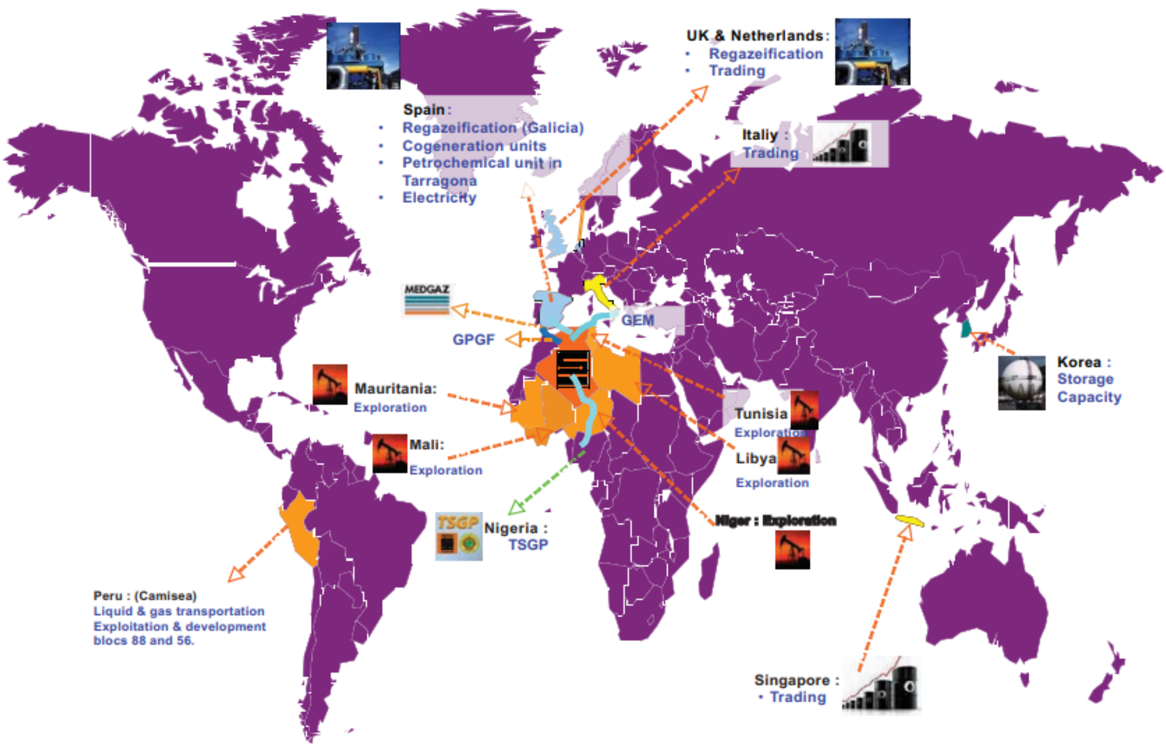

Figure 1.1: SONATRACH in the world

Alongside the historical collaboration with ENI, dating back to 1977, for the export of natural gas to Italy through the Trans-Mediterranean pipeline, other joint ventures have been created by SONATRACH to diversify its holdings and expanding internationally.
In 2000, SONATRACH entered a major joint venture with CEPSA and others to build MedGaz, a 450-kilometer gas pipeline running from Algeria to Spain.
In 2002, SONATRACH and Shell formed a partnership to explore and develop oil and gas interests and, in 2003, SONATRACH formed the joint venture Helison with Germany's Linde Engineering, but also a joint venture with BP to distribute LNG to the United Kingdom. Nowadays, SONATRACH is the first hydrocarbon group in Africa, but it is also present in Europe and Peru.
The Algerian energy system is based on the hydrocarbon sector as its economy and state balance do. Due to this strong connection, the energy sector, and consequently its largest hydrocarbon resources, is dominated by two state-owned gas and oil companies: Sonelgaz and SONATRACH.

1.3 SONATRACH An Integrated Oil & Gas Company

SONATRACH is the Algerian state company for the research, exploitation, pipeline transportation, processing and marketing of hydrocarbons and their by-products. Its purpose is to make an optimum use of its hydrocarbon national resources and create wealth serving the country's social and economic development.
As an integrated oil and gas company, SONATRACH is a major player in the oil and gas sector. SONATRACH is today the first hydrocarbons company in Africa and the Mediterranean. It carries out its activities in five main fields: Exploration-Production, Pipeline Transportation, Liquefaction and Separation, Refining and Petrochemicals, and Marketing.
It participates in various projects with different partners in Africa, Latin America and Europe.
Since its inception, SONATRACH has managed to acquire a strong capacity to integrate new technologies, to establish a proven and reliable presence in international liquid and gaseous hydrocarbons markets and to capitalize on its experience of partnership with international companies of different sizes and different geographic origins.
It has also adapted to the new global economic environment by diversifying its activities and, therefore, is expanding in other economic niches including mining, power generation, water, air and sea transport ... etc.
Today, SONATRACH is asserting itself not only as an international oil and gas-oriented

group but also as a supportive, responsible and Citizen Company.
It is committed to the economic, social and cultural development of populations; it has set essential priorities in terms of HSE, and became resolutely involved in environmental protection and ecosystems preservation.

1.4 SONATRACH Key Figures

The key figures are that SONATRACH:

- Is the 1st exporting company of the country, providing more than **95%** of foreign currency revenues.
- Owns **154** subsidiaries and affiliates.
- Contributes to **over 50%** of the country's tax revenues.
- Export turnover is **60 billion** US$
- Overall investment is **5,5 Billion** US$
- Primary production of hydrocarbons is **189.6 million** TOE
- Ensures an annual production of **1 479 million** boe or 4.1 million boe/day.
- Employs **more than 200 000** employees, at the group level.
- Has **21** main families of expertise.
- Workforce is **55 361** Permanent agents.

1.5 SONATRACH organization

Today, SONATRACH must be part of a new more agile and efficient dynamic in its organization and operation to face the challenges it must meet to serve a more prosperous Algeria,the transformation of the Company sets the course for the new policy implemented to transform the company in depth.
A Transformation Department responsible for monitoring the execution of the Company's transformation has been specially created for this purpose, it undertook to simplify the activities and modernize the management of the Group with the creation in particular of a Corporate Department to meet the new strategic challenges,the new organization thus goes from 4 to 5 Activities.

This transformation aims to place performance at the heart of the company's businesses. Among the major changes announced to strengthen the structure and skills of each activity, the Downstream division (LRP) is split into two separate entities, the first comprising the Liquefaction and Separation activities and the second, the Refining and Petrochemical activities. This creation makes it possible to erect the Refining-Petrochemicals activity into a full-fledged entity which will notably be in charge of the new projects currently being implemented and developed (HMD 3 refinery, PDHPP project, Conversion Skikda, Vapocraqueur, etc.).
As part of this in-depth reorganization, new Central Structures have been put in place within the Activities themselves, to optimize processes and support the new transformation of the Company. In particular, Group Communication has been elevated to the rank of Management.
To give more weight to cross-functional functions in the company and streamline the operating methods in this macrostructure, this transformation also involves strengthening the sovereign, cross-functional and expertise roles of the operational center. The representatives of the Central Functions in the Activities are given a mission of dual reporting intended for the hierarchical superior in the Activity (VP, Divisional, Director, etc.) and the superior in the function (DEX, DC, Function Director of the 'Activity...).

1.5.1 Operational structures

The operational structures are organized around the following activities:

- Exploration-Production.
- Pipeline Transport.
- Liquefaction and Separation.
- Refining and Petrochemicals.
- Marketing

Each activity carries out its businesses, develops its business portfolio and contributes, in its area of expertise, to the development of the Company's international activities.

1.5.1.1 Exploration-Production Activity

The Exploration-Production Activity is responsible for the development and application of policies and strategies for the exploration, development and exploitation of upstream oil and gas. within the framework of the Company's strategic objectives.

1.5.1.2 The Pipeline Transport Activity

The Pipeline Transport Activity is responsible for developing and implementing policies and strategies for the transportation of hydrocarbons by pipeline, within the framework of the Company's strategic objectives.

1.5.1.3 Liquefaction and Separation Activity

The Liquefaction and Separation Activity is responsible for developing and applying policies and strategies for the operation, management and development of gas liquefaction and separation activities, within the framework of the Company's strategic objectives.

1.5.1.4 The Refining and Petrochemicals Activity

The Refining and Petrochemicals Activity is responsible for developing and applying policies and strategies for the operation, management and development of refining and petrochemicals, within the framework of the Company's strategic objectives.

1.5.1.5 Marketing Activity

The Marketing Activity is responsible for developing and applying policies and strategies for marketing hydrocarbons abroad and on the national market, within the framework of the Company's strategic objectives.

1.5.2 The managements of SONATRACH

SONATRACH managements are :

- **Transformation Management** is responsible for coordinating and monitoring the implementation of the transformation plan for the transformation of the Company.
- **Communication Management** is responsible for developing and implementing SONATRACH's communication strategy.
- **Strategy,Planning & Économy Corporate Management**is responsible for the preparation and development in the medium and long term and for evaluating their implementation.
- **Finance Corporate Management** is responsible for developing policies and strategies in the area of Finance. It assesses their implementation and monitors the quality of financial information.

- **Business Development & Marketing Corporate Management** is responsible for formulating the growth strategy and seeking investment opportunities for the Company.
- **Human Resources Corporate Management** is responsible for developing human resources policies and strategies and monitoring their implementation.
- **Procurement & Logistics Central Management** is responsible for managing the Purchasing and Logistics processes for the Group
- **New Resources Central Management** is responsible for managing and operating, from the center, Unconventional Resources and Offshore projects.
- **Engineering & Project Management Central Management** oversees and executes the Group's major industrial projects.
- **Legal Affairs Central Management** is in charge of the development and harmonization of legal instruments and the control of their applications.
- **Digitalization & Information System Central Management**is responsible for defining and monitoring the Company's IT and digitalization policy.
- **Environment,Health & Safety Central Management** is responsible for developing policies on the environment, safety and quality of life at work. It ensures the control of their application.
- **Research & Development Central Management** is responsible for promoting and implementing the policy of applied research and development of technologies in the basic businesses of the Company.

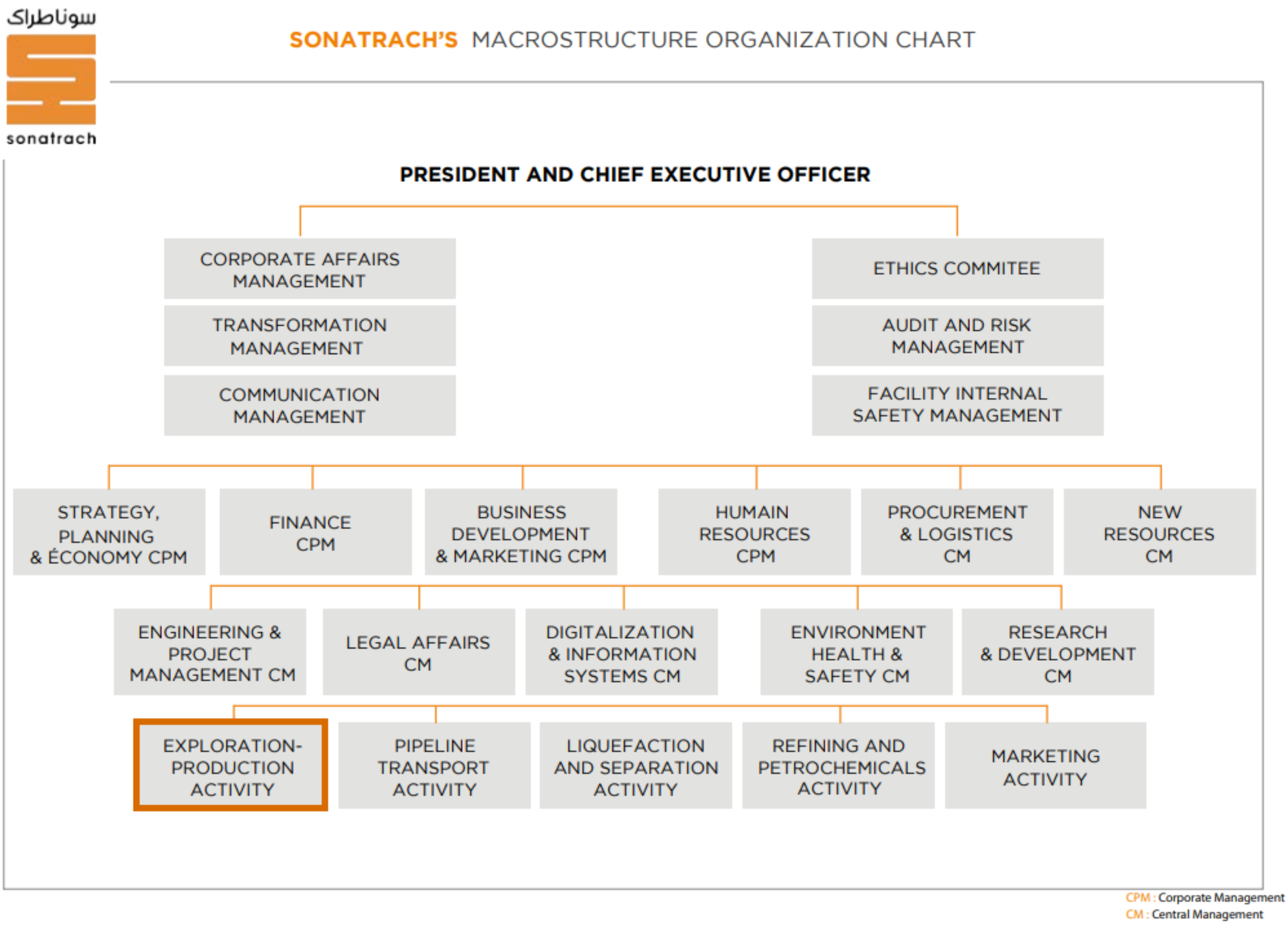

Figure 1.2: SONATRACH's Macrostructure Organization Chart

1.6 Presentation of Exploration-Production Activity

Since its creation, SONATRACH has focused its efforts on searching for new oil and gas deposits within the national territory in order to replenish its reserves and increase its production capacity.

Algeria, due to its geographical location and its rich hydrocarbon subsoil, has a vast mining domain that is still largely unexplored.

Drawing on its pioneering spirit, the Group is continuing its program to develop Algeria's hydrocarbon resources, relying on its technological expertise and know-how in exploration and production.

Producing more and producing better. This is the strategy shared by all of the Group's employees to meet the new challenges ahead.

The Exploration-Production (EP) Activity of SONATRACH is responsible for the search, development, exploitation and production of hydrocarbons.

It revolves around three axes:

1. The development and exploitation of deposits for an optimal exploitation of resources.

2. Management of activities in partnership in the exploration, development and exploitation phases of deposits.

3. Research, negotiation and development of new projects on the national territory and internationally.

1.7 Ever-increasing investments

In an international context marked by the volatility of hydrocarbon prices and the maintenance of sustained demand, both nationally and internationally, SONATRACH is committed to developing its Exploration-Production (EP) activity. The company continues to devote more than 80% of its annual investment budget to research, development, exploitation and production of hydrocarbons.
To increase the discovery of new deposits, SONATRACH has decided to devote 53 billion dollars to Exploration-Production over the period 2017-2021, including 9 billion dollars for exploration (3D seismic, exploration drilling, studies, etc.) . The objective is to achieve the drilling of some 100 wells per year. As part of its business transformation growth strategy, SONATRACH aims to double the annual volume of discoveries and increase proven reserves from 50 to 100 million TOE per year.

1.8 A new dynamic of cooperation

To accelerate its development program in Exploration-Production, SONATRACH is strengthening its policy of cooperation with major foreign oil and gas operators. Partnership has always been a key factor in the growth of SONATRACH in Algeria and around the world. It participates in the Company's overall effort to explore new territories and increase its hydrocarbon production.

1.9 Increase gas production

This growth strategy is all the more remarkable in natural gas: Europe's third gas supplier after Russia and Norway, Algeria plans to increase its gas production to more than 140 billion m^3 of here 2023.
To boost this dynamic, SONATRACH is increasing exploration projects on its own and in partnership in the gas regions of southwestern Algeria but also in the offshore. The first gas drilling in the Mediterranean will be launched in early 2019. Among the other avenues under study to find new resources, SONATRACH reaffirms its desire to develop unconventional resources. Today, Algeria ranks third in the world in terms of shale gas capacity reserves.

1.10 Preparing for the energy transition

Fully committed to the energy transition, SONATRACH is now focusing on the development of gas and solar to enrich its energy mix and be part of the national policy for the development of renewable energies. SONATRACH contributes to this ambitious program, aiming to cover by 2030 80% of the energy needs of its oil sites by solar power plants. The objective is to reduce, in a responsible and sustainable manner, the consumption of hydrocarbons on site and greenhouse gas emissions.
The Group intends to eventually acquire a production capacity of around 1.3 gigawatts. SONATRACH will allow the country, through its contribution to this program, to save

nearly 1.6 billion sm^3 of gas intended for sale by 2040.

1.11 Human Resources & Training

1.11.1 SONATRACH's Occupations

The Wealth of Expertise and Skills at the core of SONATRACH's Activities

1.11.1.1 Career opportunities to seize

SONATRACH places people at the heart of its strategy. As part of its business transformation development plan, human resources management is a high priority to meet the Group's need for qualified personnel. Also, to attract new talent and preserve the vital forces of the Company, the Company offers attractive career opportunities in more ways than one: in addition to the diversity of jobs offered in the buoyant and multidisciplinary energy sector, the collective and individual performance is recognized, encouraged and rewarded. In this regard, the Group's senior executives are mobilized to enhance the skills of SONATRACH employees, throughout their professional career, in the seven families of professions that the Company has today.

At all levels of the Company, Upstream and Downstream of the hydrocarbon value chain, the group must meet needs in seven families of businesses: the Hydrocarbons Research and exploitation of deposits family, the Hydrocarbon Transport family, the Hydrocarbon Transformation family, the Marketing family, the Studies & Development and Production family, the Maintenance family and the Security family.

1.11.1.2 High-level expertise

Undoubtedly, engineers play an important role here. They are the ones who will carry out the first field studies to estimate the potential of the reserves and choose the most suitable drilling and production technologies. Jobs related to the coordination of all operations are just as strategic in the transportation and processing of hydrocarbons. Similarly, all trades relating to the operation, maintenance and safety of industrial facilities are essential and varied throughout the oil and gas production chain. This is without counting all the positions to be filled in the development and implementation of the marketing of the product and in the monitoring of the receipt and loading flows of hydrocarbon products intended for the domestic and international markets.

1.11.2 Human Resources Policy of SONATRACH

Operational and managerial Excellence, Accountability, Equity and Transparency are the key factors of SONATRACH's Human Resources policy.
The support of employees around these corporate values strengthens the unity of the Group to build its development strategy.
Excellence is the key word of SONATRACH's new strategy,our success depends on the permanent search for operational and managerial performance at each level of the Company,the success of the Group's transformation plan is based on fairness,we need to encourage equal opportunities but also all the talents that are the lifeblood of the Company,it is everyone's responsibility,all our employees must become employee citizens by committing to act in favor of the economic and social development of Algeria and Algerians,to do this, we must work in complete transparency to encourage the support of teams around the same project: to raise SONATRACH among the five largest national oil companies in the world.

1.11.2.1 SONATRACH's Values

Simplicity and Action :
Promote the transformation of overly complex and heterogeneous procedures that hinder decision-making to better focus on value-creating action.
Delegation and Initiative :
Enhance the skills of our employees, by training them and empowering them to help them make the right decisions.
Value the initiative and risk-taking of everyone by rewarding them.
Communication and Cooperation :
Sharing our knowledge and information while joining forces within the company to be more efficient and face the problems to be solved.

1.11.3 Training

Training at the core of SONATRACH, engine of business development

1.11.3.1 A "win-win" relationship

The mission of the Human Resources activity is to reconcile the individual and collective objectives of the employees in order to achieve the support of the teams for the success of SONATRACH.
In pursuit of operational and managerial excellence, the Group makes it a point of honor

to encourage the promotion, professional mobility and in-house training of its employees throughout their careers.
Between the operational training in the field of energy and mines provided in particular by the Algerian Institute of Petroleum (IAP), and the learning of management techniques, finance, IT, legal and oil taxation or information systems, SONATRACH offers a very wide spectrum of advanced and specialization solutions to its employees.

1.11.3.2 Distance training by e-learning

The professionalization paths, targeted or even specific training and professional situations are part of this skills development system. The "SONATRACH Management Academy" Business Development Center offers new in-house training programs via e-learning. These distance learning courses make it possible to improve access to knowledge and continuous professional training for all Group employees.

1.11.3.3 Leadership Development

Developing skills and taking responsibility are among the major areas of modernization of the Company's management. This strategy aims to encourage the professional excellence of employees within the framework of a "Win-Win" relationship between the Employer and the Employees. SONATRACH Management Academy thus offers a leadership development program. As part of this movement, the "Top 200" program aims to train young talents in the Company in the most advanced management techniques. This program, limited to six months, aims to train and prepare 200 managers on the basis of "performance and excellence" to lead the future of SONATRACH. The objective is to develop targeted training programs to prepare for management succession and ensure the renewal of the skilled workforce in each of the Group's activities.

1.12 Conclusion

In Conclusion, this Chapter has provided an in-depth presentation of SONATRACH, offering insights into its history, key figures, organizational structure, activities, and management. It has also emphasized the vital role played by the Exploration-Production Activity within the organization, along with the significance of human resources and training. This foundational knowledge sets the stage for the subsequent chapters, where we delve into the theoretical and practical aspects of our research.

Chapter 2

Fundamental and Theoretical Notions

2.1 Introduction

This chapter delves into the fundamentals of Operations Research (OR) and Combinatorial Optimization, providing a solid theoretical foundation for the subsequent chapters. This chapter begins by exploring the historical origins of OR, tracing its development and evolution over time. It then proceeds to define OR and discuss its methodology and phases, highlighting the systematic approach used to solve complex problems. Furthermore, the chapter introduces the tools and techniques employed in OR, equipping the reader with the necessary knowledge to tackle real-world optimization challenges. A specific focus is placed on Combinatorial Optimization, particularly the Linear Programming Problem and the Assignment Problem. By understanding the fundamental concepts and principles covered in this chapter

2.2 The Origin of Operations Research (OR)

Operations Research (OR) started just before World War II in Britain with the establishment of teams of scientists to study the strategic and tactical problems involved in military operations,the objective was to find the most effective utilization of limited military resources by the use of quantitative techniques,following the war, numerous peacetime applications emerged, leading to the use of OR and management science in many industries and occupations.[M.G04]

2.3 What Is Operations Research (OR) ?

Operations Research (OR) is the study of mathematical models for complex organizational systems,optimization is a branch of OR which uses mathematical techniques such as linear and nonlinear programming to derive values for system variables that will optimize performance. Management science or operations research uses a logical approach to problem solving,this quantitative approach is widely employed in business,areas of application include forecasting, capital budgeting, capacity planning scheduling inventory management, project management and production planning.[M.G04]

2.4 The methodology / phases of operations research study

According to Sharma [H.S], the stages of development of OR are also known as approaches , methodology or phases and processes of operations research, OR is a scientific approach to decision-making, so it must follow the steps listed below in the order listed :

Step 1: Observe the problem environment
The first step in the process of O.R development is the problem environment observation, this step includes different activities, they are conferences, site visit, research, observations etc. . .
These activities provide sufficient information to the O.R specialists to formulate the problem.

Step 2: Analyze and define the problem
This step is analyzing and defining the problem, in this step in addition to the problem definition the objectives, uses and limitations of O.R study of the problem also defined, the outputs of this step are clear grasp of need for a solution and its nature understanding.

Step 3: Develop a model
This step develops a model, a model is a representation of some abstract or real situation. The models are basically mathematical models, which describes systems, processes in the form of equations, formula/relationships.
The different activities in this step are variables definition, formulating equations etc. . .
The model is tested in the field under different environmental constraints and modified in order to work, sometimes the model is modified to satisfy the management with the results.
A mathematical model should include mainly the following three basic sets of elements :

- Decision Variables and Parameters.
- Objective Function.
- Constraints or Restrictions.

Step 4: Select appropriate data input
A model works appropriately when there is appropriate data input, Hence, selecting appropriate input data is important step in the O.R. development stage or process.
The activities in this step include internal/external data analysis, fact analysis, and collection of opinions and use of computer data banks.
The objective of this step is to provide sufficient data input to operate and test the model developed in Step 3.

Step 5: Provide a solution and test its reasonableness
This step is to get a solution with the help of model and input data, this solution is not implemented immediately, instead the solution is used to test the model and to find if there is any limitations.
Suppose if the solution is not reasonable or the behaviour of the model is not proper, the model is updated and modified at this stage.
The output of this stage is the solution(s) that supports the current organizational objectives.

Step 6: Implement the solution
At this step the solution obtained from the previous step is implemented. The implementation of the solution involves too many behavioural issues, therefore, before implementation the implementation authority has to resolve the issues, a properly implemented solution results in quality of work and gains the support from the management.

Process Activities	Process	Process Output
Site visits, Conferences, Observations, Research	**Step 1:** Observe the problem environment	Sufficient information and support to proceed
Define: Use, Objectives,Limitations	**Step 2:** Analyze and define the problem	Clear grasp of need for and nature of solution requested
Define interrelationships,Formulate equations,Use known O.R. Model ,Search alternate Model	**Step 3:** Develop a Model	Models that works under stated environmental constraints
Analyze:internal-external data,Use computer data banks	**Step 4:** Select appropriate data input	Sufficient inputs to operate and test model
Test the model,find limitations,update the model	**Step 5:** Provide a solution and test its reasonableness	Solution(s) that support current organizational goals
Resolve behavioral issues,Sell the idea,Give explanations,Management involvement	**Step 6:** Implement the solution	Improved working and Management support for longer run operation of model

Table 2.1: The process, process activities, and process output of O.R's methodology

2.5 Tools and Techniques of Operations Research

Operations Research uses any suitable tools or techniques available. The common frequently used tools/techniques are mathematical procedures, cost analysis, electronic computation, However, operations researchers given special importance to the development and the use of techniques like linear programming, game theory, decision theory, queuing

theory, inventory models and simulation.
In addition to the above techniques, some other common tools are non-linear programming, integer programming, dynamic programming, sequencing theory, Markov process, network scheduling (PERT/CPM), symbolic Model, information theory, and value theory. There is many other Operations Research tools/techniques also exists. [Doc19]

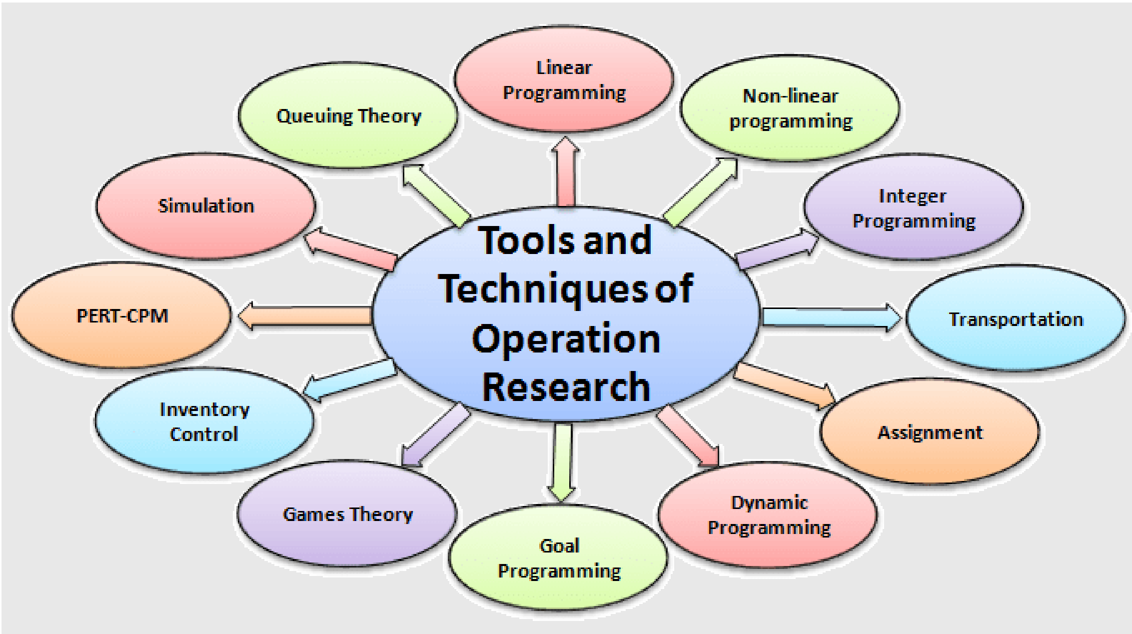

Figure 2.1: Tools and Techniques of Operations Research

2.6 Scope of Operations Research

OR is mainly concerned with the techniques of applying scientific knowledge, besides the development of science, it provides an understanding which gives the expert/manager new insights and capabilities to determine better solutions in his decision making problems, with great speed, competence and confidence.
OR has been found to be used in the following five major areas of research:

1. OR is useful to the Directing Authority in deciding optimum allocation of various limited resources such as men, machines, material, time, money, etc., for achieving the optimum goal.

2. OR is useful to Production Specialist in :

    - Designing, selecting and locating sites.
    - Determining the number and size.

- Scheduling and sequencing the production runs by proper allocation of machines.
- Calculating the optimum product mix.

3. OR is useful to the Marketing Manager (executive) in determining:
    - How to buy, how often to buy, when to buy and what to buy at the minimum possible cost.
    - Distribution points to sell the products and the choice of the customers.
    - Minimum per unit sale price.
    - The customers preference relating to the size, colour, packaging etc., for various products and the size of the stock to meet the future demand.
    - The choice of different media of advertising.
4. OR is useful to the Personnel Administrator in finding out:
    - Skilled persons at a minimum cost.
    - The number of persons to be maintained on full time basis in a variable work load like freight handling etc.
    - The optimum manner of sequencing personnel to a variety of jobs.
5. OR is useful to the Financial Controller to :
    - Find out a profit plan for the company.
    - Determine the optimum replacement policies.
    - Find out the long-range capital requirements as well as the ways and means to generate these requirements.[IAS20]

2.7 Mathematical Optimization

The field of optimization is concerned with the study of maximization and minimization of mathematical functions,very often the arguments of (i.e., variables or unknowns) in these functions are subject to side conditions or constraints,by virtue of its great utility in such diverse areas as applied science, engineering,economics, finance, medicine, and statistics, optimization holds an important place in the practical world and the scientific world, Indeed, as far back as the Eighteenth Century, the famous Swiss mathematician and physicist Leonhard Euler (1707-1783) proclaimed that " nothing at all takes place in the Universe in which some rule of maximum or minimum does not appear". [Y.Y22]

2.7.1 Combinatorial Optimization

Combinatorial optimization is a branch of mathematical optimization that has applications in artificial intelligence, theoretical computer science, applied mathematics, machine learning, software engineering, and many other domains.
It is related to computational complexity theory, algorithm theory, and operations research.
It is the process of identifying maxima or minima for an objective function that has a discreet domain with a large configuration space.
Combinatorial optimization refers mainly to the methods employed to solve optimization problems and generally does not offer guidelines on how to convert real-world problems into abstract mathematical questions, and vice versa.
The whole field of combinatorial optimization pretty much has its origin in graph theoretic concerns such as edge colorings in undirected graphs and matchings in bipartite graphs.
A lot of the early progress in combinatorial optimization is because of theorems in graph theory as well as their duals.
When **linear programming** first came into being, combinatorial optimization started getting applied on problems like **assignment**, maximal flow, and transportation.
Today, combinatorial optimization is especially used to study algorithms, particularly the ones used in artificial intelligence, machine learning, and operations research.[Eng18]

2.7.1.1 Linear Programming Problem

2.7.1.1.1 Introduction

A large number of business and economic situations are concerned with problems of planning and allocation of resources to various activities. In each case there are limited resources at our disposal and our problem is to make such a use of these resources so as to maximize production or to derive the maximum profit, or to minimize the cost of production etc, such problems are referred to as the problems of constrained optimization.
Linear programming (LP) is one of the most versatile, popular and widely used quantitative techniques. Linear Programming is a technique for determining an optimum schedule chosen from a large number of possible decisions, the technique is applicable to problem characterized by the presence of a number of decision variables, each of which can assume values within a certain range and affect their decision variables, the variables represent some physical or economic quantities which are of interest to the decision maker and whose domain are governed by a number of practical limitations or constraints which may be due to availability of resources like men, machine, material or money or may be due quality constraint or may arise from a variety of other reasons.

The most important feature of linear programming is presence of linearity in the problem. The word Linear stands for indicating that all relationships involved in a particular problem are linear. Programming is just another word for planning and refers to the process of determining a particular plan of action from amongst several alternatives.
The problem thus reduces to maximizing or minimizing a linear function subject to a number of linear inequalities.

2.7.1.1.2 Definitions of Various Terms Involved in Linear Programming

Linear Programming (LP) entails a multitude of terms that are fundamental to its comprehension and utility in optimization problems,these terms, based on the research of Malhotra and Jain [RJ12] are:

Linear :

The word linear is used to describe the relationship among two or more variables which are directly proportional, for example, if the production of a product is proportionately increased, the profit also increases proportionately, then it is a linear relationship,a linear form is meant a mathematical expression of the type,
$\mathrm{a}_1 X_1 + a_2 X_2 + ... + a_n X_n$ where $\mathrm{a}_1, a_2, ..., a_n$ are constant and $\mathrm{X}_1, X_2, ..., X_n$ are variables.

Programming :

The term Programming refers to planning of activities in a manner that achieves some optimal result with resource restrictions, a programme is optimal if it maximizes or minimizes some measure or criterion of effectiveness, such as profit, cost or sales.

Decision variables and their relationship

The decision (activity) variables refer to candidates (products, services, projects etc.) that are competing with one another for sharing the given limited resources. These variables are usually inter-related in terms of utilisation of resources and need simultaneous solutions. The relationship among these variables should be linear.
These variables are typically continuous and are represented by real numbers.However, in certain cases, integer or binary decision variables can also be used, Integer variables can only take on whole number values, while binary variables can only take on the values of 0 (false) or 1 (true). The choice of decision variable type depends on the problem's requirements and the nature of the variables being modeled.

Objective function :

The Linear Programming problem must have a well defined objective function for optimization.For example, maximization of profits or minimization of costs or total elapsed time of the system being studied. It should be expressed as linear function of

decision variables.
The general form of the objective function is expressed as :
Optimize (Maximize or Minimize) Z=$\sum C_j X_j$
Where: Z is the measure of performance variable (profit/cost), which is the function of $X_1, X_2, ..., X_n$.(Quantities of activities) , $C_j (C_1, C_2, ..., C_n)$ (parameters or coefficients) represent the contribution of a unit of the respective variables $X_1, X_2, ..., X_n$ to the measure of performance Z (the objective function).

Constraints :
There are always limitations on the resources which are to be allocated among various competing activities, these resources may be production capacity, manpower, time, space or machinery, these must be capable of being expressed as linear equalities or inequalities in terms of decision variables.
The general form of the constraints functions are expressed as :

$$\sum a_{ij} X_j \quad (<, >, \leq, =, \geq) \quad b_i$$

- a_{ij}'s are called technical coefficients and measure the per unit consumption of the resources for executing one unit of unknown variable (activities)X_j , a_{ij}'s can be positive, negative or zero in the given constraints.
- The b_i represents the total availability of the i$^{\text{th}}$ resource , it is assumed that $b_i \geq 0$ for all i. However, if any $b_i < 0$, then both sides of the constraint i can be multiplied by 1 to make $b_i > 0$ and reverse the inequality of the constraint.

Alternative courses of action :
There must be alternative courses of action. For example, there may be many processes open to a firm for producing a commodity and one process can be substituted for another.

Non-negativity restriction :
All the variables must assume non-negative values, that is, all variables must take on values equal to or greater than zero (≥ 0). Therefore, the problem should not result in negative values for the variables.

Linearity and divisibility :
All relationships (objective functions and constraints) must exhibit linearity, that is, relationships among decision variables must be directly proportional. For example, if

our resources increase by some percentage, then it should increase the outcome by the same percentage. Divisibility means that the variables are not limited to integers. It is assumed that decision variables are continuous, i.e., fractional values of these variables must be permissible in obtaining an optimal solution.

Deterministic :
In LP model (objective functions and constraints), it is assumed that the entire model coefficients are completely known (deterministic), e.g. profit per unit of each product, and amount of resources available are assumed to be fixed during the planning period.

Formulation of a Linear Programming Problem :
The formulation of the Linear Programming Problem (LPP) as mathematical model involves the following key steps:
Step 1 : Identify the decision variables to be determined and express them in terms of algebraic symbols as $X_1, X_2, ..., X_n$.
Step 2 : Identify the objective which is to be optimized (maximized or minimized) and express it as a linear function of the above defined decision variables.
Step 3 : Identify all the constraints in the given problem and then express them as linear equations or inequalities in terms of above defined decision variables.
Step 4 : Non-negativity restrictions on decision variables[RJ12].

2.7.1.1.3 Mathematical Formulation of a General Linear Programming Problem

The general formulation of LP problem can be stated as follows.
If we have n-decision variables $X_1, X_2, ..., X_n$, and m constraints in the problem , then we would have the following type of mathematical formulation of LP problem :
Optimize (Maximize or Minimize) the objective function:

$$Z = C_1X_1 + C_2X_2 + ... + C_nX_n$$

Subject to satisfaction of m-constraints:

$$\begin{cases} a_{11}X_1 + a_{12}X_2 + ... + a_{1n}X_n \ \ (\leq = \geq) \ \ b_1 \\ a_{21}X_1 + a_{22}X_2 + ... + a_{2n}X_n \ \ (\leq = \geq) \ \ b_2 \\ a_{i1}X_1 + a_{i2}X_2 + ... + a_{in}X_n \ \ (\leq = \geq) \ \ b_i \\ a_{m1}X_1 + a_{m2}X_2 + ... + a_{mn}X_n \ \ (\leq = \geq) \ \ b_m \end{cases}$$

Where the constraint may be in the form of an inequality ($\leq$ *or* $\geq$) or even in the form of an equality ($=$) and finally satisfy the non-negativity restrictions :

$$X_1, X_2, ..., X_n \geq 0$$

where C_j (j=1, 2,...,n); b_i (i=1, 2,...,m) and a_{ij} are all constants and m<n, and the decision variables $X_j \geq 0$, j=1, 2,..,n.
If b_i is the available amount of resource i then a_{ij} is amount of resource i that must be allocated (technical coefficient) to each unit of activity j.[IAS20]

Matrix form of LP problem :
the previous linear programming problem can be expressed in matrix form as :

$$\text{LP} \quad \begin{cases} Maximize \quad Z = CX \quad (objective function) \\ Subject \;\; to \\ \qquad\qquad \text{Ax} = \text{b}, \quad \text{b} \geq 0 \quad \text{(constraint equation)} \\ \qquad\qquad \text{x} \geq 0 \quad \text{(non-negativity restriction)} \end{cases}$$

Where X = $(X_1, X_2, ..., X_n)$, C = $(C_1, C_2, ..., C_n)$ and b = $(b_1, b_2, ..., b_m)$ [RJ12]

$$\text{A=} \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{21} & \dots & a_{2n} \\ a_{m1} & a_{m1} & \dots & a_{mn} \end{pmatrix}$$

2.7.1.1.4 Graphical Solution of Linear Programming Problem

According to Malhotra and Jain [RJ12] LP problems which involve only two variables can be solved graphically. Such a solution by geometric method involving only two variables is important as it gives insight into more general case with any number of variables.

Feasible solution :
A set value of the variables of a linear programming problem which satisfies the set of constraints and the non-negative restrictions is called feasible solution of the problem.

Feasible region :
The collection of all feasible solutions is known as the feasible region. Any point which does not lie in the feasible region cannot be a feasible solution to the LP problems. The feasible region does not depend on the form of the objective function in any way. If we can represent the relations of the general LP problem on an n dimensional space, we will obtain a shaded solid figure (known as Convex-polyhedron) representing the domain of the feasible solution.

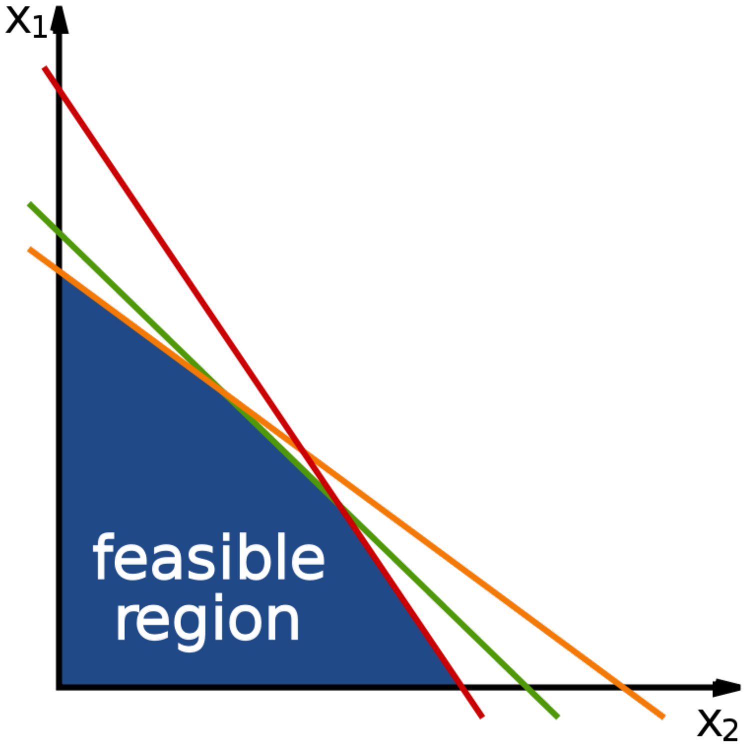

Figure 2.2: An illustration of Feasible region

Optimal solution :

A feasible solution of a linear programming problem which optimizes its objective function is called the optimal solution of the problem. Theoretically, it can be shown that objective function of a LP problem assumes its optimal value at one of the vertices (called extreme points) of this solid figure.

Steps to find graphical solution of the linear programming problem :

Step 1: Formulate the linear programming problem.

Step 2: Draw the constraint equations on XY-plane.

Step 3: Identify the feasible region which satisfies all the constraints simultaneously. For less than or equal to constraints the region is generally below the lines and for greater than or equal to constraints, the region is above the lines.

Step 4: Locate the solution points on the feasible region. These points always occur at the vertices of the feasible region.

Step 5: Evaluate the optimum value of the objective function.

Theorem:

A fundamental theorem in LP states that the feasible region of any LP is a convex polygon (that is the n dimensional version of two dimensional polygon), with a finite number of vertices, and further for any LP problem, there is at least one vertex which provides an optimal solution,whenever a LP problem has more than one optimal solution, we say that there are alternative optimal solutions. Physically, this means that the resources can be combined in more than one way to maximize profit.[IAS20]

2.8 Solution methods for Combinatorial Optimization Problems

2.8.1 Introduction

Let us consider an algorithm of solving Combinatorial Optimization (CO) problems as a procedure that either returns a subset (may be empty) of feasible solutions or indicates that the problem is insolvable.
CO algorithms can be classified according to many characteristics, of which the type of solution is the most important. Depending on the type of solution, there are exact, approximate, and heuristic algorithms.

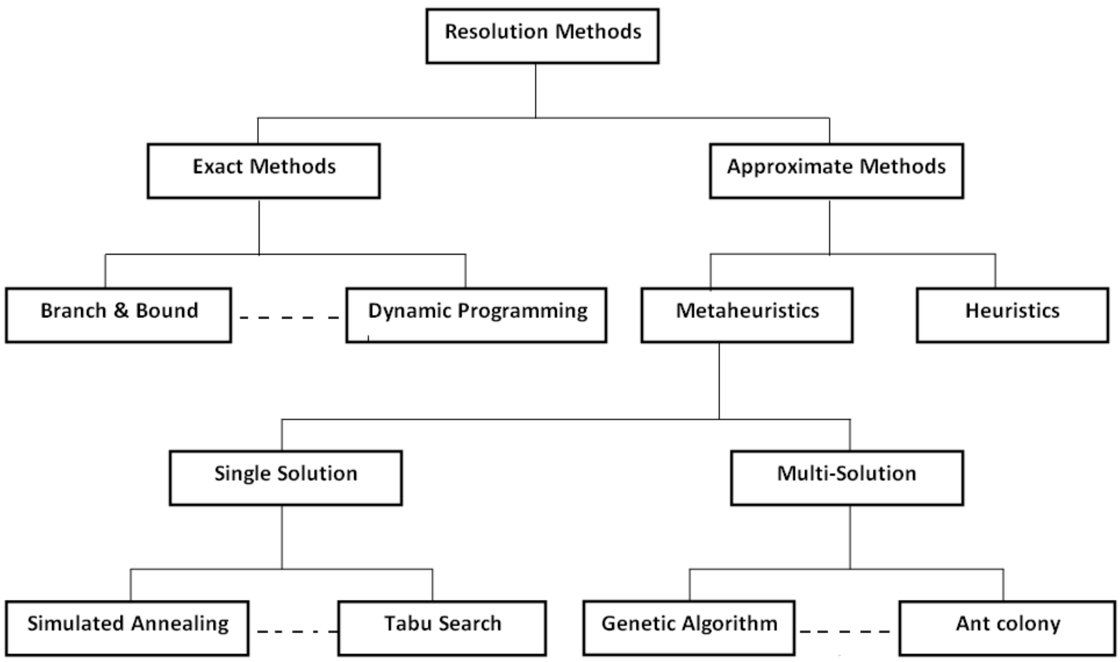

Figure 2.3: Classification of main combinatorial optimization methods

2.8.2 Exact methods

Exact algorithms guarantee the return of the optimal solution in a finite time or conclude that it does not exist if the problem is insolvable. These algorithms can be divided into two classes: general and special methods. General methods can be applied to a rather wide class of problems. Most popular are the exhaustive search, branch-and-bound algorithm,branch-and-cut method, sequential analysis and elimination of alternatives, and dynamic programming . Special methods such as the Hungarian method that is used to solve a linear assignment problem are developed for specific tasks.[VHI09]

2.8.2.1 Branch and Bound Method

Branch and bound, or B&B, is an algorithm design paradigm that solves combinatorial and discrete optimization problems. Many optimization issues, such as crew scheduling, network flow problems, and production planning, cannot be solved in polynomial time. Hence, B&B is a paradigm that is widely used to solve such problems.

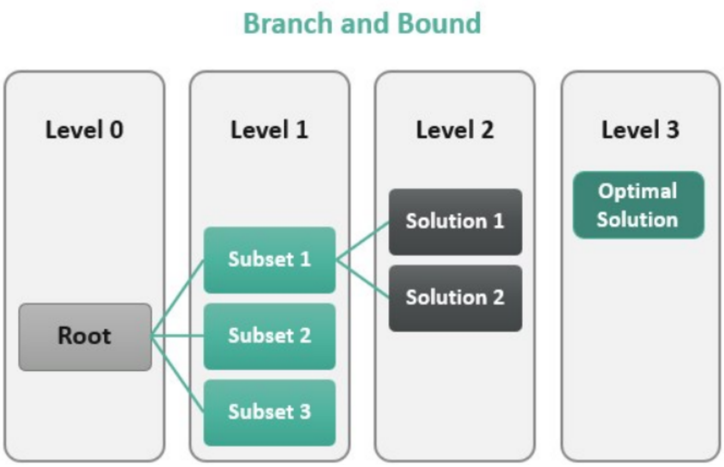

Figure 2.4: Branch And Bound Algorithm

This algorithm heavily depends on efficiently estimating the search space's lower and upper limits of branches,it degenerates to an exhaustive or thoroughgoing search if no limits are found,the branch and bound method solves these complex problems relatively faster,in most discrete optimization problems, the B&B method is a reliable choice to solve the issue.

Branch And Bound Algorithm Explained :

Branch and bound is an algorithm used to solve combinatorial optimization problems. These problems are usually augmented in terms of time complexity and issues that

require exploring all permutations and combinations possible in a worst-case scenario.
The branch and bound calculator is typically used to solve problems that are not solvable in polynomial time, such as network flow issues, crew scheduling, and production planning. However, it can be better understood as a backtracking tool using the state space tree.
As the name suggests, B&B explores branches or nodes under a particular subset of solutions,the parent node is considered to have all the possible solutions to the said problem,however, before the candidate solutions to the issue are computed or enumerated, the upper and lower limit binds the optimal solution.
The branch and bound method were put forth first by Alisa Land and Alison Doig from the London School of Economics in 1960 as a discrete programming solution[A.R21]. However, the name "B&B" was first used officially in a study on traveling salesman issues. [Moj21]

Advantages And Disadvantages :
Let us understand the advantages and disadvantages of B&B through the points below:[Moj21]
Advantages :

- **Time Complexity:** The B&B algorithm does not explore all nodes in the tree. Thereby, the time complexity is significantly lesser than most other algorithms used in exact methods.
- **Optimal Solution:** If the branching is done reasonably, the algorithm can find the optimal solution in a reasonable period.
- **Clear Pattern:** The B&B algorithm does not repeat the notes to explore the tree for the candidate solutions; instead, it follows a minimal path to derive the optimal solution.

Disadvantages :

- **Time-consuming:** Based on the size of the problem, the number of nodes that are computed might be too large in a worst-case scenario, making it a time-consuming process.
- **Parallelization:** The branching out of possible solutions provides scope for speculative parallelism. However, when alternative actions are considered for the said action, the branch and bound calculator finds difficulty.

2.8.3 Approximate methods

Approximate algorithms necessarily return an alternate solution (if exists) in a finite time, and the accuracy of these solutions can be estimated. There are two types of accuracy estimates: a priori and a posteriori. An a priori estimate of accuracy is a guaranteed estimate specified before the problem is begun to be solved. A posteriori estimates are calculated during or after the solution and allow for this solution.
By heuristic algorithms (heuristics) are usually meant algorithms with absent or unknown accuracy estimates. This means that for a specific problem, the algorithm may return a somehow “bad” (in the sense of the objective function) or no solution. At the same time, heuristics are correct in the sense that they do not return alternate solutions that are not feasible.[VHI09]
Such algorithms have shown to be efficient for many practical problems. Moreover, heuristics may often be a unique way to obtain a ’good’ solution in a reasonable time. However, it is important to note that the efficiency and effectiveness of these ’inexact’ algorithms, both those with accuracy estimates and heuristic algorithms, can vary depending on the size and complexity of the problem being solved. As the problem size increases, the time required to explore the search space or evaluate potential solutions may significantly increase, making it more challenging to find optimal or near-optimal solutions within a reasonable time frame.
In addition to their efficiency and effectiveness, metaheuristics offer flexibility and versatility in solving a wide range of problems. Their adaptive nature allows them to handle complex optimization landscapes and adapt their search strategies accordingly. Furthermore, metaheuristics often strike a balance between exploration and exploitation, exploring diverse regions of the search space while exploiting promising areas. By leveraging problem-specific knowledge and heuristics, metaheuristics can provide robust and practical solutions even in the presence of noisy or incomplete information.
All “inexact” algorithms,are often called approximate algorithms.[VHI09]

2.8.3.1 Genetic Algorithm

The genetic algorithm (GA), developed by John Holland and his collaborators in the 1960s and 1970s [S.X14] ,is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The genetic algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm selects individuals from the current population to be parents and uses them to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution. You can apply the genetic algorithm to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, nondifferentiable, stochastic, or highly nonlinear. The genetic algorithm can address problems of mixed integer programming, where some components are restricted to be integer-valued.[Mat20]

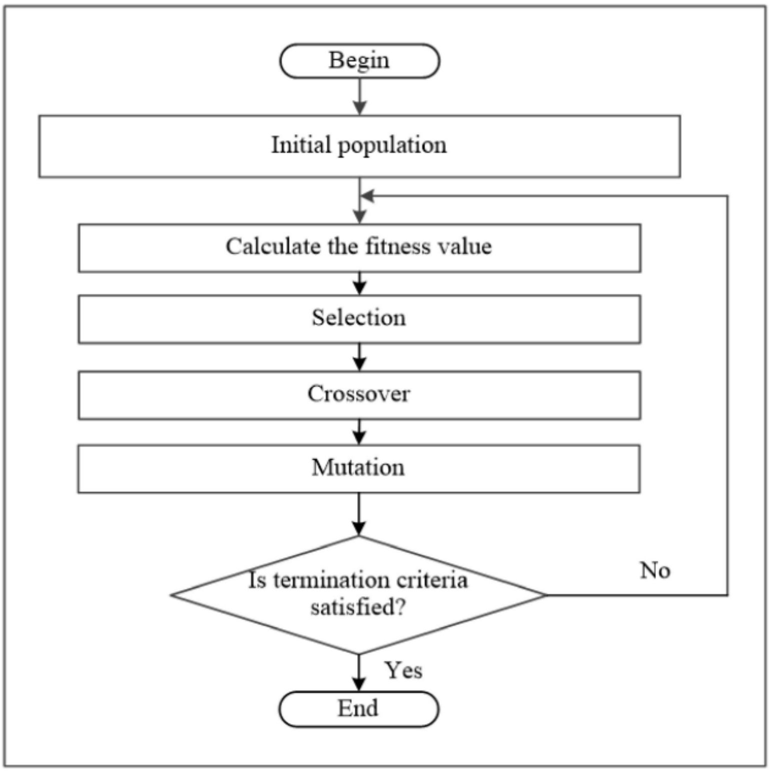

Figure 2.5: Flowchart of the standard genetic algorithm (GA)

The genetic algorithm uses three main types of rules at each step to create the next generation from the current population:[Mat20]

- Selection rules select the individuals, called parents, that contribute to the population at the next generation,the selection is generally stochastic, and can depend on the individuals' scores,there are various selection techniques used in genetic algorithms, such as tournament selection, roulette wheel selection, or rank-based selection.

- Crossover rules combine two parents to form children for the next generation,Common crossover operators include single-point crossover, two-point crossover, and uniform crossover.

- Mutation rules apply random changes to individual parents to form children,Examples of mutation operators include bit-flip mutation, swap mutation, and inversion mutation.

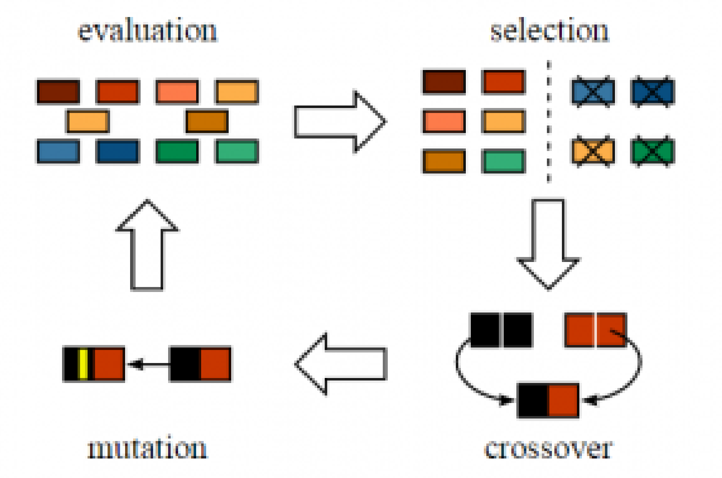

Figure 2.6: Illustration of the Genetic Algorithm

Advantages And Disadvantages : The followings are the advantages and disadvantages of Genetic Algorithm:[You22]

Advantages :

- Exploration of Search Space.
- Flexibility.
- Adaptability.
- Parallel Processing.
- Global Optimization.

Disadvantages :

- Higher computational complexity compared to other metaheuristics.
- Difficulty in Tuning Parameters.
- Dependence on Randomness.
- Risk of Premature Convergence.
- Limited Understanding of Results.

2.8.3.2 Hybrid Algorithm

Hybrid algorithms are two or more algorithms that run together and complement each other to produce a profitable synergy from their integration.
Hybridization aims to combine the advantages of each algorithm to form a hybrid algorithm, while simultaneously trying to minimize any substantial disadvantage. In general, the outcome of hybridization can usually make some improvements in terms of either computational speed or accuracy.[AM17]

Motivations for Hybridization
In a hybrid algorithm, two or more algorithms are collectively and cooperatively solving a predefined problem.[AM17]

- Unified purpose hybrids:Under this category, all sub-algorithms are utilized to solve the same problem directly; and different sub- algorithms are used in different search stages. Hybrid metaheuristic algorithms with local search is a typical example. The global search explores the search space, while the local search is utilized to refine the areas that may contain the global optimum.

- Multiple purpose hybrids: One primary algorithm is utilized to solve the problem, while the sub-algorithm is applied to tune the parameters for the primary algorithm. For example, PSO can be applied to find the optimal value of mutation rate in GAs.

Taxonomy of Hybrid Algorithms
Generally speaking, hybrid algorithms can be grouped into two categories:[XJ15]
Collaborative Hybrids
This involves the combination of two or more algorithms running either in sequential or parallel. The contributing weight of each participating algorithm can be regarded as half and half in the simplest case. The possible frameworks of the hybrid algorithms under this category are illustrated in Figure 2.6 Three structures are depicted in this figure, which are:

- Multi-stage. There are two stages involved in this case. The first algorithm acts as the global optimizer whereas the second algorithm performs local search.

- Sequential. In this structure, both algorithms are run alternatively until one of the convergence criteria is met. For simplicity, both algorithms will be run for similar number of iterations before proceeding to the next algorithm.

- Parallel. Two algorithms are run simultaneously, manipulating on the same population. One of the algorithms may be executed on a pre-specified per-centage of an algorithm

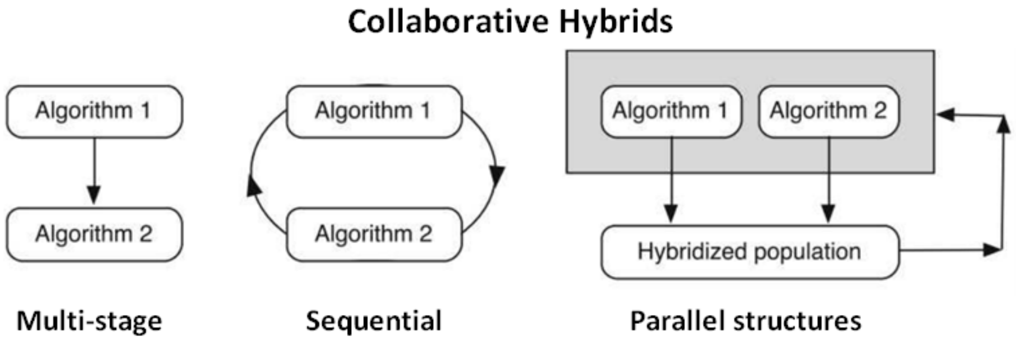

Figure 2.7: Taxonomy of Collaborative Hybrids

Integrative Hybrids

In this aspect, one algorithm is regarded as a subordinate, embedded in a master metaheuristic. For this category, the contributing weight of the secondary algorithm is approximately 10–20 %,this involves incorporation of a manipulating operator from a secondary algorithm into a primary algorithm.

- Full manipulation: The entire population is manipulated at every iteration. Such operation can be integrated inline with the existing source code, usually as a subroutine/subfunction.
- Partial manipulation: In this manipulation, only a portion of the entire population is accelerated using local search methods such as gradient methods,choosing the right portion and the right candidate to be accelerated pose a great challenge in assuring the success of this hybrid structure.

Advantages

- Increasing the diversity in a population.
- enhancing the search capability of the developed hybrid
- flexible and robust upon high dimensional data.
- enhance the search capability of the developed algorithm.

Disadvantages

- Naming Convention:Some researchers adopt very different names to their hybrid algorithms.
- Increase the complexity of the algorithm : the hybridization process usually creates extra components in the overall architecture of the hybrid algorithm.

2.9 The different classes of complexity

In computer science there exist several problems and these problems can be classified according to their hardness on various categories. These problems can be classified into four complexity classes as: P, NP, NP-Complete and NP-Hard.

2.9.1 Classification

According to Pokharel[Pok20],to classify a problem into any one of the classes the problem must be computable, there must exist some algorithm that can solve the problem. Computable problems are often referred as “solvable”, “decidable” and “recursive” problems. In other words, a problem is said to be computable if there exists a Turing machine that computes an answer for the problem. Non-computable problems are those for which there exists no algorithm to solve it.
All the computable problems can be placed in any one of the classes according to their hardness. Let’s use the real-life approach for classification using “Easy-to-Hard scale” as:

- Easy $\rightarrow P$.
- Medium $\rightarrow NP$.
- Hard $\rightarrow NP - Complete$.
- Hardest $\rightarrow NPHard$.

2.9.1.1 P (Polynomial) problems:

P problems refer to the problems where an algorithm would take a polynomial amount of time to solve the problem. P class problems can be solved in polynomial time by deterministic Turing machine.

2.9.1.2 NP (Non-deterministic Polynomial) Problems

These class of problems cannot be solved in the polynomial time. However, they can be verified in polynomial time. In other words, we can state NP problems as problems that satisfy following:

- Decision problems where a solution can be verified by a deterministic Turing machine in polynomial time.
- Decision problems where a solution can be found by non-deterministic Turing machine.

We expect these algorithms to have an exponential or factorial time complexity.
In essence, NP class problems don't have a polynomial run-time to solve, but have a polynomial run-time to verify solutions (difficult to solve, easy to check a given answer)

2.9.1.3 NP-Complete Problems

The NP-Complete class includes all the problems in class NP that satisfy an additional property of completeness. This distinction of completeness states 'for any problem that is NP-Complete, there exists a polynomial time algorithm that can transform (reduce) the problem into any other NP-Complete problem'.
Simply we can say that, NP-Complete problems belong to NP, but are among the hardest in the set. Right now, there are more than 3000 of these problems and increasing day by day.

2.9.1.4 NP-Hard Problems

The last set of problems contains the hardest, most complex problems in computer science. They are not only hard to solve but are hard to verify as well. These algorithms have a property similar to ones in NP-Complete- they can all be reduced to any problem in NP. Because of that these are in NP-Hard and are at least as hard as any other problem in NP.
Simply, A problem is classified as NP-Hard when an algorithm for solving it can be reduced to solve any NP problem. Hence NP-Hard problems are at least as hard as NP problem, but could be much harder or more complex.
The graphical representation of the complexity class can be given as follows:

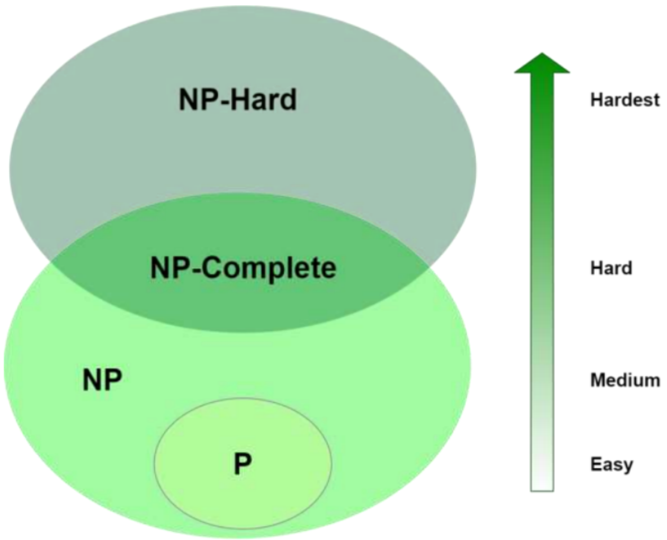

Figure 2.8: The graphical representation of the complexity class

We can generalize the findings as follows:

- P problems are quick to solve
- NP problems are quick to verify but slow to solve
- NP-Complete problems are also quick to verify but slow to solve and can be reduced to any other NP-Complete problem.
- NP-Hard problems are slow to verify, slow to solve and can be reduced to any other NP problem.

2.10 Assignment Problem

2.10.1 Introduction

Assignment problem is a special type of linear programming problem which deals with the allocation of the various resources to the various activities on one to one basis. It does it in such a way that the cost or time involved in the process is minimum and profit or sale is maximum.
The problem of assignment arises because available resources such as men, machines etc. have varying degrees of efficiency for performing different activities, therefore, cost, profit or loss of performing the different activities is different.
Thus, the problem is “How should the assignments be made so as to optimize the given

objective". Some of the problem where the assignment technique may be useful are assignment of workers to machines, salesman to different sales areas.

2.10.2 Definition of Assignment Problem:

Suppose there are n jobs to be performed and n persons are available for doing these jobs. Assume that each person can do each job at a term, though with varying degree of efficiency, let C_{ij} be the cost if the i^{th} person is assigned to the j^{th} job. The problem is to find an assignment (which job should be assigned to which person one on-one basis) So that the total cost of performing all jobs is minimum, problem of this kind are known as assignment problem.[Eng16]

The assignment problem can be stated in the form of n x n cost matrix C real members as given in the following table:

$$\begin{array}{c} \\ \textbf{Persons} \end{array} \begin{array}{c} \textbf{Jobs} \\ \left[\begin{array}{ccccccccc} & \textbf{1} & \textbf{2} & \textbf{3} & & \textbf{j} & & \textbf{n} \\ \textbf{1} & C_{11} & C_{12} & C_{13} & \ldots\ldots & C_{1j} & \ldots\ldots\ldots & C_{1n} \\ \textbf{2} & C_{21} & C_{22} & C_{23} & \ldots\ldots & C_{2j} & \ldots\ldots\ldots & C_{2n} \\ \textbf{3} & C_{31} & C_{32} & C_{33} & \ldots\ldots & C_{3j} & \ldots\ldots\ldots & C_{3n} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots \\ \textbf{i} & C_{i1} & C_{i2} & C_{i3} & \ldots\ldots & C_{ij} & \ldots\ldots\ldots & C_{in} \\ \textbf{n} & C_{n1} & C_{n2} & C_{n3} & \ldots\ldots & C_{nj} & \ldots\ldots\ldots & C_{nn} \end{array}\right] \end{array}$$

2.10.3 Mathematical Formulation of the Assignment Problem :

Mathematically an assignment Problem can be stated as follows :[Eng16]
Minimise the total cost

$$Z = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}\ X_{ij} \right)$$

where

$$X_{ij} = \begin{cases} 1 & \text{if } i^{th} \text{ person is assigned to the } j^{th} \text{ job} \\ 0 & \text{if not} \end{cases}$$

Subject to constraints

$$\text{(i).}\quad \sum_{i=1}^{n} X_{ij} = 1 \quad \text{j=1,2,...,n}$$

Which means that only one job is done by the i^{th} person, i=1,2,...,n.

$$\text{(ii).}\quad \sum_{j=1}^{n} X_{ij} = 1 \quad \text{i=1,2,...,n}$$

Which means that only one person should be assigned to the j^{th} job, j=1,2,...,n.[Eng16]

2.11 Solution Of Assignment Problem

2.11.1 Introduction

Although assignment problem can be solved either by using the techniques of Linear Programming or by the transportation method yet the assignment method developed by Dénes König a Hungarian mathematician known as the Hungarian method of assignment problem is much faster and efficient. In order to use this method, one needs to know only the cost of making all the possible assignments.

2.11.2 Hungarian Method for Solving Assignment Problem

The Hungarian method of assignment provides us with an efficient means of finding the optimal solution, according to Malhotra and Jain [RJ12], The Hungarian method is based upon the following principles:

1. If a constant is added to every element of a row and/or column of the cost matrix of an assignment problem the resulting assignment problem has the same optimum solution as the original problem or vice versa.

2. The solution having zero total cost is considered as optimum solution.

Hungarian method of assignment problem (minimization case) can be summarized in the following steps:

Step I: Subtract the minimum cost of each row of the cost (effectiveness) matrix from all the elements of the respective row so as to get first reduced matrix.

Step II: Similarly subtract the minimum cost of each column of the cost matrix from all the elements of the respective column of the first reduced matrix. This is first modified matrix.

Step III: Starting with row 1 of the first modified matrix, examine the rows one by one until a row containing exactly single zero elements is found. Make any assignment by making that zero in or enclose the zero inside a. Then cross (X) all other zeros in the column in which the assignment was made. This eliminates the possibility of making further assignments in that column.

Step IV: When the set of rows have been completely examined, an identical procedure is applied successively to columns that is examine columns one by one until a column containing exactly single zero element is found. Then make an experimental assignment in that position and cross other zeros in the row in which the assignment has been made.

Step V: Continue these successive operations on rows and columns until all zeros have been either assigned or crossed out and there is exactly one assignment in each row and in each column. In such case optimal assignment for the given problem is obtained.

Step VI: There may be some rows (or columns) without assignment i.e. the total number of marked zeros is less than the order of the matrix. In such case proceed to step VII.

Step VII: Draw the least possible number of horizontal and vertical lines to cover all zeros of the starting table. This can be done as follows:

1. Mark ($\surd$) in the rows in which assignments has not been made.
2. Mark column with ($\surd$) which have zeros in the marked rows.
3. Mark rows with ($\surd$) which contains assignment in the marked column.
4. Repeat 2 and 3 until the chain of marking is completed.
5. Draw straight lines through marked columns.
6. Draw straight lines through unmarked rows.

By this way we draw the minimum number of horizontal and vertical lines necessary to cover all zeros at least once. It should, however, be observed that in all n x n matrices less than n lines will cover the zeros only when there is no solution among them. Conversely, if the minimum number of lines is n, there is a solution.

Step VIII: In this step, we

1. Select the smallest element, say X, among all the not covered by any of the lines of the table.

2. Subtract this value X from all of the elements in the matrix not covered by lines and add X to all those elements that lie at the intersection of the horizontal and vertical lines, thus obtaining the second modified cost matrix.

Step IX: Repeat Steps IV, V and VI until we get the number of lines equal to the order of matrix I, till an optimum solution is attained.

Step X: We now have exactly one encircled zero in each row and each column of the cost matrix. The assignment schedule corresponding to these zeros is the optimum assignment.

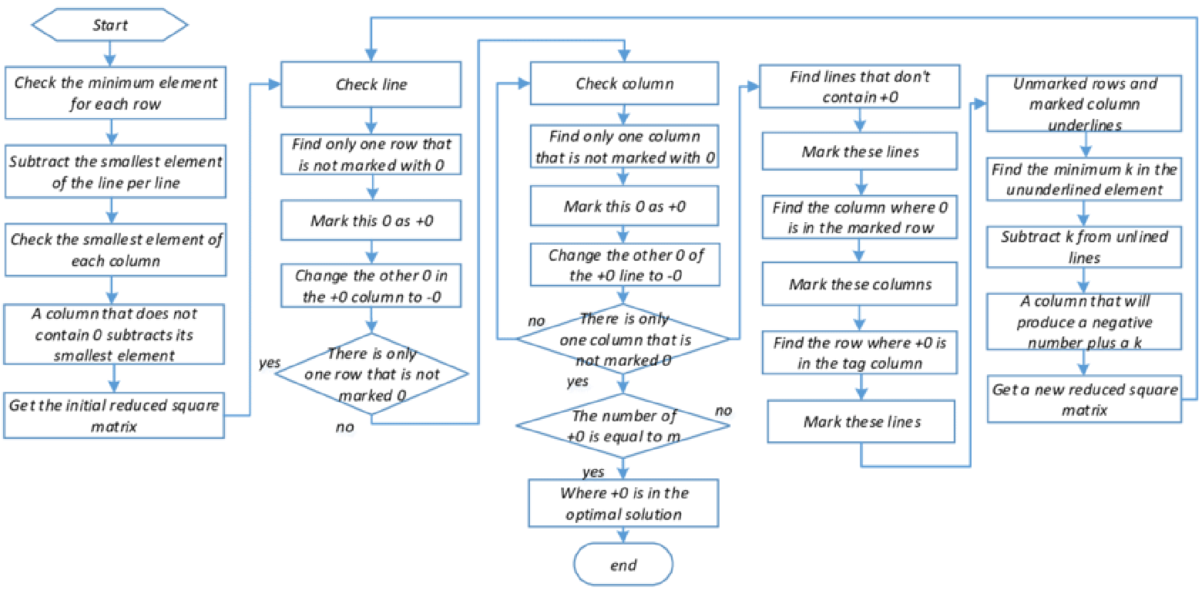

Figure 2.9: Hungarian method flow chart

2.12 Conclusion

In conclusion, This chapter has provided a comprehensive overview of the fundamentals of Operations Research (OR) and Combinatorial Optimization. The chapter began by tracing the historical origins of OR, highlighting its evolution into a systematic approach for solving complex problems. The methodology and phases of OR were discussed, emphasizing the structured process employed in conducting OR studies. The chapter also introduced a range of tools and techniques used in OR, laying the groundwork for their application in subsequent chapters. Notably, Combinatorial Optimization, including the

Linear Programming Problem and the Assignment Problem,Overall, this chapter equips readers with a solid theoretical understanding of OR and Combinatorial Optimization, setting the stage for the practical implementation and modeling discussed in the following chapters.

Chapter 3

Modeling and Development of a Computer Solution

3.1 Introduction

This Chapter focuses on the practical aspects of our research, specifically the modeling and development of a computer solution. We begin by formulating the problem at hand, establishing its objectives and constraints. This chapter presents the general mathematical model that captures the essence of the problem and guides our solution development process. Furthermore, a detailed case study is presented, showcasing the application of the mathematical model to a real-world scenario. We also explore the complexity of the model, considering its intricacies and potential challenges.

3.2 Problem Formulation

The manager of the Sonatrach petroleum site confronted a significant challenge in optimizing the operators planning for the 24-hour operation on petroleum site located in the southern of algeria.
The site manager seeks to create n teams of versatile operators (engineers) in the field in order to carry out the various tasks necessary for the site. The n teams will work on H successive shifts. The site manager must create n balanced teams in order to maximize the profitability of n teams at the same time. There are m engineers to create n teams of T operators (engineers) each.
Balancing the teams in terms of utility is crucial to ensure that each team contributes equally and maintains the same level of work quality during their shift.

3.3 The mathematical model

To formulate the objective function and constraints as a mathematical programming problem, we can define decision variables, introduce intermediate variables, and express the objective function and constraints accordingly.
Here's the formulation:

3.3.1 Decision Variables

$$x_{ij}=\begin{cases} 1 & \text{if i}^{\text{th}} \text{ engineer is assigned to the j}^{\text{th}} \text{ team} \\ 0 & \text{if not} \end{cases}$$

i = 1 to 30 and j = 1 to 3

3.3.2 Parameters:

u_i: Utility of engineer i. (i = 1 to 30) (e, f) : pairs of incompatible engineers that should not be in the same team.

3.3.3 The Objective Function

$$\min \qquad \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}|\sum_{i=1}^{m} u_i x_{ij} - \sum_{i=1}^{m} u_i x_{ik}|$$

It minimizes the utility gap between the n teams of engineers such that u_i is the utility of engineer i.

3.3.4 The Constraints

$$\sum_{i=1}^{m} x_{ij} = T \qquad \forall j = 1, .., n$$

Each team should have exactly T engineers.

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad \forall i = 1, 2, ..., m$$

Each engineer should be assigned to exactly one team.

$$x_{ej} + x_{fj} \leq 1 \qquad \forall j = 1, .., 3$$
$$\forall (e, f) \in I$$
$$I = \{1, ..., m\}$$

We assume that (e, f) are pairs of incompatible engineers that should not be in the same team.
Binary Constraints:
$x_{ij} \in \{0, 1\}$, for all i and j.

3.3.5 The General Model

$$\min \quad \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}|\sum_{i=1}^{m} u_i x_{ij} - \sum_{i=1}^{m} u_i x_{ik}| \tag{3.1}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} x_{ij} = T \qquad \forall j = 1, .., n \tag{3.2}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad \forall i = 1, 2, ..., m \tag{3.3}$$

$$x_{ej} + x_{fj} \leq 1 \qquad \forall j = 1, .., 3 \tag{3.4}$$

$$\forall (e, f) \in I \tag{3.5}$$

$$I = \{1, ..., m\} \tag{3.6}$$

$$x \in \{0, 1\} \qquad \forall i \in \{1, .., m\}, \forall j \in \{1, .., 3\} \tag{3.7}$$

Our mathematical model of the team formation problem that we made is an extension of the classic assignment problem.

3.4 Case Study :

Our case study is about assigning 30 engineers to 3 teams to work on the course of 3 shifts a day ,without assigning the pairs of incompatible engineers (e, f) into the same team.
m= 30 engineers.
n= 3 teams.

T=m/n =10
(e, f) =(1,3),(2,4),(5,2),(3,13),(18,20).

3.4.1 The mathematical model

$$\min \quad \frac{1}{2}\sum_{j=1}^{3}\sum_{k=1}^{3}|\sum_{i=1}^{30} u_i x_{ij} - \sum_{i=1}^{30} u_i x_{ik}| \tag{3.8}$$

$$\text{s.t.} \quad \sum_{i=1}^{30} x_{ij} = 10 \qquad \forall j = 1, 2, 3 \tag{3.9}$$

$$\sum_{j=1}^{3} x_{ij} = 1 \qquad \forall i = 1, 2, ..., 30 \tag{3.10}$$

$$x_{ej} + x_{fj} \leq 1 \qquad \forall j = 1, 2, 3 \tag{3.11}$$

$$\forall (e, f) \in I \tag{3.12}$$

$$I = \{1, ..., 30\} \tag{3.13}$$

$$x \in \{0, 1\} \qquad \forall i \in \{1, .., 30\}, \forall j \in \{1, 2, 3\} \tag{3.14}$$

Engineers are evaluated on the number of years of experience, the number of projects already carried out and the number of points obtained from the superiors of each engineer in order to calculate the performance bonus in the last 3 months (out of 40).the data collected is as follows:

The engineers	**Experience (Years)**	**Projects (units)**	**The bounty (out of 40 marks)**	**The utility** (u_i)
ID=1	7	2	38	47
ID=2	9	6	37	52
ID=3	4	2	30	36
ID=4	25	19	40	84
ID=5	30	26	40	96
ID=6	18	3	35	56
ID=7	23	12	40	75
ID=8	8	6	31	44
ID=9	12	5	38	55
ID=10	16	11	39	66
ID=11	28	10	40	78
ID=12	33	23	40	96
ID=13	19	9	40	68
ID=14	20	11	38	69
ID=15	6	6	32	44
ID=16	15	5	34	54
ID=17	12	11	40	63
ID=18	24	10	40	74
ID=19	12	3	37	52
ID=20	17	7	39	63
ID=21	14	1	33	48
ID=22	10	9	37	56
ID=23	22	20	40	82
ID=24	8	4	30	42
ID=25	24	8	34	66
ID=26	22	15	40	77
ID=27	28	17	40	85
ID=28	30	26	40	96
ID=29	6	5	34	45
ID=30	13	10	39	62

Table 3.1: Utility Evaluation Criteria for Engineers based on Experience, Project Participation, and Performance Bonus (Data Collected and Provided by Sonatrach HR)

Note: The Engineers' IDs serve as unique identifiers for the engineers within the list maintained by Sonatrach HR,these IDs correspond to the engineers' positions in the list. After calculating the engineers utilities, denoted as u_i (addition), , the resulting values are utilized as inputs to the developed algorithms in the upcoming chapter.

3.5 The complexity of the model

In order to analyze the complexity of the model,we need to break down the variables and constraints involved first:

Variables:
x_{ij}: Binary decision variable indicating whether engineer i is assigned to team j.
There are 30 engineers and 3 teams, so there are a total of 30 * 3 = 90 variables.

Constraints:
1. Each engineer should be assigned to exactly one team:
$\sum_{j=1}^{3} X_{ij} = 1$, $\forall i = 1, \ldots, 30$.
This constraint ensures that each engineer is assigned to only one team.
There are 30 constraints in total.
2. Each team should have exactly 10 engineers:
$\sum_{i=1}^{30} X_{ij} = 10$, $\forall j = 1, 2, 3$.
This constraint ensures that each team has exactly 10 engineers.
There are 3 constraints in total.
4. Engineers that should not be in the same team:
$X_{ej} + x_{fj} \leq 1$, $\forall j = 1, 2, 3$.
This constraint ensures that certain pairs of engineers should not be assigned to the same team,for each pair of engineers (e, f) that should not be in the same team,where e and f represent the engineers indices and j represents the team index.
In our case study, we have identified five pairs of engineers who should not be in the same team: (1,3), (2,4), (5,2), (3,13), and (18,20),to enforce these constraints, we formulate the following individual constraints:
$X_1 + X_3 \leq 1$
$X_2 + X_4 \leq 1$
$X_5 + X_2 \leq 1$
$X_3 + X_{13} \leq 1$
$X_{18} + X_{20} \leq 1$

Each constraint ensures that the corresponding pair of engineers is not assigned to the same team,**in total, there are 5 constraints**, one for each pair of engineers.
Overall , the complexity of the model can be summarized as follows:
Number of Variables: 90.
Number of Constraints: 30 + 3 + 5 =38
The matrix representation of the constraints would involve a matrix with dimensions (38 constraints) x (90 variables)

Analyzing the complexity of the model:

- Number of variables: 90 variables.
- Number of constraints: 38 constraints.
- The presence of absolute value terms in the objective function introduces nonlinearity, which may affect the computational complexity of the model.

3.6 Conclusion

In conclusion, this Chapter has provided a comprehensive overview of the modeling and development process involved in our research,we have formulated the problem at hand, defined its objectives and constraints, and presented a general mathematical model that serves as the backbone of our computer solution,the case study exemplifies the application of the model to a practical scenario, demonstrating its effectiveness. Additionally, we have discussed the complexity of the model, which sets the stage for the subsequent chapter, where we interpret the results obtained.

Chapter 4

Interpretation of Results

4.1 Introduction

This chapter delves into the practical implementation and analysis of two distinct algorithms, namely the Genetic Algorithm and the Hybrid Algorithm, in tackling the team formation problem,the chosen algorithms were carefully selected based on their suitability to address the specific constraints and objectives of the problem at hand,this chapter outlines the underlying principles and motivations behind the adoption of these algorithms, highlighting the adaptations made to enhance their effectiveness in the team formation context,extensive results obtained from the application of both algorithms are thoroughly examined, providing a comprehensive comparative analysis of their performance,through this analysis, the chapter aims to showcase the efficacy and efficiency of the Genetic Algorithm and the Hybrid Algorithm in effectively addressing the complexities associated with the team formation problem.

Note: The algorithms developed in this chapter were implemented using MATLAB, a widely-used programming language and computational tool in the field of mathematics and optimization.

4.2 The Algorithms input data

The engineers	**The utility u_i**	**The engineers**	**The utility u_i**	**The engineers**	**The utility u_i**
ID=1	47	ID=11	78	ID=21	48
ID=2	52	ID=12	96	ID=22	56
ID=3	36	ID=13	68	ID=23	82
ID=4	84	ID=14	69	ID=24	42
ID=5	96	ID=15	44	ID=25	66
ID=6	56	ID=16	54	ID=26	77
ID=7	75	ID=17	63	ID=27	85
ID=8	44	ID=18	74	ID=28	96
ID=9	55	ID=19	52	ID=29	45
ID=10	66	ID=20	63	ID=30	62

Table 4.1: The Algorithms input data

4.3 The choice of Genetic Algorithm

The decision to utilize a genetic algorithm for the team formation problem discussed in the preceding chapter is based on a thorough assessment of the problem's nature and the algorithm's appropriateness,the mathematical model presents a challenging optimization problem with nonlinear objectives and various constraints, including team size, engineers' compatibility, and restricted engineer assignments.

Genetic algorithms are highly regarded for their effectiveness in tackling complex problems, utilizing population-based search and genetic operators such as selection, crossover, and mutation,these algorithms excel at navigating nonlinear solution spaces without relying on explicit gradient information,by leveraging these characteristics, genetic algorithms demonstrate a remarkable capacity to explore vast solution spaces and uncover highly promising solutions that approach optimality.

Genetic algorithms offer a high degree of flexibility, allowing for the integration of problem-specific constraints through suitable encoding schemes and constraint-handling techniques,this adaptability enables researchers to optimize the algorithm's performance specifically for our problem, additionally, the inherent robustness of genetic algorithms makes them well-suited for addressing the uncertainties and noise often encountered in real-world data and utility calculations, further enhancing their applicability to our study's team formation problem.

Given the complexity of the problem, the nonlinearity of the objective function, and the requirement to navigate extensive solution spaces while adhering to relevant constraints, the genetic algorithm proves to be an optimal approach for solving our specific problem,it demonstrates robustness and effectiveness in producing desirable results.

4.4 The adaptation of Genetic Algorithm

During the adaptation of the genetic algorithm for the team formation problem, our primary focus was to refine and customize the standard framework to effectively handle the problem's constraints and objectives, these constraints include team size, engineers compatibility, and utility optimization for balanced team composition,our goal was to enhance the algorithm's precision in exploring the solution space,to achieve this, we made various adaptations to the genetic operators, encoding schemes, and selection strategies,these adaptations were tailored specifically to address the unique challenges posed by the team formation problem,by incorporating these modifications, our intention was to significantly improve the algorithm's performance, robustness, and efficiency in identifying optimal or near-optimal solutions, thereby providing more effective solutions to the team formation problem.

4.4.1 Initial population

The initial population plays a crucial role in the adapted genetic algorithm for the team formation problem,it serves as the starting point for the optimization process, encompassing a diverse set of potential solutions represented by individual chromosomes.

Figure 4.1: An illustration of a chromosome

In the adapted genetic algorithm, the initial population is generated through a randomized process,specifically, a predefined number of chromosomes,representing potential solutions. By employing this randomized approach, the adapted genetic algorithm ensures the exploration of a wide range of candidate solutions right from the beginning,this diversity in the initial population enables the algorithm to explore different regions of the solution space, avoiding potential biases and local optima,by encompassing a variety of potential solutions, the algorithm gains the ability to uncover promising solutions that align with the problem's constraints and objectives.

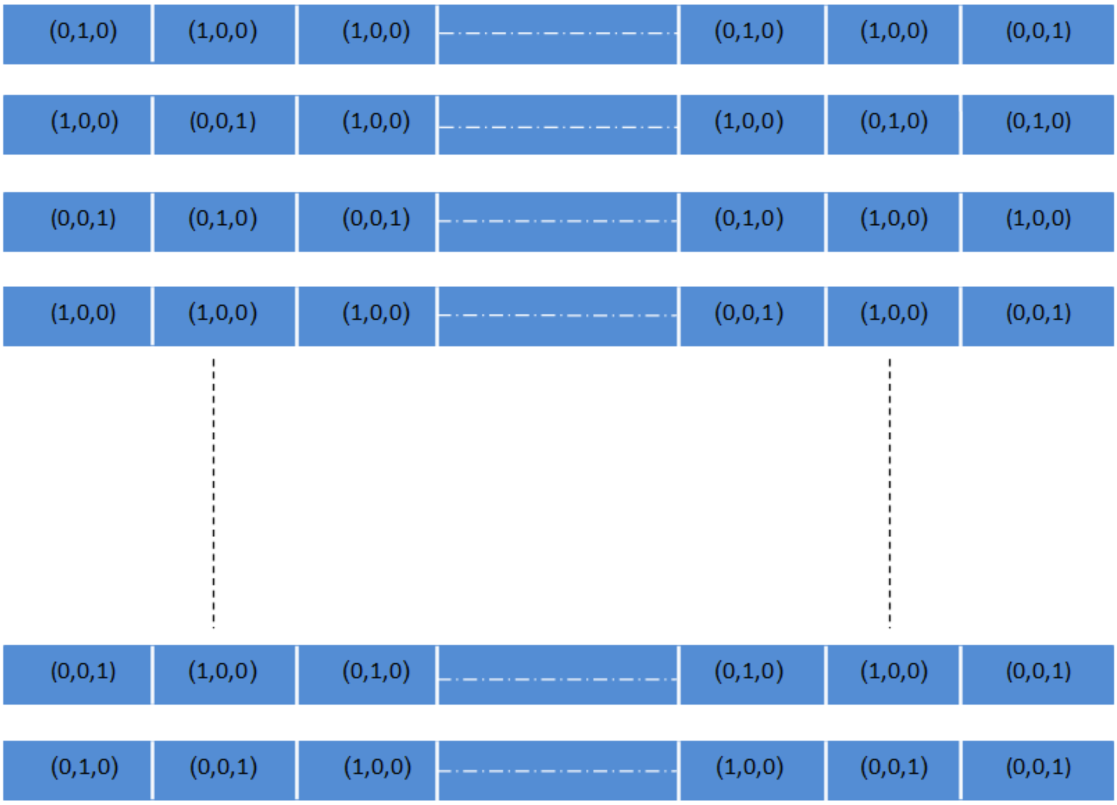

Figure 4.2: An illustration of the Initial population

4.4.2 Selection,Crossover and Reproduction

In the crossover step of the adapted genetic algorithm, our aim was to generate new offspring by combining genetic information from two parent solutions,this process served the purpose of introducing diversity and potentially enhancing the genetic makeup of the population.

To achieve this, we employed a crossover operation that involved selecting random positions within the chromosome representation and exchanging the corresponding genetic elements between selected positions of different chromosomes,this stochastic operation allowed for the exploration of different combinations of genetic information and facilitated the generation of diverse offspring.

The resulting offspring inherited characteristics from both parents, leading to a population of solutions with increased potential for exhibiting enhanced traits and qualities compared to the parents,as shown in the provided illustration, a uniform crossover strategy was employed, as evidenced by the random selection of genes and the subsequent exchange of genetic elements between those chromosomes,this implementation ensured a uniform distribution of genetic information from the parent solutions, where each gene had an equal probability of being selected for crossover,consequently, a uniform blending of genetic material occurred between the parents, contributing to the diversity and exploration within the population.

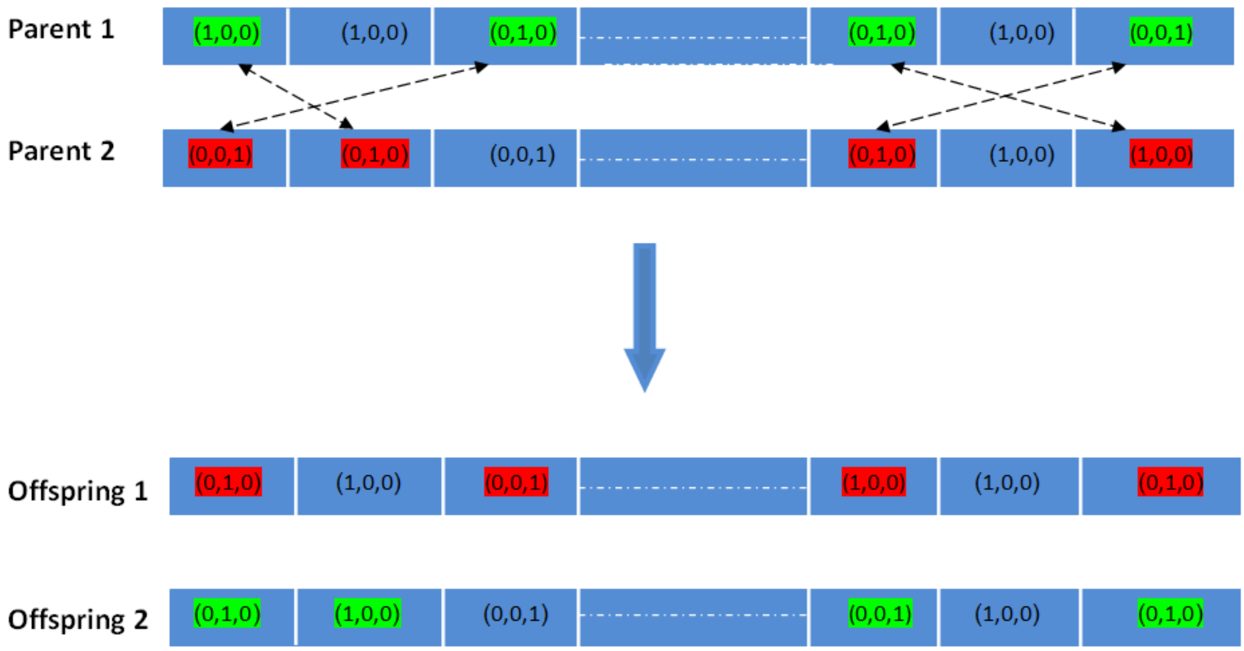

Figure 4.3: An illustration of the Initial Selection,Crossover and Reproduction processes

4.4.3 The Fitness function

The fitness function in the adapted genetic algorithm for the team formation problem plays a critical role in evaluating the effectiveness and quality of individual solutions within the population,it serves as a measure of the solution's performance and guides the algorithm in selecting the most suitable solutions for further evolution.

In our implementation, we designed a fitness function that specifically focuses on assessing the team compositions based on their utility distribution,the function aims to evaluate the balance and fairness of utility contributions across different teams of engineers.

By considering three distinct teams within each individual's chromosome representation, the fitness function calculates the utility sums for these teams,these utility sums reflect the cumulative utility scores assigned to the engineers within each team.

To determine the fitness value for an individual, the function calculates the absolute

differences between the utility sums of the three teams,these differences are then combined to derive a fitness value that represents the overall balance and fairness of the team composition in terms of utility distribution.
The objective of this fitness function is to incentivize the formation of teams where the differences in utility contributions across the three teams are minimized.

4.4.4 Merging,Sorting and selecting the best solutions processes

In the context of the adapted genetic algorithm, the merging, sorting, and selection process plays a crucial role in identifying and prioritizing the best solutions among the evaluated population, which includes both the parent solutions and the offspring solutions.
After the evaluation of fitness for each individual in the population, the merging step combines the parent solutions and offspring solutions into a single population,this consolidation ensures that all potential solutions, derived from both parents and offspring, are considered in the subsequent steps of the algorithm.
Following the merging step, the solutions are sorted in ascending order based on their fitness values,this sorting process arranges the solutions from the most minimum fitness value to the most maximum fitness value,by sorting the solutions in this manner, the algorithm identifies the best-performing individuals with the lowest fitness values, thereby positioning them at the top of the sorted list.
The sorting of solutions based on fitness values enables the algorithm to prioritize individuals that exhibit superior traits and qualities.
The sorting process emphasizes the selection of high-quality solutions with lower fitness values,this approach aims to identify the most promising candidates that are more likely to lead to optimal or near-optimal solutions for the team formation problem.

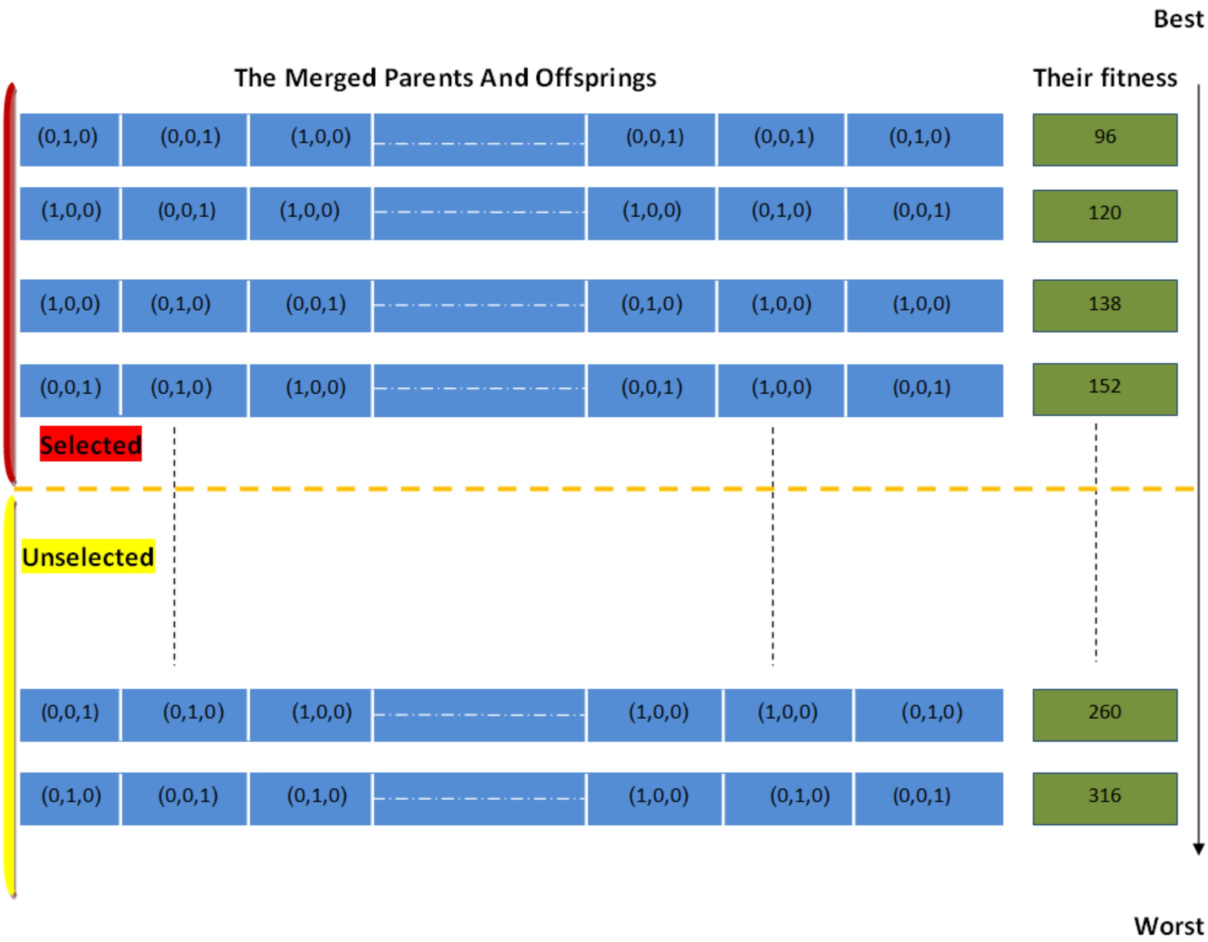

Figure 4.4: An Illustration of the Selection of Best Solutions: Merging and Sorting Based on Fitness

After the completion of the sorting process, the adapted genetic algorithm proceeds to eliminate solutions that violate the constraint of having incompatible engineers within the same team,this constraint is crucial for ensuring the formation of cohesive and effective teams. By removing solutions that contain such incompatibilities, the algorithm enhances the quality and feasibility of the generated solutions,this elimination step contributes to the optimization process by further refining the pool of potential solutions, focusing on those that satisfy the compatibility requirement and align with the problem's objectives.

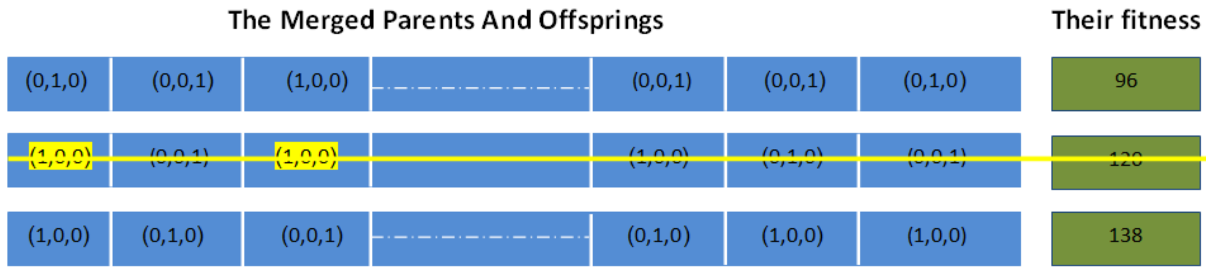

Figure 4.5: An illustration of The process of eliminating the solutions that include the incompatible engineers in the same team

Finally, After selecting the top-performing individuals from the sorted list as the best solutions,the algorithm then proceeds with another iteration, repeating the aforementioned steps of crossover, merging, fitness evaluation, sorting, and selection of the best solutions with eliminating the solutions that include the incompatible engineers in the same team ,This iterative process continues until the maximum specified number of iterations is reached.

4.4.5 Termination criteria

The termination criteria of the algorithm dictate the conditions under which the algorithm ceases its execution.
In the case of the adapted genetic algorithm for tour team formation problem, the termination criterion is defined as reaching the maximum specified number of iterations,once this predefined threshold is met, the algorithm concludes its operation and stops running. This termination condition ensures that the algorithm undergoes a sufficient number of iterations to explore and refine potential solutions while avoiding excessive computational effort or unnecessary iterations.
By reaching the specified maximum number of iterations, the algorithm achieves a balance between solution quality and computational efficiency, providing a reliable stopping point for the optimization process.

Note:
It is worth noting that the mutation process was intentionally omitted from the adapted genetic algorithm for the team formation problem,this decision was based on multiple factors:
Firstly, the variables in the problem were represented in a binary format, where each variable had only two possible values, Consequently, the potential impact of mutations on the solutions would be limited, as the binary variables could only be flipped between

these two values.
Secondly, the inclusion of the mutation process could lead to a loss of viable solutions within the population, as the mutations might introduce infeasible or undesirable configurations.
Lastly, the algorithm aimed to generate practical and achievable results, and the introduction of mutations had the potential to yield solutions that were unattainable or impractical in the given context.
By foregoing the mutation process, the algorithm focused on leveraging crossover and selection operations to refine the existing genetic material and produce viable solutions that aligned with the problem's constraints and objectives.

4.5 Genetic Algorithm Parameters

In this section, we unveil the essential parameters employed in our genetic algorithm implementation,these fundamental factors play a pivotal role in shaping the behavior and performance of the algorithm, ultimately leading us towards the discovery of optimal or near-optimal solutions,through careful parameter tuning, we strive to achieve an effective and efficient solution for the balanced teams formation problem.

Population Size	8, 16, 32, 64
Number Of Generations	1000
Crossover Probability	10% , 20% , 50%
Mutation Probability	0
Number Of Genes	30
Number Of engineers incompatibilities	5

Table 4.2: Genetic Algorithm Parameters
The table above presents the key parameters considered in our genetic algorithm framework include the population size, which determines the number of potential solutions explored in each generation,the number of generations denotes the total iterations undertaken by the algorithm, influencing the convergence and exploration capabilities,the crossover probability governs the likelihood of genetic material exchange between individuals during reproduction, promoting the sharing of favorable characteristics,the mutation probability that has been intentionally set to 0 as we made the deliberate choice to exclude mutation from our algorithm design ,the number of genes corresponds to the number of

engineers in our case,lastly, the number of engineers' incompatibilities captures the count of engineer pairs with constraints that prevent their assignment to the same team.

4.5.1 Population Size

The population size is a fundamental parameter in genetic algorithms, playing a crucial role in determining the diversity and exploration capabilities of the algorithm,a larger population size allows for a more extensive exploration of the solution space, increasing the chances of finding optimal or near-optimal solutions,in our investigation, we seek to understand the impact of population size on the behavior of our genetic algorithm for the team formation problem.,to ensure a fair comparison, we keep the Number of Generations constant at 1000 and maintain a fixed Crossover Probability of 0.2,by doing so, we can isolate the effect of population size on the algorithm's performance and assess the quality of the solutions obtained.

Population Size	**The Fitness Value**	**The Utility Of The Teams**	**Avg.Min.Iter (100 exec)**
8	4	(643 643 645)	635.2
16	2	(644 644 643)	512.36
32	2	(644 643 644)	333.54
64	0	(640 640 640)	59.8

Table 4.3: Population Size Variation and its Impact on Fitness and Avg.Min.Iter Values
Notes: Avg.Min.Iter (100 exec) = The Average Of The Minimum Iteration values over 100 executions.

Table 4.3 Analysis :
Table 4.3 presents the results of multiple executions with varying population sizes (8, 16, 32, and 64) and their corresponding fitness values and average minimum iteration values,the fitness value represents the quality of the achieved solution, while the average minimum iteration value indicates the convergence rate towards the optimal solution.
The analysis reveals an interesting relationship between population size, fitness value, and the average of the minimum iteration values over 100 executions.
Firstly, it is observed that as the population size increases, the average of the minimum iteration values decreases,this indicates that larger population sizes tend to reach the optimal solution in fewer iterations,this trend suggests that a larger pool of candidate solutions enables a more comprehensive exploration of the solution space, resulting in

quicker convergence towards the optimal solution.
Secondly, the analysis reveals a clear positive correlation between population size and fitness value,as the population size increases, there is a consistent improvement in the fitness value,notably, a fitness value of 0 signifies the attainment of the optimal solution,this implies that larger population sizes not only expedite the convergence process but also yield superior fitness values, indicating a more optimal solution.
These findings support the notion that a higher population size positively affects the optimization process,it enables more thorough exploration of the solution space, leading to improved fitness values and faster convergence.

4.5.2 Crossover Probability

The choice of crossover probability plays a vital role in the genetic algorithm, as it directly influences the production of offspring solutions through crossover operations,in our investigation, we delve into the impact of varying crossover probabilities on the fitness values attained by the algorithm,by keeping the population size fixed at 64 and maintaining all other parameters constant, we seek to gain insights into how different crossover probabilities affect the attainment of optimal solutions,this experimental exploration allows us to understand the significance of this critical parameter and its role in shaping the algorithm's performance in solving the team formation problem effectively.

Crossover Probability	**The Fitness**	**The Utilities Of The Teams**	**Avg.Min.Iter(100 exec)**
10%	2	(644 644 643)	351.01
20%	0	(647 647 647)	180.68
50%	0	(641 641 641)	49

Table 4.4: Fitness Values Obtained through Genetic Algorithm Execution with Different Crossover Probabilities

Note: It is important to mention that the results presented in this table were obtained while maintaining a fixed population size of 64,this ensures consistency in the experimental setup and allows for a direct comparison of the impact of different crossover probabilities on the fitness values.

Table 4.4 Analysis :

The table 4.4 showcases the results of executing the genetic algorithm with varying crossover probabilities, while keeping the population size fixed at 64,the focus is on analyzing the effect of crossover probability on the fitness values, aiming to understand its

impact on achieving optimal solutions.
Crossover probability is a crucial parameter in the genetic algorithm as it determines the probability of producing offspring solutions through crossover operations,by analyzing the fitness values obtained for different crossover probabilities, we can gain insights into how this parameter affects the algorithm's performance.
Upon analyzing the table, it becomes apparent that the fitness values exhibit variation across different crossover probabilities, demonstrating the effect of crossover probability on the fitness value.
Notably, the results show that with a crossover probability of 10%, the genetic algorithm achieves a fitness value of 2,however, as the crossover probability increases to 20%, the fitness value reduces to 0, similarly, setting the crossover probability to 50% also yields a fitness value of 0.
Interpreting these results, it is evident that a higher crossover probability, particularly starting from 20%, plays a significant role in generating better fitness values,this observation suggests that an increased crossover probability emphasizes the exploration of potential solutions, allowing the genetic algorithm to traverse a wider search space and uncover optimal solutions that satisfy the problem's constraints and objectives,consequently, the higher crossover probability enhances the algorithm's ability to converge towards the optimal solution with a fitness value of 0.
By considering the relationship between crossover probability and fitness values, we gain valuable insights into optimizing the genetic algorithm's performance and facilitating the achievement of optimal solutions.

4.6 The choice of Hybrid Algorithm

The selection of a hybrid algorithm as the chosen solution approach for our team formation problem is based on a rigorous evaluation of its merits and suitability.
The team formation problem is characterized by its intricate nature, featuring a nonlinear objective function and multiple constraints related to team size, engineers compatibility, and limited assignments per engineer,in order to tackle these challenges effectively, a hybrid algorithm was deemed appropriate.
By combining elements from genetic algorithms and constraint handling techniques, the hybrid algorithm offers distinct advantages, Genetic algorithms are renowned for their ability to navigate complex solution spaces without relying on explicit gradient information, making them well-suited for nonlinear optimization problems,the integration of constraint handling techniques ensures that the generated solutions conform to the con-

straints imposed by incompatible engineers assignments, thus ensuring the feasibility of the team compositions.
Additionally, the hybrid algorithm provides flexibility by allowing problem-specific constraints and encoding schemes to be seamlessly incorporated, enabling customization for the team formation problem at hand.
Furthermore,the Sturdiness of the hybrid algorithm allows it to effectively handle uncertainties and noise inherent in real-world data and utility calculations.taking into account the complexity of the team formation problem, the hybrid algorithm emerges as a suitable and robust solution approach,capable of providing effective and reliable results.

4.7 The adaptation of Hybrid Algorithm

The hybrid algorithm adopted for tackling the team formation problem integrates the strengths of both a genetic algorithm and a constraint handling approach,by leveraging these complementary techniques, the algorithm strives to achieve efficient optimization of team assignments while effectively considering the inherent constraints,this hybrid algorithm unfolds through a multi-step procedure, encompassing various stages that collectively contribute to the overall team formation process.

4.7.1 Initialization

The algorithm commences by initializing key variables and parameters pivotal to its execution,these encompass the utility values assigned to individual engineers, which quantify their respective levels of contribution or competence within the team formation problem, moreover, a matrix is established to capture the incompatibilities between engineers, reflecting the constraints that prohibit certain engineers from being assigned to the same team,this matrix serves as a fundamental representation of the incompatibility constraints inherent in the team formation problem, Additionally, the algorithm defines the maximum number of iterations, dictating the duration of the optimization process.
These initial steps play a critical role in setting the stage for subsequent operations and enable the algorithm to undertake the intricate task of team formation.

4.7.2 Genetic Algorithm Components

The hybrid algorithm incorporates essential components of a genetic algorithm to address the team formation problem, one such component is the crossover step, which plays a significant role in the assignment of engineers to teams, this step leverages the concept of

random permutations, where engineers are selected for team assignments in a randomized order, the generation of random permutations is crucial as it introduces a level of unpredictability and variability in the initial team configuration.
By shuffling the order of engineer assignments through random permutations, the algorithm ensures a fair and diverse distribution of engineers among teams,this randomness facilitates the exploration of different combinations of engineers within teams, eliminating any preconceived biases or predetermined patterns in their assignments.
During the crossover process, engineers are sequentially assigned to teams based on the shuffled sequence generated by the random permutations,this approach guarantees that each team receives an equitable representation of engineers, promoting fairness and preventing any overrepresentation or underrepresentation of particular individuals.
The synergy between the crossover process and random permutations fosters the exploration of a wide range of team configurations. By avoiding biases towards specific engineers or preconceived team compositions, the algorithm facilitates a balanced and unbiased allocation of resources,this approach enhances the algorithm's ability to generate diverse and potentially optimal team assignments, allowing for comprehensive evaluation and exploration of potential solutions to the team formation problem.

4.7.3 Fitness Evaluation

Following the crossover step , the algorithm proceeds to evaluate the fitness of each solution, which corresponds to a specific team assignment.
The fitness function plays a crucial role in quantifying the quality of a solution by incorporating the utility values associated with the engineers assigned to each team.
By analyzing the disparity in utility values between teams, the fitness function aims to minimize this difference,this evaluation process enables the algorithm to assess the effectiveness of each team composition in achieving a balanced distribution of expertise and maximizing overall team performance.
Through rigorous fitness evaluation, the algorithm can identify and prioritize solutions that exhibit optimal utility distributions, consequently guiding the subsequent stages of the optimization process.

4.7.4 incompatible engineers' constraint Handling

After generating random team assignments, the algorithm checks for conflicts between engineers in each team, this step is crucial for ensuring that the team assignments adhere to the constraints imposed by incompatible engineers.

By comparing the team configuration with the conflict matrix, which represents incompatible engineers, the code identifies if there are any conflicts present in each team.
This enables the evaluation of team compositions in terms of engineers compatibility, ensuring that conflicting engineers are not assigned to the same team.
If conflicts exist in any team, the iteration proceeds to the next iteration without updating the fitness value,this conditional statement ensures that teams with conflicts are excluded from the fitness evaluation process, by doing so, the algorithm focuses only on feasible team configurations that satisfy the constraints imposed by incompatible engineers,this constraint handling mechanism enhances the efficiency and effectiveness of the algorithm by avoiding the evaluation of infeasible or conflicting solutions.
If there are no conflicts in the team configuration, the algorithm calculates the fitness value and compares it with the current minimum fitness value.

4.7.5 Fitness Value Update and Solution Tracking

If the newly calculated fitness value is lower than the current minimum, it signifies an improvement in the team composition. Consequently, the minimum fitness value is updated, and the current team configuration, utility values, and total utility for all teams are stored,this update mechanism ensures that the algorithm keeps track of the best-performing team configuration encountered so far, allowing for potential convergence towards an optimal solution.

4.7.6 Progress Monitoring and Performance Evaluation

Throughout the iteration process, the code keeps track of the iteration count, the minimum fitness value achieved, and the utility values for each team,this information provides valuable insights into the algorithm's progress, convergence behavior, and the effectiveness of the team formation process,analyzing these metrics can aid in evaluating the algorithm's performance, identifying trends, and assessing the quality of the obtained team assignments.

4.8 Results Visualization

After the completion of the main loop, both the Genetic and Hybrid Algorithms generate plots to visualize the utility values of the three teams over generations and the fitness value over iterations.
These visualizations serve as valuable tools for analyzing the algorithm's convergence

behavior, assessing its efficiency in reaching optimal or near-optimal solutions, and identifying any patterns or trends in the fitness and utility values.
The plots provide a comprehensive overview of the optimization process, facilitating the interpretation and evaluation of the algorithm's performance.
Next, we present the results visualization for each algorithm, starting with the Genetic Algorithm and followed by the Hybrid Algorithm,for each algorithm, we provide an analysis of the respective graphs to shed light on the evolutionary progress and the achieved optimization.

4.8.1 Results Visualization Of The GENETIC Algorithm

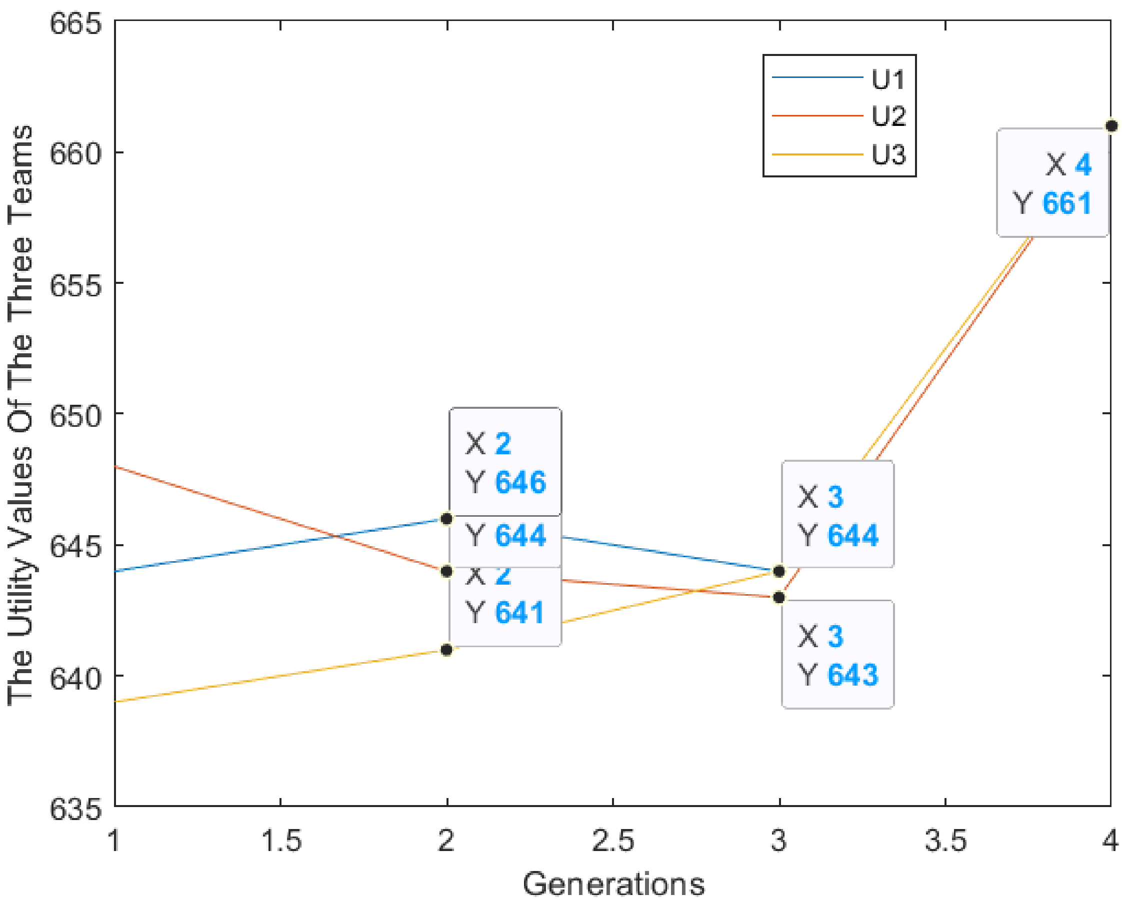

Figure 4.6: Teams utilities Evolution over generations

Figure 4.6 Analysis :
Figure 4.6 provides a visual representation of the evolutionary process of utility values

for the three teams using the Genetic Algorithm over successive generations,the x-axis denotes the generations, while the y-axis represents the utility values of the teams.
At the initial generation, distinct utility values are observed among the teams, with the first team exhibiting a utility value of 644, the second team with 648, and the third team with 639,this initial discrepancy highlights an inherent imbalance in engineers' allocation and the distribution of utilities among the teams, suggesting the need for optimization.
As the Genetic Algorithm progresses through subsequent generations, a notable convergence phenomenon unfolds,the utility values of the three teams gradually approach each other,this convergence signifies the algorithm's ability to effectively minimize the utility gap between the teams and create a more equitable arrangement.
As the algorithm progresses to the second generation, the utilities of all three teams converge towards closer values,the first team's utility decreases to 646, the second team's utility decreases to 644, and the third team's utility decreases to 641.
This convergence trend continues in subsequent generations,by the third generation, the first and third teams reach a utility value of 644, while the second team's utility is 643, still in close proximity to the other two teams,this indicates a significant improvement in balancing the utilities among the teams.
In the fourth generation, the convergence process culminates, as all three teams attain an identical utility value of 661,this signifies the successful optimization of the utility distribution, with each team having an equal share of engineers and utilities .
The observed convergence and balancing of utility values throughout the generations demonstrate the effectiveness of the Genetic Algorithm in addressing the imbalance among the teams,by iteratively adjusting the team's utilities, the algorithm optimizes the allocation of engineers and their utilities among the teams.

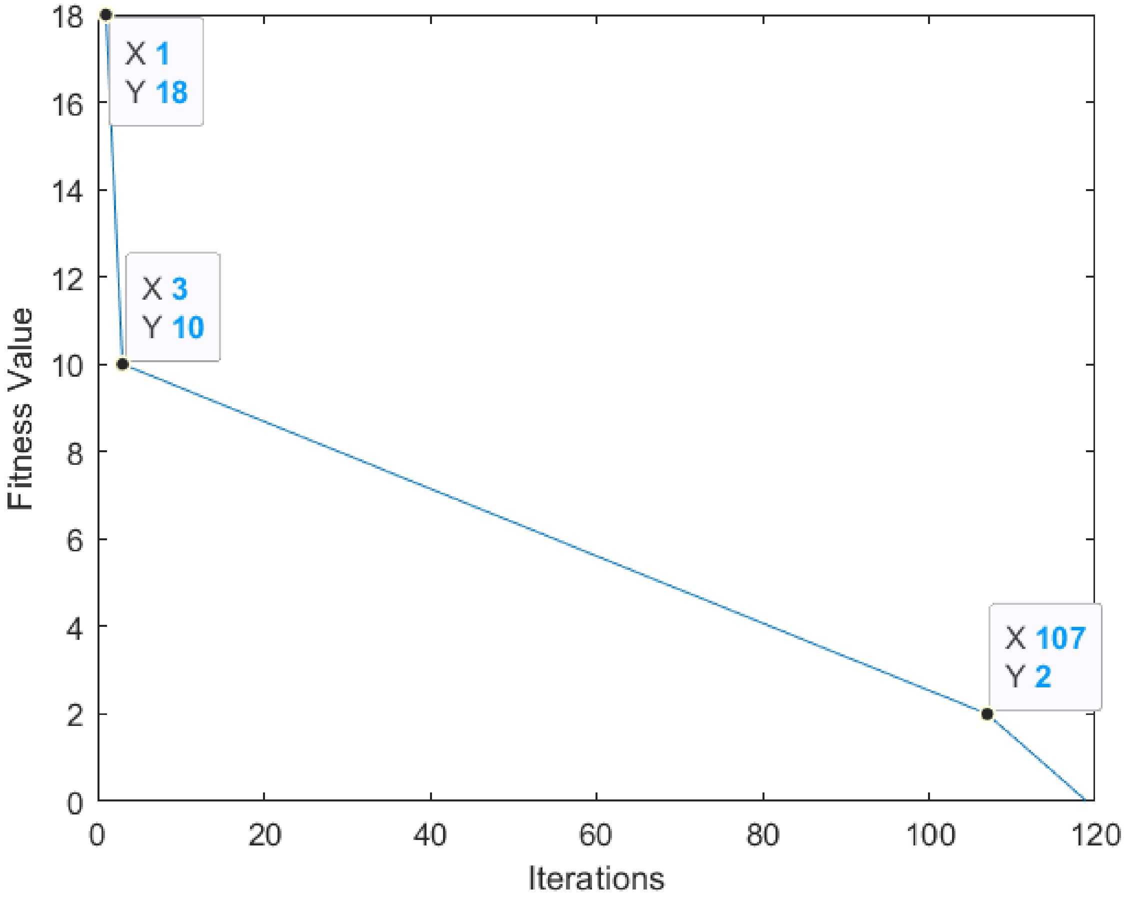

Figure 4.7: Fitness Evolution over Iterations

Figure 4.7 Analysis :

Figure 4.7 showcases the dynamic progression of fitness values produced by the Genetic Algorithm across multiple iterations,the x-axis denotes the iterations, while the y-axis represents the fitness value,this graph provides valuable insights into the algorithm's iterative search process for an optimal solution.

At the outset, the first iteration starts with a fitness value of 18, representing the initial performance of the solution, By the third iteration, a significant drop in fitness value occurs, reducing from 18 to 10, indicating a substantial improvement in the solution's quality within a single iteration,this significant improvement indicates a substantial enhancement in the solution's fitness and its alignment with the desired objectives.

As the algorithm progresses through iterations, the fitness value gradually decreases from 10 in the third iteration to 2 in the 117th iteration, this remarkable reduction indicates a substantial convergence towards an optimal solution,it exemplifies the Genetic Algorithm's ability to adapt and select fitter individuals through the evolutionary process,

ultimately leading to improved solutions.
The graph takes a notable turn at the 119th iteration, as the fitness value further decreases to zero, signifying the achievement of the optimal solution,this outcome highlights the effectiveness of the Genetic Algorithm in exploring and exploiting the solution space, enabling it to uncover the most favorable solution for the given problem.

4.8.2 Results Visualization Of The Hybrid Algorithm

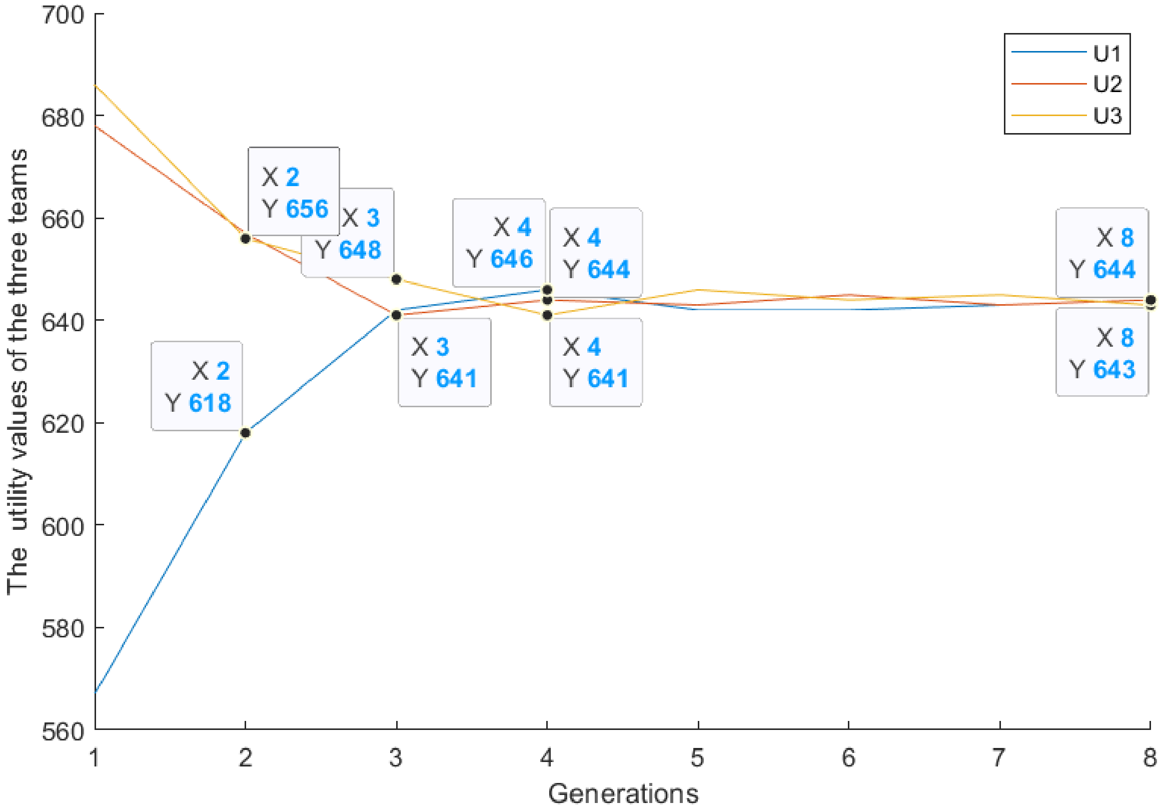

Figure 4.8: Teams utilities Evolution over generations

Figure 4.8 Analysis :
Figure 4.8,depicting the evolutionary process of utility values for the three teams using the Hybrid Algorithm reveals a compelling pattern of convergence and balance,in the initial generation, the first team starts with a utility value of 567, followed by the second team with 678, and the third team with 686,from there, a remarkable transformation occurs as the utility values gradually converge towards a more equitable distribution.

Starting from the second generation, the second and third teams demonstrate a convergence, sharing a utility value of 656, while the first team lags behind with a utility value of 618,however, in a notable turn of events in the third generation, the first team begins to catch up, steadily converging throughout the generation and ultimately reaching a utility value of 641, equivalent to that of the second team. Meanwhile, the third team achieves a utility value of 648,this progression signifies the Hybrid Algorithm's effectiveness in optimizing the allocation of engineers and their utilities among the teams, fostering a more balanced distribution.

The convergence of utility values persists in subsequent generations, leading to a significant reduction in discrepancies,in the fourth generation, the utilities of the three teams align closely, with the first team reaching a utility value of 646, the second team at 644, and the third team at 641,from the fourth generation onwards, the teams' utility values exhibit a stable convergence, with minimal fluctuations, culminating in the eighth generation,in this final stage, the utility values of the teams converge to a remarkably close range,with the first and second teams both achieving a utility value of 644 ,reflecting a high level of balance between them,the third team closely follows with a utility value of 643.

These results highlight the Hybrid Algorithm's capability to achieve a near-optimal but still very good balance in the distribution of utilities among the teams,while the utility values may not be perfectly identical, the achieved convergence signifies a highly favorable and equitable allocation of engineers and their utilities among the teams.

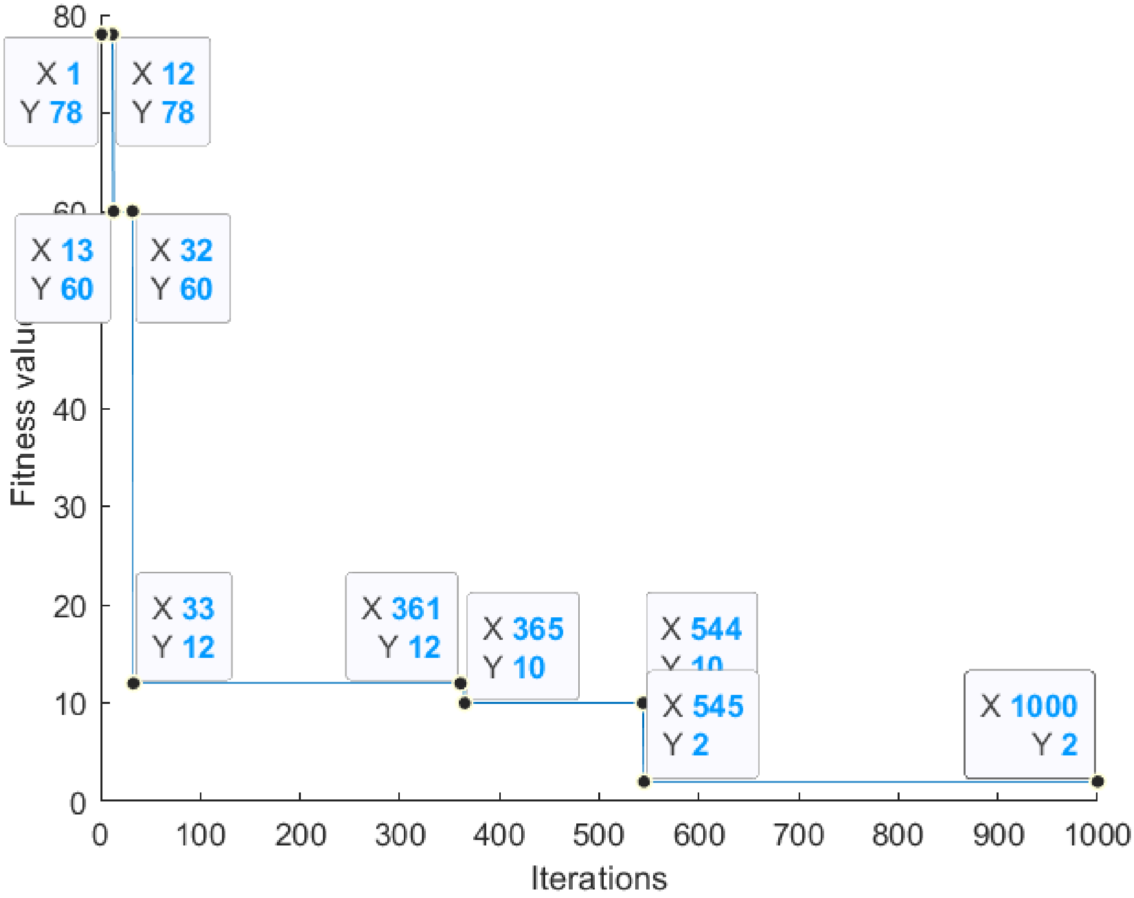

Figure 4.9: Fitness Evolution over Iterations

Figure 4.9 Analysis :

Figure 4.9 offers a comprehensive visualization of the Hybrid Algorithm's iterative process and its impact on fitness values, the x-axis signifies the iterations, while the y-axis represents the corresponding fitness values,this graph serves as a valuable tool for analyzing the algorithm's journey towards optimizing the solution and achieving the desired outcome.

The initial iteration commences with a fitness value of 78, which remains constant until the 12th iteration,however, a notable shift occurs by the 13th iteration when the fitness value significantly decreases to 60,this sudden drop suggests an improvement in the quality of the solution, indicating that the Hybrid Algorithm is adapting and refining the solution to better align with the desired objectives.

The fitness value remains stable from the 13th to the 32nd iteration, demonstrating the algorithm's ability to maintain the achieved level of optimization,however, at the 33rd iteration, an extraordinary improvement occurs as the fitness value plummets from its previous value to 12, this drastic reduction within a single iteration signifies a major breakthrough in the solution's quality, highlighting the algorithm's efficiency in exploring the solution space and identifying more favorable solutions.

Following this breakthrough, the fitness value stabilizes from the 33rd to the 361st iteration, indicating a consistent level of optimization,at the 365th iteration, the fitness value decreases further to 10, emphasizing the algorithm's ongoing fine-tuning and improvement in solution quality,the stability persists until the 544th iteration when a notable shift occurs,the fitness value decreases from 10 to its minimum value of 2, this remarkable achievement signifies that the Hybrid Algorithm has converged on a highly optimized solution, approaching the optimal outcome.

From the 545th iteration to the last iteration at 1000, the fitness value remains stable,indicating that the algorithm preserves the attained level of optimization,while the fitness value of 2 may not be at the absolute optimal solution, it reflects a highly favorable and near-optimal solution, demonstrating the effectiveness of the adapted Hybrid Algorithm.

The results from this graph underscore the capability of the Hybrid Algorithm to iteratively refine and improve the solution,the observed patterns of stability, sudden drops, and convergence towards the minimum fitness value highlight the algorithm's ability to navigate the solution space and converge on a highly optimized outcome.

4.9 Results Summary

Finally, both the Genetic and Hybrid Algorithms provide a comprehensive summary of their results, showcasing the utility values of the balanced teams, the fitness value achieved, the average of the minimum iteration values over 100 executions, the execution time, and the assignment of engineers to balanced teams.

The utility values of the balanced teams highlight the effectiveness of the algorithms in achieving team balance, reflecting the algorithms' success in allocating engineers optimally.

The fitness value achieved represents the quality of the solution obtained by the algorithms,a lower fitness value signifies a more optimal solution, indicating the algorithms' capability to generate well-balanced teams.

Furthermore, the average of the minimum iteration values over 100 executions provides insights into the convergence rate of the algorithms, a lower average value suggests that the algorithms converge faster, reaching the optimal solution within fewer iterations.

The execution time serves as a measure of the computational resources required by the algorithms to complete the optimization process,this measure enables us to assess the

efficiency and suitability of both the Genetic and Hybrid Algorithms, facilitating a meaningful comparison between the two.
Lastly, the assignment of engineers to well-balanced teams, obtained through the execution of the algorithms, showcases their practical applicability,this outcome directly impacts real-world scenarios, as it demonstrates the algorithms' capability to efficiently distribute engineers, considering their utility and compatibility, as well as other relevant constraints.
Ultimately, the utility values of balanced teams, the minimum fitness value achieved, the average of the minimum iteration values, the execution time, and the assignment of engineers to balanced teams collectively serve as a comprehensive summary of the Genetic and Hybrid Algorithms' results,these findings provide valuable insights into the algorithms' performance and their potential applicability in real-world team allocation scenarios.

4.9.1 Results Summary Of The GENETIC Algorithm

	Team 1	Team 2	Team 3
The Utilities Of The Teams	661	661	661
The Fitness Value	0		
Avg.Min.Iter (100 exec)	1824.8		
The Execution Time (Elapsed Time)	3246.664181 seconds		

Table 4.5: Results of Genetic Algorithm Execution

Avg.Min.Iter(100 exec) = The Average Of The Minimum Iteration values over 100 executions.

	The Engineers IDs									
The First Balanced Team	3	13	14	17	18	20	21	23	26	30
The second Balanced Team	4	5	7	8	12	16	19	22	24	29
The Third Balanced Team	1	2	6	9	10	11	15	25	27	28

Table 4.6: Assignment of Engineers to Balanced Teams Obtained through Genetic Algorithm Execution

Note: The numbers in the table represent the engineers' positions in the list of engineers maintained by Sonatrach HR. Each number serves as an identifier for the respective engineer.

4.9.2 Results Summary Of The Hybrid Algorithm

	Team 1	Team 2	Team 3
The Utilities Of The Teams	644	644	643
The Fitness Value	2		
Avg.Min.Iter(100 exec)	3970.2		
The Execution Time (Elapsed Time)	38.508130 seconds		

Table 4.7: Results of Hybrid Algorithm Execution

Avg.Min.Iter(100 exec) = The Average Of The Minimum Iteration values over 100 executions.

	The Engineers IDs									
The First balanced Team	1	2	6	11	15	21	22	26	28	29
The Second balanced Team	3	4	5	7	9	10	16	19	20	27
The Third balanced Team	8	12	13	14	17	18	23	24	25	30

Table 4.8: Assignment of Engineers to Balanced Teams Obtained through Hybrid Algorithm Execution

Note: The numbers in the table represent the engineers' positions in the list of engineers maintained by Sonatrach HR. Each number serves as an identifier for the respective engineer.

4.10 A Comparative Analysis The Genetic Algorithm and The Hybrid Algorithm

In our study, we conducted a comparative analysis between our adapted Genetic Algorithm (GA) and our adapted Hybrid Algorithm (HA) in terms of two key characteristics: the optimality of the solution and the execution time,the aim was to assess the performance of these algorithms and determine which one is more effective for solving our problem.

Firstly, we evaluated the optimality of the solutions produced by each algorithm,the GA yielded an optimal solution with a fitness value of 0, indicating that it achieved the best possible outcome according to the defined fitness function,on the other hand, the HA provided a near-optimal solution with a fitness value of 2, indicating that it approached the optimal solution but fell slightly short.

Additionally, we assessed the execution time of each algorithm. The HA outperformed the GA in terms of speed, as it completed its execution in 38.508130 seconds, whereas the GA took significantly longer with an execution time of 3246.664181 seconds.
This comparison analysis highlights the trade-off between optimality and execution time between the two algorithms,while the GA achieved a more optimal solution, it required substantially more time to execute,in contrast, the HA delivered a slightly less optimal solution but demonstrated superior efficiency with significantly reduced execution time.
These findings suggest that the choice of algorithm depends on the specific priorities of the problem at hand,if achieving the optimal solution is of utmost importance, despite longer execution times, the GA might be the preferred choice,however,if there are constraints on time as a priority, the HA presents a compelling alternative as it provides a near-optimal solution in significantly less time.

4.11 Conclusion

In conclusion, this chapter has shed light on the application and analysis of the Genetic Algorithm and the Hybrid Algorithm for solving the team formation problem,by adapting these algorithms to accommodate the unique constraints and objectives of the problem, we have demonstrated their effectiveness in optimizing the team allocation process,the detailed comparative analysis of their performance has provided valuable insights into the strengths and weaknesses of each algorithm,the results indicate that while the Genetic Algorithm achieves optimal solutions with a fitness value of zero, the Hybrid Algorithm yields near-optimal solutions with a fitness value of two. Moreover, the Hybrid Algorithm exhibits superior execution time compared to the Genetic Algorithm. Overall, the findings of this chapter affirm the suitability of these algorithms for addressing the complexities of the team formation problem, paving the way for further advancements in this domain.

General conclusion

In conclusion, this thesis has addressed the problem of forming balanced teams of operators in a case study conducted at Sonatrach, through a systematic approach, we have successfully developed a mathematical model that minimizes the utility gap among the teams while considering various constraints and factors such as experience, project involvement, and rewards.
By utilizing the mathematical model, we have demonstrated the effectiveness of our approach in achieving balanced teams with equal utility. The resolution process involved the use of genetic algorithms and hybrid algorithms, implemented through MATLAB programming,the results obtained from executing the algorithms have validated the capability of the model to form teams that deliver consistent performance quality.
While this thesis has provided valuable insights and solutions to the problem at hand, there is ample room for further development and improvement, future academic research should focus on enhancing the mathematical model by incorporating additional variables and constraints that may arise in real-world scenarios,exploring alternative resolution methods and algorithmic approaches can also contribute to refining the team formation process.
Furthermore, expanding the scope of the research to investigate the impact of team composition on overall team performance and productivity would be beneficial,this could involve analyzing the interplay between diverse skill sets, personalities, and communication dynamics within the teams.
Overall, this thesis has laid the foundation for further advancements in the field of team formation and optimization,by leveraging mathematical modeling techniques, we have not only addressed the specific case study at Sonatrach but have also paved the way for future research endeavors aimed at optimizing team dynamics and resource allocation in industrial settings.

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