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TOPIC

ESTIMATION OF THE LEVEL AND TEMPERATURE LIQUID OF A TWO-TANK COUPLED SYSTEM SUBJECT TO DISTURBANCES

BEFORE THE JURY:

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To every person who had a positive
impact in my life.

Appreciation

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ملخص

تستكشف هذه الأطروحة استخدام تقنيات التقدير لتحديد المتغيرات غير القابلة للقياس في نظام الخزانين المتصلين، وهو نموذج ديناميكي شائع في العمليات الصناعية. وتركز الأطروحة على خوارزمية مراقب "لونبرجر" كوسيلة فعالة للتقدير. نظرًا للقيود التي تواجه القياس المباشر، يلعب تقدير الحالة دورًا حيويًا في ضمان التحكم الدقيق في العمليات واكتشاف الأعطال. تتناول الدراسة نظرية أنظمة التحكم، نمذجة النظام الخطي وغير الخطي لنظام الخزانين، واستخدام مراقب "لونبرجر" لتقدير المتغيرات القابلة وغير القابلة للقياس. تشير النتائج إلى أن التقدير الكامل يوفر دقة عالية، بينما التقدير الجزئي يقدم تقديرات تقريبية موثوقة، مما يبرز أهمية التقدير في الحفاظ على التحكم الفعّال في ظل الاضطرابات.

الكلمات المفتاحية: تقدير الحالة، مراقب لونبرجر، نظام الخزانين، الأنظمة الديناميكية، التحكم في العمليات.

Summary

This thesis examines the application of estimation techniques to determine non-measurable state variables in a two-tank coupled system, a widely used dynamic model in industrial processes. It emphasizes the Luenberger observer algorithm as an effective estimation method. Given the inherent limitations in direct measurement, state estimation is vital for ensuring accurate process control and fault detection. The study encompasses control system theory, the modeling and linearization of the two-tank system, and the use of the Luenberger observer to estimate both measurable and non-measurable variables. Results indicate that while full-state observation delivers precise estimates, partial-state observation provides reliable approximations, highlighting the value of estimation in maintaining robust control under varying conditions and disturbances.

Keywords: State estimation, Luenberger observer, two-tank system, dynamic systems, process control.

Résumé

Cette thèse explore l'application des techniques d'estimation pour déterminer les variables d'état non mesurables dans un système couplé à deux réservoirs, un modèle dynamique largement utilisé dans les processus industriels. Elle met l'accent sur l'algorithme de l'observateur de Luenberger en tant que méthode d'estimation efficace. En raison des limitations inhérentes à la mesure directe, l'estimation des états est essentielle pour garantir un contrôle précis des processus et la détection des défauts. L'étude couvre la théorie des systèmes de contrôle, la modélisation et la linéarisation du système à deux réservoirs, ainsi que l'utilisation de l'observateur de Luenberger pour estimer les variables mesurables et non mesurables. Les résultats indiquent que l'observation à état complet fournit des estimations précises, tandis que l'observation partielle offre des approximations fiables, soulignant ainsi la valeur de l'estimation pour maintenir un contrôle robuste face aux perturbations.

Mots-clés : Estimation d'état, Observateur de Luenberger, Système à deux réservoirs, Systèmes dynamiques, Contrôle des processus.

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Σ: Sigma is the eighteenth letter of the Greek alphabet, is in general mathematic used as an operator for summation.

$\dot{\boldsymbol{x}}$**:** State space vector.

$\boldsymbol{x}$**:** State vector.

$\boldsymbol{u}$**:** Input vector, control the signals.

$\boldsymbol{y}$**:** Output vector. (refer to the signals measured by the sensors).

A: Matrix of an appropriate dimensions represent the state of linearized system.

B: Matrix of an appropriate dimensions represent the input of linearized system.

C: Matrix of an appropriate dimensions represent the output of linearized system.

$\mathbf{B}_v$: Matrix of an appropriate dimensions represent the disturbances applied on a linearized system.

$\boldsymbol{v}$: Disturbance’s vector.

$\dot{\hat{\boldsymbol{x}}}$: Estimated state space vector.

$\hat{\boldsymbol{x}}$: Estimated State.

$\boldsymbol{x}_{ss}$: Equilibrium point at the steady state condition.

$\boldsymbol{u}_{ss}$: The input of the equilibrium points at the steady state condition.

$\mathbf{X}_0$: The state vectors at the steady state conditions.

$\mathbf{U}_0$: The input vectors at the steady state conditions.

$\mathbf{V}_0$: The disturbances vectors at the steady state conditions.

$\Delta\mathbf{x}$: The variation of the state vector around the equilibrium point $\mathbf{x}_0$

$\Delta\mathbf{u}$: The variation of the input vector around the equilibrium point $\mathbf{u}_0$

$\Delta\mathbf{v}$: The variation of the disturbance vector around the equilibrium point $\mathbf{v}_0$

$\Delta\mathbf{y}$: The variation of the disturbance vector around the equilibrium point $\mathbf{y}_0$

K: Observer gain matrix.

$\boldsymbol{e}$: Observation error.

O: Observability matrix of linearized system.

K: Observer gain matrix.

$\dot{\boldsymbol{e}}$: The dynamics of the observation error.

$\mathbf{H}_1$: Liquid level of the left tank of the two-tank coupled system.

$\mathbf{H}_2$: Liquid level of the right tank of the two-tank coupled system.

$\mathbf{T}_1$: Liquid temperature of the left tank of the two-tank coupled system.

$\mathbf{T}_2$: Liquid temperature of the right tank of the two-tank coupled system.

$\mathbf{T}_w$: The temperature of the warm water.

$\mathbf{T}_c$: The temperature of the cold water.

$\mathbf{Q}_w$: Flow rate of warm water into the left tank of the two-tank coupled system.

$\mathbf{Q}_c$: Flow rate of cold water into the left tank of the two-tank coupled system.

$\mathbf{y}_1$: Liquid level measurement of the right tank (output for level control).

$\mathbf{y}_2$: Liquid temperature measurement of the right tank (output for temperature control).

$\mathbf{K}_h$: Transducer gain for level measurement.

$\mathbf{K}_t$: Transducer gain for temperature measurement.

$\mathbf{K}_a$: Flow coefficient.

$\mathbf{Q}_r$: The flow between the tanks.

$\mathbf{Q}_b$: The flow out of the outlet valve of the right tank.

$\mathbf{A}_v$: Opening area of the last valve of the two-tank coupled system.

ρ: The mass density.

c: The specific heat capacity of water.

T_0: The reference temperature at which the energy is zero.

P: The differential pressure across the orifice of the two-tank coupled system.

A_0: The area of the orifice orifice.

C_d: The constant loss coefficient.

S: The surface of the bottom of the tanks.

A_{11}: The submatrix that describes the dynamics of the measurable states.

A_{22}: The submatrix that describes the dynamics of the unmeasurable states.

A_{12} and A_{21} : The submatrices describe the interaction between the measurable and unmeasurable states.

B_1: The input submatrices corresponding to the measurable states.

B_2: The input submatrices corresponding to the unmeasurable states.

B_{v1}: The disturbances submatrices corresponding to the states.

B_{v2}: The disturbances submatrices corresponding to unmeasurable states.

T: The Coordinate Transformation.

C^T: The Moore-Penrose pseudoinverse of $\mathbf{C}$.

N: The basis of ker ($\mathbf{C}$).

R: The Gain Matrix of the Reduced Order observer.

z^+: The State space representation of the Reduced Order observer.

nx: The number of state variables.

ny: The number of system's outputs.

nu: The number of system's inputs.

nv: The number of system's disturbances.

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General Introduction

The relentless pursuit of efficiency and quality in today's industrial landscape relies on precise regulation. Industrial processes are complex systems, often involving intricate interactions among various components. To ensure these processes consistently deliver the desired outcome—whether it be specific product quality, production rate, or energy efficiency—maintaining a set of operating conditions is crucial. This is where regulation comes into play, acting as an invisible conductor, continually adjusting the process inputs based on the system outputs.

Achieving perfect regulation in industrial environments requires successfully capturing all system outputs. But this last, faces five fundamental challenges:

Physical Limitations: In many industrial systems, some variables are physically inaccessible due to their location or environment. For instance, in oil extraction, it's challenging to measure reservoir pressure and flow rates directly at deep underground levels, relying instead on surface measurements. Similarly, in offshore drilling, subsea equipment located thousands of meters underwater faces difficulties in measuring temperature, pressure, or flow rates accurately due to sensor placement challenges and harsh environmental conditions.

Sensor Costs and Complexity: Installing sensors throughout a system can be prohibitively expensive, particularly in the oil and gas industry, where monitoring a vast network of pipelines, pumps, and tanks would require thousands of costly high-quality sensors. Additionally, managing and ensuring the accuracy of data from such a large number of sensors increases system complexity and maintenance costs.

Harsh Operating Environments: The harsh conditions in the oil and gas sector for example, make sensor placement and maintenance challenging. In processes like catalytic cracking or distillation, extreme temperatures and pressures make it difficult to install sensors inside reactors or pipelines. Additionally, corrosive environments in processing plants can damage sensors, making continuous measurement impractical.

Measurement Limitations: The real sensors used to monitor industrial processes have limitations. They can introduce noise into the data or experience delays in capturing the actual state of the system. These imperfect measurements can lead to inaccurate assessments of the process and hinder effective regulation.

Process Dynamics: Industrial processes are not static. They often exhibit delays between the application of an input change and the resulting output modification. This inherent time lag makes it difficult for a purely reactive control system to maintain optimal conditions.

This is where estimation plays a crucial role in overcoming these challenges. Estimation acts as a sophisticated bridge between imperfect measurements and the desired regulation. It uses advanced mathematical techniques and algorithms to achieve two main objectives:

State Estimation: By analyzing the available sensor data, despite their limitations, estimation techniques can provide a more accurate picture of the system's current internal state. This goes beyond merely reading sensor values; it accounts for inherent noise and delays, offering a more reliable understanding of the process dynamics.

Predictive Capability: Estimation not only focuses on the present but also leverages the understanding of the system's dynamics to predict its future behavior. This allows the regulation system to anticipate how the process will react to input adjustments, considering inherent delays and ensuring timely interventions.

By providing a clearer view of the present and insights into the future, estimation enables the regulation system to make proactive adjustments, effectively mitigating the limitations of imperfect measurements and process delays.

In this thesis, we will work on estimating temperature and liquid level in a dynamic system subject to unmeasurable disturbances using a state observer.

This thesis is organized into four parts summarized as follows:

Chapter One: General Information on Regulation.

Chapter Two: Coupled Tank System.

Chapter Three: Modeling and Linearize of Two-tank Coupled system.

Chapter Four: Estimating the Liquid Level and Temperature of Two-tank Coupled System.

Chapter One: General information on the Regulation

1. Introduction

The concept of regulation is omnipresent, integrated into our natural world, and observable in living systems, as evidenced by the regulation of human body temperature. While early examples of control systems date back to antiquity, such as water level regulators, the formal study of control loops only emerged in the 19th century.

Regulation is a fundamental discipline within the field of automation, a technical science. Automation, including regulation, is generally considered to have originated in the 1930s with the advent of the first position servomechanisms and the initial definition of stability. Although "automatic" systems existed before, such as automata, they were not theoretically understood.

Following these initial advances, the field of regulation grew rapidly with the development of the first methods for synthesizing correctors in the 1940s and 1950s, and then experienced a significant expansion in the 1960s with the advent of computer science [1].

Throughout this chapter, we will establish a comprehensive foundation for understanding control systems, starting with a thorough examination of their core principles of regulation. We will also delve into the critical role of feedback loops in maintaining system stability. Furthermore, we will explore the concept of control systems in detail, dissecting the various components that work together to achieve desired outcomes. We will then turn our attention to the theory of estimation, outlining its purposes and the techniques employed for accurate state estimation. Finally, we will discuss observers and their crucial role in achieving system observability, a fundamental aspect of effective control design.

2. Regulation

2.1 Definition of Regulation

Regulation encompasses all techniques used to keep the controlled variable constant at a desired value, despite disturbances, by acting on another regulating variable. The goal is to achieve the smallest possible deviation as quickly as possible (energy efficiency) without destabilizing the response (product quality) [2].

2.2 Purpose of Regulation

The goal of regulation is to maintain a stable and desired state within a system by automatically adjusting its inputs based on its outputs. It can be likened to a thermostat managing the temperature of a room.

FIG 1. 1 - Programmable Room Thermostat that manage the temperature of a room [3]

2.3 System Control Loop

A regulation chain, also known as a control loop, is a closed-loop system that continuously monitors and adjusts the output of a system to maintain a desired setpoint or reference value. This is a fundamental concept in control theory and automation, widely used in various industries, including engineering, manufacturing, and more.

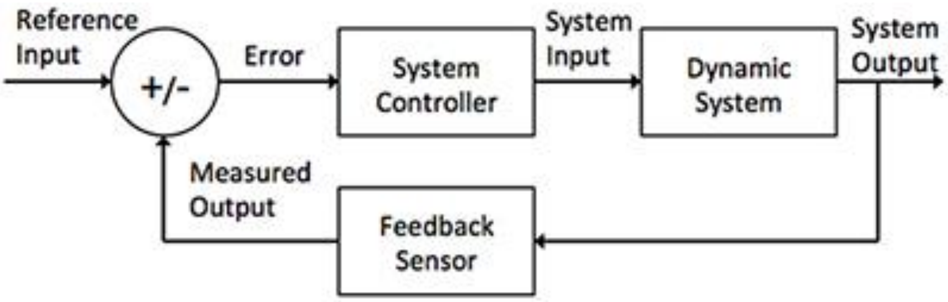

FIG 1. 2 - System Control Loop [4]

2.3.1 Operation of a System Control loop

A. Measurement: The process begins with measuring the system's output using a sensor. This sensor can be a physical device like a temperature or pressure sensor, or a more complex system like a vision sensor or a network of sensors. The sensor converts the measured physical quantity into an electrical signal that can be processed by the regulator.

B. Comparison: The measured output signal is then sent to the controller, which acts as the "brain" of the control loop. The main function of the controller is to compare the measured output to the desired setpoint or reference value. The setpoint is the value we want the system's output to maintain. The controller calculates the error, which is the difference between the measured output and the setpoint.

C. Processing: Based on the error, the controller generates a control signal using a control algorithm. The control algorithm is a set of mathematical instructions that determine how the controller should adjust the system's input to minimize the error. There are many types of control algorithms, each with its strengths and weaknesses.

Here are some common control algorithms:

- Proportional Control (P): The control signal is proportional to the error. This is a simple and effective algorithm, but it can be unstable in some systems.
- Integral Control (I): The control signal is proportional to the integral of the error over time. This algorithm helps eliminate steady-state error but can slow down the system's response to changes.
- Derivative Control (D): The control signal is proportional to the rate of change of the error. This algorithm improves the system's response to changes but can be sensitive to noise.
- PID Control (Proportional-Integral-Derivative): This algorithm combines the advantages of P, I, and D controls to provide a robust and versatile control strategy.

D. Action: The control signal is then sent to the actuator, which is the device that adjusts the system's input. The actuator can be a physical device like a valve, motor, or heating element, or it can be a more complex system like a robotic arm or a network of actuators. The actuator receives the control signal and adjusts the system's input accordingly.

F. Feedback: The new system output is then measured by the sensor, and the process repeats. This continuous cycle of measurement, comparison, processing, action, and feedback is what makes a control loop so effective in maintaining the desired output.

2.4 Level Control Systems

Level control systems are crucial for industrial processes that involve storing or using liquids. They automatically maintain the fluid level within a desired range in tanks or reservoirs. This is important for several reasons:

- Prevents overflow: An overflow can damage equipment, waste product, and create safety hazards.
- Ensures efficient operation: In many processes, consistent fluid level is necessary for optimal performance. For example, a boiler needs a certain water level to function properly.
- Protects pumps: Running a pump without sufficient liquid can damage the pump.

Example of Level control system

The diagram showed in (FIG 1.3) is a basic schematic of a water level control system for a storage tank. Here are the main components and how they work together:

Level transmitter (LT): This sensor measures the water level in the tank. It can be a float switch, a pressure sensor, or another type of device. The level transmitter sends a signal (often an electrical current) to the level controller.

Level controller (LC): This device receives the signal from the level transmitter and compares it to a setpoint, which is the desired water level in the tank. The controller then sends a signal to the level control valve.

Level control valve (LCV): This valve controls the flow of water into the tank. The signal from the level controller tells the valve to open, close, or adjust its position to maintain the water level at the setpoint.

Set point: This is the desired water level in the tank. It is typically a fixed value, but it can also be adjustable.

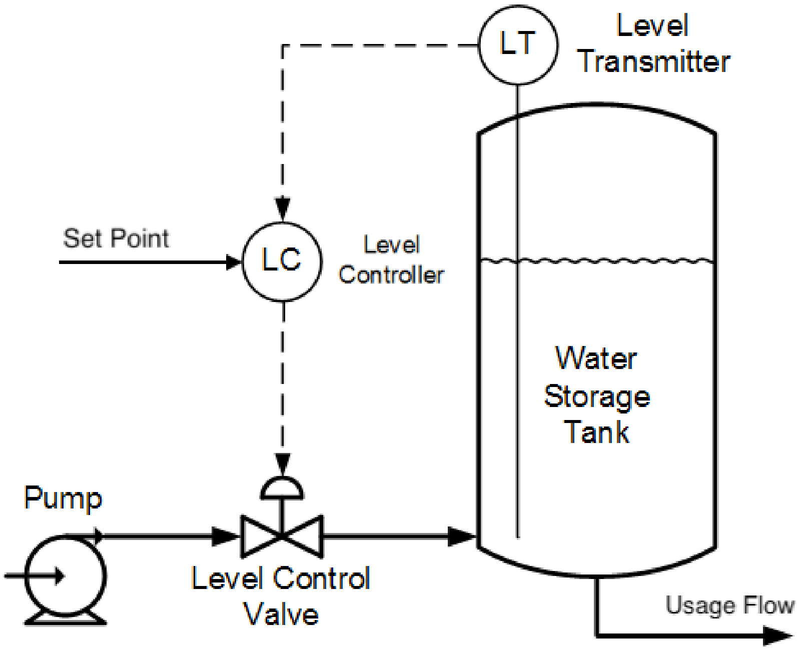

FIG 1. 3 - Level Control Diagram

Control loop of the system

The water level control system function based on a feedback loop. Here's how it works:

- The level transmitter continuously measures the water level in the tank.
- The level transmitter sends a signal corresponding to the water level to the level controller.

- The level controller compares the signal from the transmitter to the setpoint.
- If the water level is below the setpoint, the level controller sends a signal to the level control valve to open. This allows more water to flow into the tank.
- As the water level rises, the signal from the level transmitter increases.
- When the water level reaches the setpoint, the signal from the level transmitter matches the setpoint, and the level controller sends a signal to the level control valve to maintain its position.
- If the water level rises above the setpoint, the signal from the level transmitter increases further. The level controller then sends a signal to the level control valve to close or partially close, restricting the flow of water into the tank.

Level control systems are essential for ensuring safe, efficient, and reliable operation in various industries. The relatively simple feedback loop illustrated in the diagram (FIG 1.3) plays a vital role in maintaining the desired water level within a storage tank. This principle can be applied to various fluids and more complex industrial processes.

2.5 Temperature Control Systems

Throughout countless industrial processes, maintaining precise temperature is paramount. From ensuring consistent product quality to maximizing efficiency and safety, temperature control systems play a vital role.

In many applications, specific temperature ranges are crucial for a product to meet its desired characteristics. For instance, precisely controlled temperatures during chemical reactions guarantee the creation of the intended product. Furthermore, optimal temperatures can significantly enhance process efficiency. In steel production, for example, maintaining the correct temperature within furnaces optimizes metal properties while minimizing energy waste.

Finally, temperature control safeguards against potential hazards. Uncontrolled temperatures can lead to overheating, fires, or equipment damage, jeopardizing safety and causing costly downtime. Therefore, industrial operations heavily rely on these systems to guarantee consistent quality, efficient production, and a safe working environment.

Example of Temperature control system

The diagram showed in (FIG 1.4) depicts a basic temperature control system for a heat exchanger. Here are the components and how they interact:

- Temperature sensor (Outlet Temperature): This sensor measures the temperature of the fluid exiting the heat exchanger. It sends a signal (likely an electrical current) to the controller.
- Controller: This device receives the temperature signal and compares it to the setpoint, the desired temperature for the outlet fluid. The controller then determines the difference (error) between the actual and desired temperature.
- Valve: Based on the error signal, the controller sends a signal to the valve. The valve regulates the flow of the hot fluid (likely steam) entering the heat exchanger. By adjusting the flow rate, the controller can indirectly control the temperature of the outlet fluid.
- Heat exchanger: This is the equipment where heat transfer occurs between the hot and cold fluids. The hot fluid (steam) heats the cold fluid flowing through the other side of the exchanger.
- Setpoint: This is the desired temperature for the outlet fluid.

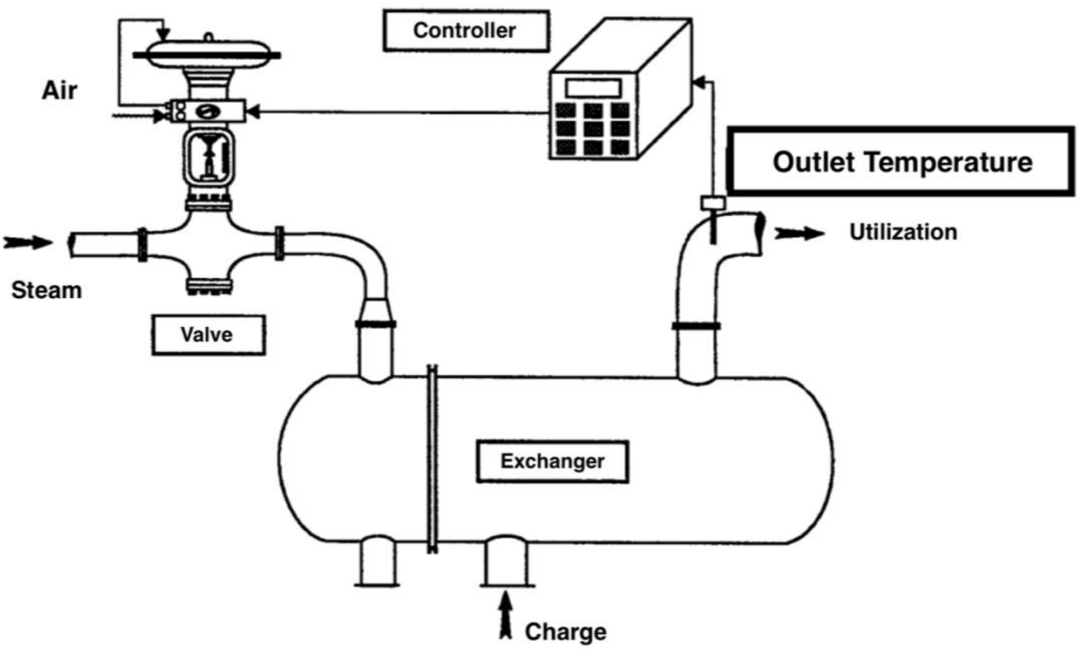

FIG 1. 4 - Diagram of a temperature control system for a heat exchanger [5]

Control loop of the system

The temperature control system function based on a feedback loop. Here's how it works:

- The temperature sensor continuously monitors the outlet fluid temperature.
- The sensor transmits a signal corresponding to the measured temperature to the controller.
- The controller compares the signal from the sensor to the setpoint.
- If the outlet temperature is lower than the setpoint (negative error), the controller sends a signal to open the valve more. This increases the flow rate of the hot fluid, raising the outlet temperature.
- Conversely, if the outlet temperature is higher than the setpoint (positive error), the controller signals the valve to close partially, reducing the hot fluid flow and lowering the outlet temperature.
- This continuous feedback loop ensures the outlet fluid temperature stays close to the desired setpoint.

The specific type of valve (on/off, throttling, etc.) and control algorithm (PID, etc.) used in real-world applications can vary depending on the process requirements.

3. Estimation

3.1 Definition of Estimation

At the heart of regulation lies estimation, an essential technique for designing and implementing high-performance control systems. Estimation involves determining the value of an unmeasured or hard-to-measure variable based on measurements of other variables and mathematical models of the system. By addressing the unknown, estimation plays a crucial role in enhancing the accuracy, stability, and overall performance of control systems [6].

3.2 The Essence of the Estimation

Estimation resembles a clever detective who, from fragmentary clues, reconstructs the complete picture. In the field of regulation, it operates similarly, utilizing accessible measurements to reveal the value of hidden or difficult-to-obtain variables. This ability to uncover the unseen is crucial for systems where direct measurement of all variables is impossible or impractical [6].

3.3 Applications of Estimation in Regulation

Estimation finds a wide range of applications in regulation, providing solutions to various challenges:

- State Estimation: Determining the values of state variables, such as position, velocity, and acceleration, from measurements of input and output variables.
- Pattern Recognition: Identifying and classifying objects or events from sensory data, such as in artificial vision or robotics systems.
- System Behavior Prediction: Anticipating the future behavior of the system by considering current conditions and past trends [7].

3.4 Common Estimation Techniques

To meet the challenges of estimation, a range of techniques is available to the regulator, each with its strengths and weaknesses:

- Kalman filters: Used to estimate the state of a dynamic system in real time, taking into account noise and measurement uncertainties.
- Observers: Mathematical models of the system are used to reconstruct the state of the system from output and input measurements.
- Regression estimation: Statistical models are used to establish a relationship between the measured variables and the unmeasured variable that is to be estimated.
- Artificial neural networks: Machine learning algorithms can be used to estimate complex variables from nonlinear data [7].

3.5 Undeniable Advantages

The integration of estimation techniques into control systems provides many advantages:

- Improved control accuracy: By estimating unmeasured variables, the controller can make more accurate and efficient ordering decisions.
- Increased robustness: Estimation helps compensate for disturbances and uncertainties in the system, improving its robustness to environmental variations.
- Better performance optimization: Estimating helps predict system behavior and optimize control parameters for optimal performance.
- Fault detection and diagnosis: Estimation can be used to detect anomalies in the system and diagnose potential failures [8].

4. Observation of dynamical systems

4.1 Observer Definition

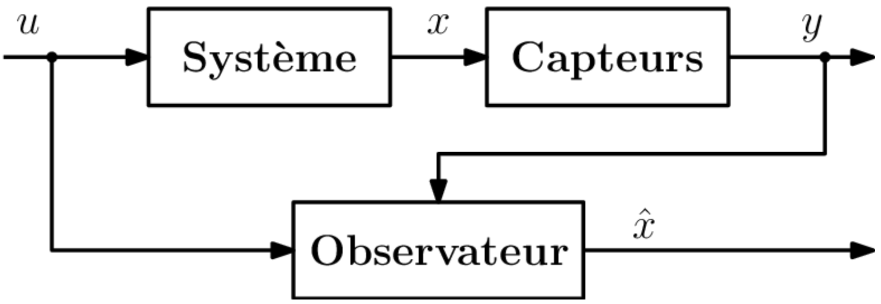

FIG 1. 5 - State Observer [9]

A state observer is a mathematical algorithm that is used to estimate (reconstruct) the x-state of a dynamical system, based on the knowledge of the available measurements (y-outputs) and the u-inputs, based on a representative model of the system (FIG 1.5).

4.2 Model of the System to be Observed

The state representation of a dynamical system is a mathematical model useful for the synthesis of observers. Depending on the physics of the system, the selection of state variables, and the modeling assumptions, this representation can be linear or non-linear. The general form of this representation for a system denoted by Σ is expressed as follows.

$$\Sigma: \begin{cases} \dot{x} = f(x,u) \\ y = h(x) \end{cases} \tag{1.1}$$

Where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector (control signals) and f(.,.) and h(.) are analytical functions and $y \in \mathbb{R}^p$ the output vector (signals measured by the sensors).

The linear system represents a specific case within dynamical systems, where the dynamics of the state are defined by a linear combination of state variables and inputs, and the output is a linear combination of state variables.

$$\dot{x} = \mathbf{A}x + \mathbf{B}u \tag{1.2}$$

$$y = \mathbf{C}x \tag{1.3}$$

A, **B**, and **C** are matrices of appropriate dimensions representing state, input, and output, respectively.

4.3 Observation Principle

Observation involves model-based estimation with measurement-based corrections. The model of the state observer mirrors that of the observed system, with an additional correction term that depends on the deviation e between the measured output y and the estimated output y. The general form of the stat observer of the system state (1.1) is as follows:

$$\dot{\hat{x}} = f(\hat{x}, u) + \mathbf{K}(.)[y - h(\hat{x})] \tag{1.4}$$

The matrix of gain **K**, called observation gain or observer gain, can be x constant or variant with time. The first term $f(\hat{x}, u)$ is the prediction term (estimate), while the second term is the correction term (or innovation).

An observer is considered to converge asymptotically when the observation error $\tilde{x} = x - \hat{x}$ tends to zero at infinity:

$$\lim_{t \to +\infty} ||\tilde{x}|| = 0 \tag{1.5}$$

If the observer converges for all initial states $x(0)$ and $\hat{x}(0)$, it is called global; otherwise, it is classified as local. If the convergence dynamics are adjustable, the observer is called adjustable; If it can be enhanced by a decreasing exponential function, the observer is called exponential.

4.4 Observability Problem

In some cases, it is difficult or impossible to fully determining the internal state of a dynamic system from its external outputs. This can pose challenges in designing and implementing effective control systems, as the controller may not have complete information about the system's behavior [6].

4.4.1 Understanding Observability

To understand the observability problem, consider a dynamic system represented by a state-space model:

$$x(t+1) = \mathbf{A}\,x(t) + \mathbf{B}\,u(t) \tag{1.6}$$

$$\mathrm{y}(t) = \mathbf{C}\,x(t) \tag{1.7}$$

where:

x(t): is the system's state vector at time t

A: is the system matrix

B: is the input matrix

u(t): is the input vector at time t

C: is the output matrix

y(t): is the output vector at time t

A system is considered observable if, for any initial state $x(0)$, the state $x(t)$ can be uniquely determined from the output sequence $y(k)$ for all $k >= 0$. In other words, the controller should be able to reconstruct the entire state of the system from its outputs [6].

4.4.2 Challenges of Unobservability

Unobservability arises when the output matrix C does not provide enough or correct information about the system's state. This can happen for various reasons, such as:

Sensor Noise: Noise refers to unwanted variations in the sensor signal, often caused by electrical interference, environmental factors (e.g., temperature fluctuations), or mechanical vibrations. Noisy data can distort the actual readings, making it harder for controllers or estimation algorithms to accurately assess the system's state.

In oil refineries, temperature or pressure sensors may pick up noise from surrounding machinery, leading to fluctuations in the data that do not reflect the true process conditions.

Measurement Delays: Sensors may have inherent delays in capturing real-time data due to processing times, communication lags, or response characteristics. These delays can create discrepancies between the actual state of the system and the measured data.

Taking natural gas compression systems as an example, flow sensors may take time to register changes in flow rate, leading to a lag in adjusting compressor output.

Sensor Drift: Over time, sensors can experience drift, where their measurements gradually deviate from the true values due to wear and tear, environmental exposure, or calibration errors. Sensor drift can lead to long-term inaccuracies in the data, affecting the quality of decisions based on that data.

For example, in a crude oil processing plant, pressure sensors exposed to harsh chemicals may degrade and provide less accurate readings.

Sensor Failures: Sensors can occasionally fail due to extreme conditions, corrosion, or electrical faults. A failed sensor might provide no data or incorrect data. This can create significant blind spots in the system's monitoring, leading to improper control responses and possible system downtime.

In oil pipelines, a failed flow meter could cause under- or over-estimation of flow rates, potentially leading to safety issues like leaks or ruptures.

Actuator limitations: Actuators may not be able to excite all state variables, making it difficult to observe their behavior from the outputs.

System dynamics: The system's dynamics may cause certain state variables to be unobservable from the outputs.

4.4.3 Consequences of Unobservability

Unobservability can have several negative consequences for control systems:

- Reduced control performance: The controller may make suboptimal control decisions due to incomplete information about the system's state.
- Increased sensitivity to disturbances: Unobservability can make the system more sensitive to disturbances and uncertainties, as the controller cannot fully compensate for them.
- Difficulty in fault detection and isolation: Unobservability can make it challenging to detect and isolate faults in the system, as the controller may not be able to distinguish between fault-induced changes and normal system behavior.

There are several approaches to address the observability problem in control systems, such as sensor placement, actuator selection, system redesign and the approach that can be used to estimate the unobservable state variables from the available outputs, and which will using in this thesis (Observer State) [7].

Example: Consider the following system, modeled by a state space [10]:

$$x(t+1) \;=\; [1\ 1]x(t) \;+\; [1;\ 0]\ u(t) \tag{1.8}$$
$$y(t) \;=\; [0\ 1]x(t) \tag{1.9}$$

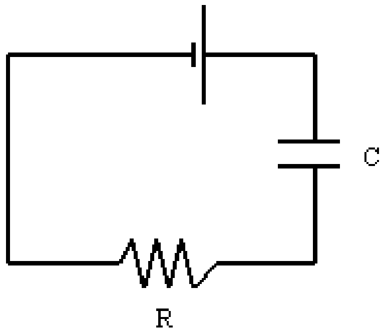

FIG 1. 6 - Simple RC circuit

System Description

This system represents a simple electrical circuit consisting of a capacitor (C) and a resistor (R) connected in series.

$x(t)$ is the state vector of the system at a given time t. In this case, it contains two elements:

- $x_1(t)$: the voltage across the capacitor at time t
- $x_2(t)$: the current flowing through the circuit at time t

$u(t)$ is the input voltage applied to the circuit at time t

$y(t)$ is the output vector of the system, which in this case contains only one element: The voltage measured across the capacitor ($x_2(t)$)

Observability analysis

Is this system observable?

Let's analyze the observability matrix (Wo) of the system:

$$Wo \;=\; [\mathbf{C}\,;\ \mathbf{C} * \mathbf{A}] \;=\; [0\ 1\ ;\ 0\ 1\ 1] \tag{1.10}$$

We calculate the rank of the observability matrix (Wo). The rank of a matrix is the maximum number of linearly independent rows.

In this case, the rank (Wo) = 2.

The rank of the observability matrix is equal to the number of states in the system (2). This means that all information about the system state (voltage and current) can be reconstructed from the measured output (voltage across the capacitor). So, the system is observable.

In this simple example, all state variables are accessible indirectly via voltage measurement. However, observability becomes a more critical issue in more complex systems, where some state variables cannot be measured directly [10].

4.5 Observer Synthesis

Designing a state observer for a linear system is relatively straightforward. On the other hand, the observation of nonlinear systems is a complex field because of their great diversity, hence the frequent use of observers for classes of nonlinear systems.

We will now present the basic structure for observing linear systems (as shown in (FIG 1.7)). Two commonly used methods for adjusting the gain matrix are pole placement and Kalman filtering, both of which are widely used in the industry. For nonlinear systems, we focus on linearization-based approaches, giving rise to the concepts of local or extended observers.

4.5.1 Luenberger observers

Luenberger observers are mathematical models that estimate the internal states of a dynamic system based on available measurements. They are particularly useful for systems where direct measurement of all state variables is impractical or impossible [11].

Theorem 4.5.1: [12] If an invariant linear system (1.2) is observable, then there exists an observer of the form:

$$\dot{\hat{x}} = \mathbf{A}\hat{x} + \mathbf{B}u + \mathbf{K}(y - \mathbf{C}\hat{x}) = (\mathbf{A} - \mathbf{KC})\hat{x} + \mathbf{B}u + \mathbf{K}y \qquad (1.11)$$

Where $\mathbf{K}$ is the observer gain matrix, can asymptotically estimate the true state x if and only if the pair $(\mathbf{A}, \mathbf{C})$ is observable.

In this case, the eigenvalues of the matrix $(\mathbf{A} - \mathbf{KC})$ can be arbitrarily placed in the left half of the complex plane by appropriately choosing the matrix $\mathbf{K}$ with dimensions n × p.

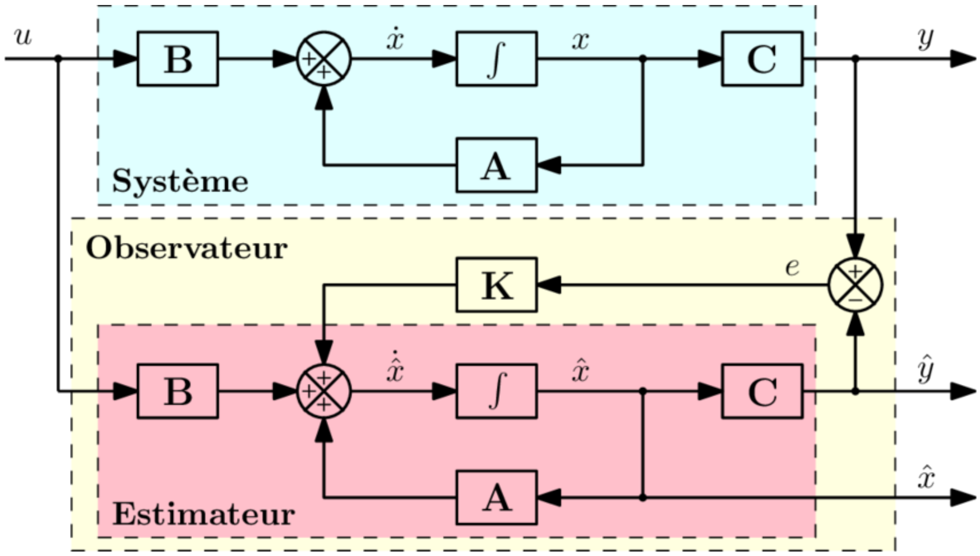

FIG 1. 7 - Status Observer for a Linear System [13]

The dynamics of the observation error $e = x - \hat{x}$ is:

$$\dot{e} = (\mathbf{A} - \mathbf{KC})e \tag{1.12}$$

The **A - KC** matrix appears both in the dynamics of the observed state and in the error. Therefore, if the observer is stable, error x tends asymptotically to 0 ($\lim_{t \to +\infty} e = 0$).

In a Luenberger-type observer, the selection of the $\mathbf{K}$ matrix is determined by the placement of the poles [13].

4.5.2 Local Observes

A local observer, such as a Luenberger observer, is applied to a nonlinear system by linearizing the system around a point of equilibrium, under certain conditions presented below [14]. Consider the nonlinear system (1.1) with its observer (1.4). The dynamics of the observation error $e = x - \hat{x}$ is:

$$\dot{e} = f(x, u) - f(\hat{x}, u) - \mathbf{K}(.)[h(x) - h(\hat{x})] \tag{1.13}$$

The aim is to get the matrix **K** which stabilizes the linearized system around the equilibrium point $x = 0$. The linearization of (1.13) gives the following system:

$$\dot{e} = \left[\frac{\partial f}{\partial x}(x,u) - \mathbf{K}\frac{\partial h}{\partial x}(x)\right] e \tag{1.14}$$

It is difficult, if not impossible, to find a constant **K** matrix that stabilizes the variable system over time (1.14). However, if we assume that the nonlinear system (1.1) has an equilibrium point at $x = x_{ss}$ for an input $u = u_{ss}$, which gives an output $y = 0$:

$$0 = f(x_{ss}, u_{ss}) \qquad ; \qquad 0 = h(x_{ss}) \tag{1.15}$$

and assuming that the vector $x(t)$, defined for all $t \geq 0$, is in a neighborhood ε of x_{ss}, in its sense. $||x(t) - x_{ss}|| \leq \varepsilon$, and that the following matrices:

$$\mathbf{A} = \frac{\partial f}{\partial x}(x_{ss}, u_{ss}) \quad ; \qquad \mathbf{C} = \frac{\partial h}{\partial x}(x_{ss}, u_{ss}) \tag{1.16}$$

satisfy the observability condition (or, more weakly, the detectability condition), then we can find a constant matrix **K** such that **A – KC** is a Hurwitz matrix, meaning its eigenvalues have negative real parts.

Lemme 4.5.2 [15] If the initial error, $||e(0)|| = ||x(0) - x_{ss}||$, is sufficiently small, and if the input $u(t)$ remains sufficiently close to u_{ss} (in its sense $sup_{t\geq 0}||u(t) - u_{ss}||$ is sufficiently small), then:

$$\lim_{t\to+\infty} e = 0 \tag{1.17}$$

5. Conclusion

Chapter One has meticulously constructed a foundation for understanding control systems. We embarked on a comprehensive exploration, dissecting the core principles of regulation. This journey encompassed the critical role of feedback loops in ensuring system stability. Furthermore, we delved into the concept of control systems themselves, including the various components that work in concert to achieve desired outcomes. The section also explored the theory of estimation, its purposes, and the techniques employed for accurate state estimation. Finally, we investigated the concept of observers and their role in achieving system observability, a cornerstone of control design. Equipped with this robust understanding of these regulatory principles, we are now prepared to go to the next chapter of this thesis.

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Chapter Two: Coupled Tank System

1. Introduction

The study of interconnected systems, where changes in one component inevitably ripple through to others, has deep roots in both engineering and the natural world. A quintessential example of this principle is the coupled tank systems, a cornerstone of control engineering dating back to its early days. While deceptively simple in appearance – often consisting of two or more tanks connected by pipes – these systems have proven to be invaluable tools for developing and refining theories and control strategies with far-reaching implications beyond fluid dynamics alone.

Coupled tank systems are particularly well-suited for investigating the dynamic behavior of various parameters, including liquid level, temperature, flow rate, and pressure. The intricate interplay of fluid flow between interconnected tanks, and its subsequent impact on factors like temperature distribution, offers a unique window into a wide array of industrial processes and natural phenomena. From their initial exploration in the 1950s and 60s to their continued relevance in modern research and educational settings, coupled tank systems remain a captivating and indispensable tool for understanding the complex dynamics of interconnected systems.

Throughout this chapter, we will delve into the construction and components of these dynamic systems, highlighting their significance in industries like oil and gas. And, we will focus on the two-tank coupled system since it will be the system that will be working on in our thesis.

2. Coupled tank system

2.1 System Definition

A coupled tank system is a dynamic system consisting of two or more tanks interconnected by pipes or channels, allowing liquid to flow between them. The system typically includes pumps or valves to control the inflow and outflow of liquid. The liquid levels in the tanks are influenced by the flow rates and the physical characteristics of the tanks and connecting pipes. The interaction between the tanks creates a complex system where changes in one tank's level or temperature affect the others, making it a valuable model for studying control theory and fluid dynamics [16].

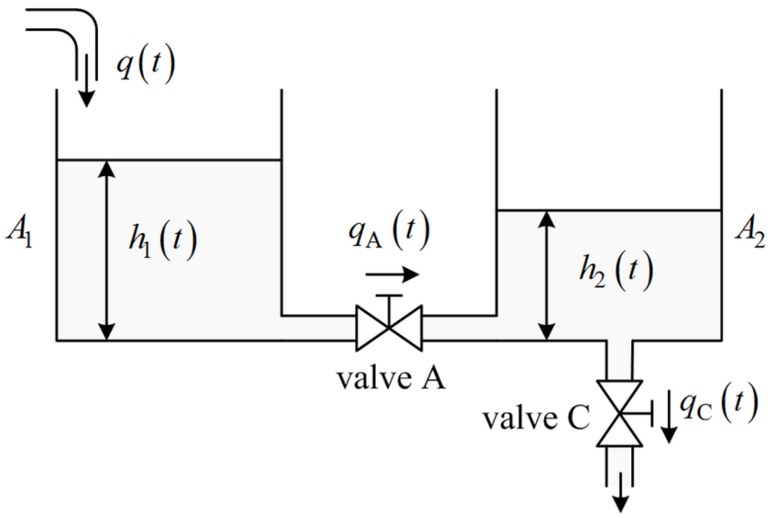

FIG 2. 1 - Example of Coupled Tank System [16]

2.2 System Construction and Components

A coupled tank system is generally constructed using the following components [6]:

Tanks: The core elements of the system, tanks are usually cylindrical containers made of materials like plastic, glass, or metal. Their size and number vary depending on the specific application.

Connecting Pipes or Channels: These conduits facilitate the flow of liquid between the tanks. They can be simple tubes, pipes with valves, or channels with adjustable gates to control the flow rate.

Pumps (Optional): In some configurations, pumps may be used to introduce liquid into the system (typically into the upper tank) or to remove liquid from the lower tank.

Level Sensors: These devices measure the liquid level in each tank, providing feedback to a control system or for monitoring purposes. Various types of level sensors can be used, including float switches, pressure sensors, or ultrasonic sensors.

Temperature Sensors: If temperature control is important, sensors are placed within the tanks to monitor the liquid temperature. These sensors may use thermocouples, thermistors, or resistance temperature detectors (RTDs).

Heaters (Optional): Heaters, such as electric heating elements or steam coils, can be used to raise the temperature of the liquid in one or both tanks.

Control System: A control system, often implemented using a programmable logic controller (PLC) or a computer, uses feedback from the sensors to adjust the flow rates, pump operation, or heater power to achieve the desired liquid levels and temperatures in the tanks.

Valves and Orifices: These devices regulate the flow of liquid between the tanks. Valves offer greater control and can be adjusted manually or automatically, while orifices provide a fixed restriction to flow.

Support Structure: A frame or structure is often used to support the tanks, pipes, and other components, ensuring stability and proper alignment.

The specific configuration and complexity of a coupled tank system (example in FIG 2.1) will depend on its intended purpose, whether it's for laboratory experiments, educational demonstrations, or industrial applications [6].

2.3 System Significance in Industries

2.3.1 Global Applications:

Coupled tank systems are invaluable tools in various applications, owing to their versatility and ability to simulate complex processes. In particular, they find significant applications in [7]:

- Water Treatment: Simulating water distribution networks, optimizing treatment processes, and ensuring consistent water levels in reservoirs.
- Chemical Manufacturing: Modeling chemical reactors, mixing processes, and separation units to ensure product quality and safety.

- Food and Beverage Processing: Simulating fermentation tanks, pasteurization processes, and mixing operations to optimize production and maintain product consistency.
- Pharmaceutical Manufacturing: Modeling drug delivery systems, bioreactors, and mixing processes to ensure precise control and quality of pharmaceutical products.

2.3.2 Oil and Gas Applications:

The oil and gas industry, in particular, benefits greatly from coupled tank systems. These systems serve as valuable tools for [8]:

- Process Simulation and Control: Coupled tank systems can model complex processes like oil refining, chemical separation, and gas processing. By simulating the behavior of interconnected vessels and the flow of fluids, engineers can optimize process parameters, design efficient control systems, and troubleshoot potential issues before they arise in actual plants.
- Level Control: Maintaining precise liquid levels in tanks is crucial for safety, efficiency, and product quality in oil and gas operations. Coupled tank systems provide a platform to test and refine level control strategies, ensuring that tanks are neither overfilled nor run dry.
- Flow Control: The flow rate of fluids through pipelines and between tanks is critical for managing production, distribution, and processing in the oil and gas industry. Coupled tank systems enable engineers to study the dynamics of flow control under various conditions, including changes in pressure, viscosity, and temperature.
- Training and Education: Coupled tank systems serve as valuable educational tools for training engineers and operators in the principles of process control. They provide hands-on experience in designing and tuning controllers, analyzing system responses, and troubleshooting common problems.

In essence, coupled tank systems (example in FIG 2.1) offer a versatile platform for understanding complex processes, mirroring the interconnected nature found in many industrial operations. This allows engineers to gain valuable insights into how larger, more intricate systems behave. Moreover, the ability to manipulate flow rates, levels, and temperatures in a controlled environment enables the development and testing of robust control strategies that can be directly applied to real-world processes [7].

Coupled tank systems also serve as effective training tools, providing hands-on experience that equips engineers and operators with the skills necessary to manage complex industrial processes efficiently and effectively.

3. Two Coupled Tanks System

3.1 System Description

Note: The Two Coupled tanks system is our dynamic system that we will work on in thesis.

Two tanks denoted L and R are connected as shown and into the left tank can be pumped warm and cold water through control valves with the two input signals U_1 and U_2. The temperatures and the volume flows are T_w, T_c, Q_w and Q_c. The water levels in the two tanks are H_1 and H_2 respectively and the tanks have the same cross sectional area A. The flow between the tanks is Q_r and the flow out of the outlet valve of tank R is Q_b. This last valve has the variable opening area A_v [17].

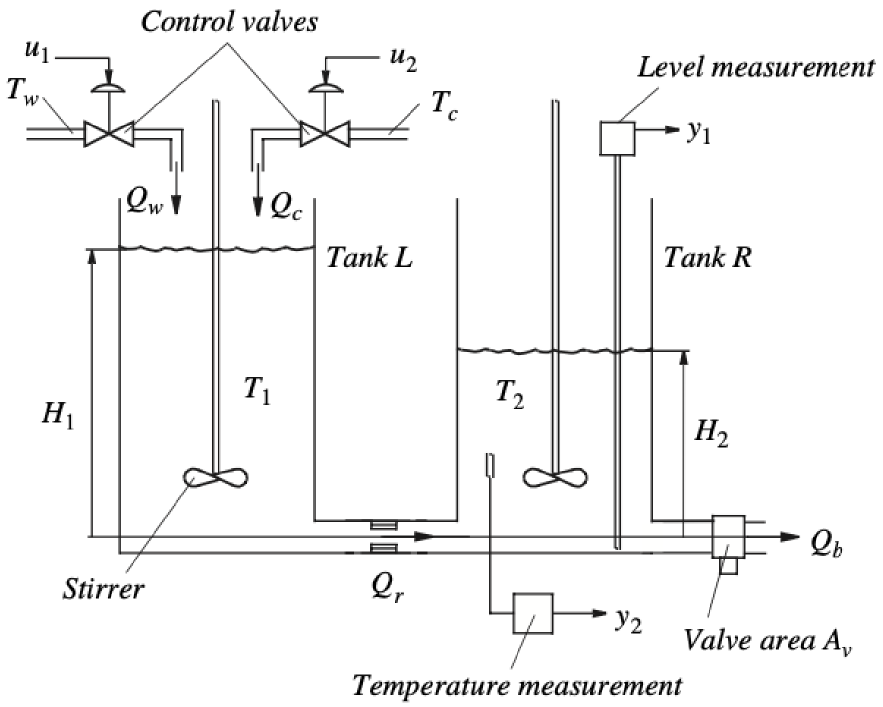

FIG 2. 2 - Two Coupled Tanks System Diagram [17]

The water is stirred rapidly in both tanks and therefore the temperature is assumed to be constant over the entire volume of each of the tanks.

3.2 System Instruments

3.2.1 Control Valves

Control valves are mechanical devices used to regulate the flow of fluids (liquids, gases, or steam) within a system by manipulating the size of a flow passage. They are essential components in industrial processes to maintain desired operating conditions such as pressure, temperature, and flow rate.

FIG 2. 5 - Electric Globe Control Valve [18]

FIG 2. 3 - Manual Control Valve [19]

FIG 2. 4 - Pneumatic Control Valve [20]

3.2.2 Outlet Valve

An outlet valve is a specific type of valve designed to control the flow of fluid exiting a vessel, tank, or pipe. It regulates the discharge rate, pressure, and sometimes the direction of the fluid leaving the system.

FIG 2. 6 - Bottom Outlet Valve [21]

FIG 2. 7 - Glass Lined Flush Valve [22]

3.2.3 Level Sensors

Level sensors are devices designed to detect and measure the level of liquids, solids, or other substances within a container or environment. They provide crucial information for process control, safety, and inventory management in various industries.

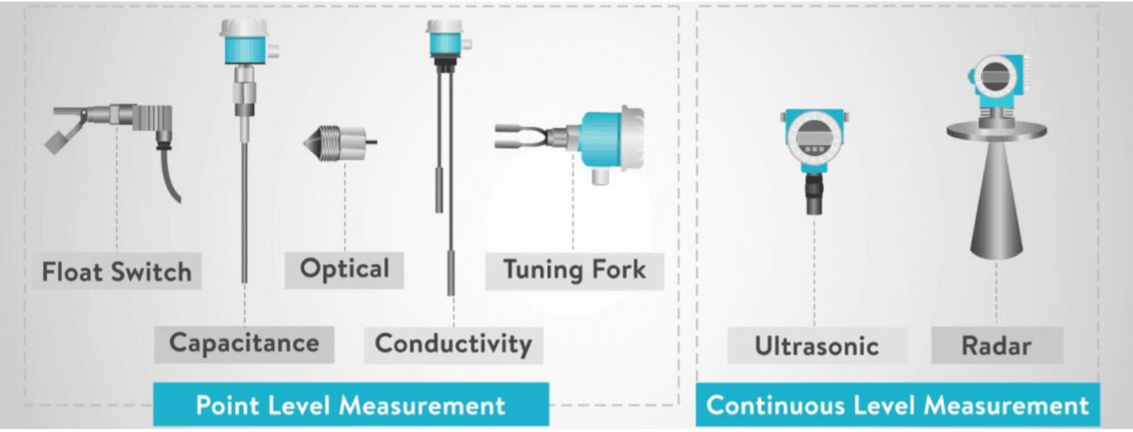

FIG 2. 8 - Different types of Level Sensors [23]

3.2.4 Temperature Sensors

Temperature sensors are devices that detect and measure the degree of hotness or coldness of a substance or environment. They convert this temperature information into an electrical signal that can be read and interpreted by a measuring instrument or control system.

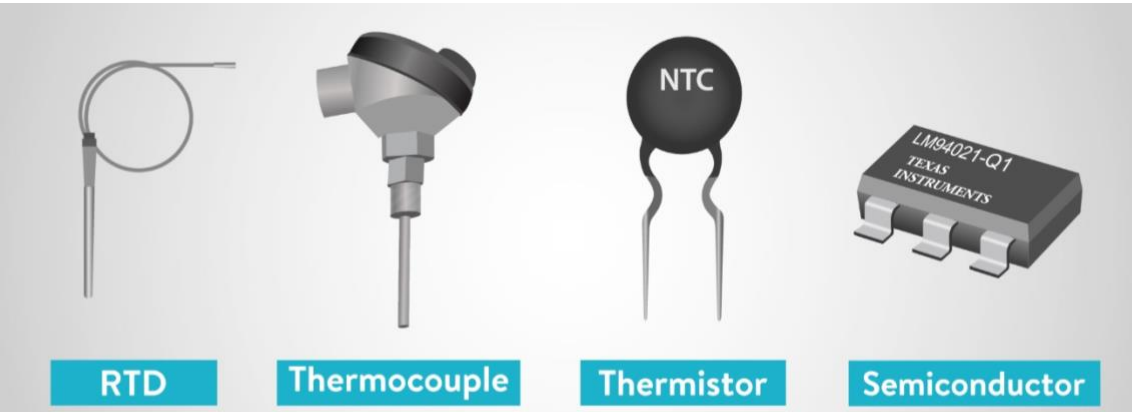

FIG 2. 9 - Different types of Temperature Sensors [24]

3.2.4 Tank Stirrer

A tank stirrer, also known as a tank agitator or tank mixer, is a mechanical device designed to mix, blend, or agitate the contents within a tank or vessel. It is used to create a homogenous mixture, promote heat transfer, enhance chemical reactions, or maintain suspensions of solids in liquids.

FIG 2. 11 - Tank Stirrer with one group of Blades [25]

FIG 2. 10 - Tank Stirrer with two groups of Blades [26]

3.2.5 Other System Instruments

While the pervious instruments that we showed in the pages 35, 36, and 37 are the necessary components that we need in our dynamic System (FIG 2.5) for our work purposes, the Two Coupled Tanks System (FIG 2.5) can be equipped with other instruments depend on the goal that we want to achieve.

From those other instruments we can find for example Pressure sensors, Flow Rate sensors, Pumps, …etc.

FIG 2. 13 - Digital Pressure Gauge (Manometer) [27]

FIG 2. 12 - Digital Liquid Magnetic Flow Meter [28]

FIG 2. 14 - Centrifugal Pump [29]

3.3 System Operation

The coupled-tank system shown in (FIG 2.5) operates based on the interaction of several key components and processes:

Inlet Flow (Q_w and Q_c): Two control valves regulate the flow rates (Q_w and Q_c) of incoming liquids depend on the desired value of the two inputs (U_1 and U_2) with temperatures T_w and T_c into the left (L) and right (R) tanks, respectively. These valves are the primary means of manipulating the system's behavior.

Tanks and Liquid Levels (H_1 and H_2): The two tanks store the incoming liquids, and their levels (H_1 in the left tank and H_2 in the right tank). The level H_2 of the right tank is monitoring by a level sensor, while the level H_1 of the left tank is not.

Inter-tank Flow (Qr): A pipe with valve area A connects the bottom of the two tanks, allowing liquid to flow from the left tank to the right tank at a rate (Q_r) determined by the pressure difference and the valve area.

Outlet Flow (Q_b): A valve at the bottom of the right tank controls the outlet flow (Q_b), which discharges the liquid from the system.

Stirrers: Stirrers in both tanks ensure that the liquids are well-mixed, maintaining a uniform temperature distribution within each tank.

Tanks Temperature (T_1 and T_2): A temperature sensor measure the temperature of the liquids in the right tank only, while the left tank has no temperature sensor.

By manipulating the control valves and potentially the outlet valve, the system can be operated in various modes to achieve different objectives, such as maintaining specific liquid levels, temperatures, or flow rates. This coupled-tank system (FIG 2.5) is a valuable tool for studying and demonstrating various control strategies in a practical and intuitive way.

In order to control the system properly and get the desired outputs, all the system variables (T_1, T_2, H_1, H_2) should be measurable. But in our case the system variables (T_1, H_1) are not measurable, due to the absence of level sensor and temperature sensor in the left tank.

To solve this problem, we will use the Observe State Algorithm shown in (FIG 1.5) to estimate the non-measurable variables (T_1, H_1) so we can control the system effectively.

4. Conclusion

This chapter provided a comprehensive overview of coupled tank systems, emphasizing their significance in various industries as versatile tools for studying and demonstrating control strategies. We began by defining the system, highlighting its key components, exploring its applications across different sectors and getting a little view on some similar dynamic systems. We delved into the specific case of a two-tank coupled system, detailing its construction, instrumentation, and operational principles.

The knowledge we gained from exploring and studying the two-tank coupled system help us to go further more with our work on this dynamic system.

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Chapter Three: Modeling and Linearize of a Two-tank Coupled system

1. Introduction

The study of interconnected systems, where changes in one component inevitably impact others, has deep roots in both engineering and the natural world. A quintessential example is coupled tank systems, a cornerstone of control engineering from its early days. Though deceptively simple in appearance, often consisting of two or more tanks connected by pipes, these systems have proven invaluable for developing and refining theories and control strategies with far-reaching implications beyond fluid dynamics. However, to optimize their performance and ensure stability, we need a deeper understanding of their behavior. This is where the powerful tools of modeling and linearization come into play.

Modeling provides a framework to represent the complex dynamics of dynamic systems in a simplified yet informative way. By constructing a mathematical model, we can gain valuable insights into how these systems respond to changes in operating conditions. This allows us to predict their behavior, analyze their efficiency, and ultimately optimize their design for specific industrial needs.

However, real-world dynamic systems often exhibit non-linear characteristics, making them challenging to analyze directly. This is where linearization steps in. This technique approximates the non-linear behavior around a specific operating point with a linear model. By leveraging the well-understood principles of linear systems, we can effectively analyze and design control systems for these non-linear systems.

Throughout this chapter, we will mention some general information about modeling and linearization of dynamic systems. Furthermore, we will start constructing a mathematical model of our dynamic system “Two-tank Coupled System”. After that we’ll approximates the non-linear behavior of our system around a specific operating point with a linear model. That we allow us to analyze and design control system for this non-linear dynamic system.

2. Modeling a Dynamic System

2.1 Definition of Modelisation

Modeling is the process of creating a simplified mathematical representation of a real-world system or phenomenon to understand its behavior. It involves identifying key variables, their relationships, and expressing them in equations to make predictions or design interventions to improve the system's performance. This can be applied to various domains, including physics, engineering, economics, and biology.

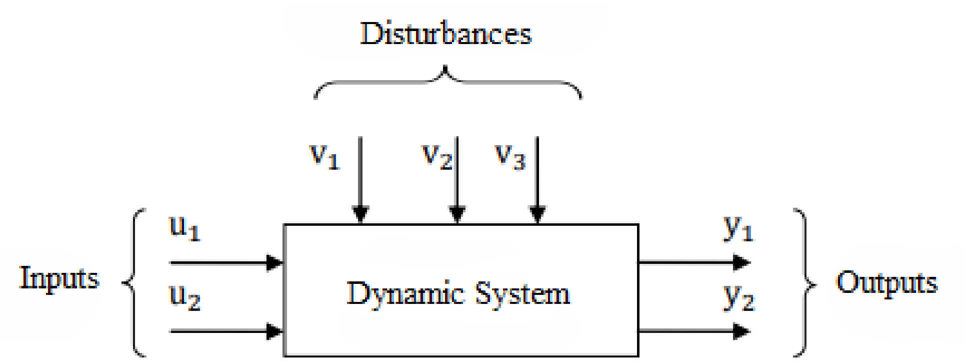

FIG 3. 1 - Dynamic System Multi Variables

2.2 Types of Dynamic System Models

Linear vs. Nonlinear: Linear models assume linear relationships between variables, while nonlinear models capture more complex behavior.

Continuous-Time vs. Discrete-Time: Continuous-time models represent systems that evolve continuously over time, while discrete-time models describe systems that change at discrete time steps.

State-Space vs. Transfer Function: State-space models describe the system's internal state variables and their evolution, while transfer function models relate the system's input and output without explicitly considering the internal state.

2.3 Uses of Dynamic System Models

Simulation and Prediction: Simulate the system's behavior under different conditions to predict future states.

Analysis and Understanding: Gain insights into the system's underlying dynamics, stability, and response to disturbances.

Control Design: Design controllers to achieve desired performance and stability.

Optimization: Optimize system parameters to maximize or minimize specific objectives.

2.4 Modeling the Two-tank Coupled System

2.4.1 Identifying Key Variables

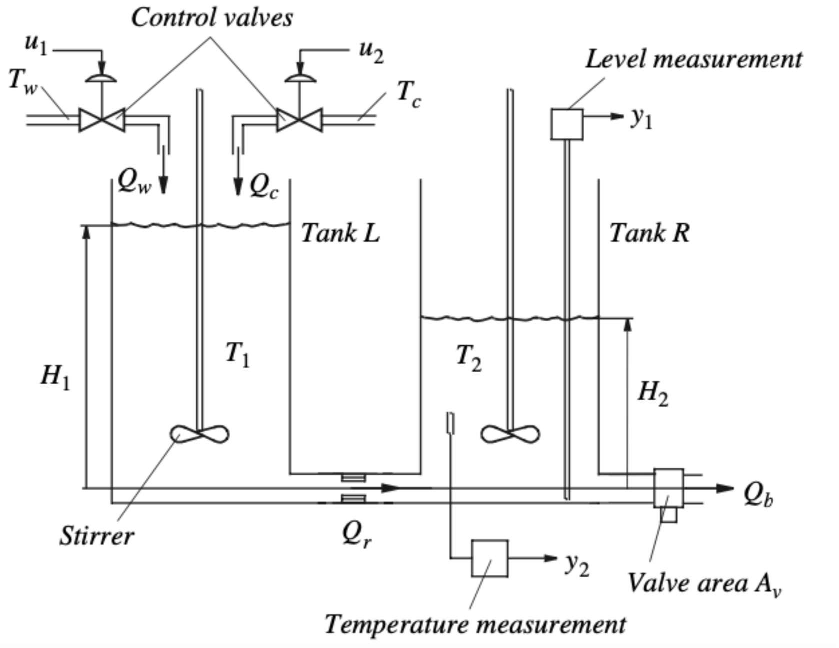

FIG 3. 2 - Two Coupled Tanks System [17]

From this diagram we can identify the key variables of our dynamic system. And we have three types of variables:

State Variables:

- H_1: Liquid level in Tank L
- H_2: Liquid level in Tank R
- T_1: Temperature in Tank L
- T_2: Temperature in Tank R

Input Variables:

- Q_w: Flow rate of warm water into Tank L
- Q_c: Flow rate of cold water into Tank L

Output Variables:

- y_1: Liquid level measurement of Tank R (output for level control)
- y_2: Temperature measurement of Tank R (output for temperature control)

2.4.2 Defining Relationships

Two quantities can be measured on our system: the level and the temperature of tank R. For these two measurement systems, it is known that:

$$y_1 = \mathbf{K}_h H_2 \tag{3.1}$$

$$y_2 = \mathbf{K}_t T_2 \tag{3.2}$$

where $\mathbf{K}_h$ and $\mathbf{K}_t$ are the transducer gains.

The two control valves have identical flow characteristics, and the following two relationships are assumed to be valid:

$$Q_w = \mathbf{K}_a u_1 \tag{3.3}$$

$$Q_c = \mathbf{K}_a u_2 \tag{3.4}$$

where $\mathbf{K}_a$ is the flow coefficient.

The mathematical model of our system involves volume and energy conservation laws, as well as suitable equations describing flow through orifices. Conservation of fluid volume for the two tanks gives:

$$\mathbf{S}\dot{H}_1 = Q_w + Q_c - Q_r \tag{3.5}$$

$$\mathbf{S}\dot{H}_2 = Q_r - Q_b \tag{3.6}$$

The energy content in the water volumes can be expressed as:

$$E_L = \mathbf{S}H_1\boldsymbol{\rho}\mathbf{c}(T_1 - T_0) \tag{3.7}$$

$$E_R = \mathbf{S}H_2\boldsymbol{\rho}\mathbf{c}(T_2 - T_0) \tag{3.8}$$

where $\boldsymbol{\rho}$ is the mass density and c is the specific heat capacity of water. T_0 is the reference temperature at which the energy is zero. It is easy to show that one can set T_0, reducing the energy equations to:

$$E_L = \mathbf{S} H_1 \boldsymbol{\rho} \mathbf{c} T_1 \tag{3.9}$$
$$E_R = \mathbf{S} H_1 \boldsymbol{\rho} \mathbf{c} T_2 \tag{3.10}$$

For the flow through the orifices, it is reasonable to assume that a square root relation is valid. This can be expressed by the formula:

$$Q = \mathbf{C}_d \mathbf{A}_0 \sqrt{\frac{2}{\boldsymbol{\rho}} \Delta P} \tag{3.11}$$

where P is the differential pressure across the orifice, $\boldsymbol{\rho}$ is the mass density, $\mathbf{A_0}$ is the area of the orifice, and $\mathbf{C_d}$ is a constant loss coefficient. The hydrostatic pressure in a liquid at a level H below the surface is given by $P_h = \boldsymbol{\rho} \mathbf{g} H + P_a$ (where P_a is the atmospheric pressure, and $\mathbf{g}$ is the acceleration due to gravity). Thus, the flow through the outlet valve can be written as:

$$Q_b = \mathbf{D}_v \mathbf{A}_v \sqrt{H_2} \tag{3.12}$$

Where $\mathbf{D}_v = \mathbf{C}_d \sqrt{2\mathbf{g}}$.
The orifice between the tanks has a constant flow area, and one can write:

$$\mathrm{Q}_r = \mathbf{C}_0 \sqrt{H_1 - H_2} \tag{3.13}$$

Where $\mathbf{C}_0 = \mathbf{K}_0 \mathbf{A}_0 \sqrt{2\mathbf{g}}$, and it is assumed that $H_1 > H_2$.

2.4.3 Formulating Equations

Now, the fact that the net power flux into the tanks equals the accumulated energy per time unit can be utilized. Thus, the time derivatives of the energy expressions (3.9) and (3.10) provide the left-hand sides of two new equations:

$$\frac{dE_L}{dt} = \mathbf{S} \boldsymbol{\rho} \mathbf{c} \frac{d(H_1 T_1)}{dt} = Q_w \boldsymbol{\rho} \mathbf{c} T_w + Q_c \boldsymbol{\rho} \mathbf{c} T_c - Q_r \boldsymbol{\rho} \mathbf{c} T_1 \tag{3.14}$$
$$\frac{dE_R}{dt} = \mathbf{S} \boldsymbol{\rho} \mathbf{c} \frac{d(H_2 T_2)}{dt} = Q_r \boldsymbol{\rho} \mathbf{c} T_1 - Q_b \boldsymbol{\rho} \mathbf{c} T_2 \tag{3.15}$$

Dividing all terms by $\boldsymbol{\rho c}$ and differentiating the product yields:

$$\mathbf{S}(\dot{H}_1 T_1 + H_1 \dot{T}_1) = Q_w T_w + Q_c T_c - Q_r T_1 \tag{3.16}$$
$$\mathbf{S}(\dot{H}_2 T_2 + H_2 \dot{T}_2) = Q_r T_1 - Q_b T_2 \tag{3.17}$$

Inserting Eqs. (3.5) and (3.6) into Eqs. (3.16) and (3.17) respectively, yields:

$$\mathbf{S} H_1 \dot{T}_1 = Q_w T_w + Q_c T_c - Q_c T_1 - Q_w T_1 \tag{3.18}$$
$$\mathbf{S} H_2 \dot{T}_2 = Q_r T_1 - Q_r T_2 \tag{3.19}$$

By inserting Eqs. (3.12) and (3.13) into Eqs. (3.5), (3.6), (3.18) and (3.19), we will get the final system equations:

$$\begin{cases} \dot{H}_1 = \frac{1}{\mathbf{S}}(Q_w + Q_c - \mathbf{C}_0\sqrt{H_1 - H_2}) \\ \dot{H}_2 = \frac{1}{\mathbf{S}}(\mathbf{C}_0\sqrt{H_1 - H_2} - \mathbf{D}_v \mathbf{A}_v \sqrt{H_2}) \\ \dot{T}_1 = \frac{1}{\mathbf{S}H_1}(T_w - T_1)Q_w + (T_c - T_1)Q_c \\ \dot{T}_2 = \frac{1}{\mathbf{S}H_2}(T_1 - T_2)\mathbf{C}_0\sqrt{H_1 - H_2} \end{cases} \tag{3.20}$$

The natural choice of states is the output variables of the four integrators. The outlet valve area and the two inlet temperatures are disturbances, and the two control valve voltages are the manipulable inputs. Therefore, the state, input, and disturbance vectors will be:

$$\mathbf{x}(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ T_1 \\ T_2 \end{bmatrix}, \ \mathbf{u}(t) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \ \mathbf{v}(t) = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} A_v \\ T_w \\ T_c \end{bmatrix} \tag{3.21}$$

The system output is considered to be the height H_2 and the temperature T_2 of the right tant. So, the output vectors will be:

$$y(t) = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} H_2 \\ T_2 \end{bmatrix} = \mathbf{C}\mathbf{x}(t) \tag{3.22}$$

Now by replacing what's in (3.21) into (3.20), we will get our non-linear model representation:

$$\begin{cases} \dot{x}_1 = \frac{1}{\mathrm{S}}[\mathbf{K}_a(u_1 + u_2) - \mathbf{C}_0\sqrt{x_1 - x_2}] \\ \dot{x}_2 = \frac{1}{\mathrm{S}}[\mathbf{C}_0\sqrt{x_1 - x_2} - \mathbf{D}_v v_1 \sqrt{x_2}] \\ \dot{x}_3 = \frac{1}{\mathrm{S}x_1}[(v_2 - x_3)\mathbf{K}_a u_1 + (v_3 - x_3)\mathbf{K}_a u_2] \\ \dot{x}_4 = \frac{1}{\mathrm{S}x_2}(x_3 - x_4)\mathbf{C}_0\sqrt{x_1 - x_2} \end{cases} \tag{3.23}$$

$\mathbf{K}_a u_1$ represent the flow rate of warm water, while $\mathbf{K}_a u_2$ represent the flow rate of cold water as is shown in (3.3) and (3.4) respectively.

The output is already linear in the states. And by simplifying (3.22) we will obtain:

$$y(t) = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{K}_h & 0 & 0 \\ 0 & 0 & 0 & \mathbf{K}_t \end{bmatrix} \mathbf{x}(t) \tag{3.24}$$

Note: since we want to consider y_1 and y_2 respectively on meter (m) and degree Celsius (°C), $\mathbf{K}_h$ and $\mathbf{K}_t$ will equal one.

3. Linearize a Dynamic System

3.1 Definition of Linearization

Linearization is the process of finding the linear approximation of a nonlinear system around a specific operating point. In the context of dynamic systems, it involves approximating the nonlinear equations that describe the system with linear equations that are valid near a particular equilibrium point.

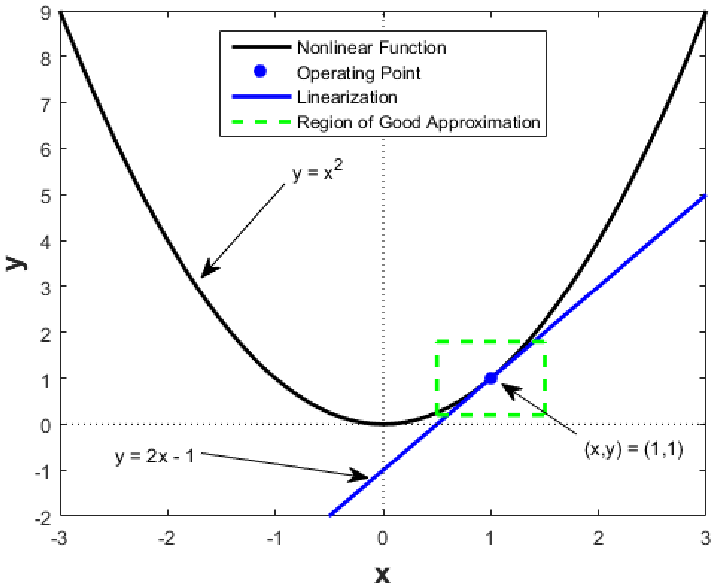

FIG 3. 3 - Linearize Non-linear Model [30]

3.2 Purpose of Linearization

Simplification: Linear models are generally easier to analyze and design controllers for than nonlinear models.

Local Behavior: Linearization allows us to study the local behavior of a nonlinear system around an equilibrium point. This can be useful for understanding stability properties and designing controllers that work well near the operating point.

3.3 Linearize our Two-tank Coupled System

3.3.1 Choosing an Operating Point

In order to choose an operating point, our system need to be in the steady-state condition. This refers to a situation where the system's variables remain constant over time, and the rates of change of all state variables are zero. In other words, the system has reached a state of equilibrium where there is no further change in its behavior as long as the external inputs and conditions remain constant.

Mathematically, this means:

$$\begin{cases} 0 = \frac{1}{\mathbf{S}}[\mathbf{K}_a(u_{10} + u_{20}) - \mathbf{C}_0\sqrt{x_{10} - x_{20}}] \\ 0 = \frac{1}{\mathbf{S}}[\mathbf{C}_0\sqrt{x_{10} - x_{20}} - \mathbf{D}_v v_{10}\sqrt{x_{20}}] \\ 0 = \frac{1}{\mathbf{S}x_{10}}[(v_{20} - x_{30})\mathbf{K}_a u_{10} + (v_{30} - x_{30})\mathbf{K}_a u_{20}] \\ 0 = \frac{1}{\mathbf{S}x_{20}}(x_{30} - x_{40})\mathbf{C}_0\sqrt{x_{10} - x_{20}} \end{cases} \tag{3.25}$$

And by simplifying (3.25) we will get:

$$\begin{cases} \mathbf{K}_a(u_{10} + u_{20}) = \mathbf{C}_0\sqrt{x_{10} - x_{20}} \\ \mathbf{C}_0\sqrt{x_{10} - x_{20}} = \mathbf{D}_v v_{10}\sqrt{x_{20}} \\ (v_{20} - x_{30})u_{10} = -(v_{30} - x_{30})u_{20} \\ (x_{30} - x_{40})\mathbf{C}_0\sqrt{x_{10} - x_{20}} = 0 \end{cases} \tag{3.26}$$

The four equations in (3.26) contain nine variables. If, for example, values for the two inputs and the three disturbance variables are selected, the four stationery states (Operating Points at the steady-state condition) can be determined.

The system parameter values will be assumed:

$$\mathbf{S} = 0.785\ \text{m}^2 \qquad \mathbf{K}_a = 0.004\ \text{m}^3/\text{volt}.\text{sec}$$
$$\mathbf{D}_v = 2.66\ \text{m}^{1/2}/\text{sec} \qquad \mathbf{K}_h = 1\ \text{volt/m}$$
$$\mathbf{C}_0 = 0.056\ \text{m}^{5/2}/\text{sec} \qquad \mathbf{K}_t = 1\ \text{volt/°C}$$

assumingly that the input voltages can take on values in the interval 0–10 volts and choosing $u_{10} = u_{20} = 5$ volts. Also, setting $\mathbf{A}_{v0} = 0.0122\ \text{m}^2$, $\text{T}_{w0} = 60°\text{C}$, $\text{T}_{c0} = 30°\text{C}$, the values of the operating points at the steady-state condition will be:

$x_{10} = 2.03$ m
$x_{20} = 1.519$ m
$x_{30} = x_{40} = 45°\text{C}$

3.3.2 Taylor Series Expansion

The linearization of nonlinear systems is performed using a Taylor series expansion around the operating points $(\mathbf{X_0}, \mathbf{U_0}, \mathbf{V_0})$, where higher-order terms are negligible. It can be represented as follows:

$$\dot{\mathbf{x}} \approx f(\mathbf{x}_0, \mathbf{u}_0, \mathbf{v}_0) + \frac{df(\mathbf{x}_0,\mathbf{u}_0,\mathbf{v}_0)}{d\mathbf{x}}\Delta\mathbf{x} + \frac{df(\mathbf{x}_0,\mathbf{u}_0,\mathbf{v}_0)}{d\mathbf{u}}\Delta\mathbf{u} + \frac{df(\mathbf{x}_0,\mathbf{u}_0,\mathbf{v}_0)}{d\mathbf{v}}\Delta\mathbf{v} \qquad (3.27)$$

Where

$$\frac{df(\mathbf{x}_0,\mathbf{u}_0,\mathbf{v}_0)}{d\mathbf{x}} = \mathbf{A},\ \frac{df(\mathbf{x}_0,\mathbf{u}_0,\mathbf{v}_0)}{d\mathbf{u}} = \mathbf{B},\ \frac{df(\mathbf{x}_0,\mathbf{u}_0,\mathbf{v}_0)}{d\mathbf{v}} = \mathbf{B}_v \qquad (3.28)$$

$(\mathbf{A}, \mathbf{B}, \mathbf{B}_v)$ are constant matrices.

The linearization of the output function follows exactly the same lines. So, it also can be represented as follows:

$$\mathbf{y}(t) \approx f(\mathbf{x}_0) + \frac{df(\mathbf{x}_0)}{d\mathbf{x}}\Delta\mathbf{x}(t), \text{ where } \frac{df(\mathbf{x}_0)}{d\mathbf{x}} = \mathbf{C} \qquad (3.29)$$

But since the outputs are already linear, we don't need to calculate the matrix **C**.

From (3.27), (3.28) and (3.29), the linearized state, and output equations can write in form:

$$\begin{cases} \Delta\dot{\mathbf{x}}(t) = \mathbf{A}\Delta \mathrm{x} + \mathbf{B}\Delta \mathrm{u} + \mathbf{B}_v \Delta \mathrm{v} \\ \Delta\mathbf{y}(t) = \mathbf{C}\Delta \mathrm{x} \end{cases} \tag{3.30}$$

For small variations around the equilibrium points $\mathbf{x}_0, \mathbf{u}_0, \mathbf{v}_0, \mathbf{y}_0$ we obtain:

$$\begin{cases} \mathbf{x} = \mathrm{x}_0 + \Delta \mathrm{x} \\ \mathbf{u} = \mathrm{u}_0 + \Delta \mathrm{u} \\ \mathbf{v} = \mathrm{v}_0 + \Delta \mathrm{v} \\ \mathbf{y} = \mathrm{y}_0 + \Delta \mathrm{y} \end{cases} \tag{3.31}$$

The Δ-variables above denote the (small) deviations from the stationary (constant) values and are called the incremental states, inputs, disturbances, and outputs.

The calculation of the matrices $(\mathbf{A}, \mathbf{B}, \mathbf{B}_v)$ will be like this:

$$\mathbf{A} = \begin{bmatrix} \frac{d\dot{x}_1}{dx_1} & \frac{d\dot{x}_1}{dx_2} & \frac{d\dot{x}_1}{dx_3} & \frac{d\dot{x}_1}{dx_4} \\ \frac{d\dot{x}_2}{dx_1} & \frac{d\dot{x}_2}{dx_2} & \frac{d\dot{x}_2}{dx_3} & \frac{d\dot{x}_2}{dx_4} \\ \frac{d\dot{x}_3}{dx_1} & \frac{d\dot{x}_3}{dx_2} & \frac{d\dot{x}_3}{dx_3} & \frac{d\dot{x}_3}{dx_4} \\ \frac{d\dot{x}_4}{dx_1} & \frac{d\dot{x}_4}{dx_2} & \frac{d\dot{x}_4}{dx_3} & \frac{d\dot{x}_4}{dx_4} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} \frac{d\dot{x}_1}{du_1} & \frac{d\dot{x}_1}{du_2} \\ \frac{d\dot{x}_2}{du_1} & \frac{d\dot{x}_2}{du_2} \\ \frac{d\dot{x}_3}{du_1} & \frac{d\dot{x}_3}{du_2} \\ \frac{d\dot{x}_4}{du_1} & \frac{d\dot{x}_4}{du_2} \end{bmatrix}, \ \mathbf{B}_v = \begin{bmatrix} \frac{d\dot{x}_1}{dv_1} & \frac{d\dot{x}_1}{dv_2} & \frac{d\dot{x}_1}{dv_3} \\ \frac{d\dot{x}_2}{dv_1} & \frac{d\dot{x}_2}{dv_2} & \frac{d\dot{x}_2}{dv_3} \\ \frac{d\dot{x}_3}{dv_1} & \frac{d\dot{x}_3}{dv_2} & \frac{d\dot{x}_3}{dv_3} \\ \frac{d\dot{x}_4}{dv_1} & \frac{d\dot{x}_4}{dv_2} & \frac{d\dot{x}_4}{dv_3} \end{bmatrix} \tag{3.32}$$

$$\mathbf{A} = \begin{bmatrix} \frac{-\mathrm{C}_0}{2\mathrm{S}\sqrt{x_{10}-x_{20}}} & \frac{\mathrm{C}_0}{2\mathrm{S}\sqrt{x_{10}-x_{20}}} & 0 & 0 \\ \frac{\mathrm{C}_0}{2\mathrm{S}\sqrt{x_{10}-x_{20}}} & \frac{\mathrm{C}_0}{2\mathrm{S}\sqrt{x_{10}-x_{20}}} - \frac{\mathrm{D}_v v_{10}}{2\mathrm{S}\sqrt{x_{20}}} & 0 & 0 \\ 0 & 0 & -\frac{\mathrm{K}_a(u_{10}+u_{20})}{\mathrm{S}x_{10}} & 0 \\ 0 & 0 & \frac{\mathrm{C}_0\sqrt{x_{10}-x_{20}}}{\mathrm{S}x_{20}} & -\frac{\mathrm{C}_0\sqrt{x_{10}-x_{20}}}{\mathrm{S}x_{20}} \end{bmatrix} \tag{3.33}$$

$$\mathbf{B} = \begin{bmatrix} \frac{\mathrm{K}_a}{\mathrm{S}} & \frac{\mathrm{K}_a}{\mathrm{S}} \\ 0 & 0 \\ \frac{\mathrm{K}_a(v_{20}+x_{30})}{\mathrm{S}x_{10}} & \frac{\mathrm{K}_a(v_{30}+x_{30})}{\mathrm{S}x_{10}} \\ 0 & 0 \end{bmatrix}, \ \mathbf{B}_v = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{\mathrm{D}_v\sqrt{x_{20}}}{\mathrm{S}} & 0 & 0 \\ 0 & \frac{\mathrm{K}_a u_{10}}{\mathrm{S}x_{10}} & \frac{\mathrm{K}_a u_{20}}{\mathrm{S}x_{10}} \\ 0 & 0 & 0 \end{bmatrix} \tag{3.34}$$

With the parameter and stationary variable values above, we can find:

$$\mathbf{A} = \begin{bmatrix} -0.0499 & 0.0499 & 0 & 0 \\ 0.0499 & -0.0667 & 0 & 0 \\ 0 & 0 & -0.0251 & 0 \\ 0 & 0 & 0.0335 & -0.0335 \end{bmatrix} \tag{3.34}$$

$$\mathbf{B} = \begin{bmatrix} 0.00510 & 0.00510 \\ 0 & 0 \\ 0.0377 & -0.0377 \\ 0 & 0 \end{bmatrix}, \ \mathbf{B}_v = \begin{bmatrix} 0 & 0 & 0 \\ -4.177 & 0 & 0 \\ 0 & 0.01255 & 0.01255 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.35}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.36}$$

The system poles in open loop are:

$$\begin{cases} P_1 = -0.0077 \\ P_2 = -0.1089 \\ P_3 = -0.0335 \\ P_4 = -0.0251 \end{cases} \tag{3.37}$$

Since they all have negative real parts, the linear model is stable around this operating point.

The linearized model describes the behavior of deviations from the stationary values. If the incremental system vectors are defined as in equation (3.31), then:

$$\Delta\mathbf{x}(t) = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix} = \begin{bmatrix} \Delta H_1 \\ \Delta H_2 \\ \Delta T_1 \\ \Delta T_2 \end{bmatrix}, \ \Delta\mathbf{u}(t) = \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix}, \ \Delta\mathbf{v}(t) = \begin{bmatrix} \Delta v_1 \\ \Delta v_2 \\ \Delta v_3 \end{bmatrix} \tag{3.38}$$

Where

$$\begin{cases} H_1(t) = H_{10} + \Delta H_1 \\ H_2(t) = H_{20} + \Delta H_2 \\ T_1(t) = T_{10} + \Delta T_1 \\ T_2(t) = T_{20} + \Delta T_2 \end{cases} \tag{3.39}$$

4. Simulate a Dynamic System

4.1 Simulation definition

Simulation of a dynamic model is using a computer program to mimic the behavior of a changing system over time, based on mathematical equations representing the system's dynamics. It involves numerical methods to solve these equations and track the evolution of system variables. This virtual experimentation allows for prediction, optimization, and deeper understanding of complex interactions without real-world consequences. It has wide applications in fields like engineering, physics, biology, economics, and social sciences.

4.2 Simulation Software

Simulation software refers to computer programs designed to create virtual representations of real-world systems or processes, enabling users to observe, analyze, and experiment with their behavior under various conditions. These software tools leverage mathematical models and algorithms to mimic the dynamics of these systems, facilitating predictions, optimizations, and deeper understanding of complex interactions without the need for costly or risky real-world experimentation [31].

4.2.1 Key Features and Applications [31]

Modeling: Simulation software enables the creation of detailed models of systems, incorporating components, relationships, and behaviors. These models can be represented visually or through code, depending on the specific software.

Experimentation: Once a model is built, simulation software allows users to run virtual experiments by changing inputs, parameters, or scenarios and observing the resulting outputs. This facilitates what-if analysis and scenario planning.

Analysis: Simulation software provides tools for analyzing results, including statistical analysis, visualization, and optimization. This helps identify trends, bottlenecks, and opportunities for improvement.

Applications: Simulation software finds extensive use across various fields, including:

- Engineering: Design, testing, and optimization of complex systems like manufacturing processes, supply chains, and transportation networks.
- Healthcare: Modeling patient flows, resource allocation, and disease outbreaks to improve efficiency and decision-making.
- Business: Analyzing market trends, customer behavior, and financial risk to support strategic planning and decision-making.
- Education: Providing interactive and engaging learning experiences through simulations of scientific phenomena and real-world scenarios.

4.2.2 Popular Simulation Software

AnyLogic: A versatile simulation platform supporting various modeling approaches like discrete-event, agent-based, and system dynamics.

MATLAB Simulink: A graphical environment for modeling, simulating, and analyzing multi-domain dynamic systems.

Arena: A specialized simulation software for discrete-event systems, widely used in manufacturing and service industries.

Simul8: A user-friendly simulation tool for process modeling and optimization, particularly useful for visualizing and improving workflows.

FlexSim: A 3D simulation software for modeling and optimizing complex systems, particularly those involving material handling and logistics.

Simulation software has become an indispensable tool for modern problem-solving and decision-making, providing a powerful means to explore complex systems, test ideas, and optimize solutions in a virtual environment before implementing them in the real world [32].

4.2.3 MATLAB Simulink

Matlab Simulink is a powerful graphical programming environment that enables engineers and scientists to model, simulate, and analyze multi-domain dynamic systems using a visual block diagramming approach. It supports model-based design, automatic code generation, and analysis tools, making it valuable for diverse applications in industries like automotive, aerospace, robotics, industrial automation, renewable energy, and biomedical engineering [33].

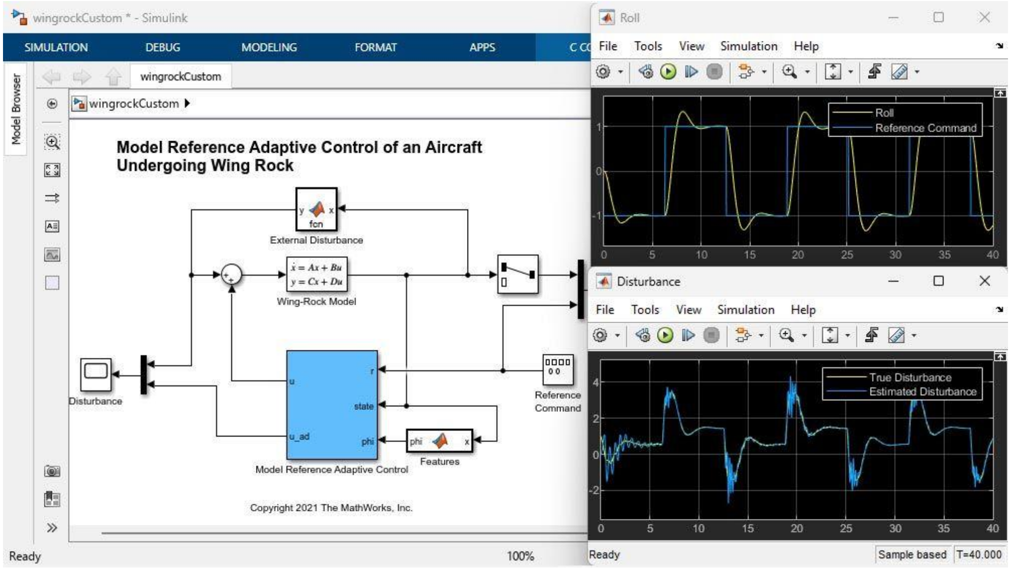

FIG 3. 4 - MATLAB Simulink Interface [33]

4.3 Dynamic System Models on Matlab Simulink

4.3.1 Non-Linear Model

it is a mathematical representation that captures the full complexity of the two-tank coupled system's dynamics, including the nonlinear relationships between the liquid levels, flow rates, and temperatures. It is described by a set of nonlinear differential equations derived from mass and energy balance principles.

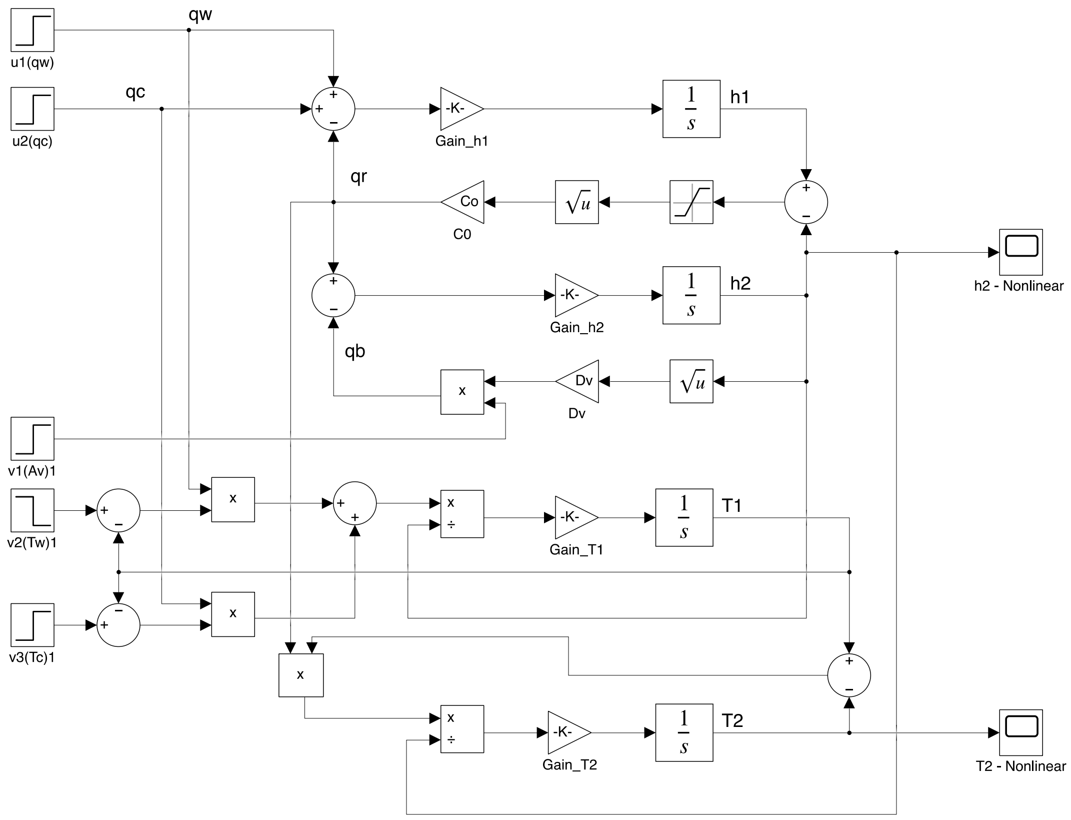

FIG 3. 5 - Non-Linear Model of Two-tank Coupled System

4.3.2 Linearized Model

It is a simplified approximation of the nonlinear model obtained by linearizing the nonlinear equations around a specific operating point (steady-state condition). It uses linear differential equations to represent the system's behavior in a small region around the operating point.

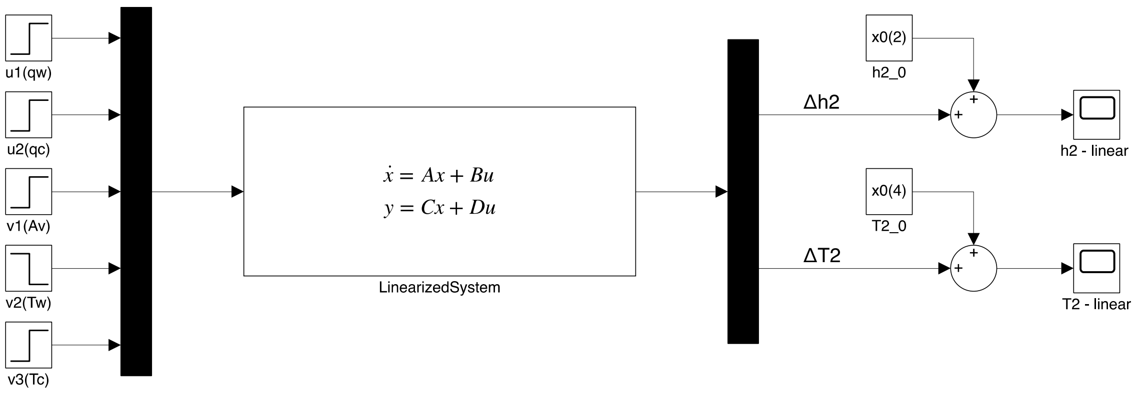

FIG 3. 6 - Linearized Model of Two-tank Coupled System

4.3 Simulation Purpose

The purpose of our simulation is to compare the responses of the nonlinear (FIG 3.5) and linearized (FIG 3.6) models of the two-tank system under different inputs and disturbances conditions.

Inputs Changes: We'll apply step changes to the warm and cold water flow rates (Q_w and Q_c) to observe how the liquid levels and temperatures in the tanks respond.

Disturbances: We'll introduce disturbances in the form of changes to the valve area ($\mathbf{A}_v$), warm water temperature (T_w), and cold water temperature (T_c) to see how the system reacts to these external variations.

By comparing the responses of the nonlinear and linearized models, we can assess the accuracy and limitations of the linearized model as an approximation of the nonlinear system's behavior.

We're particularly interested in the liquid level (H_2) and temperature (T_2) in the right tank, as these are the measurable outputs of the system.

The simulation aims to provide insights into:

- How well the linearized model captures the dynamics of the nonlinear system, especially under different input and disturbance conditions.
- The range of operating conditions where the linearized model provides a good approximation.
- The potential limitations of using a linearized model for control design or analysis of this system.

4.4 Simulation Results

4.4.1 Input Change Response (No Disturbances, Changed Inputs)

In the first case, we'll evaluate how well the linearized model captures the system's response to changes in the input flow rates.

Conditions:

- Simulation time interval: $t \in [0\,;3000s]$
- Sample time (Pas) equal 0.1s

- No disturbances are applied ($\mathbf{A}_v$, T_w, and T_c remain at their operating point values).
- Step changes are introduced in the input flow rates Q_w and Q_c.

The linearized model should provide a reasonable approximation of the nonlinear model's response, especially for small input changes. Results of this comparison are represented in (FIG 3.7) and (FIG 3.8).

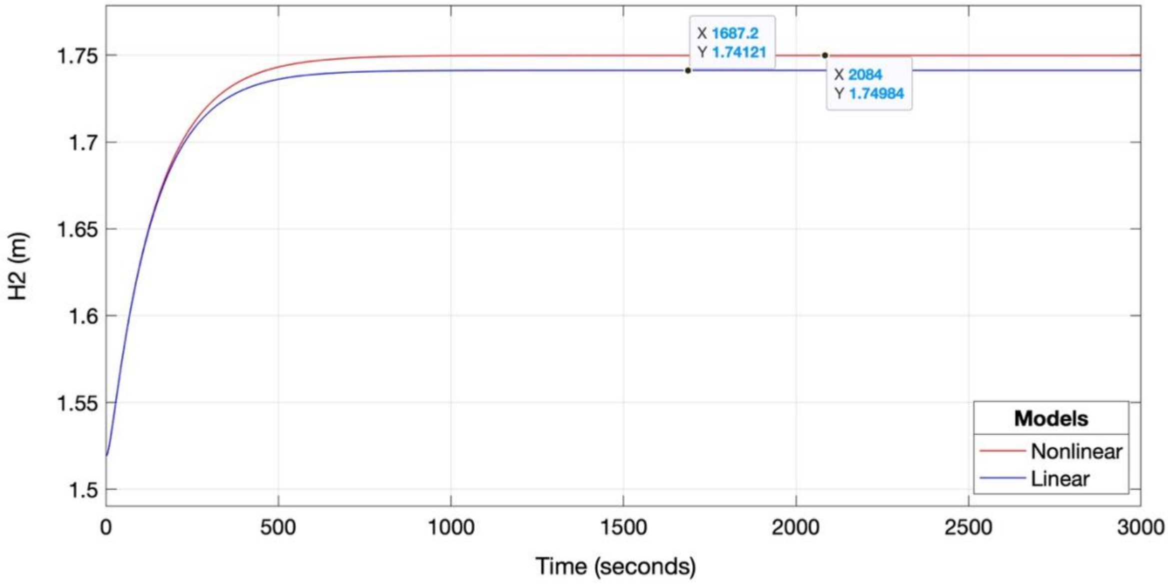

FIG 3. 8 - Comparison between Non-linear and Linearized Models for liquid level H2 of Two-tank Coupled System in the case of Input Change Response

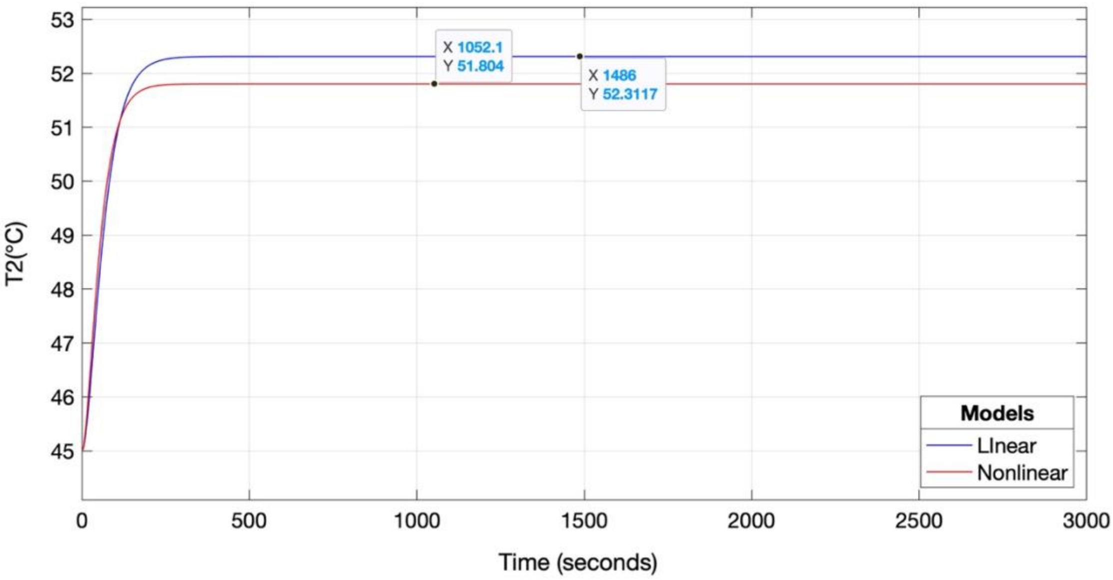

FIG 3. 7 - Comparison between Non-linear and Linearized Models for Temperature T2 of Two-tank Coupled System in the case of Input Change Response

From the results we can say that our expectation was right. The linearized model provided a reasonable approximation of the nonlinear model's response after due the changing in the inputs, especially for the liquid level H2.

The difference between these two models at steady state is caused by the large distance from the operating points to the desired outputs ($H_2 = 1.75$ m and $T_2 = 51.8$ ℃)

The value of the static error between the nonlinear model and linearized model is shown in the table below.

System Outputs	Nonlinear Model	Linearized Model	Static Error
H_2	0.23 m	0.22 m	– 4.34 %
T_2	6.8 °C	7.3 °C	+ 7.35 %

TAB 3. 1 - The Static error between Non-linear and Linearized Model's response in the case of Input Change Response

From the table, we can see that the static error does not exceed 5% for H_2 and does not exceed 8% for T_2, despite the significant variation on the inputs. So, we can now say that our linearized model is valid.

4.4.2 Disturbances Change Response (With Disturbances, Same Inputs)

In the second case we will assess the ability of the linearized model to predict the system's behavior in the presence of disturbances while the inputs remain constant.

Conditions:

- Simulation time interval: $t \in [0\,;3000s]$
- Sample time (Pas) equal 0.1s
- Step changes are introduced in the disturbances ($\mathbf{A}_v$ by +3%, T_w by –3% and T_c by +2%) in the time interval $t \in [800s\,;3000s]$.
- The input flow rates are kept constant at their operating point values.

The disturbances might cause a considerable change in the system's states, and we might see a small deviation of the linearized model from the nonlinear model. Results of this comparison are represented in (FIG 3.9) and (FIG 3.10).

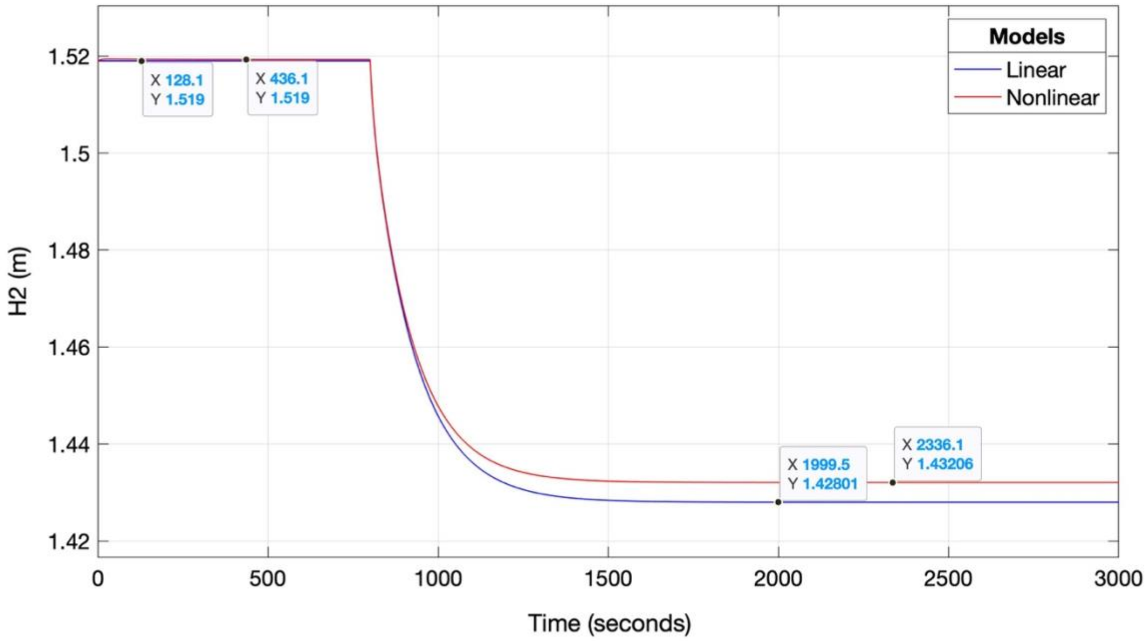

FIG 3. 9 - Comparison between Non-linear and Linearized Models for liquid level H2 of Two-tank Coupled System in the case of Disturbances Change Response

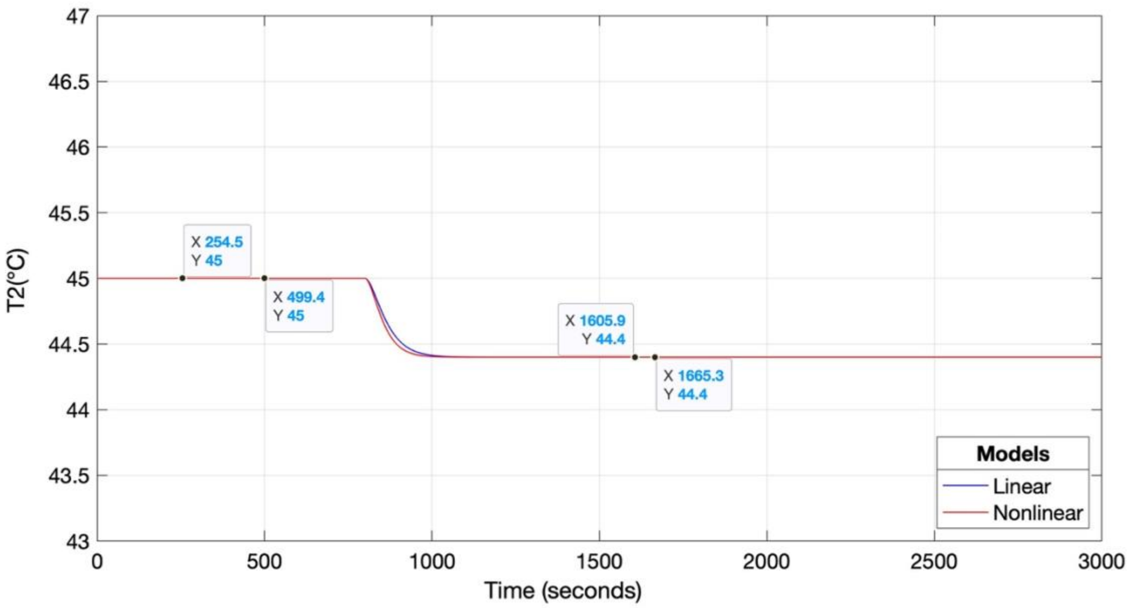

FIG 3. 10 - Comparison between Non-linear and Linearized Models for Temperature T2 of Two-tank Coupled System in the case of Disturbances Change Response

From the results we see a considerable change in the system's states as we expected, so that we can see a significant decrease in the liquid level H_2 (due to the effect of the disturbance $\mathbf{A}_v$) and liquid temperature T_2 (due to the effect of the disturbances T_w and T_c) of the two-tank coupled system, for both nonlinear and linearized models.

We can notice that there is no deviation of the linearized model from the nonlinear model for the temperature T_2. And that is because of the small difference between the output in the operating point and the actual one that caused by the disturbances (T_w and T_c).

In the other hand, there is a very small deviation of the linearized model from the nonlinear model for the liquid level H_2. And the reason of that is the considerable difference between the output in the operating point and the actual one that caused by the disturbance $\mathbf{A}_v$).

The effect of the disturbances on the nonlinear and linearized models and the static error between them is shown in the table below.

System Outputs	Applied Disturbance	Before		After		
		Nonlinear Model	Linearized Model	Nonlinear Model	Linearized Model	Static Error
H_2	A_v	1.52 m	1.52 m	– 0.088 m	– 0.092 m	– 4.54 %
T_2	T_w T_c	45 °C	45 °C	– 0.6 °C	– 0.6 °C	0 %

TAB 3. 2 - The Effect of the Disturbances on the Non-linear and Linearized Models and the Static Error between their responses in the case of Disturbances Change Response

From the table, we can see that in this case the linearized model reacts to the applied disturbances closely same as the nonlinear model reacts. Also, if we take a look to the static error, we can see that is does not exceed 5 % for H_2 and is null for T_2. So, we can now say again that our linearized model is valid.

4.4.3 Combined Scenario (With Disturbances, Changed Inputs)

In the third case we'll evaluate the overall performance of the linearized model under a more realistic scenario where both input changes and disturbances are present.

Conditions:

- Simulation time interval: $t \in [0\,;3000s]$
- Sample time (Pas) equal 0.1s
- Step changes are introduced in the disturbances ($\mathbf{A}_v$ by +3%, T_w by –3% and T_c by +2%) in the time interval $t \in [1300s\,;3000s]$.
- Step changes are introduced in the input flow rates.

This scenario will likely reveal the most significant differences between the nonlinear and linearized models, highlighting the limitations of the linear approximation under combined input and disturbance variations. Results of this comparison are represented in (FIG 3.11) and (FIG 3.12).

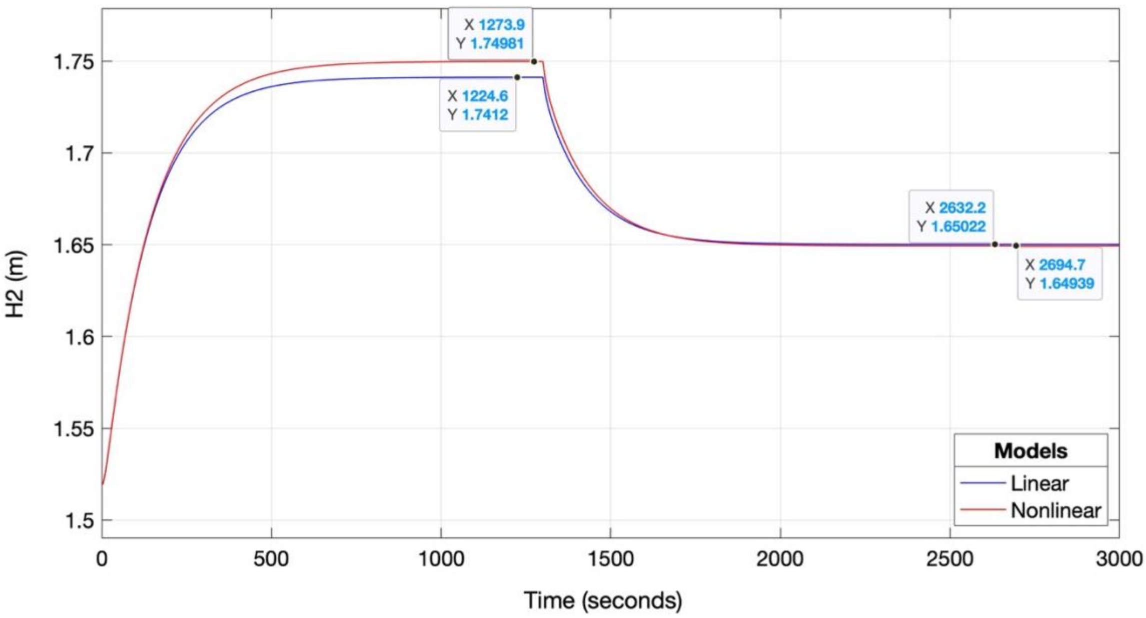

FIG 3. 11 - Comparison between Non-linear and Linearized Models for liquid level H2 of Two-tank Coupled System in the case of Combined Scenario

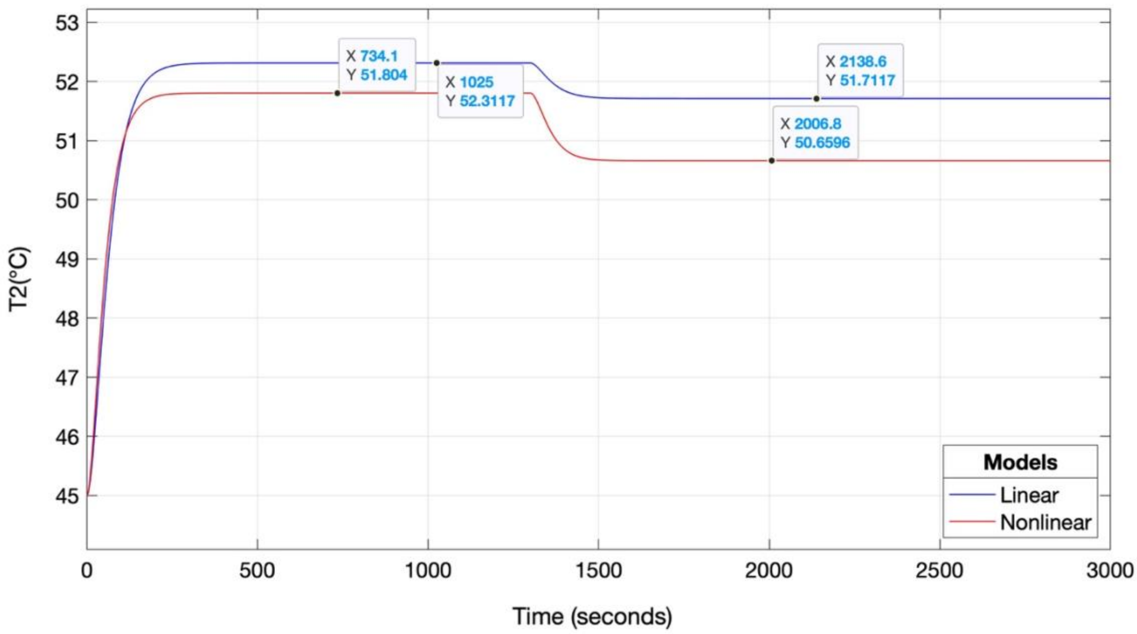

FIG 3. 12 - Comparison between Non-linear and Linearized Models for Temperature T2 of Two-tank Coupled System in the case of Combined Scenario

Based on the results of the comparison between the nonlinear model and the linearized model in the combined scenario, we can confirm that the linearized model provides a good approximation of the nonlinear model.

We can notice that there is a small deviation of the linearized model from the nonlinear model for the temperature T_2. And that is because of the considerable difference between the desired output and the actual one that caused by the disturbances (T_w and T_c).

In the other hand, there is almost no deviation of the linearized model from the nonlinear model for the liquid level H_2. And the reason of that is the small difference between the desired output and the actual one that caused by the disturbance $\mathbf{A}_v$).

The effect of the disturbances on the nonlinear and linearized models is shown in the table below.

System Outputs	Applied Disturbance	Before		After	
		Nonlinear Model	Linearized Model	Nonlinear Model	Linearized Model
H_2	A_v	1.75 m	1.74 m	– 0.101 m	– 0.09 m
T_2	T_w T_c	51.8 °C	52.3 °C	– 1.141 °C	– 0.59 °C

TAB 3. 3 - The Effect of the Disturbances on the Non-linear and Linearized Models and the Static Error between their responses in the case of Combined Scenario

From the table, we can see that in this case the linearized model reacts to the applied disturbances closely same as the nonlinear model reacts for liquid level H_2. In the other hand, the linearized system didn't react closely much to the applied disturbances like the reaction of nonlinear model. And we can say that the reason of that is the significant difference between the output in the operating point and the desired output.

From all the results we saw before, we can be confirmed that our linearized model is valid.

4. Conclusion

In this chapter, we provided a general overview of the modeling and linearization of dynamic systems. We then developed the mathematical model for our nonlinear two-tank coupled system. Due to the inherent complexity of nonlinear systems, it is challenging to study, analyze, optimize them and …etc. Therefore, we needed to linearize our nonlinear model to facilitate a more straightforward application.

To ensure that our linearized model accurately represents the original nonlinear system, we compared the behavior of both models. This comparison was conducted by simulating the two systems using MATLAB Simulink—one model for the nonlinear system and another for the linearized version.

The results of this comparison demonstrated that the linearized model approximates the nonlinear system, making it suitable for further applications, analysis, optimization of the two-tank coupled system.

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Chapter Four: Estimating the Liquid Level and Temperature of Two-tank Coupled System

1. Introduction

Accurate estimation of non-measurable variables and disturbances in a dynamic system (e.g., a two-tank coupled system) is crucial for effective process control and fault detection. However, direct measurement of these quantities is often impractical or impossible due to limitations in available sensors or the complexity of the system dynamics. In this chapter we will apply the Luenberger observer algorithm to address these challenges and provide reliable estimates of the unobservable system states and disturbances of our two-tank coupled system.

Luenberger observer are mathematical model that estimate the internal states of a dynamic system based on available measurements. By leveraging the system's dynamics and input-output relationships, they can reconstruct unmeasured states with reasonable accuracy.

In the context of our work on the two-tank coupled system, we will use the Luenberger observer to estimate both the full system state (including measurable and non-measurable variables) and the partial system state (consisting of only the non-measurable variables) of our linearized dynamic system.

The goal of using the both full state observation and partial state observation is to compare those two and see if the partial state observation can give us the same results as the full state observation, or at least an approximate estimation result.

That will determine if we can rely on the partial state observation as an accurate estimation of dynamic systems' state method in the future applications.

2. Observability of Dynamic System

Observability is a fundamental property of a dynamical system that determines whether it is possible to estimate the internal state of the system based solely on its input and output measurements. In other words, a system is observable if it is possible to uniquely reconstruct the initial state of the system from its input and output history over a finite time interval [34].

This is crucial for many control system applications, such as state feedback control, model predictive control, and fault detection.

2.1 Observability of Linearized System

To analyze the observability of a linearized system, we can use the concept of observability matrix.

2.1.1 Observability Matrix [34]

The observability matrix for a linear system is defined as:

$$\mathbf{O} = [\mathbf{C};\ \mathbf{CA};\ \mathbf{CA^2};\ \ldots;\ \mathbf{CA}^{(n-1)}] \tag{4.1}$$

where:

- $\mathbf{C}$ is the output matrix.
- $\mathbf{A}$ is the system matrix.
- n is the dimension of the state vector.

The system is observable if and only if the observability matrix has full rank, meaning its rank is equal to the dimension of the state vector.

2.1.2 Observability of Linearized Two-tank Coupled System

To determine the observability of a linearized system, we can use Matlab command to calculate the rank of the observability matrix (4.1) .

From (3.34) and (3.36) we can get matrix **A** and **C** of our system respectively, and by using the code "rank(obsv(A,C)) " we find our rank.

```
>> rank(obsv(A,C))

ans =

     4
```

(4.2)

We can see that the rank of the observability matrix equal 4, so it's equal to the dimension of the state vector. That means that our linearized system is observable.

3. Luenberger Observer

3.1 Reminder about the Luenberger Observer [9]

By leveraging the system's dynamics and input-output relationships, Luenberger observers can reconstruct the unmeasured states with reasonable accuracy.

Luenberger proposes the following observer for the system (3.37):

$$\begin{cases} \dot{\hat{x}} = \mathbf{A}\hat{x} + \mathbf{B}u + \mathbf{B}_v v + \mathbf{K}(y - \mathbf{C}\hat{x}) \\ \hat{y} = \mathbf{C}\hat{x} \end{cases} \tag{4.3}$$

Where,

$$\begin{cases} \dot{\hat{x}} = (\mathbf{A} - \mathbf{KC})\hat{x} + \mathbf{B}u + \mathbf{B}_v v + \mathbf{K}y \\ \hat{y} = \mathbf{C}\hat{x} \end{cases} \tag{4.4}$$

The dynamics of the observation error $e = x - \hat{x}$ is:

$$\dot{e} = (\mathbf{A} - \mathbf{KC})e \tag{4.5}$$

3.2 Calculating the Luenberger Observer Gain Matrix

Using a pole placement technique, it is sufficient to calculate the observer gain matrix **K** such that the eigenvalues of the matrix $(\mathbf{A} - \mathbf{KC})$ are positioned in the left half of the complex plane.

The '**place**' function is used for pole placement. Since **K** is used in the equation $\mathbf{A} - \mathbf{KC}$, and MATLAB places poles using the system $\mathbf{A}'$ and $\mathbf{C}'$, we transpose the matrices and then transpose the result to get the correct **K** .

The '**place**' function is written on matlab in this form:

$$\mathbf{K} = place(\mathbf{A}', \mathbf{C}', desired\_poles)'; \tag{4.6}$$

Desired poles are a vector containing the eigenvalues where you want the observer's poles to be located. The number of poles should match the number of states in your system.

The desired poles we choose are 3.5 times faster than the system poles (3.38).

$$desired_{poles} = [-0.0269;\ -0.3812;\ -0.1173;\ -0.0878] \tag{4.7}$$

And after using the '**place**' function, the gain matrix **K** will be :

$$\mathbf{K} = \begin{bmatrix} 0.3016 & -0.0129 \\ 0.3519 & -0.0150 \\ -0.0166 & 0.0058 \\ -0.0119 & 0.0861 \end{bmatrix} \tag{4.8}$$

The Luenberger observer gain matrix **K** plays a crucial role in determining the performance of the observer. A well-chosen **K** can ensure that the observer's error dynamics are asymptotically stable, leading to accurate state estimation.

4. Simulating the System's State Estimation

4.1 Linearized Model associated with Luenberger Observer

The figure below represents the Linearized model (FIG 3.6) of the two-tank Coupled system associated with Luenberger observer model in Matlab Simulink. The goal of connecting the observer is to estimate the state variables of our system.

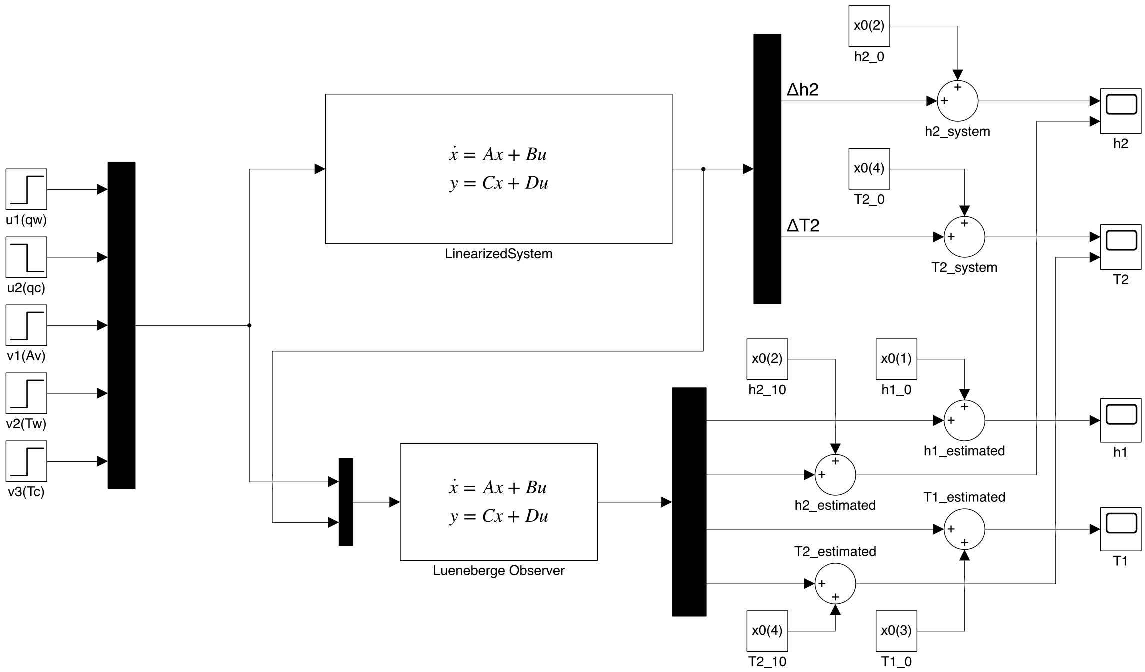

FIG 4. 1 - The Linearized Model of Two-tank Coupled System associated with Luenberger Observer Model

4.2 Full State Observation

Full-state observation involves estimating all the state variables of a system, both measurable and unmeasurable. This gives a complete picture of the system's dynamics and allows more accurate control and monitoring.

Luenberger observers are a best tool for achieving this goal.

4.2.1 Estimating System State variables (Without Disturbances)

In this case, we'll evaluate the accuracy and effectivity of the luenberger observer in estimating the four state variables of the system in the absence of the disturbances.

Conditions:

- Simulation time interval: $t \in [0\,;850s]$
- Sample time (Pas) equal 0.1s
- No disturbances are applied ($\mathbf{A}_v$, T_w, and T_c remain at their operating point values).
- Step changes are introduced in the input flow rates.
- The initial conditions of the observer are $x_0 = [0.1, 0.1, 0.2, 0.2]$, while the initial conditions of the linearized model are kept zero.

The Luenberger observer should provide a good estimation for the four state variables, and the estimated values should match the linearized model's values in the steady state condition. The results of the estimation are represented in the (FIG 4.2), (FIG 4.3), (FIG 4.4) and (FIG 4.5).

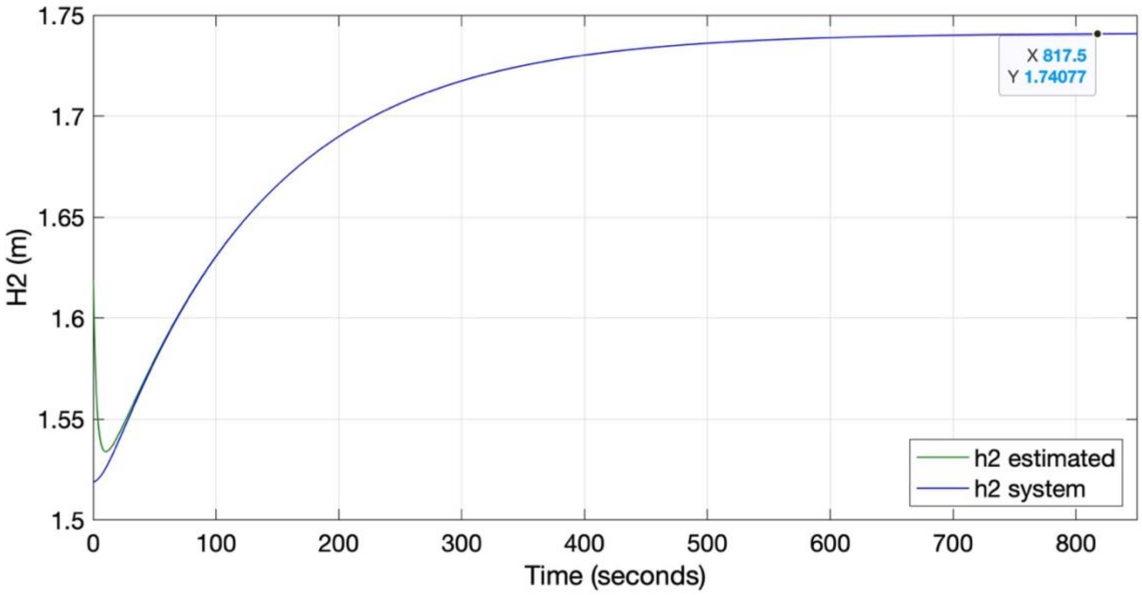

FIG 4. 2 - The Estimated Liquid level H2 compared to the measurable value of H2 of the Linearized Model (Without Disturbances)

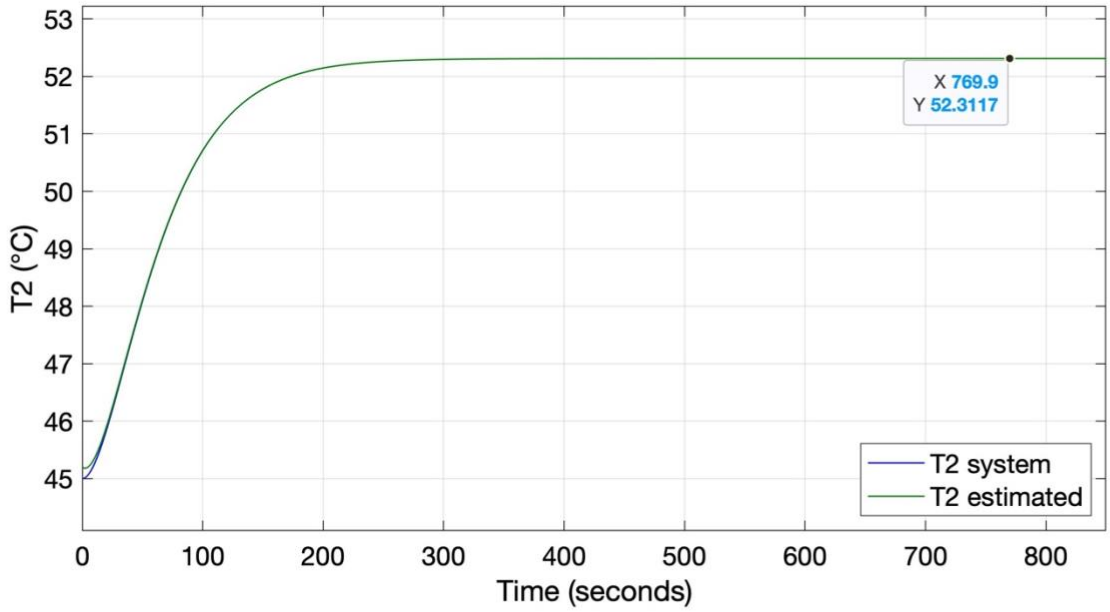

FIG 4. 3 - The Estimated Liquide Temperature T2 compared to the measurable value of T2 of the Linearized Model (Without Disturbances)

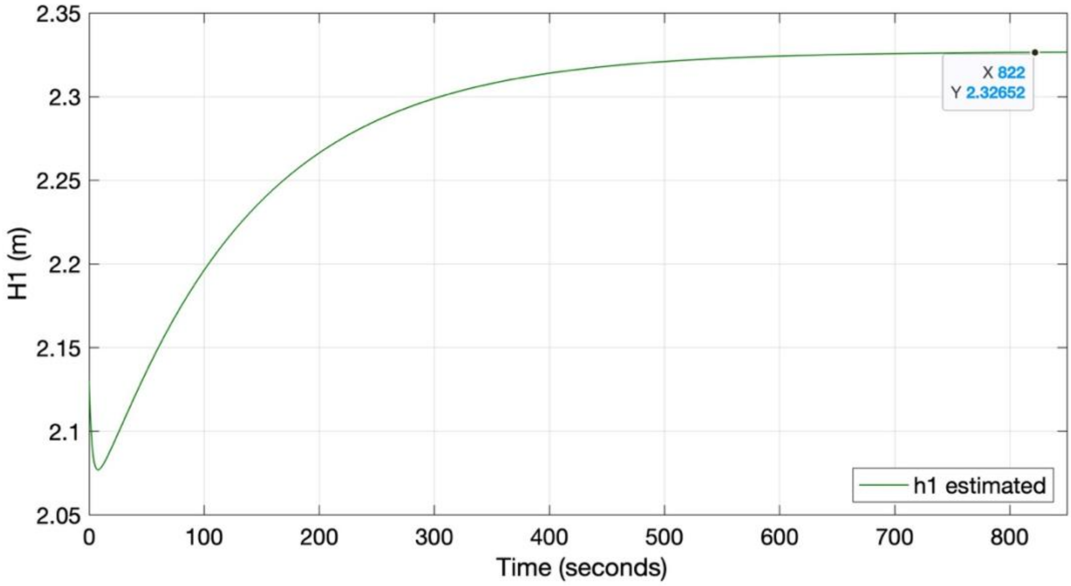

FIG 4. 4 - The Estimated Liquide level H1 compared to the non-measurable value of H1 of the Linearized Model (Without Disturbances)

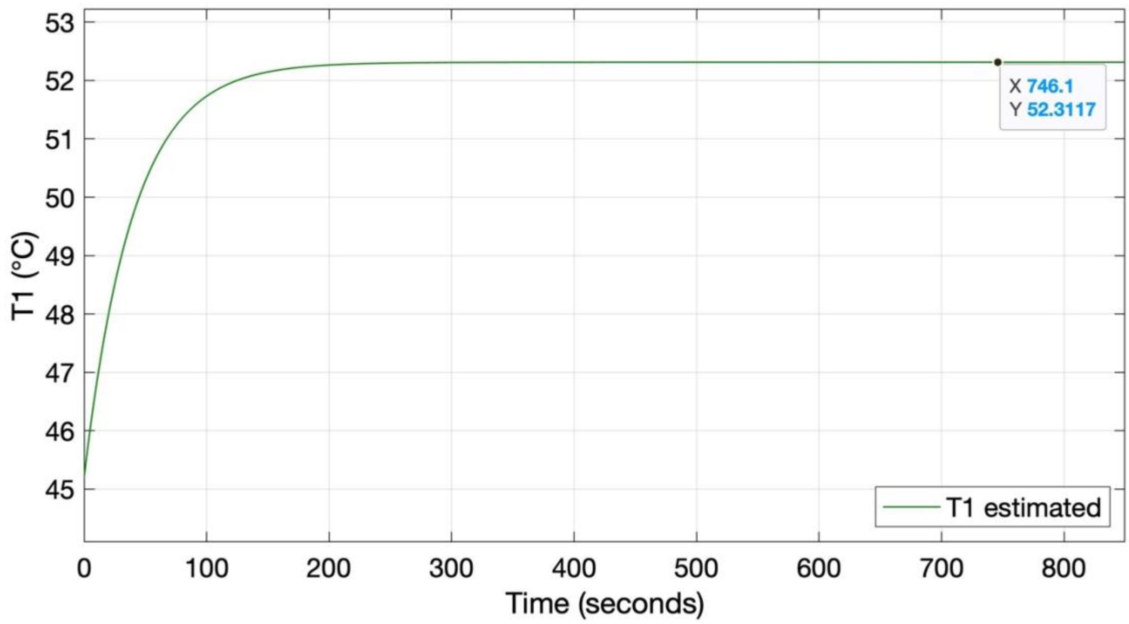

FIG 4. 5 - The Estimated Liquide Temperature T1 compared to the non-measurable value of T1 of Linearized Model (Without Disturbances)

From comparing the estimated measurable state variables (H_2 and T_2) by the luenberger observer with the state variables of our linearized model, and by noting the estimated non-measurable state variables, we can say that the observer is working effectively, since the estimated state variables match the state variables of the system in the steady state condition.

Also, we can notice that the estimation error tends to zero quickly after the simulation started. which implies the accuracy and the rapidity of the associated observer to the system.

4.2.2 Estimating System State variables (With Disturbances)

In this case, we'll evaluate the accuracy and effectivity of the luenberger observer in estimating the four state variables of the system in the presence of the disturbances.

Conditions:

- Simulation time interval: $t \in [0\,;1500s]$
- Sample time (Pas) equal 0.1s
- Step changes are introduced in the disturbances ($\mathbf{A}_v$ by +3%, T_w by –3% and T_c by +2%) in the time interval $t \in [800s\,;1500s]$.
- Step changes are introduced in the input flow rates.
- The initial conditions of the observer are $x_0 = [0.1, 0.1, 0.2, 0.2]$, while the initial conditions of the linearized model are kept zero.

The Luenberger observer should estimate the four state variables and the effect of the disturbances applied to the system normally, given to the effectivity and accuracy of the observer. The results of the estimation are represented in the (FIG 4.6), (FIG 4.7), (FIG 4.8) and (FIG 4.9).

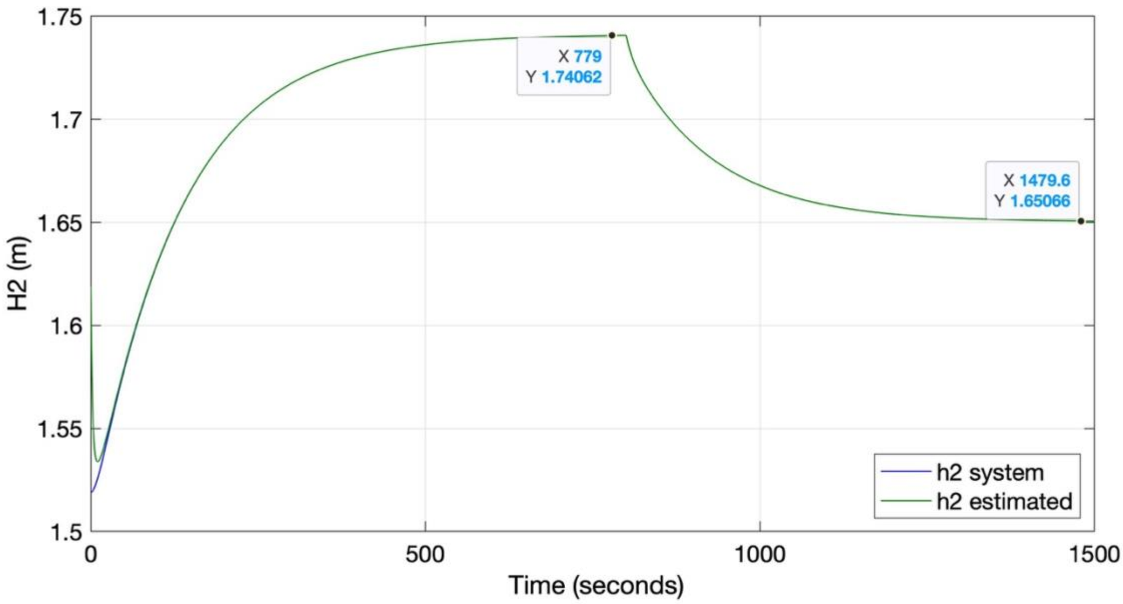

FIG 4. 6 - The Estimated Liquid level H2 compared to the measurable value of H2 of the Linearized Model (With Disturbances)

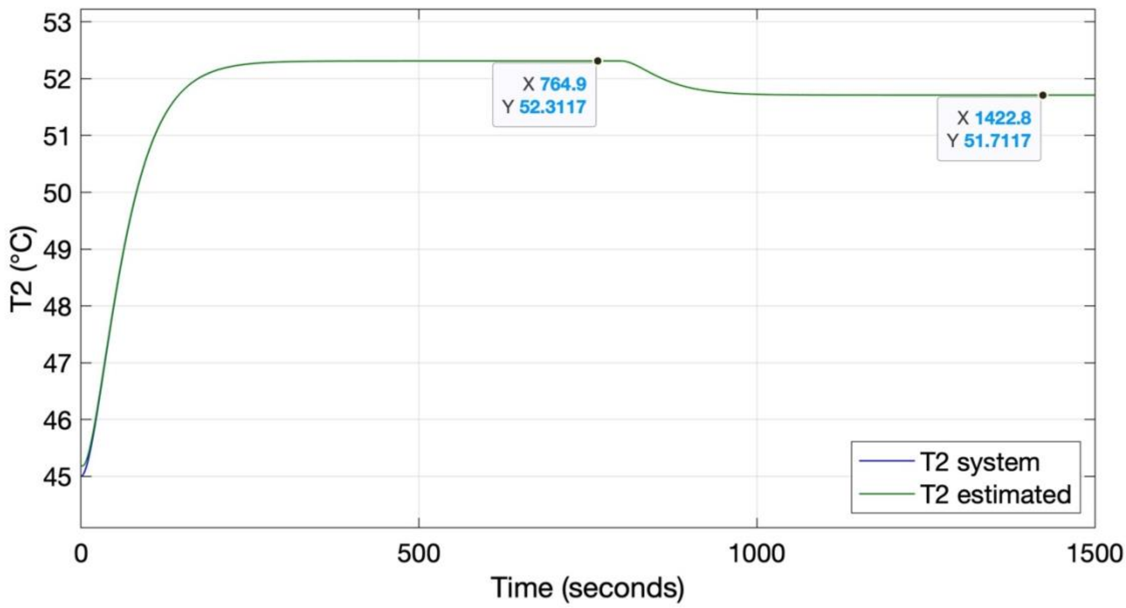

FIG 4. 7 - The Estimated Liquide Temperature T2 compared to the measurable value of T2 of the Linearized Model (With Disturbances)

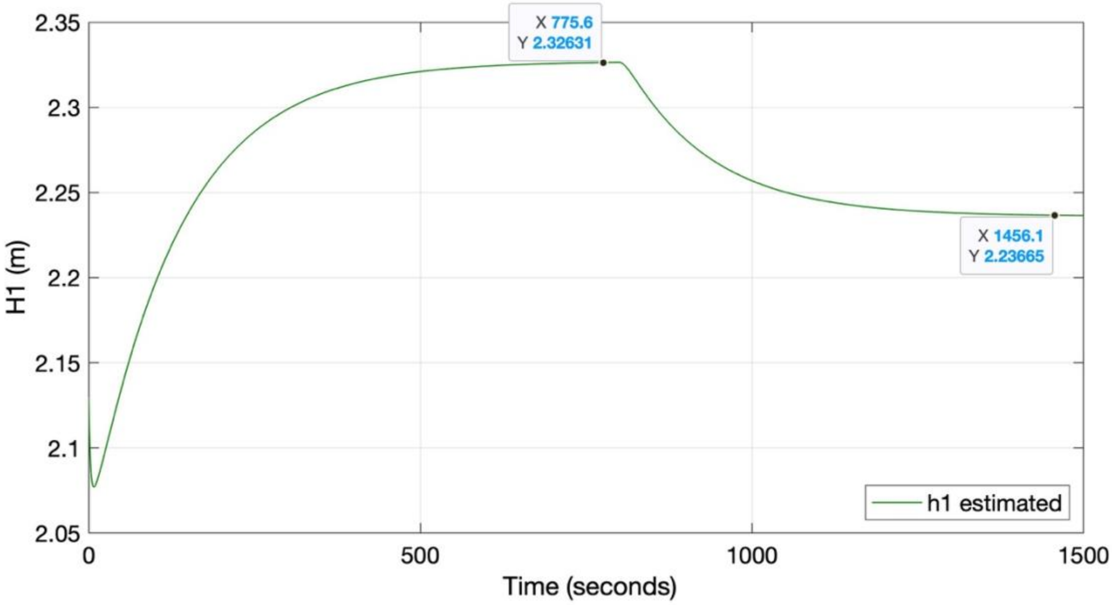

FIG 4. 8 - The Estimated Non-measurable Liquide level H1 of the Linearized Model (With Disturbances)

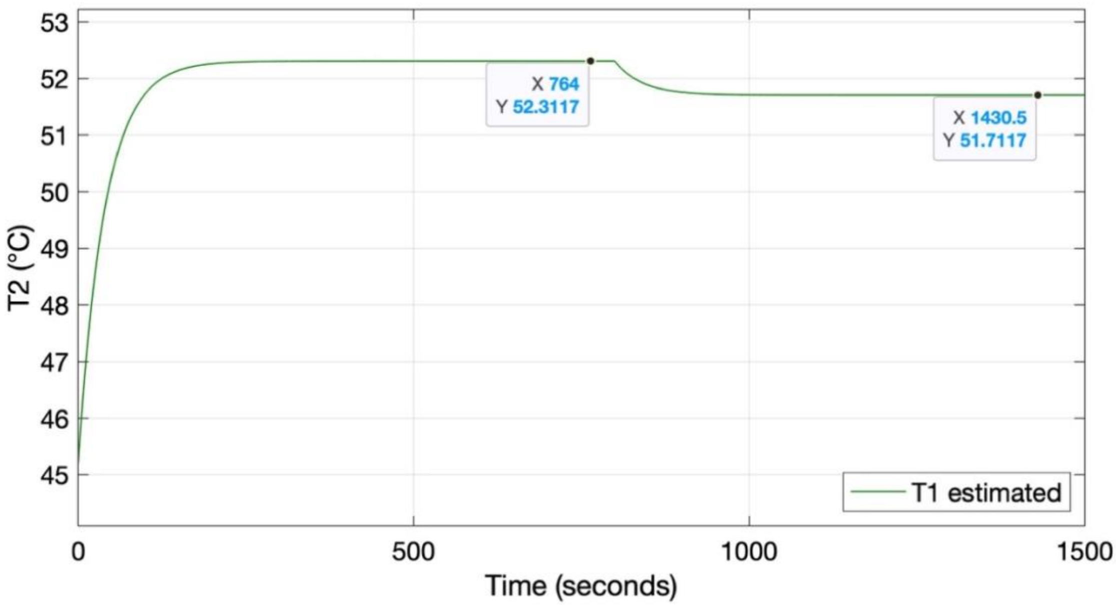

FIG 4. 9 - The Estimated Non-measurable Liquide Temperature T1 of Linearized Model (With Disturbances)

From the results we can see that the luenberger observer estimate the four state variables of the system (measurable and non-measurable) with the effect of the disturbances applied effectively as we expected.

This can let us be more confirmed with the accuracy and the efficacity of the observer.

4.3 Partial State Observation

Partial state estimation refers to the process of estimating only a subset of the state variables in a dynamic system, typically when not all the states are directly measurable. This approach focuses on reconstructing the non-measurable states $\hat{x}_{unmeasurable}$ based on the available system outputs ($x_{measurable}$) and a mathematical model of the system.

4.3.1 Reduced Order Observer [11]

The reduced-order observer is used to estimate only the unmeasurable states of a system when some states are already measurable. This approach simplifies the estimation process by focusing on reconstructing only the unknown (unmeasurable) states.

To apply the reduced-order observer, we partition the system states x of our system (3.30) into measurable states and unmeasurable states.

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{4.9}$$

Where, x_1 are the measurable states (directly observed as $y = \mathbf{C}x$), and x_2 are the unmeasurable states (which need to be estimated).

Now partition the system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_v$, and $\mathbf{C}$:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix}, \mathbf{B}_v = \begin{bmatrix} \mathbf{B}_{v1} \\ \mathbf{B}_{v2} \end{bmatrix}, \mathbf{C} = [I_2 \quad 0] \tag{4.10}$$

Where,

- $\mathbf{A}_{11}$ describes the dynamics of the measurable states.
- $\mathbf{A}_{22}$ describes the dynamics of the unmeasurable states.
- $\mathbf{A}_{12}$ and $\mathbf{A}_{21}$ describe the interaction between the measurable and unmeasurable states.
- $\mathbf{B}_1$ and $\mathbf{B}_2$ are the input matrices corresponding to the measurable and unmeasurable states.
- $\mathbf{B}_{v1}$ and $\mathbf{B}_{v2}$ are the disturbances matrices corresponding to the measurable and unmeasurable states.
- $\mathbf{C}$ is the identity matrix of the measurable outputs.

The partition of the system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_v$, and $\mathbf{C}$ is a transformation that help us to separate the measurable states from the non-measurables state inside those matrices. And in order to do that, will use the Coordinate Transformation T.

$$\mathbf{T} = [\mathbf{C}^{\mathrm{T}} \,|\mathbf{N}] \tag{4.11}$$

Where,

- $\mathbf{C}^{\mathrm{T}}$ is the Moore-Penrose pseudoinverse of $\mathbf{C}$.
- $\mathbf{N}$ is a basis of ker ($\mathbf{C}$).

By using the Matlab Codes below we can calculate $\mathbf{T}$, $\mathbf{A}_{11}$, $\mathbf{A}_{22}$, $\mathbf{A}_{12}$ and $\mathbf{A}_{21}$, $\mathbf{B}_1$and $\mathbf{B}_2$, $\mathbf{B}_{v1}$ and $\mathbf{B}_{v2}$.

Calculate the coordinate transformation T

$\mathbf{N} = null(\mathbf{C});$

$\mathbf{T} = [pinv(\mathbf{C}), \mathbf{N}];\ inv\mathbf{T} = inv(\mathbf{T});$

Transform the system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_v$.

$\mathbf{A}bar = inv\mathbf{T} * \mathbf{A} * \mathbf{T};$

$\mathbf{B}bar = inv\mathbf{T} * \mathbf{B};$

$\mathbf{B}\boldsymbol{v}bar = inv\mathbf{T} * \mathbf{B}\boldsymbol{v};$

Extract the submatrices for the reduced-order observer

$\mathbf{A}_{11} = \mathbf{A}bar(1{:}\,ny, 1{:}\,ny);$

$\mathbf{A}_{12} = \mathbf{A}bar(1{:}\,ny, nx - ny + 1{:}\,nx);$

$\mathbf{A}_{21} = \mathbf{A}bar(nx - ny + 1{:}\,nx, 1{:}\,ny);$

$\mathbf{A}_{22} = \mathbf{A}bar\big(nx - ny + 1{:}\,nx, nx - ny + 1{:}\,nx\big);$

Partition $\mathbf{B}$ into $\mathbf{B}_1$ and $\mathbf{B}_2$ based on which states are measurable/non-measurable

$\mathbf{B}_1 = \mathbf{B}bar(1{:}\,ny, 1{:}\,nu);$

$\mathbf{B}_2 = \mathbf{B}bar(nx - ny + 1{:}\,nx, 1{:}\,nu);$

Partition $\mathbf{B}_v$ into $\mathbf{B}_{v1}$ and $\mathbf{B}_{v2}$ based on which states are measurable/non-measurable

$\mathbf{B}_{v1} = \mathbf{B}\boldsymbol{v}bar(1{:}\,ny, 1{:}\,nv);$

$\mathbf{B}_{v2} = \mathbf{B}\boldsymbol{v}bar(nx - ny + 1{:}\,nx, 1{:}\,nv);$

Where,

- nx is the number of state variables.
- ny is the number of system's outputs.
- nu is the number of system's inputs.
- nv is the number of system's disturbances.

Now the description of our system depending on (4.9) and (4.10) will be :

$$\begin{cases} x_1^+ = \mathbf{A}_{11}x_1 + \mathbf{A}_{12}x_2 + \mathbf{B}_1 u + \mathbf{B}_{v1} v \\ x_2^+ = \mathbf{A}_{21}x_1 + \mathbf{A}_{22}x_2 + \mathbf{B}_2 u + \mathbf{B}_{v2} v \\ y = x_1 \end{cases} \tag{4.11}$$

Where x_1^+ is the state vector correspond to the measurable states variables, and x_2^+ is the state vector correspond to the non-measurable states variables.

The reduced order observer can be described as follow:

$$\begin{cases} z^+ = (\mathbf{A}_{22} - \mathbf{R}\mathbf{A}_{12})z + (\mathbf{A}_{22}\mathbf{R} - \mathbf{R}\mathbf{A}_{12}\mathbf{R} + \mathbf{A}_{21} - \mathbf{R}\mathbf{A}_{11})y + (\mathbf{B}_2 - \mathbf{R}\mathbf{B}_1)u \\ y = x_1 \end{cases} \tag{4.12}$$

Where **R** is the gain matrix that will be calculated later.
To estimate the non-measurable state variables, x_2 has to be written as follow:

$$x_2 = z + \mathbf{R}x_1 \tag{4.13}$$

4.3.2 Calculating the Reduced Order Observer Gain Matrix

By replacing the original matrices of our system with their submatrices, the '**place**' function used to calculate **K** will be updated to:

$$\mathbf{R} = place(\mathbf{A}_{22}', \mathbf{C}', desired\_poles)'; \tag{4.14}$$

The new desired poles will remain 3.5 times faster than the system poles. But in this case, the system poles will be updated too, since we are using the submatrices instead of the original ones.

The updated system poles in open loop will be :

$$\begin{cases} \mathbf{P}_1 = -0.4990 \\ \mathbf{P}_2 = -0.0251 \end{cases} \tag{4.15}$$

Now the desired poles will be :

$$desired_{poles} = [-0.1746;\ -0.0878] \tag{4.16}$$

Using the '**place**', we find the value of the gain matrix of the updated luenberger observer:

$$\mathbf{R} = \begin{bmatrix} 0.1495 & 0 \\ 0 & 0.0379 \end{bmatrix} \tag{4.17}$$

4.3.3 Linearized Model associated with the Updated Luenberger Observer

The figure below represents the Linearized model (FIG 3.6) of the two-tank Coupled system associated with updated Luenberger observer model in Matlab Simulink. The goal of connecting the observer is to estimate the non-measurable state variables of our system only.

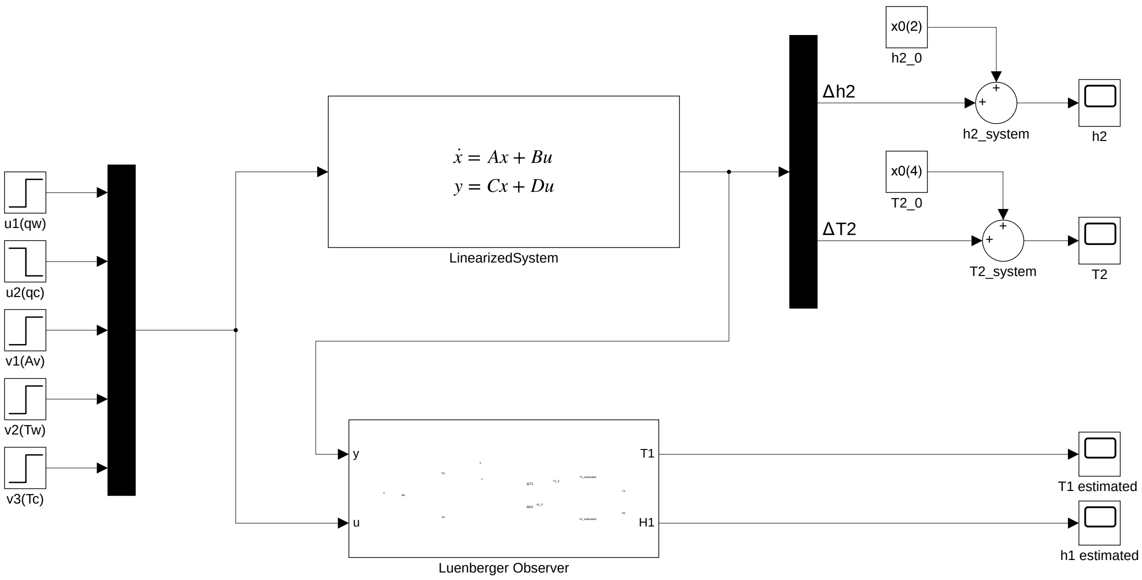

FIG 4. 10 - The Linearized Model of Two-tank Coupled System associated with updated Luenberger Observer Model

4.3.4 Estimating Non-measurable System State variables (Without Disturbances)

In this case, we'll evaluate the accuracy and effectivity of the updated luenberger observer in estimating only the non-measurable state variables of the system in the absence of the disturbances.

Conditions:

- Simulation time interval: $t \in [0\,;1000s]$
- Sample time (Pas) equal 0.1s
- No disturbances are applied ($\mathbf{A}_v$, T_w, and T_c remain at their operating point values).
- Step changes are introduced in the input flow.
- The initial conditions of the observer are $x_0 = [0.1, 1]$, while the initial conditions of the linearized model are kept zero.

The updated Luenberger observer should provide the exact estimation for the non-measurable state variables as we saw in the full state observation. The results of the estimation are represented in the (FIG 4.11), (FIG 4.12).

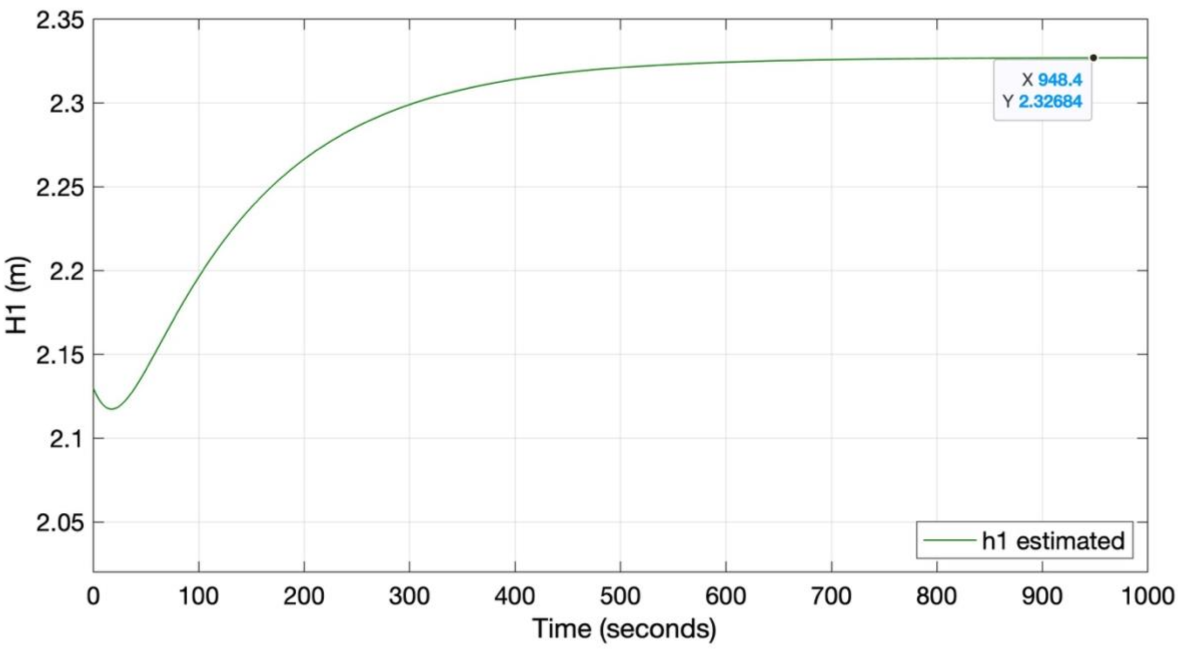

FIG 4. 11 - The Estimated Non-measurable Liquide Level H1 of Linearized Model (Without Disturbances)

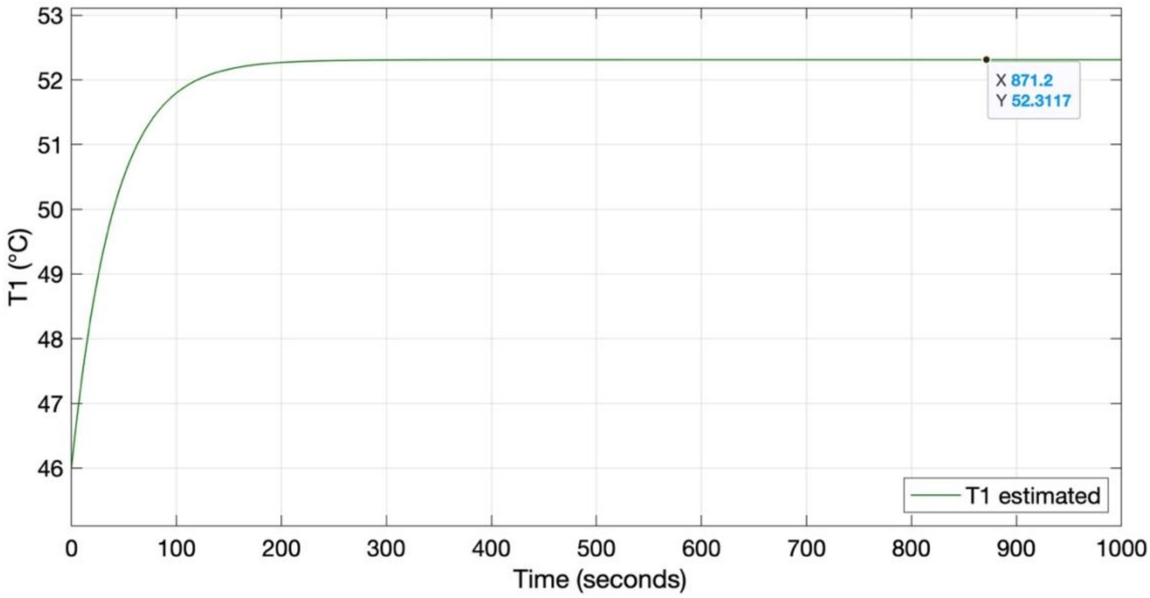

FIG 4. 12 - The Estimated Non-measurable Liquide Temperature T1 of Linearized Model (Without Disturbances)

Form the results we can see that the estimated value of the liquid level $H1$ match the value of the liquid level $H1$ of the linearized model in the steady state condition. Also, the estimated value of the liquid temperature T match exactly its counterpart T of the linearized model.

These results show us that the updated luenberger observer is accurate and effective to estimate the non-measurable state variables of the system based on the measured state variables.

4.3.5 Estimating Non-measurable System State variables (With Disturbances)

In this case, we'll evaluate the accuracy and effectivity of the updated luenberger observer in estimating only the non-measurable state variables of the system in the presence of the disturbances.

Conditions:

- Simulation time interval: $t \in [0\,; 1500s]$
- Sample time (Pas) equal 0.1s
- Step changes are introduced in the disturbances ($\mathbf{A}_v$ by +3%, T_w by –3% and T_c by +2%) in the time interval $t \in [800s\,; 1500s]$.
- Step changes are introduced in the input flow rates.
- The initial conditions of the observer are $x_0 = [0.1, 1]$, while the initial conditions of the linearized model are kept zero.

Depending on the results in (FIG 4.11) and (FIG 4.12) that showed us the accuracy of the observer in estimating the non-measurable state variable , we expect that the observer will estimate the non-measurable states variables in during the presence of the disturbances effectively.

The results of the estimation are represented in the (FIG 4.13), (FIG 4.14).

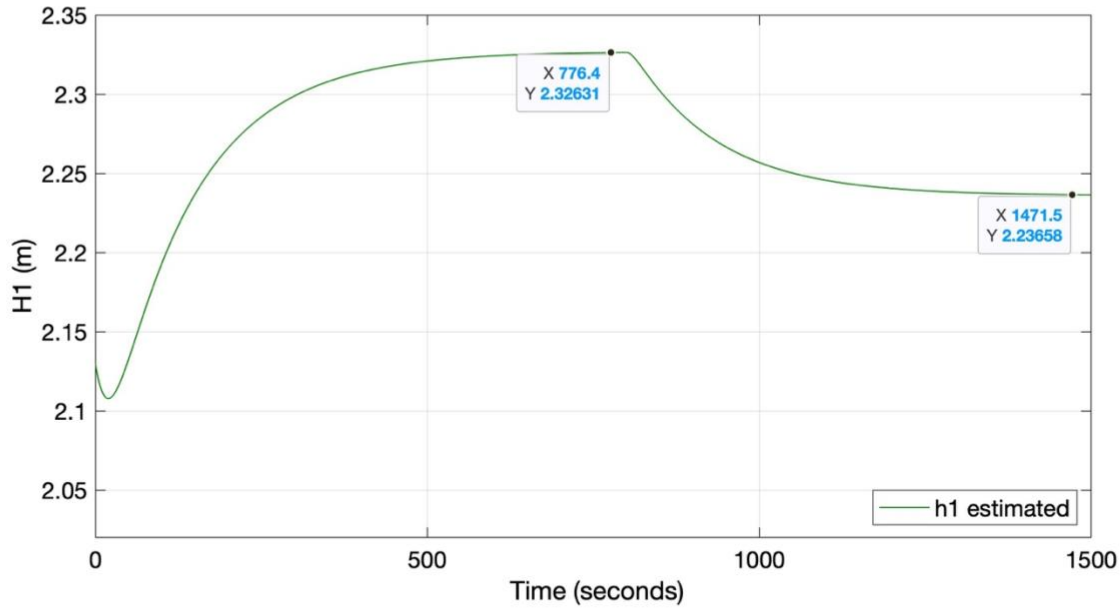

FIG 4. 6 - The Estimated Non-measurable Liquide Level H1 of Linearized Model (With Disturbances)

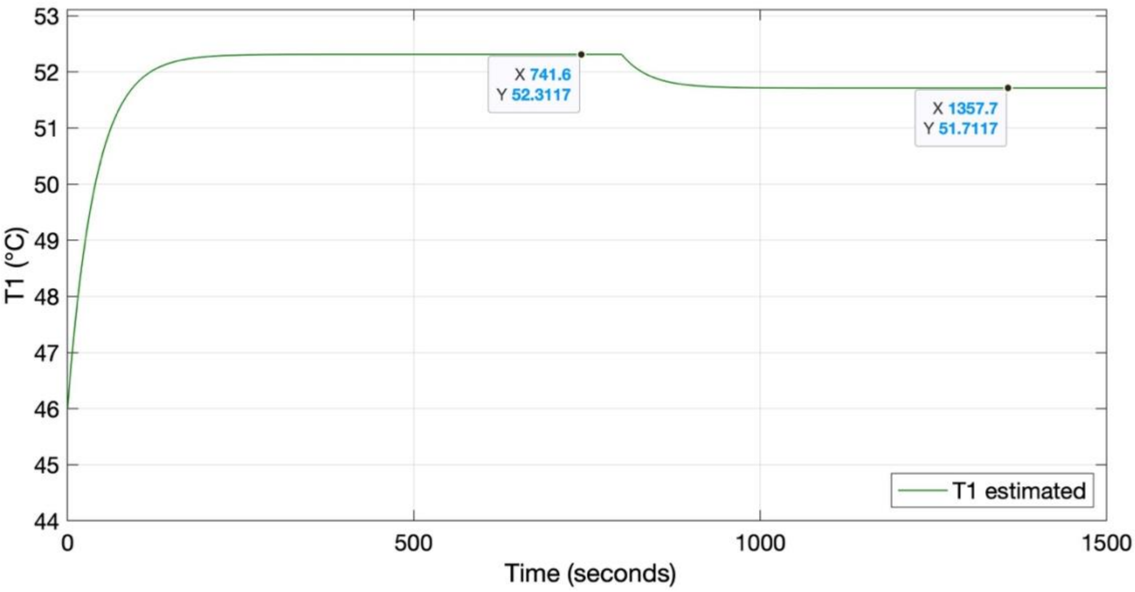

FIG 4. 14 - The Estimated Non-measurable Liquide Temperature T1 of Linearized Model (With Disturbances)

In the (FIG 4.13) and (FIG 4.14) we can see that the updated luenberger observer estimates the non-measurable state variable of our system in the presence of disturbances effectively, also the observer gives the exact estimation of the effect of these disturbances on our linearized model like we saw in the model itself.

The results made us more confirmed about the reliability of the partial state observation as an estimation method for our two-coupled tank system.

4.4 Comparison between the Full State and Partial State Observation

We gonna compare the results of the estimation of the non-measurable states variables of the linearized model of the both methods, full state observation and partial state observation.

The goal of this comparison is to show the diffrence between the full state and the partial state observation. The results of the comparison are represented in the (FIG 4.15), (FIG 4.16), (FIG 4.17), and (FIG 4.18).

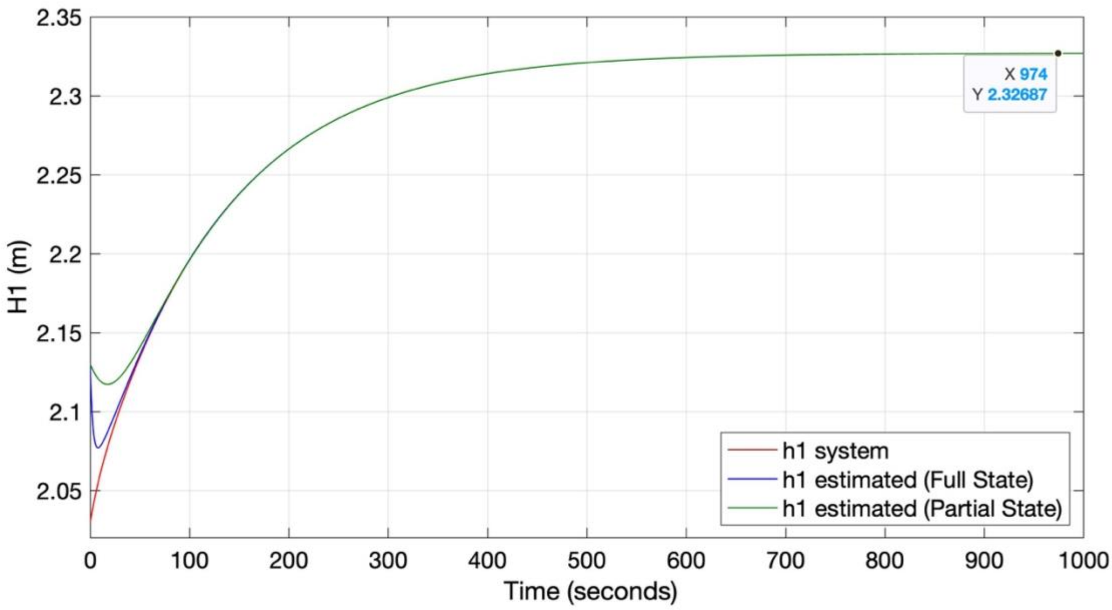

FIG 4. 15 - Comparison between the Full State Observation and Partial State Observation for the estimation of Liquide Level H1 of Linearized Model (Without Disturbances)

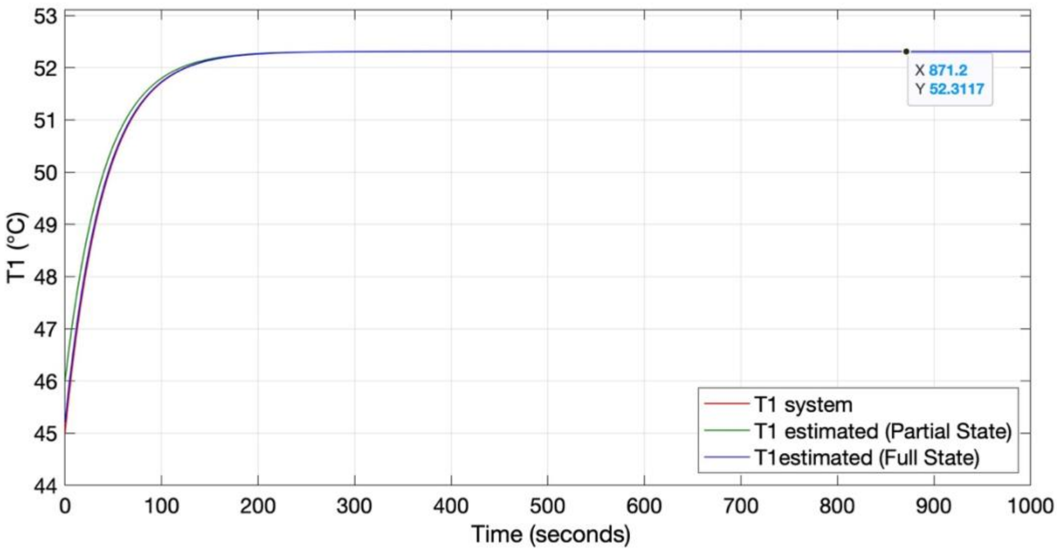

FIG 4. 7 - Comparison between the Full State Observation and Partial State Observation for the estimation of Liquide Temperature T1 of Linearized Model (Without Disturbances)

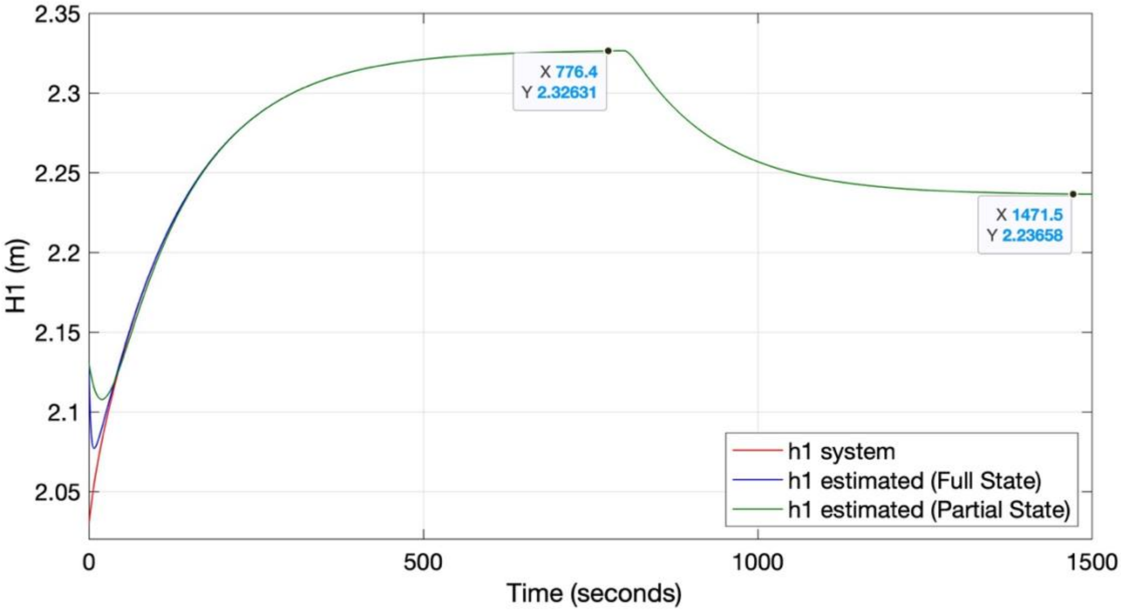

FIG 4. 17 - Comparison between the Full State Observation and Partial State Observation for the estimation of Liquide Level H1 of Linearized Model (With Disturbances)

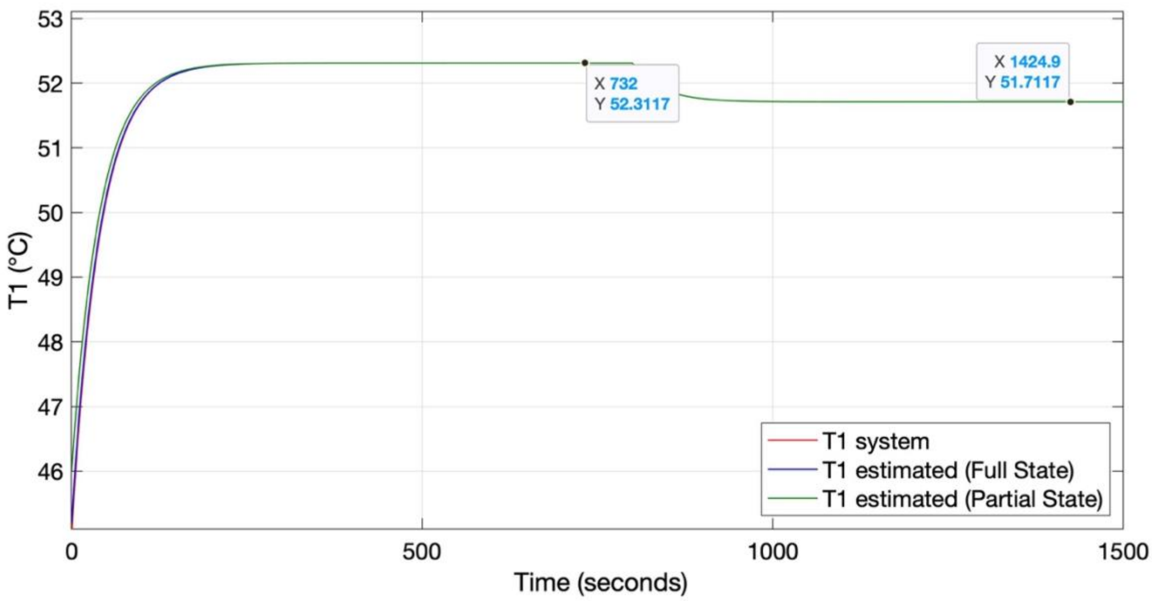

FIG 4. 18 - Comparison between the Full State Observation and Partial State Observation for the estimation of Liquide Temperature T1 of Linearized Model (With Disturbances)

From the results in the (FIG 4.15), (FIG 4.16) , (FIG 4.17), and (FIG 4.18) we can see that the partial state observation has the same accuracy and effectivity as the full state observation in estimating the non-measurables state variables of our two-coupled tank system.

5. Conclusion

In this chapter, we explored the application of the Luenberger observer algorithm to our linearized two-tank coupled system. We began by estimating both measurable and non-measurable state variables using full state and partial state observation methods. The observer's performance was evaluated by comparing the estimated state variables with those of our linearized model.

The results showed that the Luenberger observer effectively estimates the system's state variables, including the impact of disturbances in the full state observation and the partial state observation. Confirming to us that the partial state observation is reliable as an estimation method for our dynamic systems, same as is the full state observation method.

Overall, this chapter demonstrates that the Luenberger observer is a valuable tool for estimating unobservable system states, offering a viable method for enhancing process control and fault detection in dynamic systems like our two-tank coupled system.

General Conclusion

In this thesis, we have systematically addressed the challenge of accurate state estimation in dynamic systems, with a specific focus on the two-tank coupled system. Through a detailed exploration of control systems, coupled tank dynamics, and state estimation methodologies, we have developed an approach that confirms the reliability of existing techniques in the monitoring and control of complex industrial processes.

The journey began with a thorough investigation into the fundamental principles of control systems, establishing a strong foundation for understanding the intricate dynamics involved. Building on this, we explored the specific case of the two-tank coupled system, which served as a practical model for demonstrating the significance of advanced estimation techniques in industrial applications.

Our work progressed into the modeling and linearization of this dynamic system, ensuring that our linearized model closely approximated the nonlinear system, thereby making it suitable for further analysis and application. This step was crucial as it allowed for the effective application of the Luenberger observer, a mathematical tool designed to estimate unmeasurable states within the system.

The final phase of our research involved applying the Luenberger observer algorithm to the linearized model. The results confirmed that the observer effectively estimates both measurable and non-measurable state variables, even in the presence of disturbances. The comparison between full and partial state observations showed that while the full state observation provided precise results, the partial state observation provided the same accurate results. Confirming to us that the partial state observation is reliable as an estimation method for our dynamic systems, same as the full state observation method.

Overall, this thesis reinforces the applicability and effectiveness of estimation as a solution for knowing the value of non-measurable state variables within dynamic systems. The insights gained from this study confirm the robustness of established methodologies for enhancing process control and fault detection in various industrial settings. As industrial processes continue to evolve in complexity, this work affirms the continued relevance and applicability of existing techniques in dynamic system estimation and control.

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