**People's Democratic Republic of Algeria**

**Ministry of Higher Education and Scientific Research**

**University M'Hamed BOUGARA – Boumerdès**

**Institute of Electrical and Electronic Engineering**

**Department of Power and Control**

Final Year Project Report Presented in Partial Fulfilment of the

Requirements of the Degree of

# 'MASTER'

**In Electrical and Electronic Engineering**

Option: **Control Engineering**

Title:

# Simulation Study of Model Predictive Control Applied to Binary Distillation Column


Presented By:

**TAOULI Houria Ghoufranellah**

**ASSELAH Lamia**

Supervisor:

**Dr.ROUANI Lahcene**

Co-Supervisor:

**Mr.OUAZEN Hamza**


Registration Number…… /2024

# ACKNOWLEDGMENTS

First and foremost, praises and gratefulness are to Allah you given us the guidance and the power to believe in ourselves, to complete this work successfully.

Secondly, we would like to extend our heartfelt gratitude to all those who have supported and guided us throughout the course of this thesis. We wish to thank our supervisor Mr. Rouani and Co. Supervisor Mr. OUAZENE, for their unwavering support, insightful guidance, and continuous encouragement.

We are also grateful to the administrative and technical staff at IAP-Boumerdes & RA1D Refinery of Adrar-Sonatrach, for their assistance and support in various aspects of our research. Their efforts have facilitated the smooth progress of our work.

# DEDICATION

*This thesis is dedicated to my beloved family, whose unwavering support, encouragement, and love have been my constant source of inspiration.*

*To my parents, for their endless sacrifices and belief in my dreams. To my sibling & cousins, for their encouragement and understanding.*

*To my friends, colleagues, and my closest ones.*

*Last but not least, I want to thank me.*
*I want to thank me for believing in me, I want to thank me for achieving this project, I want to thank me for all those sleepless nights, I want to thank me for having the continuous will to learn and not giving up on improving my knowledge in the technical field, I want to thank me for my hard work and making myself and my family proud.*

***GHOUFRANE***

*I dedicate this thesis to my family. A special feeling of gratitude goes to my loving parents, who have always encouraged me and pushed me toward success in life. To my mother, who has sacrificed everything for me during my years of study. To my sisters and my brother, who have never left my side.*

*I also dedicate this work to my friends and colleagues who have supported me throughout this process. I deeply appreciate all they have done, especially Katia and Nouha. Additionally, I want to thank my partner for sharing this incredible experience with me. Your partnership and support have been invaluable.*

***LAMIA***

# ABSTRACT

Distillation columns are essential in many industries for separating liquids based on the volatility of their components. However, traditional control methods often struggle to achieve
high component purity due to the process's complexity. This project investigates the use of Model Predictive Control (MPC) on a binary distillation column, comparing its performance to conventional control techniques. Guidelines for tuning the MPC controller are also provided. Simulation results show that MPC greatly improves setpoint tracking and disturbance rejection compared to traditional methods, demonstrating its potential to enhance process efficiency and product quality.

# TABLE OF CONTENT

# List of figures

# List of Tables

# List of Abbreviations

| | |
|---|---|
| CARIMA | Controlled Autoregressive Integrated Moving Average |
| CV | Controlled Variable |
| DB | Distillate and Bottoms Structure |
| DCS | Distributed Control Systems |
| DMC | Dynamic Matrix Control |
| DQ | Distillate and Reboiler Duty Structure |
| $DQ_T$ | Reboiler Duty Controlling the Tray Temperature |
| $D_TQ$ | Distillate Controlling the Tray Temperature |
| D-Q/B | Distillate and Reboiler Ratio Structure |
| DV | Disturbance Variable |
| FC | Flow Controller |
| FOPDT | First Order Plus Dead Time |
| GPC | Generalized Predictive Control |
| IMC | Internal Model Control |
| ISA | International Society of Automation |
| LB | Reflux and Bottoms Structure |
| LC | Level Controller |
| LQ | Reflux and Reboiler duty Structure |
| $LQ_T$ | Reboiler Duty Controlling the Tray Temperature |
| $L_TQ$ | Reflux Controlling the Tray Temperature |
| L/D-B | Reflux Ratio and Bottoms Structure |

| | |
|---|---|
| L/D-Q | Reflux Ratio and reboiler Duty Structure |
| LQR | Linear Quadratic Regulator |
| MIMO | Multi-Input Multi-Output |
| MPC | Model Predictive Control |
| P | Proportional |
| PB | Proportional Band |
| PI | Proportional-Integral |
| PID | Proportional-Integral-Derivative |
| PV | Process Variable |
| RGA | Relative Gain Array |
| RMPC | Robust Model Predictive Control |
| SISO | Single-Input Single-Output |
| SP | Setpoint |
| TC | Temperature Controller |
| TIC | Temperature Indicating Control |

# General Introduction

Distillation columns are critical components in the oil refining and petrochemical industries, facilitating the separation of liquid mixtures into their vapor and liquid phases based on different boiling points. Maintaining these columns at optimal operating conditions is essential for maximizing plant efficiency and profitability. Effective control of distillation columns must address several key factors, including safety, operational constraints, energy consumption, and product specifications. Traditional PID (Proportional-Integral-Derivative) controllers often struggle with the inherent nonlinearities and time-varying dynamics of distillation processes [1].

In contrast, Model Predictive Control (MPC) is a highly effective strategy for monitoring binary distillation columns. MPC employs a mathematical model of the process to predict future behavior and optimize control actions. This capability is particularly advantageous for binary distillation columns, as it can accommodate the nonlinear dynamics and multivariable nature of these systems. By leveraging MPC, it is possible to achieve more precise control over the distillation process, resulting in higher-quality products and reduced energy consumption [2].

This project presents a practical study about MPC of an industrial distillation column based on applying MPC on both single-input single-output and multi-input multi-output systems. The primary focus is to investigate the performance of PID and PID feedforward control strategies on MIMO (Multiple-Input Multiple-Output) systems. Using such controllers may lead to interaction between control loops, decreased system performance, instability, and difficulty in achieving the desired setpoints. Such problems and challenges make the MPC a good candidate to be used in the distillation column. In this thesis, Aspen HYSYS software has been utilized to simulate and test various control strategies applied on the distillation column.

This project is divided into four chapters organized as follows:

Chapter Ⅰ: introduces the fundamental principles of distillation columns and the control strategies employed to manage them, and defines briefly the depropanizer column.

Chapter Ⅱ: explores the different conventional strategies that are used for controlling the distillation columns. It also introduces the RGA (Relative Gain array) method for identifying the interaction between loops and finally proposes solutions for such problems by either using decouplers or model predictive control.

Chapter Ⅲ: gives a comprehensive overview of Model Predictive Control, from its historical roots to its theoretical foundations and the implementation of unconstrained MPC on a SISO system (separator). It also provides actionable insights on how to tune MPC parameters effectively.

Chapter Ⅳ: focuses on the implementation of MPC on a MIMO system that is a binary distillation column (depropanizer) using HYSYS, analyzing its performance in comparison to PID and PID feedforward controllers. Finally, it offers guidelines for applying MPC to real columns with similar specifications to the studied column.

# Chapter Ⅰ: Basic principles of distillation columns

# I.1 Introduction

Distillation is a fundamental process in chemical engineering that separates components in a liquid mixture based on their varying boiling points. By vaporizing the mixture and then condensing vapor, distillation enables the purification and separation of valuable products. Understanding the basics of distillation, including its principles, components, and control strategies, is crucial for optimizing separation processes and ensuring product quality. In this chapter, we will explore the basics of distillation columns and the various strategies utilized for their control.

# I.2 Distillation system and its working principal

As shown in Figure I-1 distillation towers also known as distillation columns consist of: A vertical shell where the separation of components is carried out , column internals such as trays or plates or packings that are used to enhance component separation, a reboiler that provides the necessary heat for vaporization of the liquid mixture, a condenser for cooling and condensing the vapor leaving the top of the tower, and a reflux drum to hold the condensed vapor from the top of the column so that liquid (reflux) can be recycled back to the column [3].

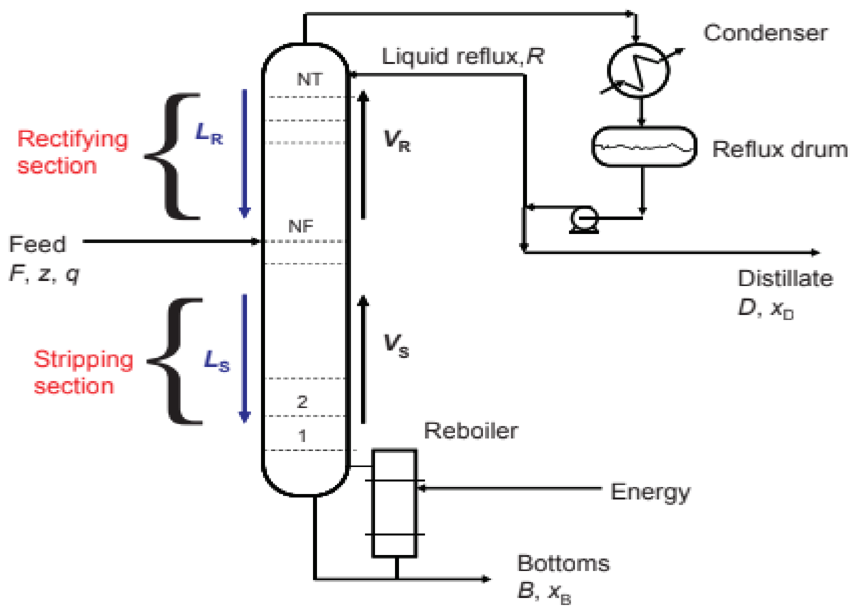



**Figure I- 1 : Representation of distillation column system [4]**

In a distillation unit, the feedstock is introduced near the middle of the column at a feed tray, dividing the column into enriching (top) and stripping (bottom) sections. The feed flows down to the reboiler at the bottom, where heat is applied to generate vapor. The vapor is reintroduced at the bottom of the column, while the liquid from the reboiler (bottoms) is removed.

As the vapor moves up the column and exits at the top, it is condensed by the condenser, with the liquid collected in the reflux drum. Some liquid is recycled as reflux, while the rest is removed as distillate or top product. This process involves internal flows of vapor and liquid within the column, as well as external flows of feeds and product streams into and out of the column [4].

# I.3 Types of distillation columns

Distillation has many different types designed for specific needs; it is categorized based on many factors. Each type offers unique advantages in terms of efficiency, capacity and suitability for different separation tasks.

## I.3.1 The feed composition

Distillation can be categorized based on the nature of the feed that enters the column.

- **Binary distillation:** The feed contains only two components [3].
- **Multi-component distillation:** The feed contains more than two components [3].

## I.3.2 The product streams

Distillation columns can be classified according to the number of product streams into:

- **Multi-product distillation columns**

For applications where achieving extremely high purity for intermediate products is not crucial, multi-product columns can be designed to recover more than two products simultaneously. This is achieved by incorporating a series of lateral withdrawal points along the length of the column, where either liquid or vapor phases are extracted [5].

## I.3.3 The treatment method

Distillation columns are primarily classified into two types based on the treatment method:

- **Batch distillation columns**

. In batch operation, the feed to the column is introduced batch-wise. That is, the column is charged with ‘batch’, and then the distillation process is carried out. When the desired task is achieved, the next batch of feed is introduced [3].

- **Continuous Columns**

In contrast, continuous columns process a continuous feed stream, no interruptions occur unless there is a problem with the column or surrounding process units. They are capable of handling high throughputs and are the most common of the two types [3].

## I.3.4 The column internals

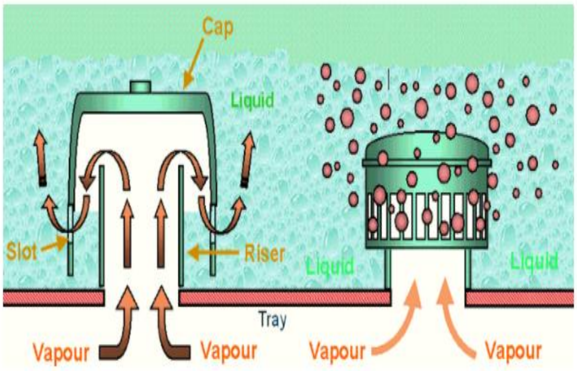
Distillation towers can be divided into two types according to the tray’s composition:

### I.3.4.1 Tray tower

Trays of various designs used to hold up the liquid to provide better contact between vapor and liquid. Two common types of trays used in distillation columns are sieve trays and bubble cap trays [3].

- **Bubble cap trays**

Bubble cap tray features individual risers or chimneys filled over each hole, each capped with a cap. These caps are designed with space to allow vapor to ascend through the chimney, then directed downwards by the cap, finally discharging through slots in the cap, and bubbling through the liquid on the tray [3].




(a)

(b)

**Figure I- 2 : Bubble cap trays: (a) Vapor flow within a bubble cap (b) Bubble cap design with downcomers and weir**

- **Sieve trays**

Sieve trays are a simpler design compared to bubble cap trays. They consist of metal plates with spaced holes. During distillation, vapor rises directly up through these holes and bubbles through the liquid resting on the tray. The effectiveness of a sieve tray depends on the design parameters, which include the number, size, and arrangement of the holes in the plate [3].

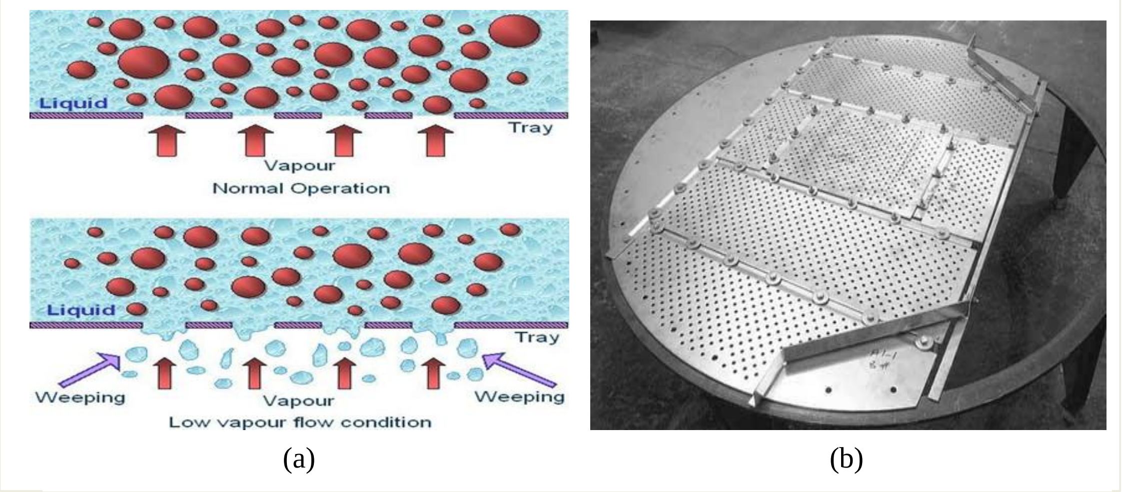



**Figure I- 3 : Sieve Trays (a) weeping phenomenon in seive trays (b) Seive tray design**

**Figure I- 4 : Structured packing**

### I.3.4.2 Packed column

Packing are devices designed to enhance vapor-liquid contact. They minimize the pressure drop across a packed section, which is important because a high-pressure drop would mean that more energy required to drive the vapor up the distillation column [3].

## I.4 Product Streams composition

The distillate refers to the product collected from the top of the column**.** It contains the lighter components which are the more volatile. There are two possibilities for the distillate:

- **Fully liquid:** this requires a total condenser, where all the incoming vapor is condensed, providing liquid for both the distillate and reflux streams.The distillate‘s composition matches that of overhead vapor.
- **Fully vapor:** A partial condenser is needed. Only a portion of the overhead vapor is condensed to form the reflux stream, leaving the uncondensed vapor as the distillate. The distillate’s composition differs from that of the overhead vapor.

The bottoms product is always a liquid stream collected from the bottom of the column. It contains the heavier components, which are less volatile [6].

## I.5 Material and energy balance

Material and energy balances are fundamental operations in the analysis of any process, based on conservation laws. Material balance in a distillation tower ensures that the amount of feed entering the column equals the total amount of products leaving the tower. Any imbalance results in material accumulating or depleting within the tower, disrupting the separation process. Conversely, energy balance asserts that the energy flow rate (input power) into a process must equal the energy flow rate (output power) out of the process, ensuring that the process neither gains nor loses internal energy [6] [7].

## I.6 Control strategies

To maintain balance, control loops are used as shown in Figure (I-6). A simple distillation column with a total condenser has six control valves: feed, reflux, distillate, bottoms, reboiler steam, and condenser cooling duty [8].

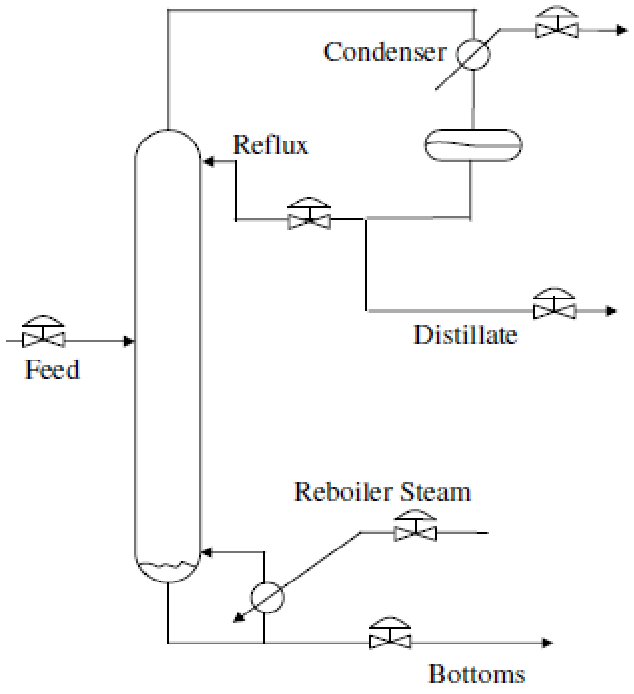



**Figure I- 5 : Schematic of a simple distillation columnalong with the control valves [10]**

### I.6.1 Basic control structures

In a simple distillation tower, the feed valve is commonly controlled by an upstream unit in the process. Additionally, two valves are necessary to manage the reflux drum level and the reboiler level since liquid levels are not self-regulating. Another valve is utilized to maintain the column pressure, which signifies the vapor inventory within the column. Typically, the cooling duty valve in the condenser is employed for pressure regulation. Once the three inventory loops are established, the operator or a controller can adjust the positions of the remaining two control valves to oversee the separation process. This results in an operational degree of freedom of two for a basic distillation column. There are four basic control structures depending on which valves are used for level control: LQ structure uses distillate for reflux drum level and bottoms for reboiler level which leaves the reflux (L) and reboiler duty (Q) free to control separation. DQ structure uses reflux for reflux drum level while distillate and reboiler duty are used to control separation. In the LB structure, the bottoms level is controlled using the reboiler duty. Finally, in the DB structure reboiler and condenser levels are controlled using the reboiler duty and reflux respectively, this structure is rarely used as it violates the mass balance constraint [8].

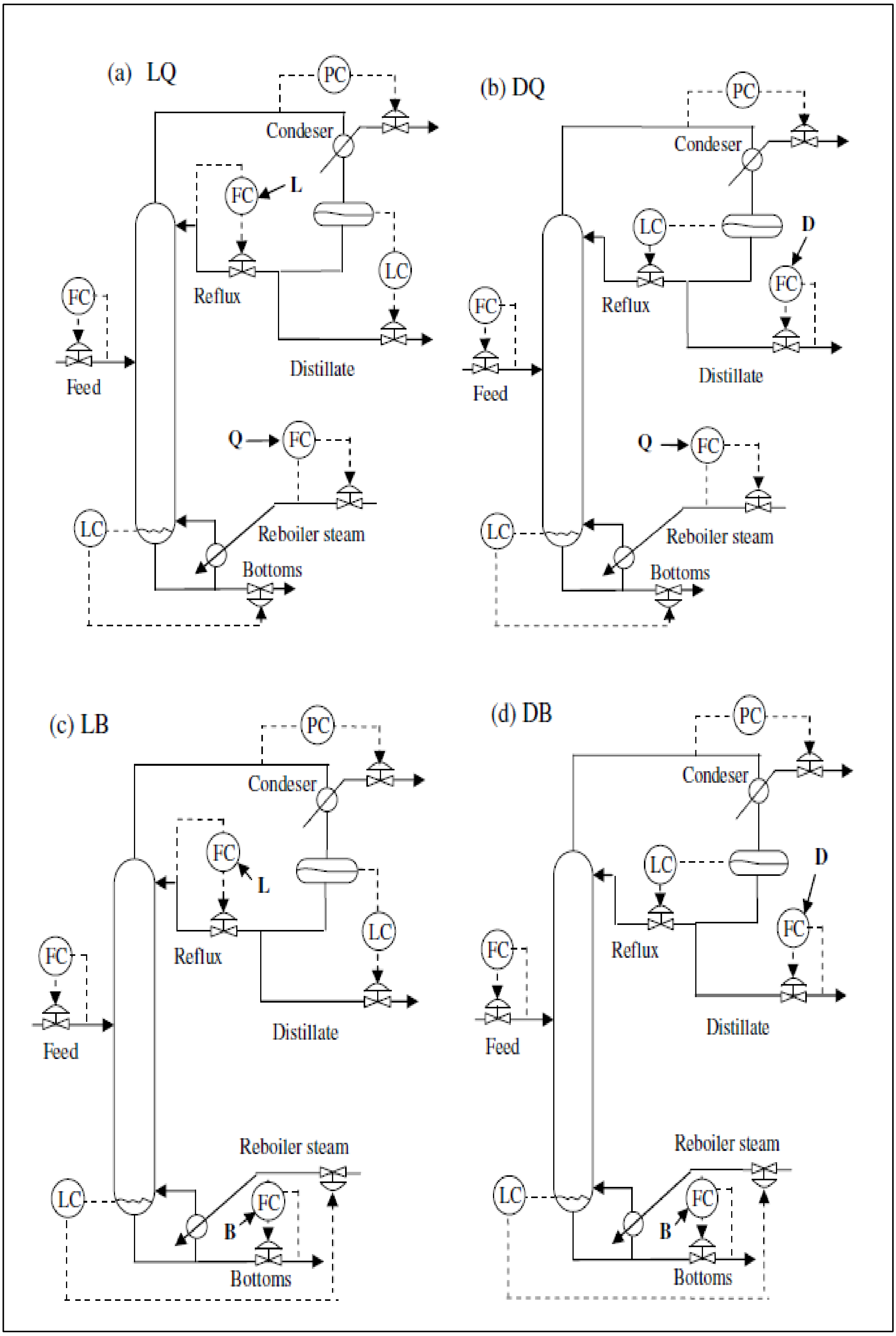



**Figure I- 6 : Schematics of LQ, DQ, LB and DB control structures**

#### I.6.1.1 The Energy Balance (LQ) Structure

The LQ control structure is the most common approach because it naturally accounts for how separation occurs in the column. Separation is driven by successive condensation and vaporization of the counter-current vapor and liquid streams as they flow through the column. The LQ structure controls the column's energy balance by manipulating the cold reflux (L), which provides condensation, and the reboiler duty (Q), which provides vaporization. These are logical controls because condensation and vaporization are the main drivers of separation. Adjusting L and Q alters the column's energy balance and thereby changes the distillate-bottoms split by influencing how components are distributed between the vapor and liquid phases as they rise and fall through the column [8].

#### I.6.1.2 Material Balance Structures

Material balance control structures directly adjust the product split by changing the distillate or bottoms stream flow rate. This type of structure is needed when a conventional LQ structure using levels would be ineffective. The DQ structure is suitable for columns with a very high reflux ratio ($L/D > 4$). In this case, the distillate flow is a small fraction of the large reflux stream, making it difficult to control the reflux drum level with just the distillate flow. Therefore, the level must be controlled using the reflux flow instead. The LB structure is used for columns where the bottoms flow rate is much smaller than the boil-up. In such cases, the small bottoms stream cannot effectively control the reboiler level. So, the reboiler duty is manipulated instead to control the level. The DB structure is rarely used in steady state because the distillate and bottoms flows cannot be set independently due to the material balance constraint. However, in dynamic situations where the reflux and reboil are much greater than the distillate and bottoms, the DB structure may provide adequate control by manipulating both flows [8].

### I.6.2 Other control structure variants

Other structure variants in distillation columns involve: The L/D-Q, L/D-B, and D-Q/B control structures adjust the reflux ratio, reboil ratio, or both for controlling the separation. In the L/D-Q and L/D-B structures, the distillate stream can be used to control the reflux drum level even if it is small relative to the reflux, and the bottoms stream can control the reboiler level even if small relative to reboil. This is done by adjusting the respective streams in ratio to the reflux or reboil. Maintaining the reflux ratio through feedforward control is common as it helps compensate for changes in the distillate rate before column dynamics affect purity. This is because changes in reflux due to liquid hydraulics between trays have a time constant of 15-30 seconds. Maintaining the reboil ratio through feedback control alone is not as popular because tray temperatures and compositions respond almost immediately to reboil changes due to fast vapor dynamics. Adjusting the reboiler duty in feedback is usually sufficient for effective control [8].

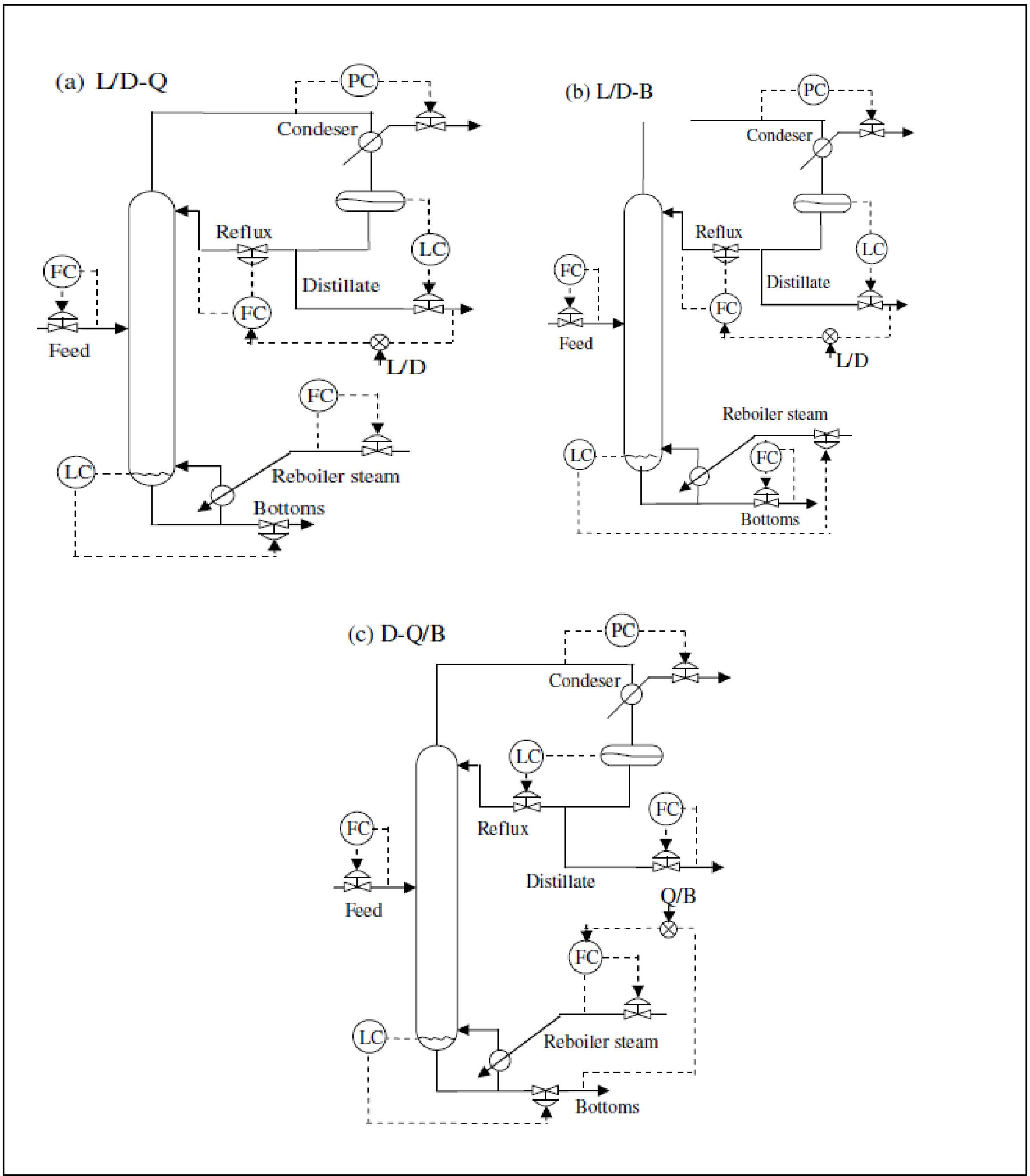



**Figure I- 7 : Schematics of L/D-Q, L/D-B, and D-Q/B control structures[10]**

## I.6.3 Temperature based inferential control

The main control objective is to ensure that the impurity levels of light key and heavy key components in the distillate and bottoms streams remain below their design specifications. Directly controlling these impurity levels using composition measurements is not feasible due to long time delays between a disturbance and its impact on product purity. The individual trays in the column would respond faster to load disturbances like changes in feed flow or composition compared to the response time of the product streams. Therefore, controlling an inferential variable like tray temperature allows for tighter control of impurities in the distillate and bottoms. In a feedback system, adjustments to reflux or reboil can be made after a deviation in product purity to maintain the desired separation.

The tray temperature can be used as an indirect measurement of the tray composition. At a constant pressure, the boiling point of a liquid mixture depends on its composition, heavier components boil at higher temperatures. For a binary mixture, the relationship between tray temperature and composition is exact, with the tray temperature increasing as the concentration of heavier components increases, and vice versa. For a multi-component mixture, the relationship is only approximate [8].

### I.6.3.1 Single-ended temperature control

Single-ended temperature control refers to controlling the temperature of a single tray in the distillation column. This can be done by manipulating one of the two available degrees of freedom, which are typically the reflux rate or reboiler duty. Manipulating the reboiler duty (Q) is preferred as it provides a fast response, since all tray temperatures respond quickly to changes in Q. The reflux rate can also be used, as long as the controlled tray is not too far below the reflux (within about 10 trays). This strategy can be applied to different common control structures. In the LQ structure, either the reflux or reboiler duty can be used. In the DQ structure, if distillate rate (D) controls the temperature, it is nested with the reflux drum level controller. Similarly, in the LB structure if bottoms rate (B) controls temperature, it is nested with the reboiler level controller. The level controllers in these nested cases must be tuned tightly as PI controllers; otherwise, the temperature control will be very sluggish. Ultimately, both the reflux and reboiler are the only variables that directly affect tray temperature, so any control scheme must manipulate one of these to change temperature [8].

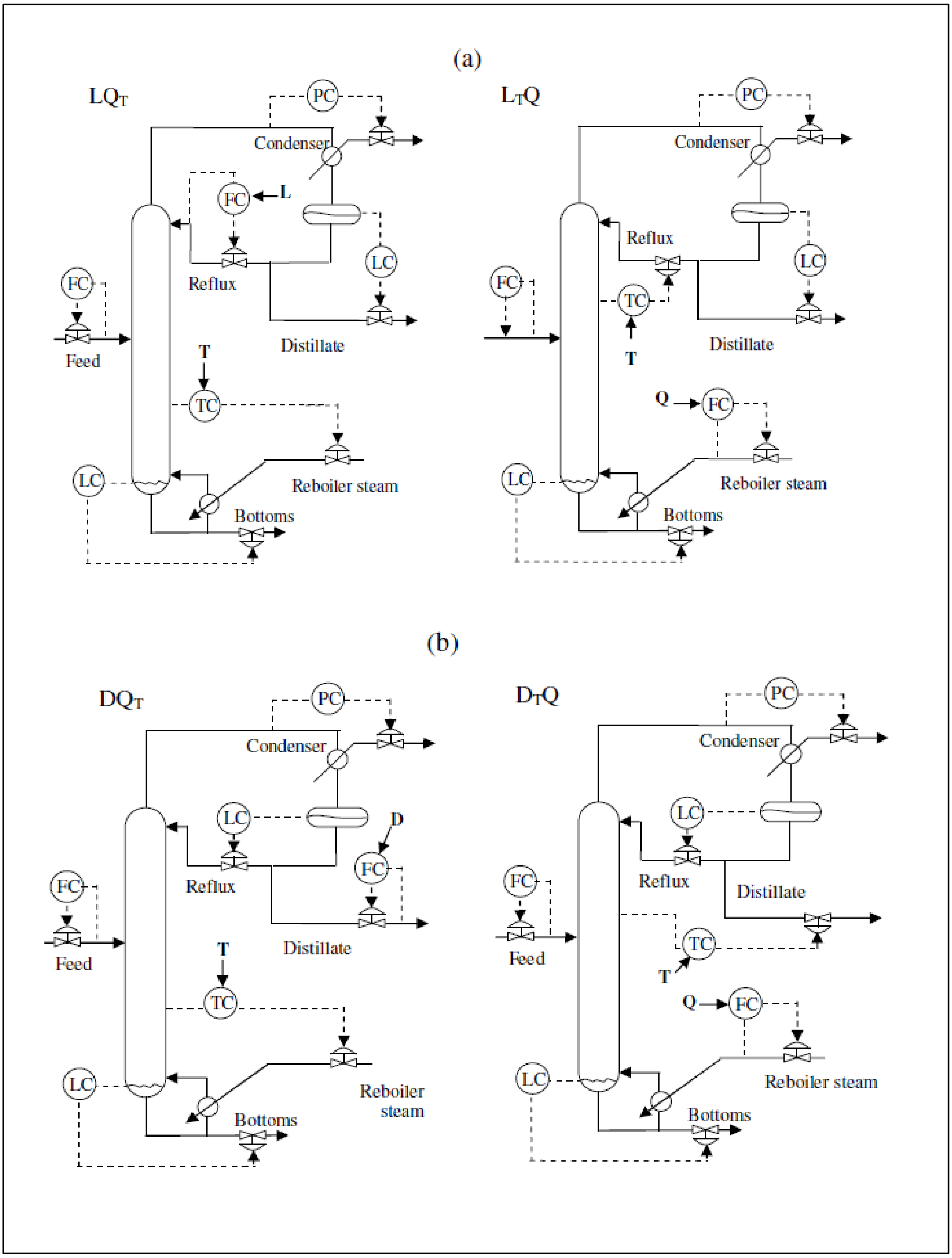



**Figure I- 8 : Single ended temperature control structures using LQ and DQ scheme[ 10]**

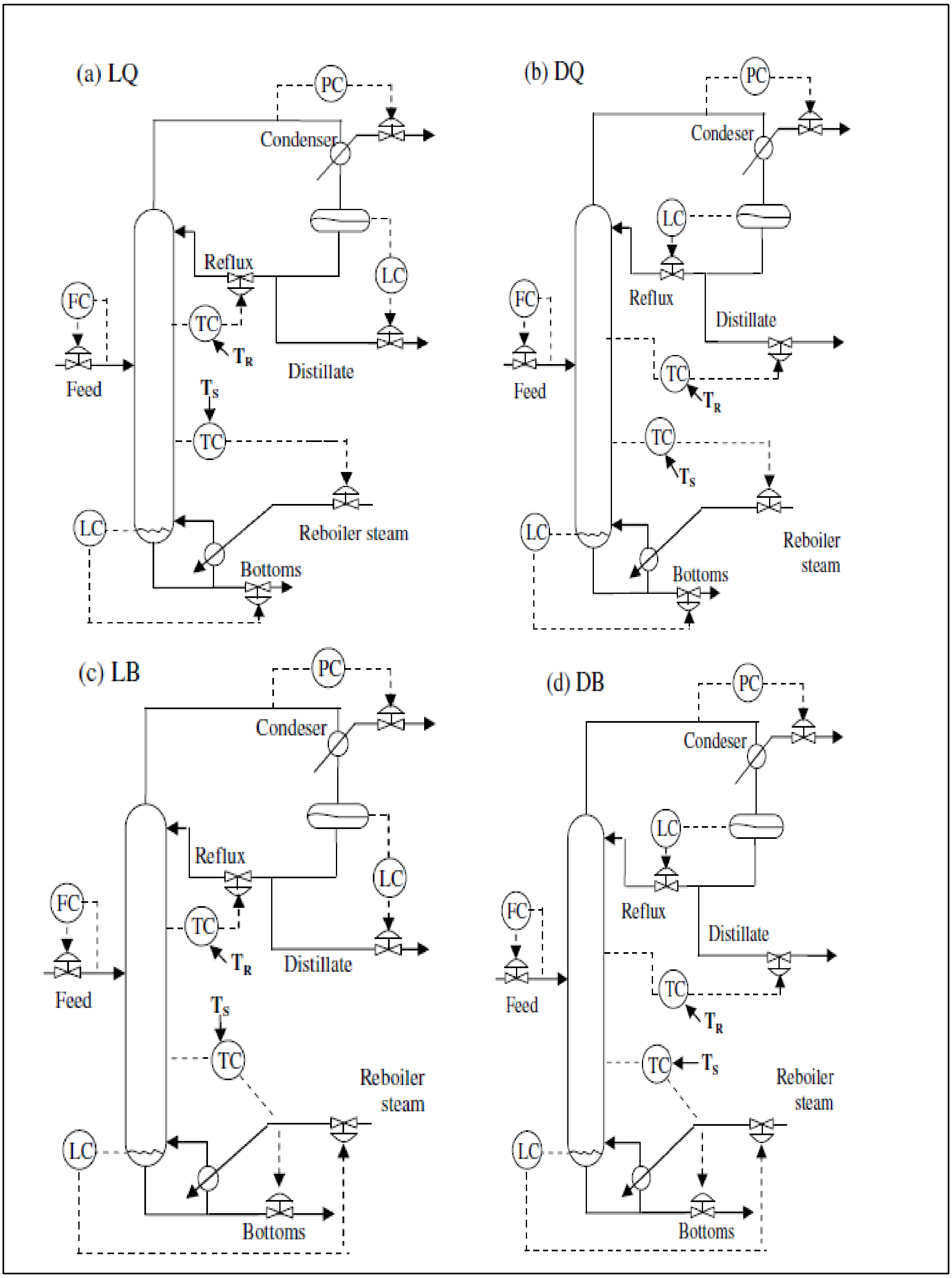



**Figure I- 9 : Dual ended temperature control structures using LQ, DQ, LB and DB schemes [10]**

### I.6.3.2 Dual-ended temperature control

Theoretically, since a distillation column has two degrees of freedom, it is possible to control two tray temperatures simultaneously. This is known as dual-ended temperature control. For example, in the LQ control structure the reflux rate could control a rectifying tray temperature while the reboiler duty controls a stripping tray temperature. However, in industrial practice it is more common to control a single tray temperature. This is because controlling two temperatures requires detuning the temperature controllers to account for interaction between the loops. More importantly, the two controlled tray temperatures may not be truly independent, meaning the controllers could end up seeking infeasible set points as they interact. Dual temperature control is feasible for very long distillation columns where the controlled trays are far enough apart to be essentially independent. However, for most columns, single temperature control is preferable to avoid interaction issues [8].

## I.7 Depropanizer column

A depropanizer is a multi-component distillation column, commonly used in oil refineries. It separates propane from a mixture of components containing ethane, propane, butane, and pentane. The control of a depropanizer is challenging. In addition, set points may fluctuate for various reasons, which may depend on the processing units. This can also result from technical issues like insufficient condensation capacity, the market demand, the available products [9].

## I.8 Conclusion

This chapter aims to provide a general overview of distillation columns, introducing their principle, fundamental types and the control strategies used to control a simple distillation column for effective performance.
The next chapter will delve deeper into the theoretical background, focusing primarily on conventional control strategies and pairing problems encountered in distillation processes.

# Chapter Ⅱ: Control strategies

## II.1 Introduction

The technology used in developing the control configuration for the distillation column involves various control strategies and methods. An essential concept is the PID control, the most common feedback control algorithm used in industry. Understanding the process dynamics is essential before tuning the controller parameters. Additionally, the feedforward control is implemented to counteract disturbance, while the cascade control is used when there is a hierarchical relationship between variables in the system such as in multivariable processes. To address loop interaction and pairing issues decouplers and model predictive controllers are used to optimize the control system’s efficiency in managing the distillation column.

## II.2 Feedback Control

In control theory and engineering, feedback control refers to a technique in which a system's output is continuously observed and compared with a desired reference or setpoint. Any difference between the intended and actual outputs—referred to as errors—causes the system to behave differently and requires correction. A controller decides these corrective measures and uses feedback data to direct the system in the desired direction of the intended state. Feedback control systems may maintain stability, regulate performance, and react to changes in the environment or operational conditions by continuously sensing, comparing, and changing.

Conventional feedback controllers use one, two, or three methods to determine the value of the controller output. These methods, called the modes of control, are as follows:

- Proportional (P)
- Integral (I)
- Derivative (D)

These modes can be used singly or in combination. P, PI, and PID cover almost all the actual feedback controller applications, with the PI combination being the most prevalent [10]

### II.2.1 Proportional Mode

When operating in the proportional mode, a controller's output and the position of the final control element are directly proportional to the current measurement value. Unlike other control modes, this mode does not take into account the past behavior of the measurement values or their rate of change.

$$m = K_c.e + b \qquad \text{(II-1)}$$

Equation II-1 shows the behavior of a proportional controller (where m is the controller output, $K_c$ is the controller gain, e is the error and b is the output bias) [10].

The general block diagram is shown on the following figure:

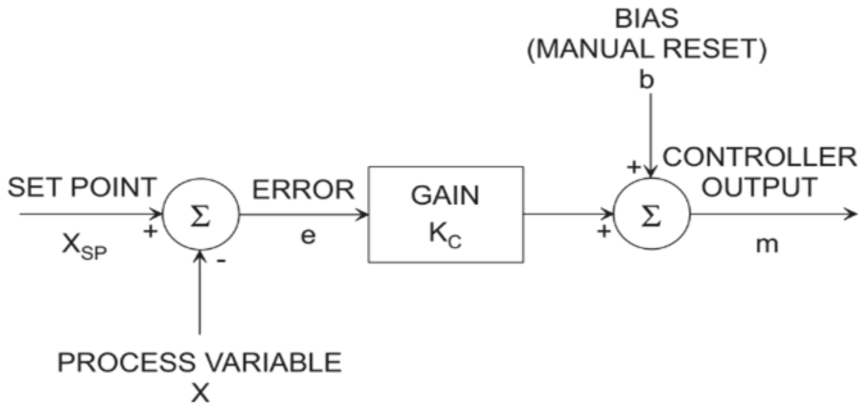



**Figure II- 1 : Block diagram of a proportional controller[12]**

Tuning such a controller for desired performance is relatively straightforward, typically requiring just one or two adjustments. However, despite its simplicity, the proportional controller suffers from a significant limitation: it tends to introduce an offset between the setpoint and the actual measurement value, particularly under varying load conditions [10].

## II.2.2 Integral Mode

An integrator is the perfect tool for automating the adjustment of the controller output bias. It is called "automatic reset." This is often shortened to simply "reset." The behavior of an integral controller is given by the following equations:

$$m = K_C \left(e + \frac{1}{T_I} \int e \, dt\right) \qquad \text{(II-2)}$$

$$M = K_C \left(1 + \frac{1}{T_I S}\right) E \quad \text{(Laplace)} \qquad \text{(II-3)}$$

The PI tuning parameters are the controller gain $K_C$ and the integral time $T_I$.

The block diagram of a typical commercial PI controller is shown in the following figure:

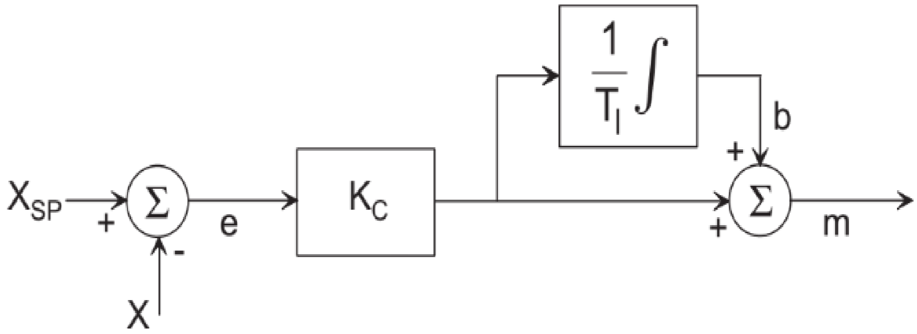



**Figure II- 2 : Block diagram representation of a typical commercial PI controller [12]**

**The Integral Time $T_I$ :** $T_I$ is the reaction of a PI controller to a change in an error that is impacted by the reset parameter ($T_I$). It represents the time required for the integral response to repeat the proportional response, which is dependent on the $T_I$ setting. A large $T_I$ values result in a slower integral response, while a small $T_I$ value leads to a faster response. However, many manufacturers use the reciprocal of $T_I$ ($1/T_I$ ). So, a higher value on the dial for $1/T_I$ corresponds to a slower integral response, and vice versa [10].

## II.2.3 Derivative Mode

The improvement of control-loop performance is made by adding a component to the controller output proportional to the measurement's rate of change. This reduces variations by helping to predict changes in load. Equation II-4 shows how the derivative mode is added to the proportional and integral modes, with its contribution based on the rate of change of the product of the controller gain times error [10].

The tuning parameter $T_D$ adjusts the relative effect of this mode of control:

$$m = K_C \left(e + \frac{1}{T_I} \int e\, dt + T_D \frac{de}{dt}\right) \qquad \text{(II-4)}$$

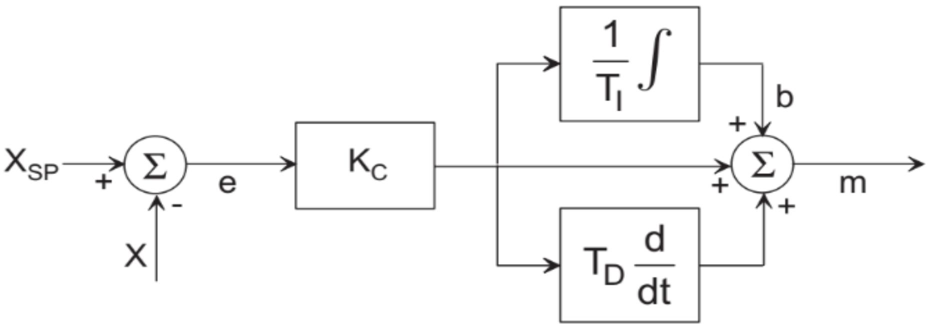



**Figure II- 3 : Block diagram representation of an “ideal” PID controller**

Proportional-plus-derivative (PD) control adds a term that considers the rate of change of the error (difference between desired and actual value). This anticipation, characterized by a lead time of $T_D$ minutes, allows the controller to react faster to disturbances compared to using only proportional control. This faster response is one of the key benefits of using the derivative mode in PD control. With proportional-plus-derivative control, the controller predicts the measurement $T_D$ minutes into the future and uses this predicted value to compute the controller output, rather than the current measurement. PD controllers are now common on modern controllers and ideal for temperature loops with low response times and minimal noise [10] .

The table shown next demonstrates the difference between the previously discussed modes:

**Table II - 1: Summary of feedback control modes [12]**

| **Mode** | **Common Name (s)** | **Tuning Parameter** | **Application** |
|---|---|---|---|
| Proportional | Proportional | Gain, $\mathbf{K_C}$<br>or<br>Proportional Band PB | Used when:<br>Simple form of control is desired, load does not change significantly or offset is acceptable.<br>Also used when the control loop dynamics permit a relatively high gain to be set without causing excessive oscillation. Then, even if significant load changes are present, offset is only minimal |
| Integral | Reset<br>Automatic Reset | Min./Repeat, $\mathbf{T_I}$<br>or<br>Repeats/min.1/$\mathbf{T_I}$ | Used almost always in conjunction with proportional mode to eliminate steady state offset.<br>Occasionally used alone; known as Integral controller. For most applications, I-only controller provides inferior performance when compared with PI modes. |
| Derivative | Rate Action<br>Pre-Act | Deriv. Time, $\mathbf{T_D}$ | Used usually in combination with P and I modes to improve loop performance by anticipating the effect of load changes.<br>Used mainly on temperature loops or other loops that have similar characteristics (low noise level, fairly slow response). |

## II.3 PID Controller Algorithms

The proportional, integral and derivative modes in PID controllers are arranged into three distinct controllers algorithms or forms known as interactive, noninteractive, and parallel forms.

### II.3.1 Interactive (series PID)

Commercially available analog controllers were developed with pneumatic mechanisms to achieve proportional, integral, and derivative control objectives. However, the mathematical analysis of these mechanisms did not match the ideal PID controller's mathematical form. Instead, they could be represented by an interactive block diagram, and by the following equation, written in Laplace form:

$$M(s) = \hat{k}_C \left( \frac{(1 + \hat{T}_I\, s)\,(1 + \hat{T}_D\, s)}{\hat{T}_I\, s} \right) E(s) \qquad \text{(II-5)}$$

The "^" over the symbols indicates the entered value for the tuning parameters.

In the interactive controller, the controller gain, integral time and derivative time are not directly set by entering the parameters $\hat{k}_C$, $\hat{T}_I$ and $\hat{T}_D$ but by a combination of them. If any of the parameters is adjusted, it will affect the other remaining parameters [10].

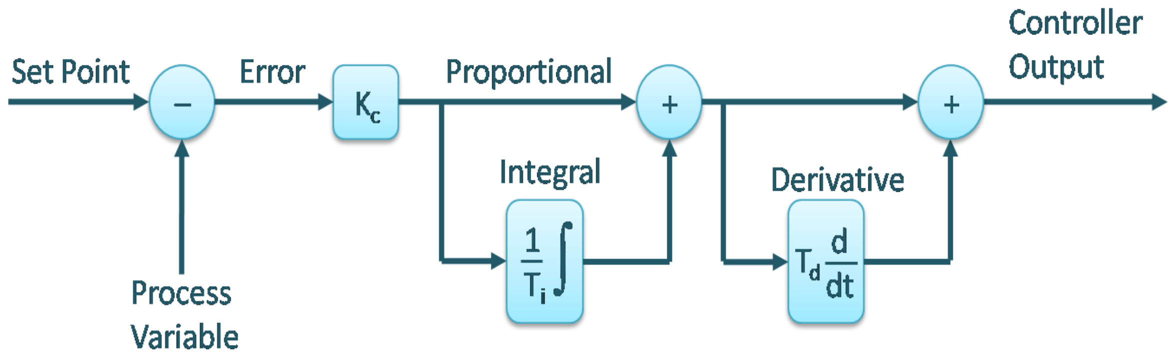



**Figure II- 4 : Block diagram of interactive controller**

## II.3.2 Noninteractive (ideal PID)

This algorithm is used to establish robust and stable control systems that can effectively manage disturbances. If the time difference is $T_D = 0$, the interactive and noninteractive forms are identical. Equation II-4 shows the mathematical representation of noninteractive controller [11].

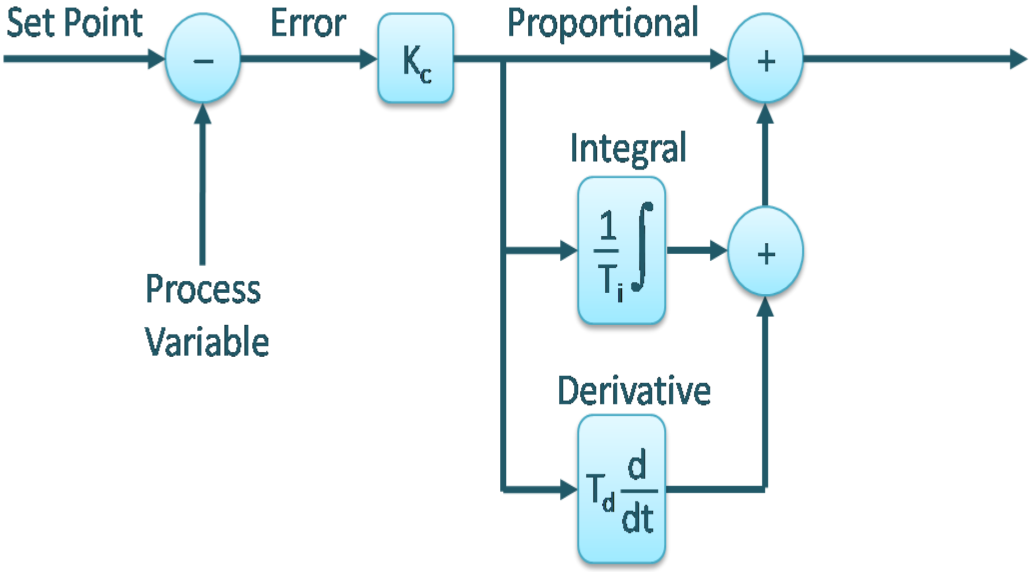



**Figure II- 5 : Block diagram representation of noninteractive controller**

## II.3.3 Parallel PID

In the concept of parallel PID, also known as independent gains, the controller is structured such that each mode (proportional, derivative, and integral) has its own gain. This differs from the traditional PID controller, where a single gain parameter affects the three modes simultaneously. The equation and figure below are used to show this concept, as illustrated in [10].

$$m = k_c e + k_I \int e\, dt + K_D \frac{de}{dt} \qquad \text{(II-6)}$$

where:

The proportional gain $k_p = k_c$

The integral time is $T_I$ and $k_I = \frac{k_c}{T_I}$

The derivative time is $T_D$ and $k_D = k_c\, T_D$

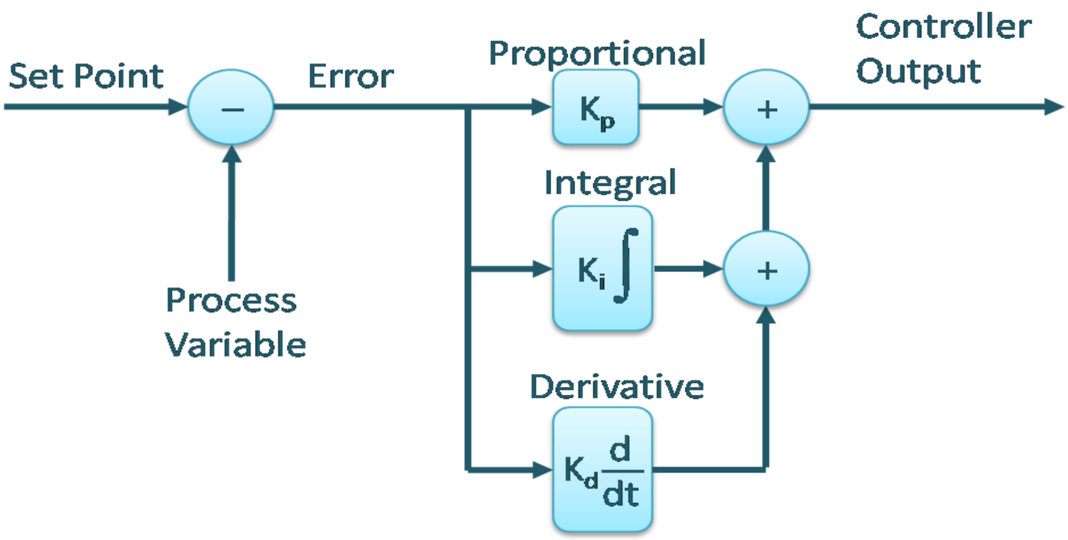



**Figure II- 6 : Block diagram of parallel controller**

- The equations below provide equivalent parameters for a noninteracting controller based on an interacting controller's parameters [10]:

$$\begin{cases} k_c = \hat{k}_c \left( \dfrac{\hat{T}_I + \hat{T}_D}{\hat{T}_I} \right) \\ T_I = \hat{T}_I + \hat{T}_D \\ T_D = \dfrac{\hat{T}_I\, \hat{T}_D}{\hat{T}_I + \hat{T}_D} \end{cases} \tag{II-7}$$

➢ While the equations below provide equivalent parameters for an interacting controller based on a noninteracting controller's parameters [10] :

$$\begin{cases} \hat{k}_c = \lambda\, k_c \\ \hat{T}_I = \lambda\, T_I \\ \hat{T}_D = \dfrac{T_D}{\lambda} \end{cases} \tag{II-8}$$

$$\text{where: } \lambda = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{T_D}{T_I}} \tag{II-9}$$

## II.4. Process Characterizations

In industry, there are three primary types of process responses: self-regulating, integrating, and runaway. These responses are defined based on how the process reacts to a step change when the PID controller is not actively correcting the process. By understanding these types, one can draw general conclusions about the appropriate proportional (P), integral (I), and derivative (D) settings needed for effective control [7].

- **Self -regulating process**

Self-regulating processes can maintain a new stable output on their own after a change in input or load. They require an integral controller to achieve a perfect match between the desired output (setpoint) and the actual output (process variable). More aggressive integral control eliminates offset faster, but can cause instability (oscillations) in slow processes [7].

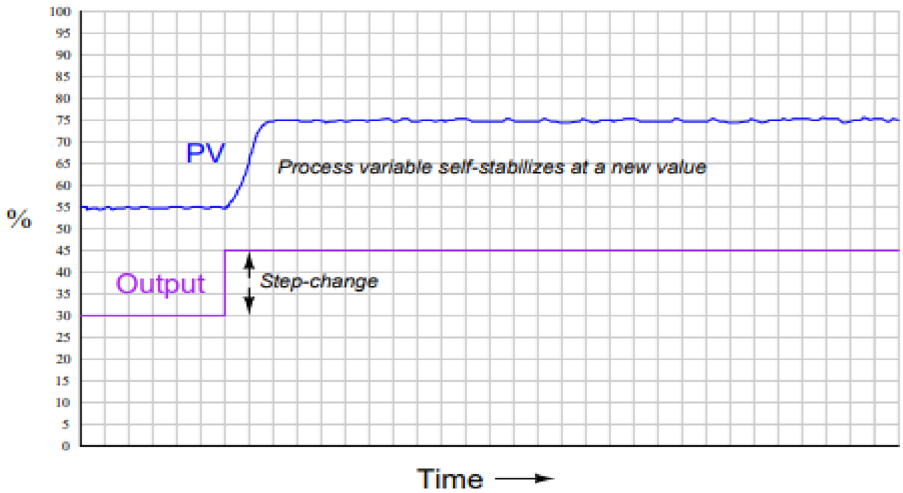



**Figure II- 7 : Self-regulating response to a step-change [9]**

- **Integrating processes**

Liquid level control is a classic example where the liquid level ramps at a rate proportional to the difference between flow rates in and out. Unlike self-regulating processes, integrating processes reach new setpoints without offset using only proportional control. However, too much proportional will lead to oscillations and noise. Integral control action is not necessary to eliminate offset but can be helpful for handling load changes. Integrating processes can be turned into self-regulating processes by natural internal negative feedback, which brings the system to equilibrium [7]

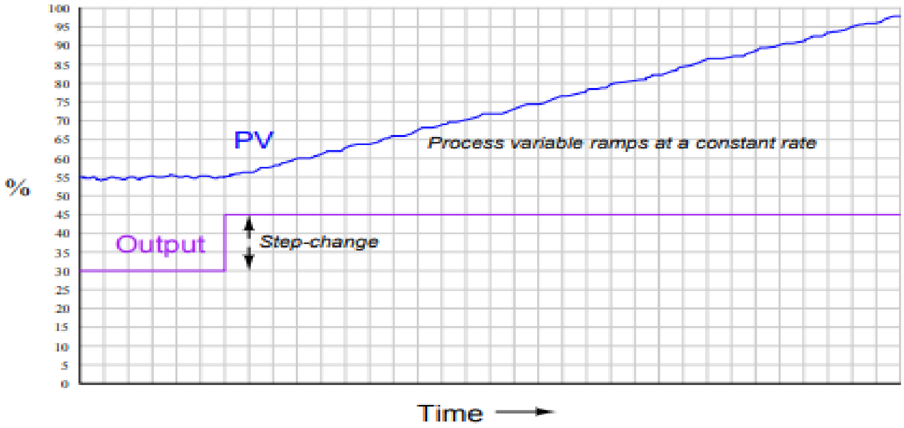



**Figure II- 8 : Integrating process's response to a step-change [9]**

- **Runway processes (negative self-regulation or negative lag)**

Inverted pendulums represent a classic illustration of runaway processes, where instability ensues upon tipping, causing them to continuously deviate from the vertical position. Such processes demonstrate exponential growth in reaction to load changes due to positive feedback loops. Derivative control is crucial for runaway processes, while proportional and integral actions alone are insufficient. Natural negative feedback mechanisms can transform a runway process into a self-regulating one, as observed in water-cooled nuclear reactors [7].

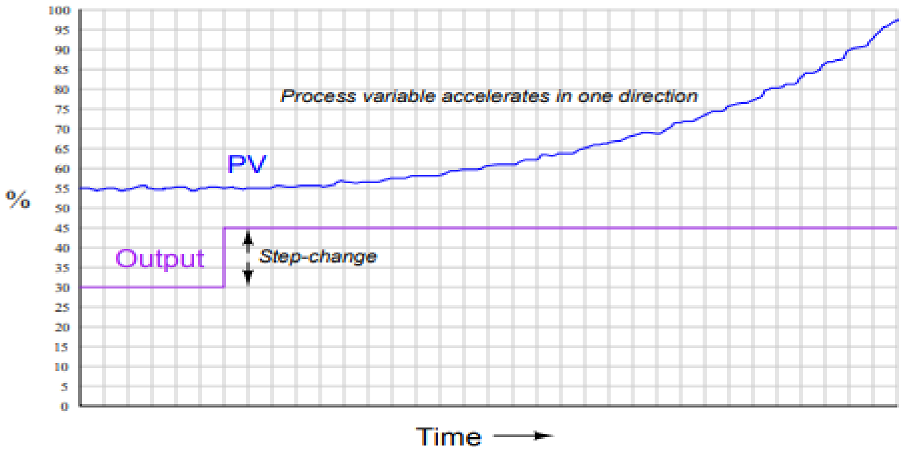



**Figure II- 9 Response of runaway process to an open-loop step change [9]**

## II.5 Tuning

Tuning controllers involves adjusting their parameters to optimize their performance. Traditional tuning techniques, like Ziegler-Nichols, are accessible to people with limited computational abilities, while automated methods are executed by the controller and processed by a personal computer [12].

In order to ensure a good tuning, the following steps are applied as mentioned in [12] :

- Study the process characterization by performing a process test (Open loop test or Closed Loop test.)
- Quantify the behavior of the process by defining the Process gain, time constant and dead time.
- Calculate the controller parameters according to the process defined in the previous step.
- Implement the controller parameters calculated in the previous step.
- Test the controller performance by making a step change in the set point, and the disturbance rejection test.
- In the case of unsatisfied performance, go back to the previous steps.

The model of this study can be approximated by a FOPDT (first-order system plus dead time), represented by the following transfer function:

$$G(s) = \frac{K_p e^{-\theta s}}{1 + \tau s} \qquad \text{(II-10)}$$

$K_p$: Process gain.

$\theta$ : Process dead time (time delay)

$\tau$: Time constant

### II.5.1 Open-Loop Tuning methods

Open loop tuning methods perform the following steps provided by Broida and explained in [12] :

- Observe the process response to a known change in the controller output.
- Analyze the response data ( $K_p = \frac{\Delta y}{\Delta u}$ , dead time and time constant).
- Calculate controller tuning coefficients using the parameters obtained from the analysis.

### II.5.1.1 ZIEGLER – NICHOLS OPEN - LOOP METHOD

The Ziegler-Nichols method as described in [13] is a classic approach, involves deriving $K_C$ and $T_i$ and $T_d$ from the parameters of a first-order- plus-deadtime model. These calculations are influenced by the type of controller being used (P, PI, PID). Table Ⅱ-2 summarizes these calculations.

**Table II - 2 : ZIEGLER-NICHOLS Open loop coefficient [16]**

| Mode | $K_C$ | $T_i$ | $T_d$ |
|---|---|---|---|
| P | $\tau / K_p\theta$ | - | - |
| PI | $0.9\ \tau/K_p\theta$ | $3.33\theta$ | - |
| PID Series | $1.2\ \tau/K_p\theta$ | $2\theta$ | $0.5\theta$ |
| PID Parallel | $1.5\ \tau/K_p\theta$ | $2.5\theta$ | $0.4\theta$ |

### II.5.1.2 ISA (Greg McMillan)

Greg McMillan's approach to tuning PID controllers presented in [14] emphasizes the importance of considering the performance objectives in the tuning process. He classified the performance objectives into three levels: Aggressive, moderate and slow. The coefficients calculation is shown in the table below:

**Table II - 3 : Greg McMillan Open loop coefficient [16]**

| Performance | $K_C$ | $T_i$ | $T_d$ |
|---|---|---|---|
| Aggressive | $\tau / K_p\theta$ | $3\theta$ | $\leq \frac{1}{2}\theta$ |
| Moderate | $\frac{1}{2}\tau / K_p\theta$ | $3\theta$ | $\leq \frac{1}{2}\theta$ |
| Slow | $\frac{1}{4}\tau / K_p\theta$ | $3\theta$ | $\leq \frac{1}{2}\theta$ |

### II.5.1.3 Internal Model Control (IMC)

Internal Model Control (IMC) is a controller synthesis method that uses the process model and performance specifications to derive a control equation. It includes compensation for dead time in the process. When the dead time is relatively small compared to the process time constant, an approximation can be used to simplify the control equation [12] .The parameters are calculated according the desired performance to be met.

**Table II - 4 : Internal Model Control coefficient [16]**

| Performance | $K_C$ | $T_i$ | $T_d$ |
|---|---|---|---|
| Disturbances | $\tau / K_p\theta$ | $\tau$ | $\frac{1}{2}\theta$ |
| SP Change | (PI) 0.6 $\tau / K_p\theta$<br>(PID) 0.83 $\tau / K_p\theta$ | $\tau$ | $\frac{1}{2}\theta$ |
| 5% Overshoot | $\frac{1}{2}\tau / K_p\theta$ | $\tau$ | $\frac{1}{2}\theta$ |

### II.5.1.4 the Lambda Method

The Lambda tuning method is a synthesis method commonly used in the paper industry. It involves designing a controller specifically for the process. To do this, two key pieces of information are needed:

1. A process model, including all relevant parameters such as gain and time constant.
2. Performance specifications for the control loop, such as how the loop should respond to a unit step change in the set point.

The Lambda tuning technique uses open-loop data $k_p$, $\tau$, and $\theta$, with the addition of a parameter $\lambda$ to determine the desired time constant for closed-loop response to set point changes. Here is how it works:

- When there is a high confidence in process parameters, set $\lambda = \tau$.
- In cases of low confidence in process parameters, set $\lambda$ to be within the range $2\tau < \lambda < 4\tau$.

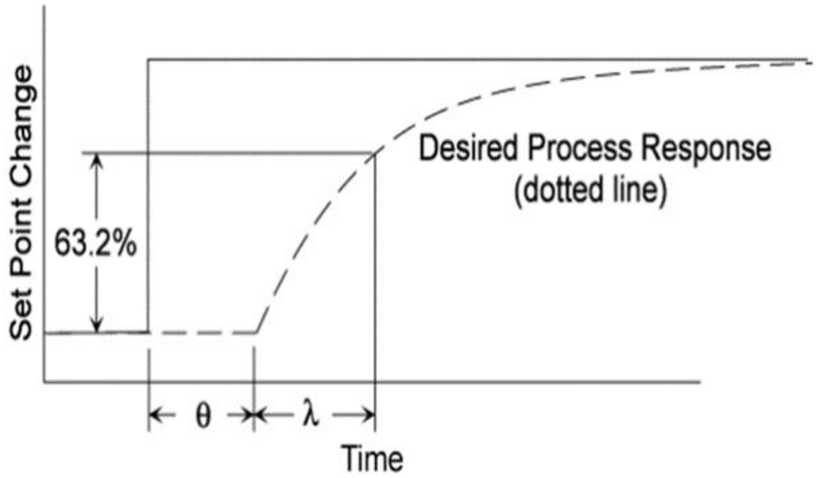



**Figure II- 10 : Desired closed-loop response to set point change: the basis of lambda tuning [12]**

The Lambda method equation for controller tuning according to [12] depends on the process model:

• PI when the model is time - constant - plus - dead - time.

• PID when the model is two - time - constants - plus - dead - time.

**Table II - 5 : Lambda Method Tuning Equations [14]**

| Control mode | Proportional | Integral | Derivative |
|---|---|---|---|
| PI | $KK_C = \frac{\tau}{\tau_{CL}+\theta}$ | $T_I = \tau$ | - |
| PID-Series | $KK_C = \frac{\tau_1}{\tau_{CL}+\theta}$ | $T_I = \tau_1$ | $T_D = \tau_2$ |
| PID-Parallel | $KK_C = \frac{\tau_1+\tau_2}{\tau_{CL}+\theta}$ | $T_I = \tau_1 + \tau_2$ | $T_D = \frac{\tau_1\tau_2}{\tau_1+\tau_2}$ |

The key parameters of lambda tuning are the process time constant (τ), process time delay (θ), the process gain (K), the proportional gain ($K_c$), the closed loop time constant ($\tau_{CL}$) and Time constants used for series and parallel PID controllers ($\tau_1$and $\tau_2$ ).

## II.5.2 Ziegler – Nichols Closed - Loop Method

Ziegler-Nichols closed-loop method is used to tune PID controllers. It involves finding the process's behavior under proportional-only control by:

1. Disabling the integral and derivative terms of the controller.
2. Increasing the controller gain until the system starts oscillating continuously.

3. Recording the gain value (ultimate gain $K_u$) and the oscillation period (ultimate period $P_u$) at this point.

These values are then plugged into specific equations provided in and shown in Table II-6 to calculate the PID controller settings that achieve a desired response. The method offers settings for P-only, PI, PID (series structure), and PID (parallel structure) controllers [12].

**Table II - 6 : Ziegler–Nichols closed-loop tuning equations [14]**

| Control mode | $K_C$ | $T_I$ | $T_D$ |
|---|---|---|---|
| P-only | $K_C = 0.45\ K_U$ | - | - |
| PI | $K_C = 0.5\ K_U$ | $T_I = P_U/1.2$ | - |
| PID-Series | $K_C = 0.6\ K_U$ | $T_I = 0.5\ P_U$ | $T_D = P_U/8$ |
| PID-Parallel | $K_C = 0.75\ K_U$ | $T_I = 0.625\ P_U$ | $T_D = P_U/10$ |

# II.6 Feedforward Control

Feedforward control involves adjusting the control input preemptively to compensate for the effect of a measured disturbance on the output, aiming to maintain the output at its setpoint despite the disturbance. This proactive compensation contrasts with feedback control, which adjusts the control input based on the output error.

The design of a feedforward compensator is illustrated using block diagrams in figure II-12, where $G_d$ represents the disturbance to output transfer function and $G_p$ represents the control input to output transfer function. The control input u is adjusted by the feedforward compensator with the transfer function $G_{ff}$ such that $G_p.u + G_d.d = 0$. Solving for $G_{ff}$ yields $G_{ff} = -\ G_d/\ G_p$. Assuming $G_d$ and $G_p$ are first-order plus dead time transfer functions, the feedforward compensator takes the form of a lead-lag plus dead time transfer function.

Modern Distributed Control Systems (DCS) allow for the configuration of lead- lag plus dead time blocks into the control system, facilitating the implementation of feedforward compensators [7].

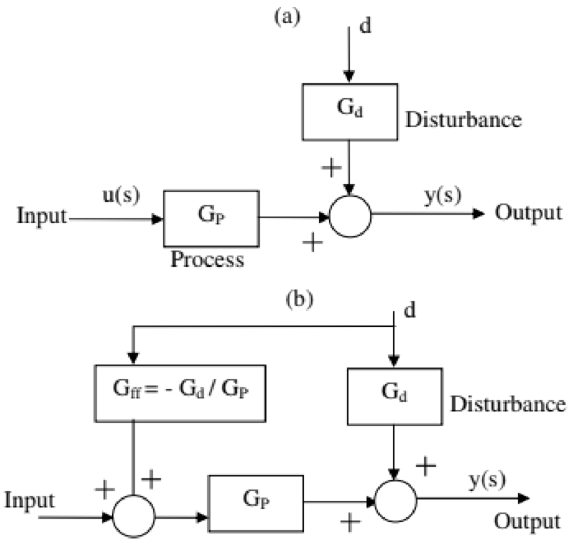



**Figure II- 11 : Design of feedforward compensator: (a) A process (b) Process with feed forward compensator [9]**

# II.7 Cascade Control

To cascade controllers means to connect the output signal of one controller to the setpoint of another controller. Both controllers monitor distinct parameters within the same process. The primary controller acts as the master loop, it provides the desired operating point for the secondary loop (slave loop) through the manipulation of its setpoint [7]. The key advantage of the cascade control is the ability to separate the control of critical process parameters from disturbances such as those caused by control valves exhibiting hysteresis or stiction. Responding quickly to certain disturbances will enhance system's overall performance [15].

Tuning a cascade involves tuning the inner loop first, followed by tuning the outer loop. Tuning a cascade is generally no more difficult than tuning two simple feedback loops, as it involves tuning two controllers. Two potential issues are inadequate dynamic separation between the loops, and interacting stages within the process. For dynamic separation, the inner loop needs to be at least 5 times faster than the outer loop. Less separation makes the outer loop difficult to tune [15].

# II.8 Loop Interaction

Multivariable control deals with processes involving multiple variables; while a single-loop PID controller can be used for each variable, tuning can be difficult due to interactions between the loops [12].

## II.8.1 Multivariable Processes

Figure II-13 depicts a multivariable process, with inputs being either manipulated variables or disturbances. The control system determines the values of manipulated variables, while external factors determine disturbance values. Outputs are categorized as controlled variables, which should be maintained near their targets (setpoint), and dependent variables that are influenced by both manipulated variables and disturbances and are not targeted but may have constraints in some applications [12].

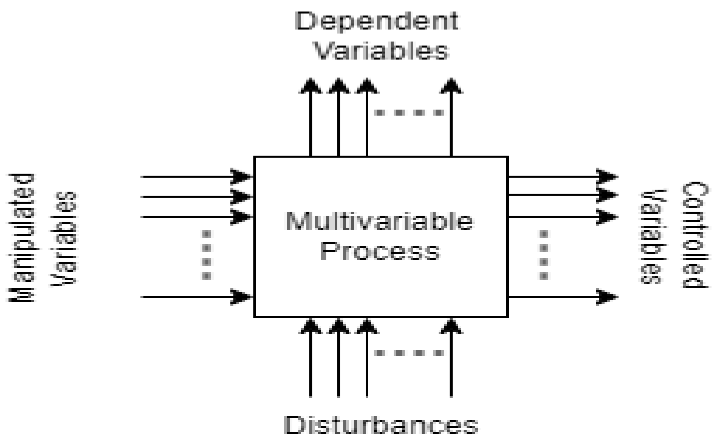



**Figure II- 12 : Multivariable process [14]**

## II.8.2 Multivariable Processes Representation

For a 2 × 2 process, a simplified representation is shown in Figure II-14, where a block with arrows shows how inputs affect outputs, in which the manipulated variables are $u_1$ and $u_2$, while $y_1$ and $y_2$ are the controlled variables:

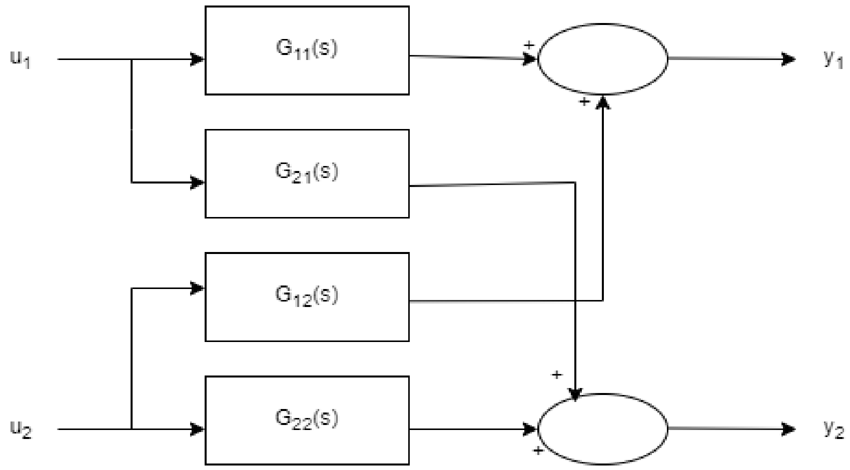



**Figure II- 13 : A block diagram of a 2x2 multivariable process**

A 2 × 2 multivariable process is expressed by the following equations:

$$\begin{cases} y_1(s) = G_{11}(s)\, u_1(s) + G_{12}(s) u_2(s) \\ y_2(s) = G_{21}(s) u_1(s) + G_{22}(s) u_2(s) \end{cases} \qquad \text{(II-11)}$$

Thus:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad \text{(II-12)}$$

Then:

$$\mathrm{Y} = \mathrm{G.U} \qquad \text{(II-13)}$$

The equations provided in [12] for a multivariable process are most conveniently expressed using vectors and matrices in which:

Y: Vector of controlled variables

G: Transfer Matrix

U: Vector of manipulated variables

## II.8.3 RGA Method

RGA (relative Gain Array) was introduced by Bristol in 1966 [16]. It offers a jointly-conditioned relative measure of input-output interactions within a multi-input multi-output (MIMO) system [17]. A widely-used metric, it quantifies the interaction of a control loop with other loops by comparing the steady state process gain observed by the controller when the other loops by comparing the steady state process gain observed by the controller when the other loops are inactive to the process gain when all other loops are active (with all other outputs at their setpoints). Mathematically, if the $\mathbf{i^{th}}$ output is regulated by the $\mathbf{j^{th}}$ input, its relative gain is defined as:

$$\boldsymbol{\lambda}_{\mathbf{ij}} = \frac{\left(\frac{\partial y_i}{\partial u_j}\right)_{u_k=\text{constant},k\neq j}}{\left(\frac{\partial y_i}{\partial u_j}\right)_{y_k=\text{constant},k\neq i}} \qquad \text{(II-14)}$$

Negative λ values imply inconsistent process gain, advising against certain input-output pairings. Pairings with λ close to 1 are preferred, indicating independence from other loop states.

For the nonsingular matrix G, the relative gain array is defined in [16] as follows:

$$\text{RGA} = G(s) \otimes (G(s)^{-1})^T \qquad \text{(II-15)}$$

$\otimes$: represents the Hadamard multiplication.

The corresponding RGA matrix as indicated in [18]:

$$k = \lim_{s=0} G(s) = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \qquad \text{(II-16)}$$

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} \frac{k_{11}k_{22}}{k_{11}k_{22}-k_{12}k_{21}} & \frac{-k_{12}k_{21}}{k_{11}k_{22}-k_{12}k_{21}} \\ \frac{-k_{12}k_{21}}{k_{11}k_{22}-k_{12}k_{21}} & \frac{k_{11}k_{22}}{k_{11}k_{22}-k_{12}k_{21}} \end{bmatrix} \qquad \text{(II-17)}$$

If the diagonal elements of the RGA are close to 1, then the interaction level in the process is very low. Otherwise (less than or greater than 1) the interactions are strong.

For a 2x2 matrix if $\lambda_{11}$=1, $\lambda_{22}$=1 and $\lambda_{12}$=$\lambda_{21}$=0. This is the case of partial interaction, therefore the input $u_1$can control the output$y_1$ and input $u_2$ controls the output$y_2$. If $\lambda_{12}$= 1, $y_2$ must be controlled by $u_1$ and $y_1$ by $u_2$.

If $\boldsymbol{\lambda_{ij}}$ is negative, the corresponding loop response may change direction of variation (reverse response method), if the other loops are closed. In addition, the loop itself can be unstable or the overall process becomes unstable if the loop considered will not open [16].

## II.8.4 Addressing interaction problems

To confirm that interaction is the issue, ensure each loop performs well when the other is manually controlled. If both loops function properly individually but cycle excessively together in automatic mode, the problem is likely interaction [12].

The most used methods to eliminate the effect of interactions are:

### II.8.4.1 Decoupling

Decoupling involves integrating additional intelligence between primary controllers and final control elements to create the impression of independent loops. Typically, the final control elements are valves, although lower-level flow-control loops are preferred. The traditional form is known as "forward" decoupling.

Our approach to forward decoupling is similar to the method utilized in feedforward control. The block diagram shown in figure represents a process with two inputs and two outputs involving a decoupler [10]:

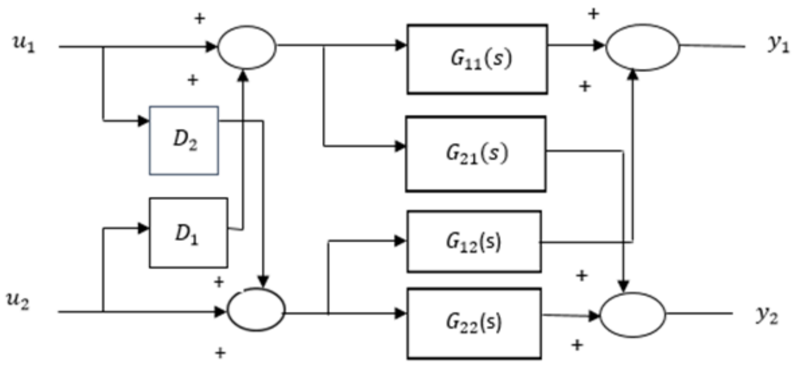



**Figure II- 14 : Block diagram of forward decoupling**

$D_1$, $D_2$ are the decoupler gains. From figure.

The corresponding transfer functions for $y_1$ and $y_2$ are:

$$y_1(s) = ( G_{11}(s)+ D_2 . G_{12} (s)) u_1(s) + ( G_{12}(s)+D_1 . G_{11}(s) )\ u_2(s)$$
$$y_2(s) = ( G_{21}(s) +D_2 . G_{22}(s) ) u_1(s) + ( G_{22}(s)+ D_1 . G_{21}(s) )\ u_2(s) \quad \text{(II-18)}$$

As indicated in [10] , to eliminate the effect of $u_2$ on $y_1$ , we choose $D_1$ according to the following equation

$$D_1 = -\frac{G_{12}(s)}{G_{11}(s)} \qquad \text{(II-19)}$$

And the effect of $u_1$ on $y_2$, we choose $D_2$ as follows:

$$D_2 = -\frac{G_{21}(s)}{G_{22}(s)} \qquad \text{(II-20)}$$

### II.8.4.2 Model Predictive Control (MPC)

It is developed by Shell Oil, it enhances control of complex processes by integrating decouplers, dead time compensation, and constraint control. MPC replaces PID controllers and adjusts all manipulated variables to meet setpoints for each controlled variable. It can control final elements directly or act as the outer loop of a cascade.

Originally, MPC enabled optimization package decisions on multivariable processes. However, if optimization decisions were directed to single-loop controllers, process interaction could lead to control problems. Process optimization is often necessary to justify installing MPC, as the cost may not be justified for simpler processes like the flow-pressure process [12].

## II.9 Conclusion

This chapter aims to elucidate the most beneficial control strategies in industries, encompassing feedback, feedforward, and cascade control. We also delve into control modes and PID algorithms, discussing tuning methods and the impact of process dynamics on controller mode choice. Finally, we explore multivariable control, highlighting the significance of addressing interactions as our primary motivation for introducing model predictive control.

.

# Chapter Ⅲ: Model predictive Control

## Ⅲ.1 Introduction

This chapter introduces MPC (model predictive control) as a type of modern controller and provides a compact overview of its essential elements. Moreover, it demonstrates the underpinning theory of predictive control along with the computational algorithm. Then we will simulate a single-input single-output system to investigate the effect of different parameters used to tune the MPC.

## Ⅲ.2 History of Predictive control

Model Predictive Control (MPC) emerged in industry to overcome complex control problems. Initially presented in practical applications, it later gained theoretical grounding from academics. Commercial software, requiring significant computing power, made MPC accessible, with early adoption in petroleum refining, particularly for catalytic cracking units. As technology improved and expertise grew, MPC spread to other industries, including batch processes, and is now widely used across the entire industrial processing sector [10].

Traditional control methods, like PID controllers, excelled at single-loop regulation but struggled with complex, interconnected industrial processes. In response, engineers turned to Predictive Control techniques specifically designed for the demands of industrial practice.

These Predictive Control techniques, like Dynamic Matrix Control (1980), Generalized Predictive Control (1987), have significantly affected industries. While some companies develop custom solutions, these commercially available options represent the forefront of modern Model Predictive Control technology [19].

- **Dynamic Matrix Control (DMC):** DMC stands out as a popular method for controlling chemical processes using Model Predictive Control (MPC). Its strength lies in its model simplicity and error minimization. DMC calculates optimal control adjustments over a time horizon, similar to solving the least-squares problem. DMC is an effective approach for the chemical processing industry [20].

- **Generalized Predictive Control (GPC)**: is one of the most popular methods of model predictive control (MPC), widely implemented in industrial applications. It effectively handles different control problems with a reasonable number of variables design. It calculates a sequence of future control signals that minimizes a defined cost function over a prediction horizon optimizes a functional cost using a receding horizon to compute the output. The model for predictions is the Controlled Autoregressive Integrated Moving Average (CARIMA), which can be efficiently identified and adjusted online by sampling input and output data. GPC controllers offer stable control for processes with time-varying parameters and delays. It requires a dynamic model, which can be developed through an identification process based on step response data [21].

**Linear Quadratic Regulator (LQR)**: LQR controller is a powerful tool for optimization performance. It relies on a full understanding of the system's state, which can be challenging to obtain in practice. To address this, high-gain observers can be used to estimate the missing state values. However, for LQR controller to function effectively, the system must be fully controllable. Meaning the ability to maneuver the system to any desired state using available controls. The controllability can be easily verified using softwares like MATLAB [22].

Although LQR controller performs well, MPC emerges as the superior choice due to its dynamic nature. This likely refers to MPC's ability to adapt to changing conditions and handle constraints, which LQR's fixed control strategy may not handle as effectively. This adaptability makes MPC more suitable for complex systems [23].

## Ⅲ.2 Model Predictive control Philosophy

From a philosophical standpoint, MPC mimics how humans choose control actions that we believe will produce the best-anticipated result (or output) within a certain period. We employ an internal model of the relevant process to arrive at this decision. We constantly revise our decisions with the help of new observations [24]

The most important components of the MPC are presented on the following section:

- **Process Model**

MPC requires a mathematical model that describes the dynamics of the system being controlled, that can be identified from experimental data. It typically includes equations that describe how the system's states evolve over time in response to control inputs and disturbances [10].

- **Prediction Horizon**

The prediction horizon specifies the number of sample values of the process output (controlled values), in the step response model into the future over which predictions are made [10].

- **Control Horizon**

The control horizon determines the number of the controller output (Manipulated variables). It is typically shorter than the prediction horizon. [10].

- **Feedback Correction**

It is done by calculating the difference between the actual control variable and the predicted one. This calculation at every sample helps with the correction for the next predicted control variable [10].

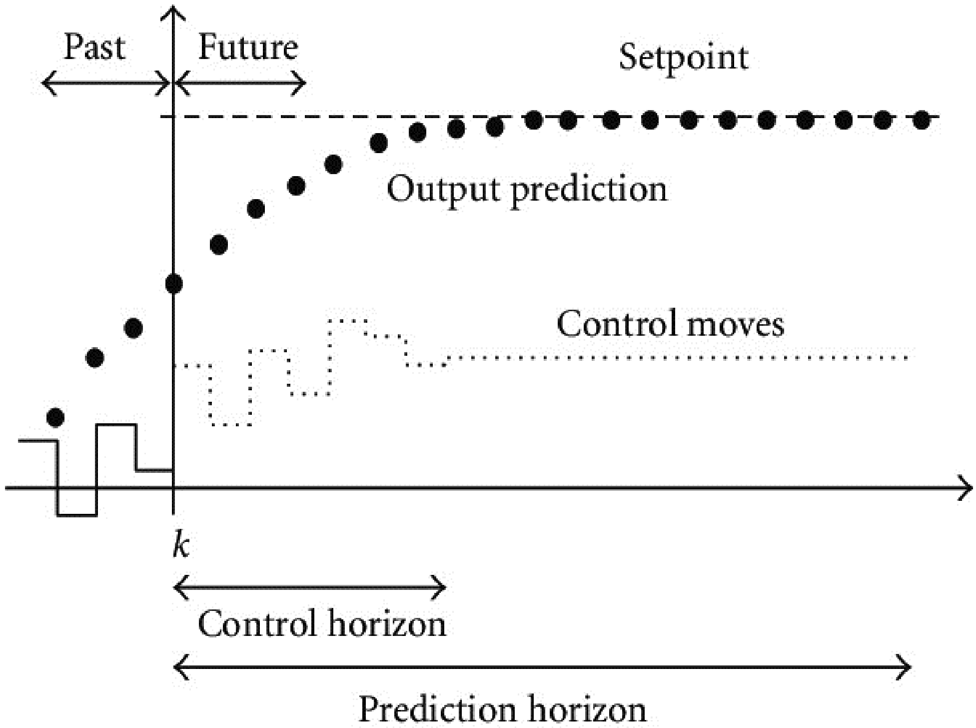



**Figure III- 1 : Model Predictive Control concept**

# Ⅲ.3 Implementation of MPC

This section shows the computational steps made by an MPC, in both constrained and unconstrained cases.

## Ⅲ.3.1 Unconstrained MPC on Single-Input Single-Output System

This section demonstrates the mathematical expressions for a single-input, single-output system (SISO) as explained in [10]:

### Ⅲ.3.1.1 Process Model

The MPC model is based on a step-response model. represented by the sequence P:

$$P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ \vdots \\ P_N \end{bmatrix} \quad (\text{III-1})$$

### III.3.1.2 Prediction

The sequence $\boldsymbol{x}^{\wedge}$ represent the current prediction of *CV* (controlled variable), for the next N sample periods.

$$\begin{bmatrix} x_1^{\wedge} \\ x_2^{\wedge} \\ x_3^{\wedge} \\ \vdots \\ x_K^{\wedge} \\ \vdots \\ x_N^{\wedge} \end{bmatrix} = \begin{bmatrix} x_0 \\ x_0 \\ x_0 \\ \vdots \\ \vdots \\ \vdots \\ x_0 \end{bmatrix} + \begin{bmatrix} P_1 & 0 & 0 & \cdots & 0 \\ P_2 & P_1 & 0 & \cdots & 0 \\ P_3 & P_2 & P_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_K & P_{K-1} & \vdots & P_2 & P_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_N & P_{N-1} & \cdots & & P_{N-K+1} \end{bmatrix} \begin{bmatrix} \Delta m_0 \\ \Delta m_1 \\ \vdots \\ \Delta m_{K-1} \end{bmatrix} \quad (\text{III-2})$$

$$x^{\wedge} = x_0 + P\Delta m \quad (\text{III-3})$$

N: Is the Prediction horizon.

K: Is the Control horizon.

Δm: Is the sequence of moves observed in the controller output (MV).

$\boldsymbol{x_0}$ : the current value of the *CV*.

If the set point is known during the prediction horizon, then the error values in the future can be predicted:

$$\mathbf{e_i^{\wedge}} = \mathbf{x_{sp,i}} - \mathbf{x_i^{\wedge}} \quad (\text{III-4})$$

$$e_0^{\wedge} = x_{sp} - x_0^{\wedge} \quad (\text{III-5})$$

By substituting equation (III-3) into (III-4):

$$e_i^{\wedge} = e_0 - P\Delta m \quad (\text{III-6})$$

### III.3.1.3 Calculating Control moves

The control moves are calculated through minimizing the cost functional *J:*

$$J = \sum_{1}^{N} (e_i^{\wedge})^2 = e^{\wedge T} e^{\wedge} \quad (\text{III-7})$$

The Substitution of (Ⅲ-6) into (Ⅲ-7) results in equation (Ⅲ-8):

$$J = e_0^T e_0 - 2e_0^T P\Delta m + \Delta m^T P^T P\Delta m \qquad (Ⅲ\text{-}6)$$

The minimization procedure is made by setting the cost functional derivative equals to zero:

$$\frac{\partial j}{\partial \Delta m} = \begin{vmatrix} \frac{\partial j}{\partial \Delta m_0} \\ \frac{\partial j}{\partial \Delta m_1} \\ \vdots \\ \vdots \\ \frac{\partial j}{\partial \Delta m_{k-1}} \end{vmatrix} = 0 \qquad (Ⅲ\text{-}7)$$

The simplification of equation (Ⅲ-9) result in equation (Ⅲ-10):

$$\Delta m = [P^T P]^{-1} P^T e_0 \qquad (Ⅲ\text{-}10)$$

Consider the KxN matrix W defined as: $W = [P^T P]^{-1} P^T$

The current control move $\Delta m_0$ to be made is:

$$\Delta m_0 = [W_1^T] e_0 \qquad (Ⅲ\text{-}8)$$

### Ⅲ.3.1.4 Incorporating Feedback

Equation (Ⅲ-9) is the difference between the measured present value $x_0$ and the predicted present value $\hat{x}_0$ :

$$\Delta x_0 = x_0 - \hat{x}_0 \qquad (Ⅲ\text{-}12)$$

Now, the entire profile, including the predicted current value, is shifted by this difference, as shown in equation (Ⅲ-13). For i=0, 1…N

$$\hat{x}_i \leftarrow \hat{x}_i + \Delta x_0 \qquad (Ⅲ\text{-}13)$$

### Ⅲ.3.1.5 Incorporating Feedforward Control

Consider the disturbance sequence D Obtained from a unit step-response model on the process.

$$D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_K \\ \vdots \\ d_N \end{bmatrix} \tag{III-10}$$

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \vdots \\ \hat{x}_k \\ \vdots \\ \hat{x}_N \end{bmatrix} = \begin{bmatrix} x_0 \\ x_0 \\ x_0 \\ \vdots \\ \vdots \\ \vdots \\ x_0 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_K \\ \vdots \\ d_N \end{bmatrix} \Delta u + \begin{bmatrix} P_1 & 0 & 0 & \cdots & 0 \\ P_2 & P_1 & 0 & \cdots & 0 \\ P_3 & P_2 & P_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_K & P_{K-1} & \vdots & P_2 & P_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_N & P_{N-1} & \cdots & & P_{N-K+1} \end{bmatrix} \begin{bmatrix} \Delta m_0 \\ \Delta m_1 \\ \vdots \\ \Delta m_{K-1} \end{bmatrix} \tag{III-11}$$

$$\hat{x} = x_0 + D\,\Delta u + P\,\Delta m \tag{III-12}$$

$$e_0 = x_{sp} - x_0 - D\Delta u \tag{III-13}$$

## III.3.1.6 Tuning MPC

- **Trajectory tuning:** is used. This method sets up a path for the system to return to a desired value (set point) over time. The speed of this return is controlled by a parameter called lambda (λ). A small value for λ will cause the controller to be aggressive; a larger value will result in a more conservative controller [10].

$$x_{sp,i} = (x_{sp,i} - x_0)\left(1 - \exp\left(-\frac{i\Delta t}{\lambda}\right)\right) \tag{III-14}$$

- **Move Suppression**: the cost functional J is augmented by the weighted sum of squares of the control moves [10].

$$J = \sum_{1}^{N} \hat{e}_i^{\,2} + \sum_{1}^{K} q_i\,\Delta m_{i-1}^2 \tag{III-15}$$

Simultaneously:

$$\Delta m = [P^T P + Q]^{-1} P^T e_0 \tag{III-16}$$

$$W = [P^T P + Q]^{-1} P^T \tag{III-17}$$

Figure III-2 summarizes the MPC algorithm:

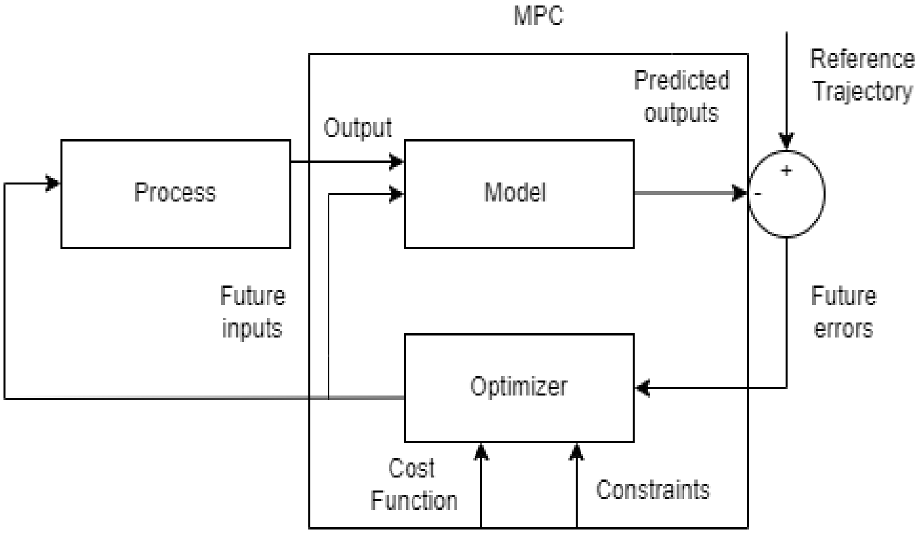



**Figure III- 2 : Model Predictive Control concept**

## III.3.2 Unconstrained MPC on MIMO System

Assuming R as number of CVs, S as number of MVs and T as number of DVs:

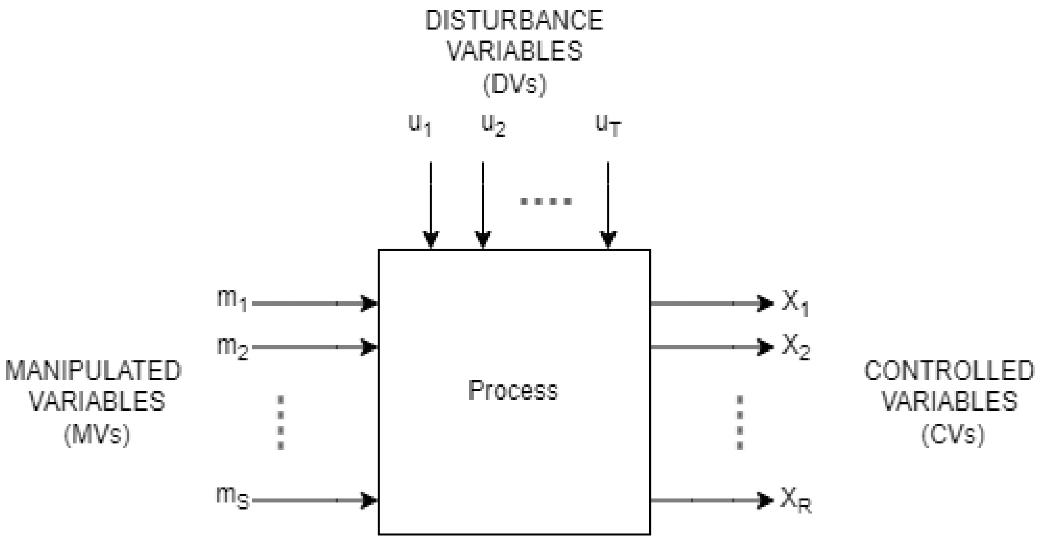



**Figure III- 3 : Multiple-Input, Multiple-Output Process**

Based on Figure III-3, from each MV to each CV there will be a step-response model (Some may be null; that is, not every MV will affect all of the CVs.) These models are designated $\boldsymbol{P_{ij}}$, where subscript “i” represents “to CV” and subscript “j” represents “from MV” [10]. Hence: For i = 1... R; and j = 1…, S

$$P_{ij} = \begin{bmatrix} P_{ij,1} \\ P_{ij,2} \\ \vdots \\ \vdots \\ P_{ij,N} \end{bmatrix} \qquad (Ⅲ\text{-}18)$$

From each DV to each CV there will be a similar step-response model (some may be null), where subscript "i" represents "to CV" and subscript "k" represents "from DV. For i = 1…, R; k = 1…, T

$$d_{ik} = \begin{bmatrix} d_{ik,1} \\ d_{ik,2} \\ \vdots \\ \vdots \\ d_{ik,N} \end{bmatrix} \qquad (Ⅲ\text{-}19)$$

The vectors representing the current values and predicted profiles of the CVs illustrated in (Ⅲ-20) and (Ⅲ-21), respectively. For i = 1…, R

$$x_{i,0} = \begin{bmatrix} x_{i,0} \\ x_{i,0} \\ \vdots \\ \vdots \\ x_{i,0} \end{bmatrix} \qquad (Ⅲ\text{-}24)$$

$$x_i^{\wedge} = \begin{bmatrix} x_{i,1}^{\wedge} \\ x_{i,2}^{\wedge} \\ \vdots \\ \vdots \\ x_{i,N}^{\wedge} \end{bmatrix} \qquad (Ⅲ\text{-}25)$$

The vectors representing future control moves in equation (Ⅲ-26). For j = 1 to S:

$$\Delta m_j = \begin{bmatrix} \Delta m_{j,1} \\ \Delta m_{j,2} \\ \vdots \\ \vdots \\ \Delta m_{j,K-1} \end{bmatrix} \qquad (Ⅲ\text{-}26)$$

The predicted profile for each of the CVs shown in equation (Ⅲ-27) for i = 1 to R:

$$x_i^{\wedge} = x_{i,0} + \sum_{k=1}^{T} d_{ik}\Delta u_k + \sum_{j=1}^{S} P_{ij}\Delta m_j \qquad (Ⅲ\text{-}27)$$

## Ⅲ.3.3 Constrained MPC

In real-world applications, constraints on process variables, manipulated variables, and auxiliary variables may be hard or soft. Physical limits define hard constraints, whereas soft constraints are determined by process design, equipment limitations, and safety considerations. If a feasible solution satisfies all constraints simultaneously, they can be treated equally. However, if no feasible solution can satisfy all constraints at the same time, it may be preferable to plan a "smooth violation" of each limit [10].

## Ⅲ.4 Rahul Shridhar and Douglas J. Cooper equations

In order to calculate the MPC controller parameters Rahul Shridhar and Douglas J. Cooper [25] suggest set of equations presented in this section. For this purpose, the process dynamics are approximated by first order plus dead time (FOPDT) model:

$$G(s) = \frac{K_p e^{-\theta_p s}}{\tau_p\, s+1}$$

- Sample time:"$T_s$" is the largest value that satisfies $T_s \leq 0.1\tau_p$ and $T_s \leq 0.5\theta_p$.

- Response horizon “N”, where $Tr$ is the rise time.

$$N = \frac{\theta_p}{Tr} + 1 \quad (Ⅲ\text{-}28)$$

.

- Prediction Horizon “P”:

$$P = int(\frac{\theta_p}{T_s}) + int\,(\frac{5\tau_p}{T_s}) + 1 \quad (Ⅲ\text{-}29)$$

- Control Horizon “M”:

$$M = int\,(\frac{\theta_p}{T_s}) + int\,(\frac{\tau_p}{T_s}) + 1 \quad (Ⅲ\text{-}30)$$

- Weighting factor: according to Iglesias [26], equation (Ⅲ-31) ensures that tuning is always smooth.

$$r = 1.631 K_p\,(\frac{\tau_p}{\theta_p})^{0.409} \quad (Ⅲ\text{-}31)$$

## Ⅲ.5 Simulation of a Single-Input Single-Output System

The simulated process is a separator having the steady state model shown on Figure Ⅲ-4 and its data illustrated in Table Ⅲ-1. Whereas this model and its data were provided at the internship at IAP-Boumerdes (Institut Algerien de Petrol).

**Table III - 1: Separator specification and mixture data**

| | |
|---|---|
| Separator | |
| Type | Vertical – Flat cylinder |
| Diameter (m) | 1.067 |
| Height (m) | 3.734 |
| Volume ($\mathbf{m^3}$) | 3.337 |
| Liquid Volume ($\mathbf{m^3}$) | 0.9501 |
| Temperature (°C) | 72.49 |
| Pressure (KPa) | 103.5 |
| Feed Flow (Kgmole/h) | 457.5 |
| Feed composition (mole fraction) | 0.199 Acetone / 0.3166 2-Propanol / 0.4844 H2O |
| Liquid composition (mole fraction) | 0.1409 Acetone / 0.3203 2-Propanol / 0.5388 H2O |
| Vapor composition (mole fraction) | 0.4132 Acetone / 0.303 2-Propanol / 0.2838 H2O |

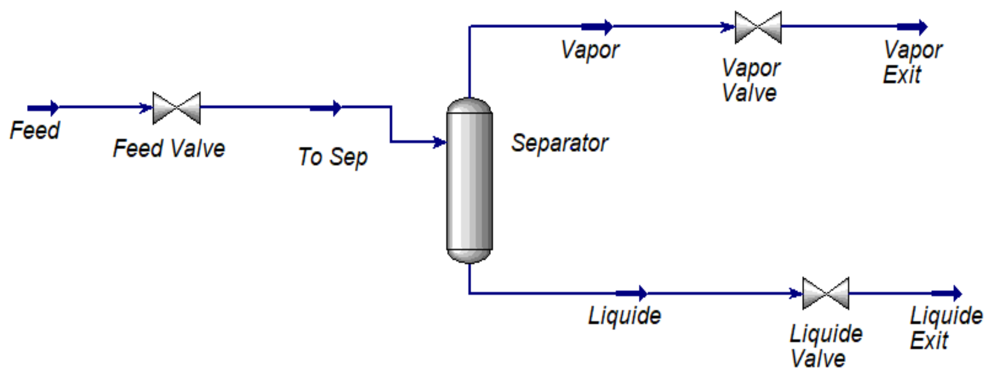



**Figure III- 4 : Steady state model of a Separator**

## Ⅲ.5.1 Applying MPC Controller

MPC controller is used to control the liquid percent level of the separator. TRF-1 Block is used for an approach of the process to the real case (3rd order system response).

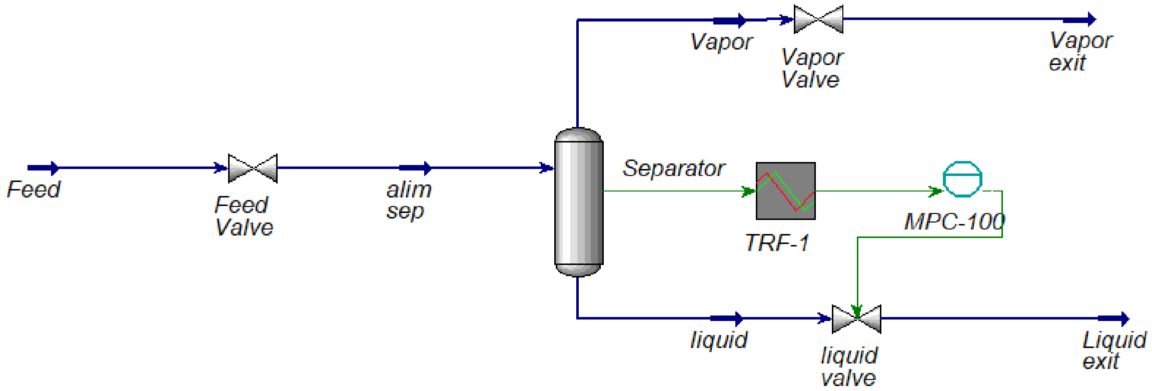



**Figure III- 5 : MPC controller applied to the Separator**

**Process model:**

The process model obtained from the step change test is shown on Table III - 2:

**Table III - 2 : Level Process Model**

| | |
|---|---|
| Process gain | -7.37 |
| Process time constant | 58 |
| Delay | 10 |

## Model mismatch:

It refers to a situation where the mathematical model used in the simulation does not accurately represent the actual system being controlled. In this case, a random model is substituted in the process model tab.

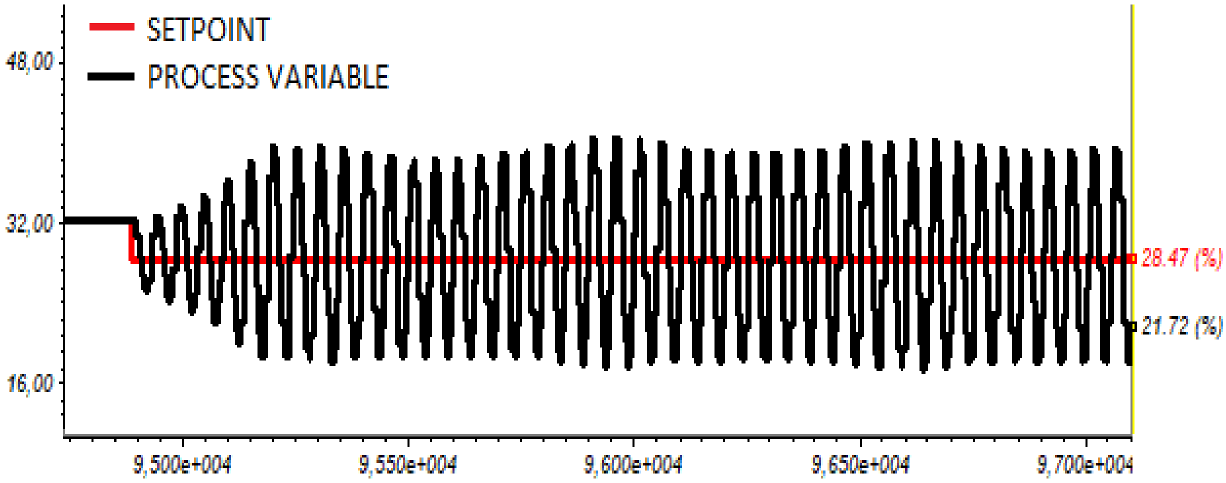



**Figure III- 6 : MPC Controller response to a model mismatch**

For a better understanding of process dynamics effect on the system, additional test is made.

The process model is given by: $G(s) = \frac{-7.37e^{-10s}}{58\,s+1}$

To study the effect of each dynamic on its own; we make changes on one at a time. The recorded results are shown on the following tables.

- **Process gain**: where $K_p$ = -7.37. The initial performance is shown in the colored column.

**Table III - 3 : The process gain analysis**

| k | $K_p$ | $\pm5\%$ | $\pm10\%$ | $\pm20\%$ | $\pm50\%$ | $\pm70\%$ | $\pm90\%$ |
|---|---|---|---|---|---|---|---|
| Disturbance rejection (%) | 1.53% | 1.54% | 1.58% | 1.58% | 1.6% | 1.63% | 1.66% |
| Oscillations | No oscillation | No oscillation | No oscillation | No oscillation | No oscillation | No oscillation | No oscillation |
| $T_r$(min) | 30 | 33 | 28 | 32 | 36 | 40 | 49 |
| $T_s$(min) | 36 | 35 | 37 | 38 | 41 | 47 | 58 |

It can be noticed from Table Ⅲ-3, that the process gain has an impact on the controller performance as the error value increases. For a $\pm10\%$ error in the process gain, an increase of 3% in settling time and a 3.27% in the disturbance rejection overshoot. While a $\pm50\%$ error gain leads to 16.7% increase in settling time and 4.57% overshoot, which is unacceptable. starting from a 50% error in the process gain value will lead to slowing down the process; that increases both the settling time and the disturbance rejection percentage overshoot.

As for $\pm90\%$ will result in a massive increase in the settling time (73.33%) and in disturbance rejection 8.5%.

- **Time constant**: where $\tau_p$ =58min. The initial performance shown in the colored column:

**Table III - 4: The process time constant analysis**

| $\tau$ | $\frac{1}{2}\tau_p$ | $\frac{3}{4}\tau_p$ | $\boldsymbol{\tau_p}$ | $2.\tau_p$ | $3.\tau_p$ | $4.\tau_p$ |
|---|---|---|---|---|---|---|
| Disturbance rejection (%) | 1.58% | 1.57% | 1.53% | 1.43% | 1.39% | 1.4% |
| Oscillations | Small Oscillations | Small Oscillations | No oscillation | Small Oscillations | More oscillation | More Oscillations |
| $T_r$(min) | 27 | 28 | 30 | 20 | 18 | 17 |
| $T_s$(min) | 32 | 32 | 36 | 25 | 22 | 22 |

It can be noticed from Table Ⅲ-4**,** that the increase and decrease of process time constant will result in oscillations. Decreasing the time constant by half results in a decrease of 11% in settling time and 3.27% increase in disturbance rejection overshoot, however, small oscillations are induced.

Increasing the time constant by 4-times the initial value will result in decreasing settling time by 38.9% and 9.2% disturbance overshoot, but massive oscillations occur leading to system instability (Limit Cycle) as shown in Figure Ⅲ-6.

The results obtained from table’) show how the accuracy of identifying a process model is crucial for an MPC controller. It is safer to state that a gain error up to 20% and no more than 10% for the time constant can be tolerated in identifying the process dynamics.

### MPC Parameters:

Calculating the parameters of the MPC according to the previous equations in section (Ⅲ.4):

**Table III - 5: The Model predictive control initial parameters**

| N | $T_s$(min) | P | M | GammaU | GammaY |
|---|---|---|---|---|---|
| 102 | 5 | 61 | 14 | 1 | 1 |

## Ⅲ.5.2 Tuning MPC:

In this section, we will study the effect of each parameter to conclude the right way to tune MPC for the desired response. Start by observing the effect of each parameter on the performance. Start changing each parameter at a time while keeping other parameters constants. The disturbance is made by opening the Feed valve by 3% (50%-53%) and record the overshooting.

While set-point change is made by changing, the controller output by 3% as well (28.47%-31.47%) and record the results. The colored row is the initial parameters performance.

1. **Prediction horizon**

**Table III - 6: The prediction horizon analysis**

| P | Disturbance rejection (%) | $T_r$(min) | $T_s$(min) |
|---|---|---|---|
| 45 | 1.6% | 29 | 35 |
| 49 | 1.6% | 30 | 35 |
| 54 | 1.57% | 31 | 37 |
| 57 | 1.57% | 28 | 36 |
| 59 | 1.55% | 30 | 36 |
| 61 | 1.53% | 30 | 36 |
| 64 | 1.55% | 30 | 35 |
| 67 | 1.56% | 30 | 36 |
| 70 | 1.56% | 33 | 37 |
| 76 | 1.6% | 32 | 36 |

From results obtained in Table Ⅲ-6, it can be observed that the prediction horizon does not have a great impact on the process performances; however, increasing its value would increase the computational steps that may affect control quality (±35.5% prediction error results in 4.5% increase in disturbance overshooting and a 2.8% rise in settling time).

### 2. Control horizon

**Table III - 7: The control horizon analysis**

| M | Disturbance rejection (%) | $T_r$(min) | $T_s$(min) |
|---|---|---|---|
| 1 | 2.54% | 150 | 169 |
| 4 | 1.6% | 30 | 50 |
| 6 | 1.61% | 28 | 37 |
| 8 | 1.61% | 27 | 37 |
| 10 | 1.62% | 28 | 37 |
| 12 | 1.61% | 29 | 35 |
| 14 | 1.53% | 30 | 36 |
| 16 | 1.62% | 28 | 37 |
| 18 | 1.61% | 31 | 39 |
| 20 | 1.61% | 30 | 39 |
| 22 | 1.61% | 30 | 37 |
| 24 | 1.62% | 31 | 35 |
| 26 | 1.61% | 34 | 40 |
| 28 | 1.62% | 35 | 42 |
| 30 | 1.62% | 33 | 42 |
| 35 | 1.62% | 35 | 44 |
| 59 | 1.61% | 36 | 49 |

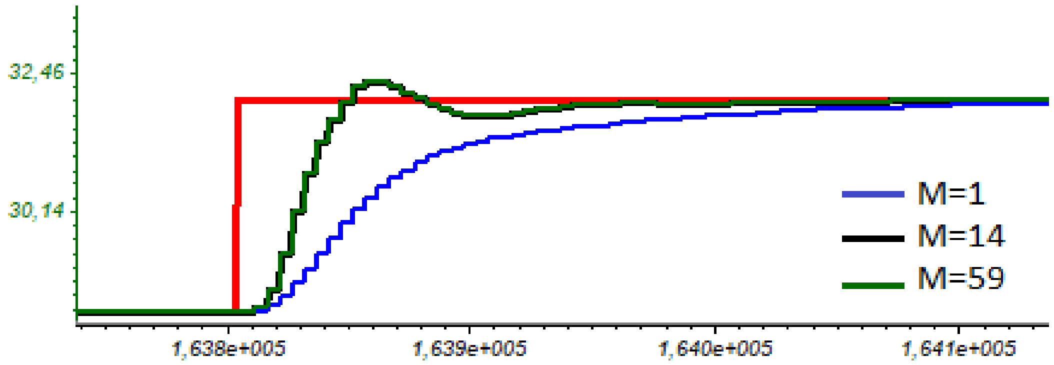



**Figure III- 7 : Responses to step change with different control horizon**

From results shown in Table Ⅲ-7, and response in Figure Ⅲ-7, it can be observed that the control horizon has a remarkable impact on the process performances. Decreasing its value would decrease the rise time. Decreasing the control horizon by 57% (8 steps) reduces the rise time by 6.7%. However, the response is more aggressive leading to less stable control actions. This explains the time gap between settling time and the rising time.

Decrease in the control horizon to the minimum value (1-2) have a massive drawback, For a value of 1, a 66% rise in disturbance overshoot and increase in settling time (about 5-times more than the initial value).

Increasing the control horizon (about 4-times greater than initial value) does not have a great impact on disturbance rejection (a 5% increase in disturbance overshoot) in comparison to reducing it, but it will also increase the process settling time by 36%, in addition to increasing computational complexity.

**3. Response horizon “N”:**

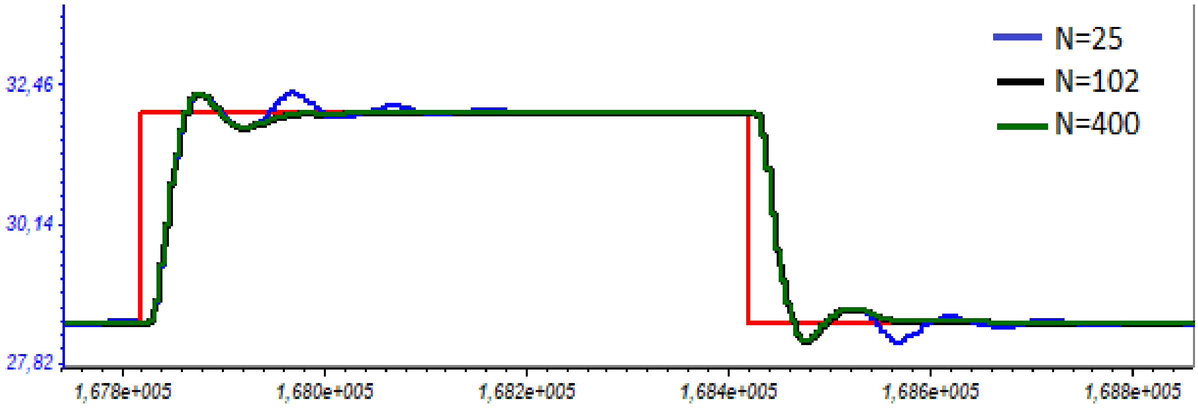



**Figure III- 8 : Responses to step change with different Response horizon**

**Table III - 8: the response horizon analysis**

| N | Disturbance rejection (%) | $T_r$(min) | $T_s$(min) |
|---|---|---|---|
| $\frac{1}{4}N$ | 1.54% Oscillations | 28 | 32 |
| $\frac{1}{2}N$ | 1.54% Oscillations | 32 | 33 |
| $\frac{3}{4}N$ | 1.6% Reduced Oscillations | 30 | 35 |
| N=102 | 1.53% No Oscillations | 30 | 36 |
| 2.N | 1.62% No Oscillations | 34 | 39 |
| 3.N | 1.61% No Oscillations | 29 | 28 |
| 4.N | 1.61% No Oscillations | 24 | 33 |

From results obtained in Table Ⅲ-8 and Figure Ⅲ-8, small values of response horizon make the system oscillates. While increasing it does not have a remarkable impact on the performance. However, increases the computational complexity.

### 4. weighting functions:

**Gamma_U**: This weighting function represents the priority given to minimizing changes in the control input. The initial performance is shown in the colored column.

**Table III - 9: The weighting function Gamma_U analysis**

| Gamma_U | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|---|
| Disturbance rejection (%) | Unstable system (Big Oscillations) | 1.38% Oscillations | 1.45% No Oscillations | 1.53% No Oscillations | 1.54% No Oscillations | 1.53% |
| $T_r$(min) | 30 | 27 | 27 | 29 | 28 | 30 |
| $T_s$(min) | 34 | 29 | 30 | 30 | 30 | 36 |

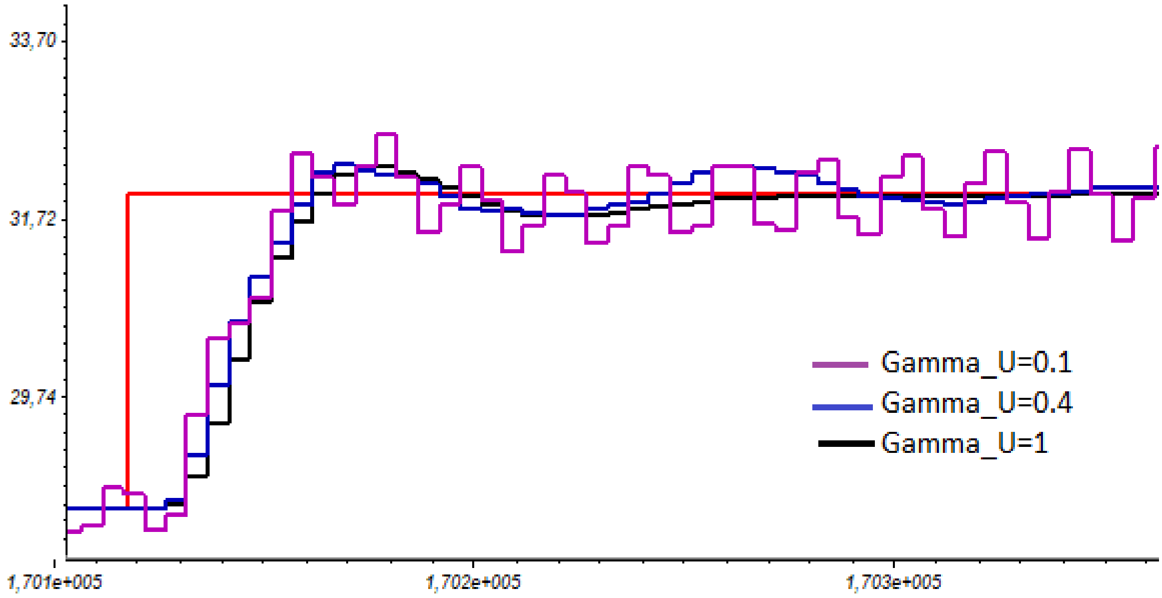



**Figure III- 9 : Responses to step change with different Gamma_U values**

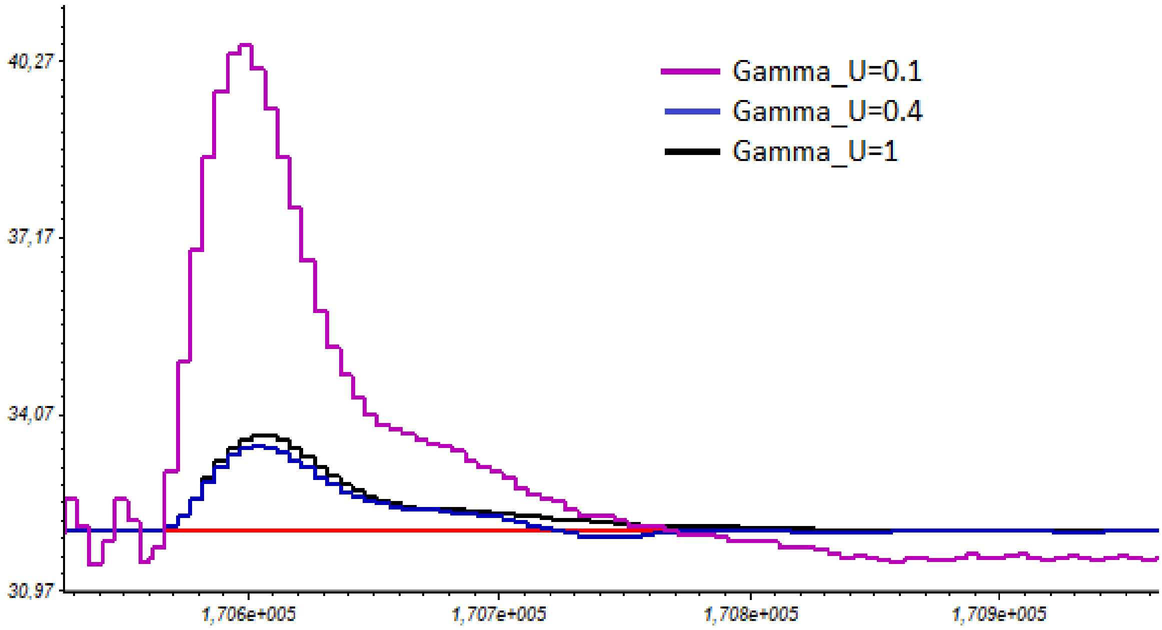



**Figure III- 10 : Responses to disturbance with different Gamma_U values**

Based on obtained results from Table Ⅲ-9**,** Figure Ⅲ-9, Figure Ⅲ-10; Smaller value for Gamma_U (0.1-0.3) make the process oscillate leading to instability (limit cycle) However, for values (0.4-0.9) leads to (5.22%-0.65%) reduction in disturbance rejection overshoot, and the settling time is decreased as well (by 16.7%) for an optimized performance.

**Gamma_Y:** This weighting function represents the priority given to tracking the desired setpoint or reference trajectory for the output. The initial performance shown in the colored column.

**Table III -10: The weighting function Gamma_Y analysis**

| Gamma_Y | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|---|
| Disturbance rejection (%) | 2.69% | 2.14% | 1.88% | 1.76% | 1.67% | 1.53% |
| $T_r$(min) | 45 | 41 | 29 | 32 | 29 | 30 |
| $T_s$(min) | 57 | 54 | 40 | 36 | 36 | 36 |

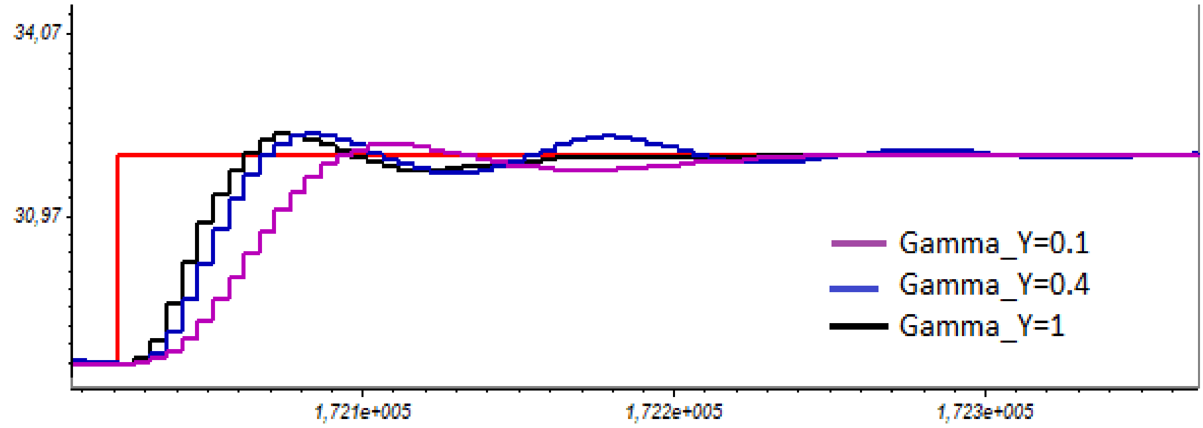



**Figure III- 11 : Responses to step change with different Gamma_Y values**

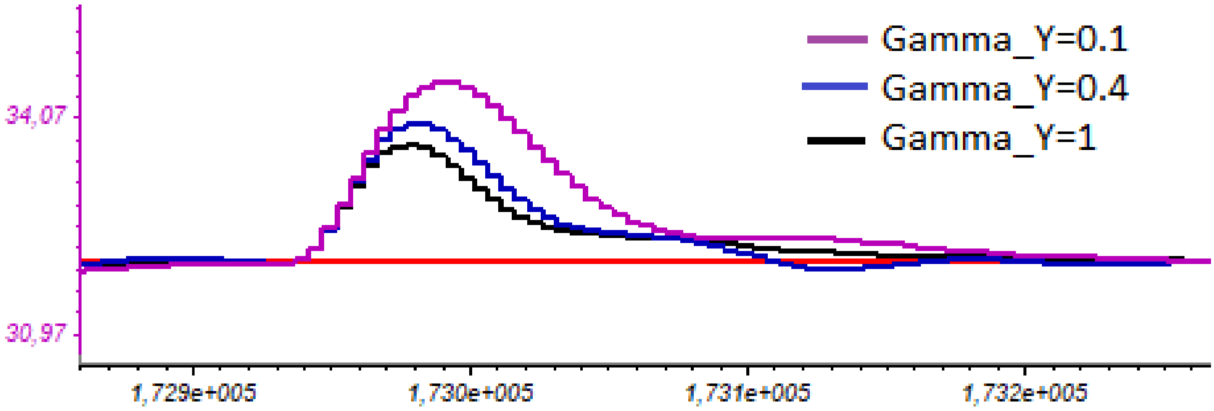



**Figure III- 12 : Responses to disturbance with different Gamma_Y values**

According to results from Table Ⅲ-10, Figure Ⅲ-11, Figure Ⅲ-12, smaller value for Gamma_Y leads to slowing down the system and increasing the disturbance rejection. For values (0.1-0.3), a massive drawback can be noticed; (24.2%-75.2%) increase in disturbance rejection and (58.3%-11%) rise in settling time.

## Final Parameters

The MPC parameters calculated at first are not necessarily the optimal ones, in order to determine it; a several trial-and-error are made based on the results obtained from the previous analysis. Tuning the MPC, starts off by the control and prediction horizon tuning; in attempt to reduce the settling time and disturbance rejection overshoot.

Once this step is done, we move to optimizing the system through the weighting functions tuning (tuning Gamma_U and Gamma_Y).

Final parameters are shown on Table III - 11:

**Table III - 11: The Model predictive control tuned parameters**

| N | $T_s$(min) | P | M | GammaU | GammaY |
|---|---|---|---|---|---|
| 115 | 5 | 60 | 14 | 0.4 | 1 |

## Ground Rules:

After the study, the tuning of each parameter on its own, and simulation of the separator process, the following steps are recommended when tuning MPC:

- **Process Model**: the accuracy of the process model is crucial for a good control. Considering the operating region while identifying the system is very important, the dynamics of a system may change when changing the operating conditions. In other words, the dynamics of the system at the top of the distillation column are not the same as at the bottom.
  No more than 20% for gain error 6% for the time constant are accepted, that is why several tests for process identification are required; also called "MODEL TUNING".
- **Response horizon N**: its value must be large enough such that the last step response model parameter is equal to the steady state gain.
- **Prediction Horizon:** Must be at a range where the control horizon is always less than the prediction horizon. However, it should not be too large; attention is required by the time the prediction value is 10-12 steps greater than the initial one.
- **Control horizon:** should choose a value that guarantees a smooth response with small disturbance rejection overshoot, it is about 1/4 to 1/2 of the prediction horizon.
- **Weighting Functions**: A value of one implies that objective of the function, is as important as other objectives in the optimization problem. A value closer to zero would mean that the objective is less important relative to other objectives. Gamma_U & Gamma_Y need to be selected conversely; increasing the value of Gamma_U implies decreasing the value of Gamma_Y and vise-versa, according to the desired performance.

## III.6 Conclusion

This chapter has discussed the MPC theory and philosophy, along with a simulation of a distillation column that shows the MPC tuning procedure that needs to be followed when dealing with a Single-Input Single-Output system.

Next chapter will show a detailed application of MPC on a binary distillation column along a comparison with some traditional controllers.

# Chapter IV: Results and Discussion

## IV.1 Introduction

In this chapter, a study is made on a 2x2 process with loop interactions and disturbance rejection problems, using different controllers in attempt for an optimal performance.
First, an overview of the software used for the simulation, the studied process, and its data.
Next; a deep study on performance of each controller, and finally we will dive into the new method of control (MPC) by comparing its results with the ones from the conventional methods; a rule of thumb is presented for the process controlled by the end of the chapter.

## IV.2 Simplifications in Depropanizer Processes

In depropanizer operations, although the initial feed contains a complex mixture of components (such as N2, CO2, C1, C2, C3, i-C4, n-C4, i-C5, n-C5, C6, and H2O), the process can be simplified by focusing on the two key components: propane (C3) and iso-butane (i-C4). This simplification involves preliminary filtering stages to remove non-essential components and impurities, allowing the system to be approximated as a binary mixture. Assumptions such as constant relative volatility and ideal gas behavior are made to further streamline the analysis. The primary goal of these simplifications is to concentrate on the control aspects of the depropanizer, ensuring efficient separation and product purity.

## IV.3 Steady State Model

The column configuration chosen for the simulation (using Aspen Hysys) is a depropanizer that consists of a total condenser, 30 stages. The column pressure was 1389 Kpa and the column had two feed streams: 40 % propane and 60% i-butane are fed at tray 14 at a 48°C temperature. At standard operating points, the temperature at tray 5 and tray 25 was 45°C and 70°C, respectively. The reflux ratio was set to 3.095 and the distillate flow rate 0.4 kg mole.

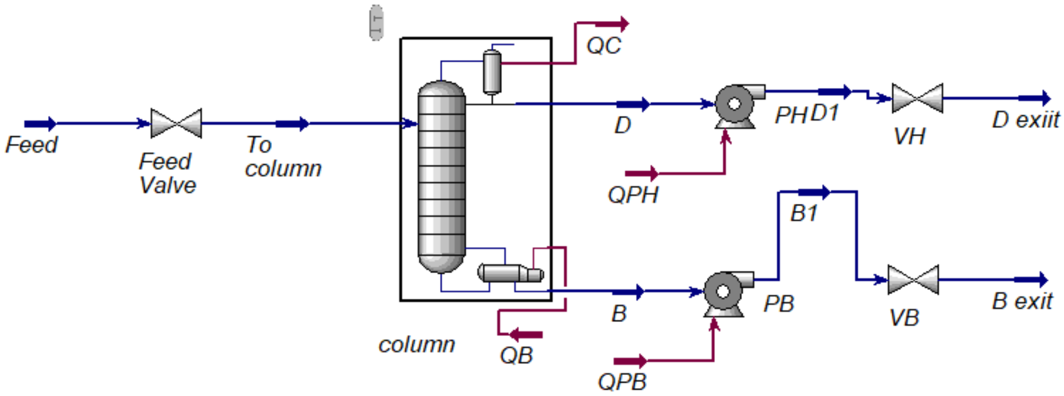



**Figure IV - 1 : Steady-state model of depropanizer distillation column**

## IV.4 Dynamic Model

Before transitioning to dynamic mode, the dynamic characteristics of the equipment must be specified. This process uses the nominal operating conditions from the steady-state regime as a starting point. Additionally, PID controllers are added to identify the system's transfer matrix. For the dynamic model, the temperatures at tray 5 and tray 25 are considered as outputs, while the reflux ratio and reboiler heat duty are considered as inputs. Whereas this model and its data were provided at the internship at IAP-Boumerdes (Institut Algerien de Petrol).

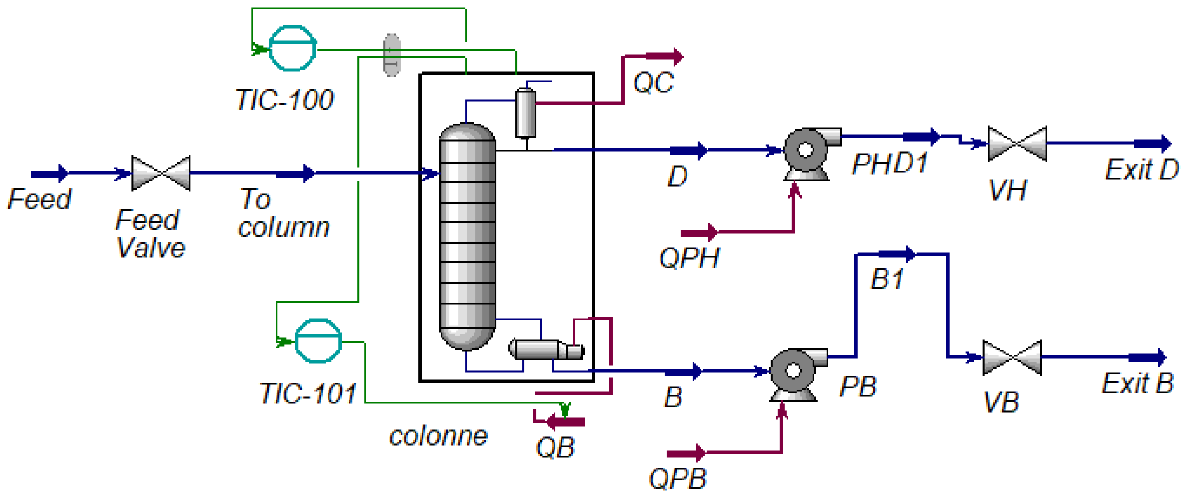



**Figure IV - 2 : PID-controlled distillation column system**

By applying a 3% step change to both inputs and observing the system's response. Using the Broida method, we obtained the following results:

$$\begin{bmatrix} T_5 \\ T_{25} \end{bmatrix} = \begin{bmatrix} \dfrac{-2.18}{1+127s} & \dfrac{3.12}{1+193s} \\ \dfrac{-2.17}{1+95s} & \dfrac{3.25}{1+111s} \end{bmatrix} \begin{bmatrix} L \\ Q \end{bmatrix}$$

Where:

$T_5$ : is the temperature at tray 5

$T_{25}$ : is the temperature at tray 25

L: is the reflux flow rate

Q: is the reboiler heat duty

## IV.4.1 Design of PID Controllers

The PID controllers are designed according to the following tables:

**Table IV - 1 : TIC-100 controller settings**

| | |
|---|---|
| **Connections** | |
| Name | TIC-100 |
| Process variable | Main Tower/ Stage Temperature 5 |
| Output target | Reflux/ control valve |
| **Configuration** | |
| Action | Direct |
| PV minimum | 0 |
| PV maximum | 100 |

**Table IV - 2 : TIC-101 controller settings**

| | |
|---|---|
| **Connections** | |
| Name | TIC-101 |
| Process variable | Main Tower/ Stage Temperature 25 |
| Output target | QB / control valve |
| **Configuration** | |
| Action | Reverse |
| PV minimum | 30 |
| PV maximum | 130 |

## IV.4.2 Applying PI Controller

Initially PI parameters obtained using the methods shown in section II-5

**Table IV - 3 : PI controller initial parameters**

| PI controller | $K_C$ | $T_i$(min) |
|---|---|---|
| TIC-100 | 0.45 | 127 |
| TIC-101 | 0.3 | 111 |

### Setpoint change on TIC 100

An increase in the setpoint of TIC-100 controller from 45°C to 50°C will cause the condenser reflux valve to close. This, in turn, will lead to an increase in the temperature of tray 5 to the operator's desired setpoint value as shown in Figure IV-3.

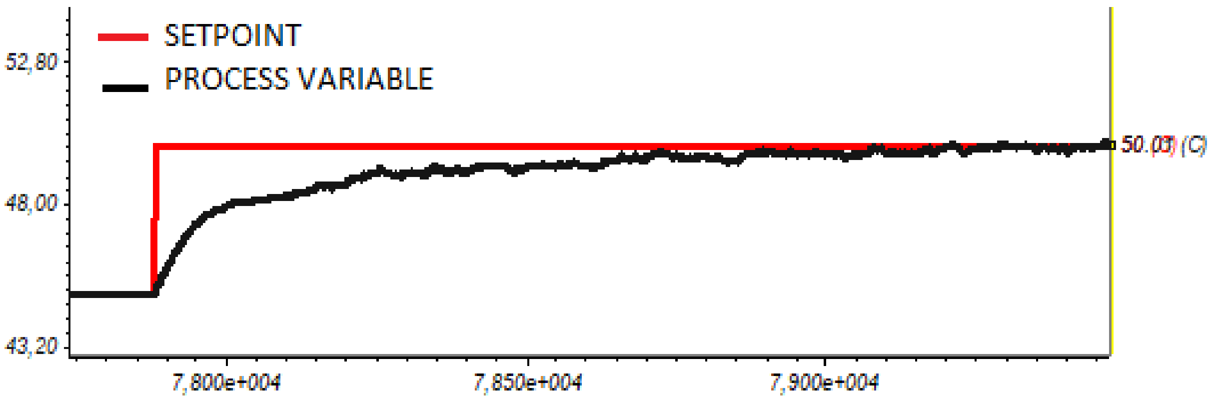



**Figure IV - 3 : Setpoint change in TIC-100 with initial PI parameters**

This change in temperature at TIC-100 affects the temperature of the tray 25 with an overshoot of 4% in the response as shown in the Figure IV-4.

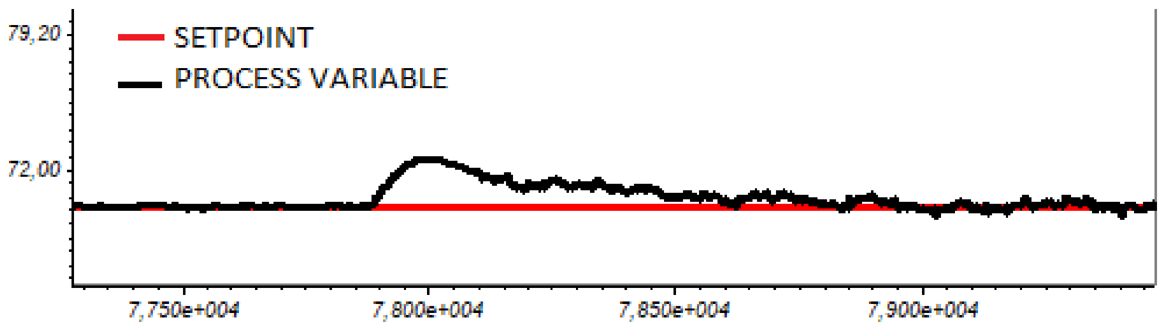



**Figure IV - 4 : TIC-101 response to a setpoint change in TIC-100 with initial PI parameters**

## Setpoint change on TIC 101

A setpoint change of TIC-101 controller from 70°C to 75°C causes the opening of the reboiler valve, thus the temperature at tray 25 will increase up to the operator required setpoint value as shown in IV-5.

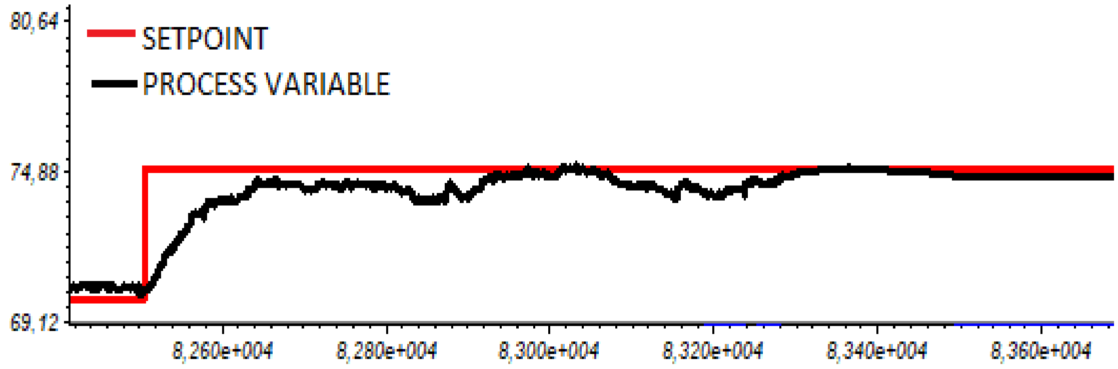



**Figure IV - 5 : Setpoint change in TIC-101 with initial PI parameters**

This change in temperature at tray 25 affects the temperature of the tray 5 with an overshoot of 4.9% in the response as shown in the Figure IV-6.

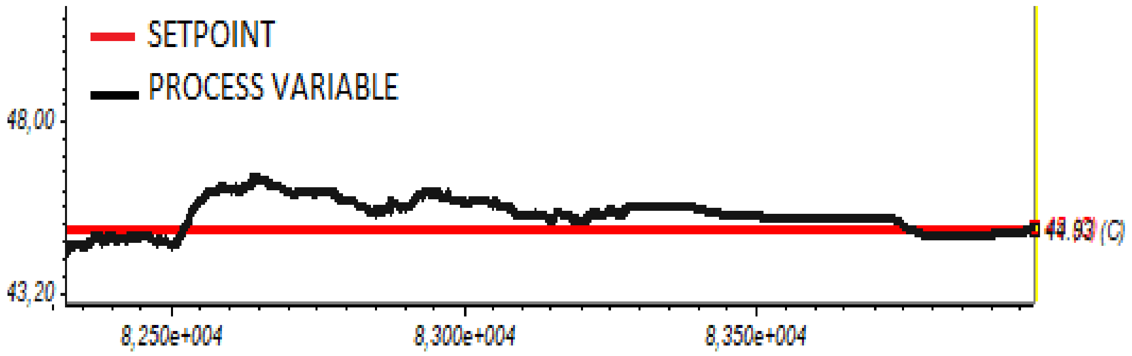



**Figure IV - 6 : TIC-100 response to a setpoint change in TIC-101 with initial PI parameters**

The obtained results are summarized in Table IV-4:

**Table IV - 4: Performance analysis of the controllers with PI initial parameters**

| PI Controller | $T_s$(min) | $T_r$(min) | Disturbance rejection | SP change in TIC-100 | SP change In TIC-101 |
|---|---|---|---|---|---|
| TIC-100 | 783 | 585 | 6.31% | No overshoot | 4.9% |
| TIC-101 | 931 | 634 | 4.18% | 4% | No overshoot |

Temperature in tray-5 takes a long time to settle (783 min), for minimization purpose, we increase the controller gain.

As for TIC-101 controller, oscillations can be noticed on the response curve, a decrease in the integral time can eliminate it. Table IV-5 is proposed for a better performance.

**Table IV - 5: PI Controller modified parameters**

| PI controller | $K_C$ | $T_i$(min) |
|---|---|---|
| TIC-100 | 0.55 | 127 |
| TIC-101 | 0.3 | 55 |

## Setpoint change on TIC-100

Change in TIC-100 result in the response shown in Figure IV-7:

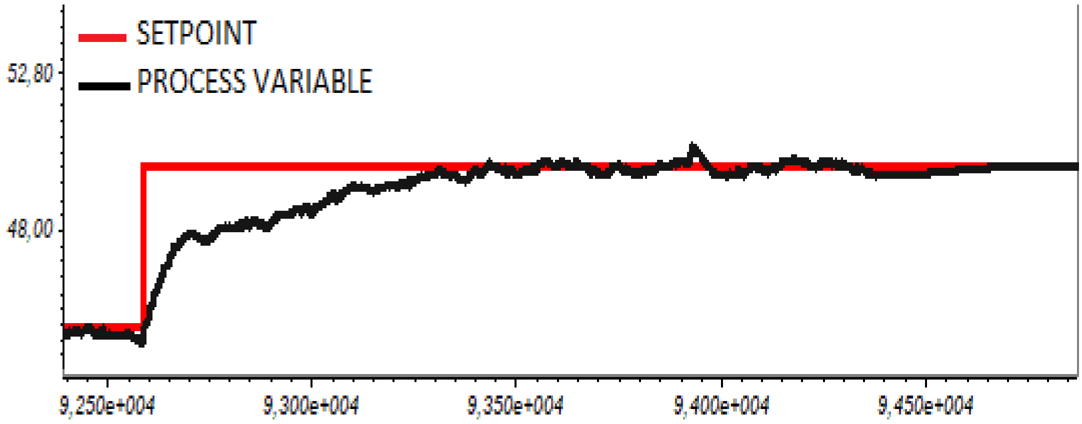



**Figure IV - 7 : Setpoint change in TIC-100 with Table IV-4 parameters**

This change in temperature at TIC-100 affects the temperature of the tray 25 with an overshoot of 2.97°C (4.2%) in the response as shown in the Figure IV-8:

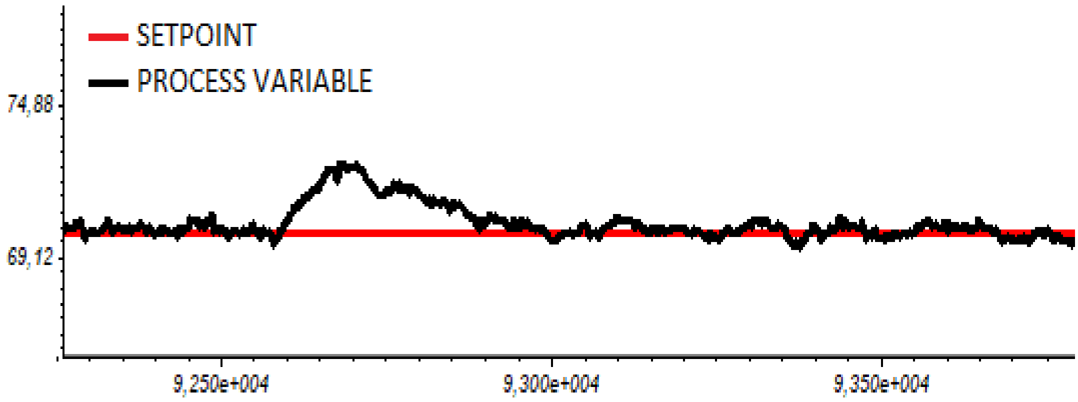



**Figure IV - 8 : TIC-101 response to a setpoint change in TIC-100 with Table IV-4 parameters**

## Setpoint change on TIC-101

Setpoint change in TIC-101 result in the response shown in Figure IV-9:

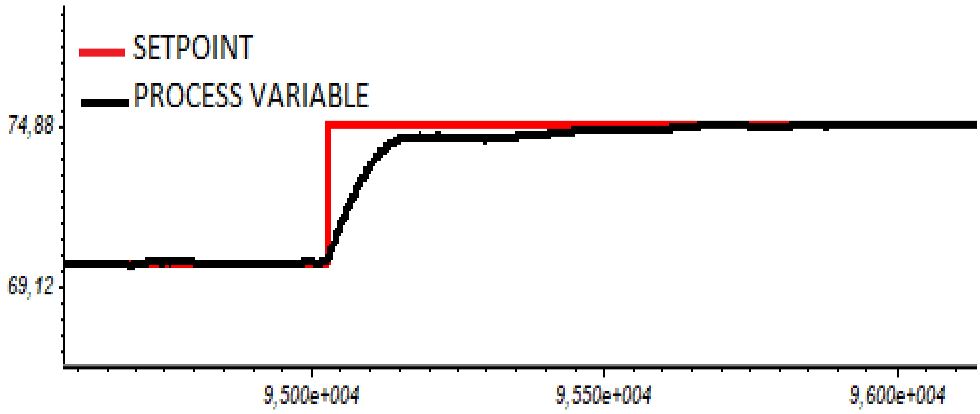



**Figure IV - 9 : Setpoint change in TIC-101 with Table Ⅳ-4 parameters**

It affects TIC-100 by a 1.94°C (4.3%) response overshoot, as shown in Figure Ⅳ-10:

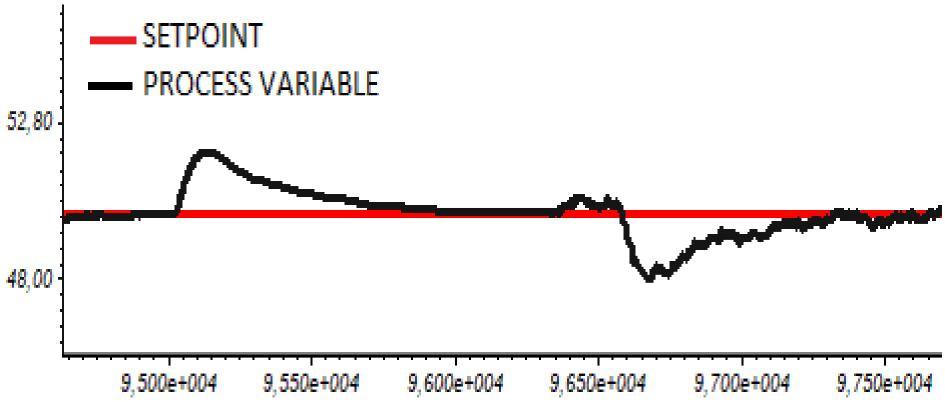



**Figure IV - 10 :TIC-100 response to a setpoint change in TIC-101 with Table Ⅳ-4 parameters**

The settling time in tray-5 temperature has decreased by 6.7%. As for TIC-101 controller; oscillations are still noticed on the response curve. A further tuning is required.

After a continues error trial tests, the obtained parameters are illustrated in Table Ⅳ-6:

**Table IV - 6: PI Controller Tuning Parameters**

| PI controller | $K_C$ | $T_i$(min) |
|---|---|---|
| TIC-100 | 0.56 | 45.7 |
| TIC-101 | 0.5 | 40 |

In order to test the performance of the PID parameters shown in Table Ⅳ-6. A Setpoint change and disturbance rejection tests are made on both controllers.

## Setpoint change in TIC-100

An increase in the setpoint of TIC-100 controller from 45°C to 50°C will cause the condenser reflux valve to close. This, in turn, will lead to an increase in the temperature of tray 5 to the operator's desired setpoint value as shown in Figure Ⅳ-11.

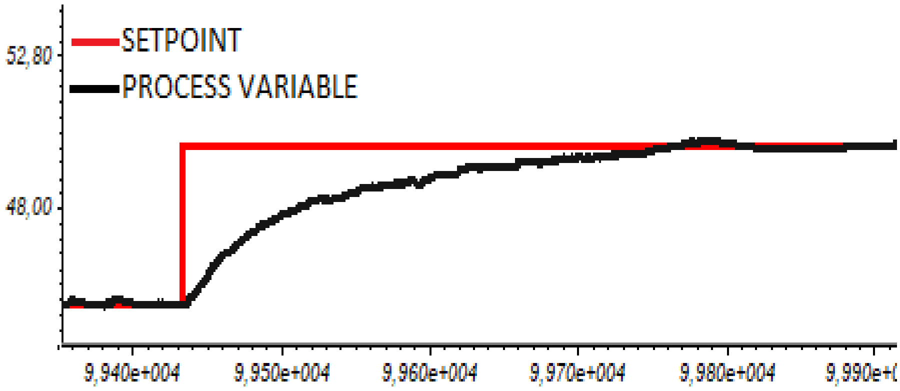



**Figure IV - 11 : Setpoint change in TIC-100 with PI tuned parameters**

This change in temperature at tray 5 affects the temperature of the tray 25; with an overshoot of 2.71°C (3.87%) in the response as shown in the Figure Ⅳ-12.

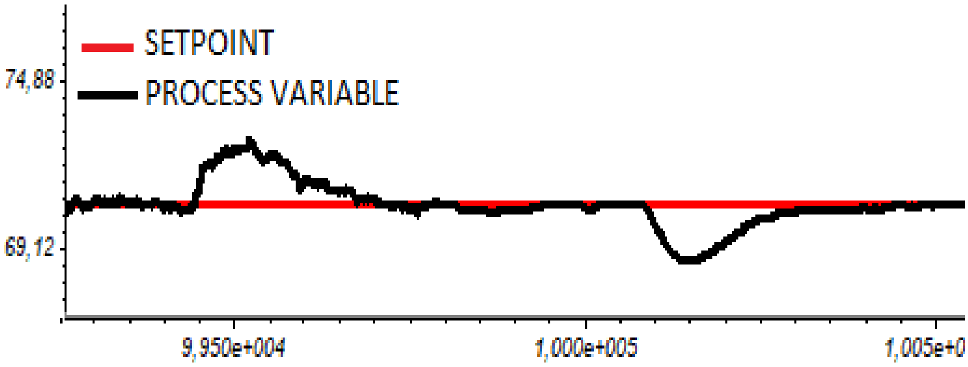



**Figure IV - 12 : TIC-101 response to a setpoint change in TIC-100 with PI tuned parameters**

## Setpoint change on TIC-101

An increase in the setpoint of TIC-101 controller from 70°C to 75°C causes the opening of the reboiler valve, thus the temperature at tray 25 will increase up to the operator required setpoint value as shown in Ⅳ-13.

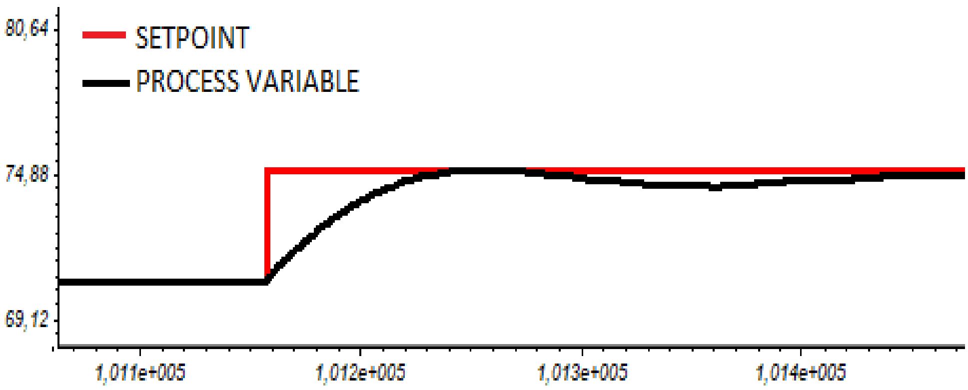



**Figure IV - 13 : Setpoint change in TIC-101 with PI tuned parameters**

This change in temperature at tray 25 affects the temperature of the tray 5; with an overshoot of 2.06 °C (4.58%) in the response as shown in the Figure IV-14.

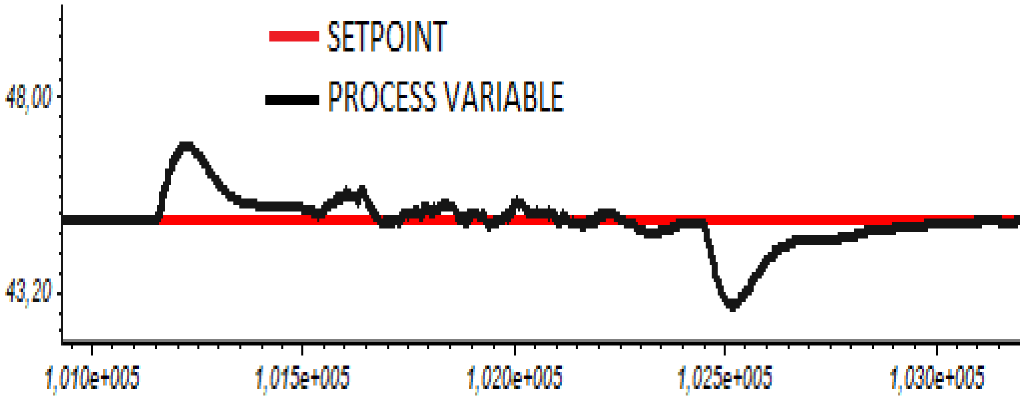



**Figure IV - 14 : TIC-100 response to setpoint change in TIC-101 with PI tuned parameters**

## Disturbance rejection

An increase in the feed valve opening from 50% to 60% will result in a change in temperature on both tray 5 and 25, causing an overshooting of 2.7°C (6%) for TIC-100 and 2°C (2.86%) for TIC-101 as shown in Figure IV-15 and Figure IV-16, respectively.

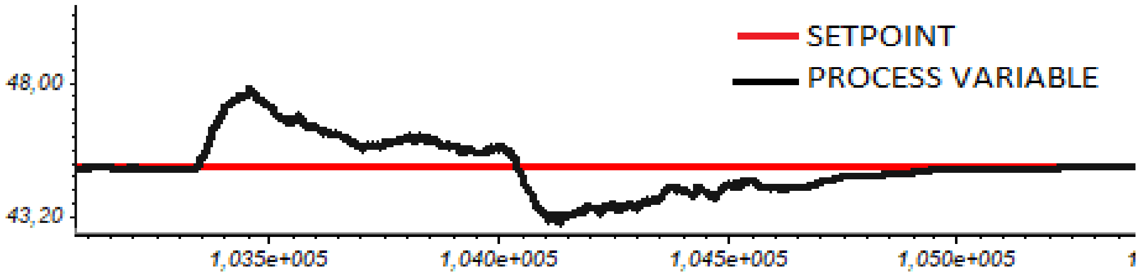



**Figure IV - 15 : TIC-100 response to a 10% disturbance with PI tuned parameters**

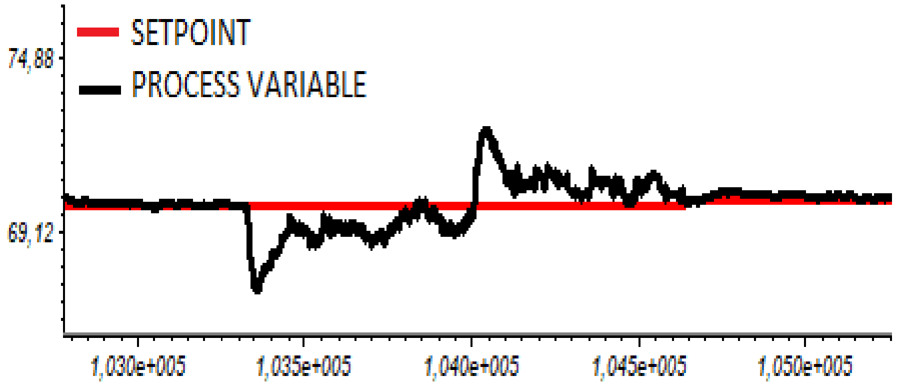



**Figure IV - 16 : TIC-101 response to a 10% disturbance with PI tuned parameters**

The performance of the two controllers is shown in Table IV-7:

**Table IV - 7: Performance analysis of the controllers with PI tuned parameters**

| PI Controller | $T_s$(min) | $T_r$(min) | Disturbance rejection | SP change in TIC-100 | SP change In TIC-101 |
|---|---|---|---|---|---|
| TIC-100 | 327 | 254 | 6% | No overshoot | 4.58% |
| TIC-101 | 236 | 150 | 2.86% | 3.87% | No overshoot |

According to the results obtained, The PI controller demonstrates a faster rise time compared to the system operating without any control, even though the rise time remains relatively large. This indicates that the PI controller improved the response speed.

It was observed that temperature fluctuations at tray 5 directly influenced the temperature at tray 25, and conversely, changes in temperature at tray 25 affected the temperature at tray 5. This bidirectional interaction is problematic because the chemical process is highly sensitive to temperature variations, which will affect the rate of temperature change, potentially compromising the quality of the distillate.

The disturbance rejection test caused an overshooting of (6%) and (2.86%) for TIC-100 and TIC-101, respectively. This is due to the increased feed flow having a temperature of 48°C that causes a rise of 2.7°C in tray 5 temperature and decrease of 2°C in tray25.

Although the PI parameters were tuned, the interactions and disturbance problems were not solved yet. The derivative action will be added next, in attempt to minimize the set of problems stated.

## IV.4.3 Applying PID Controller

The PID parameters are illustrated in Table IV-8:

**Table IV - 8: PID Controller Parameters**

| PID controller | $K_C$ | $T_i$(min) | $T_d$(min) |
|---|---|---|---|
| TIC-100 | 0.56 | 45.7 | 5 |
| TIC-101 | 0.5 | 40 | 4 |

### Setpoint change in TIC-100

An increase in the setpoint of TIC-100 controller from 45°C to 50°C will cause the condenser reflux valve to close. The response is shown in Figure IV-17.

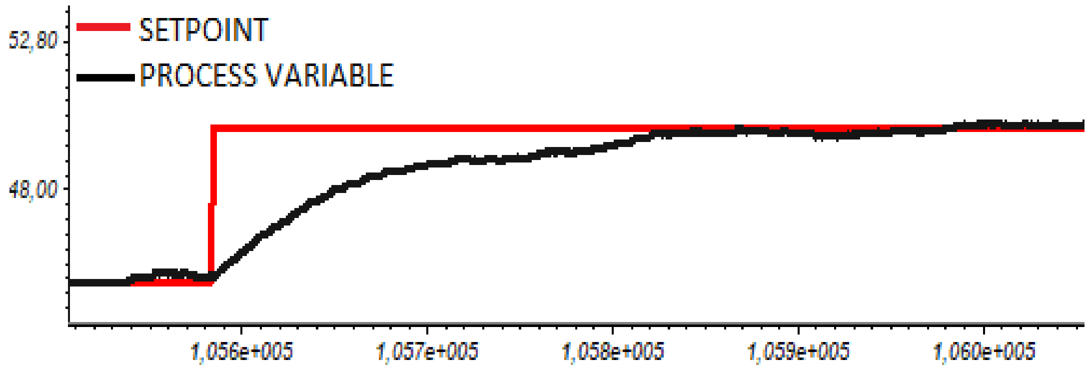



**Figure IV - 17 : Setpoint change in TIC-100 with PID parameters**

This change in temperature at tray 25 affects the temperature of the tray 5; with an overshoot of 2.22 °C (3.17%) in the response as shown in the Figure IV-18.

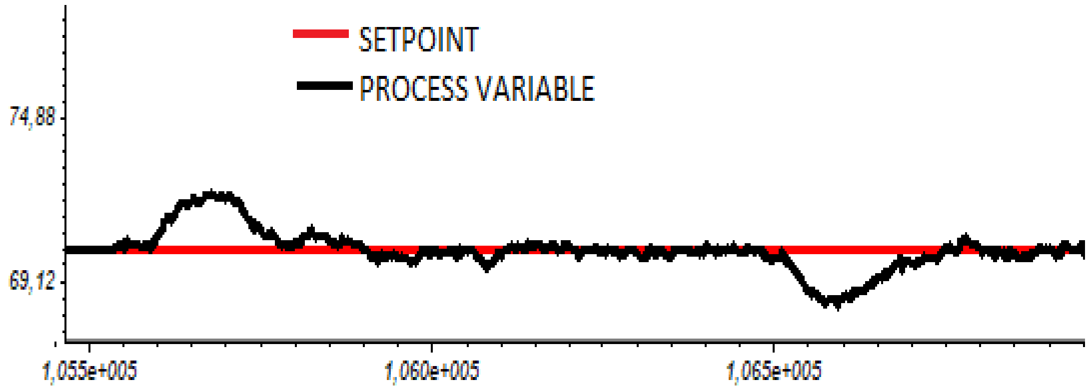



**Figure IV - 18 : TIC-101 response to a setpoint change in TIC-100 with PID parameters**

## Setpoint change on TIC-101

An increase in the setpoint of TIC-101 controller from 70°C to 75°C causes the opening of the reboiler valve, thus the temperature at tray 25 will increase up to the operator required setpoint value as shown in IV-19.

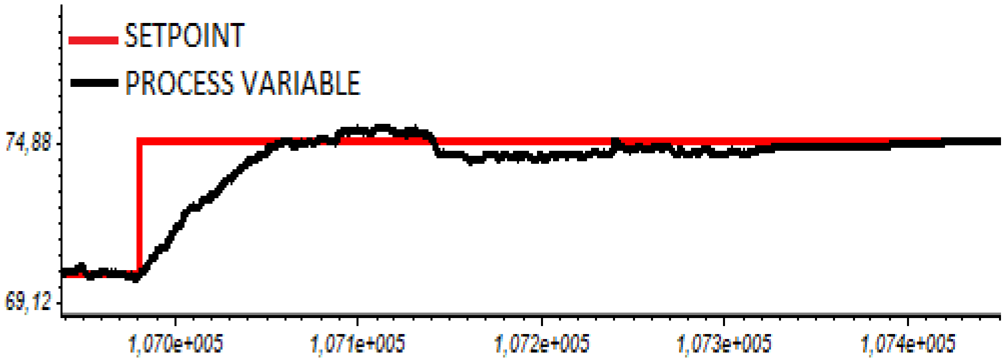



**Figure IV - 19 : Setpoint change in TIC-101 with PID parameters**

This change in temperature at tray 25 affects the temperature of the tray 5; with an overshoot of 2°C (4.4%) in the response as shown in the Figure Ⅳ-20.

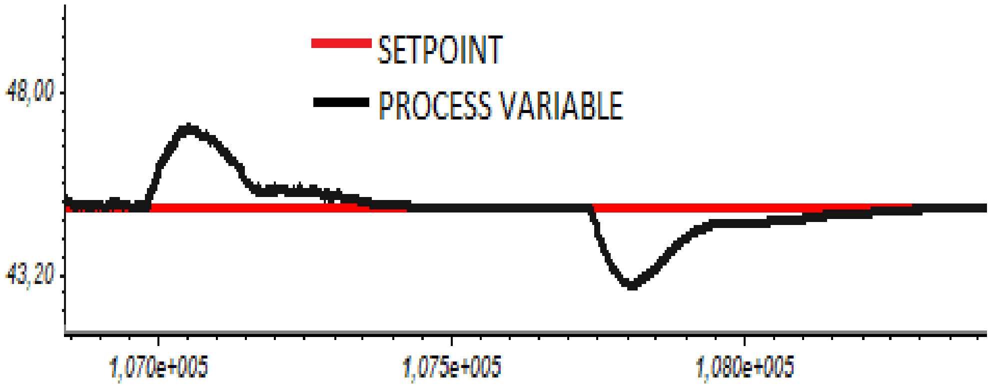



**Figure IV - 20 : TIC-100 response to setpoint change in TIC-101 with PID parameters**

## Disturbance rejection

An increase in the feed valve opening from 50% to 60% will result in a change in temperature on both trays 5 and 25, causing an overshooting of 2.46°C (5.5%) for TIC-100 and 2.36°C (3.37%) for TIC-101 as shown in Figure Ⅳ-21 and Figure Ⅳ-22, respectively.

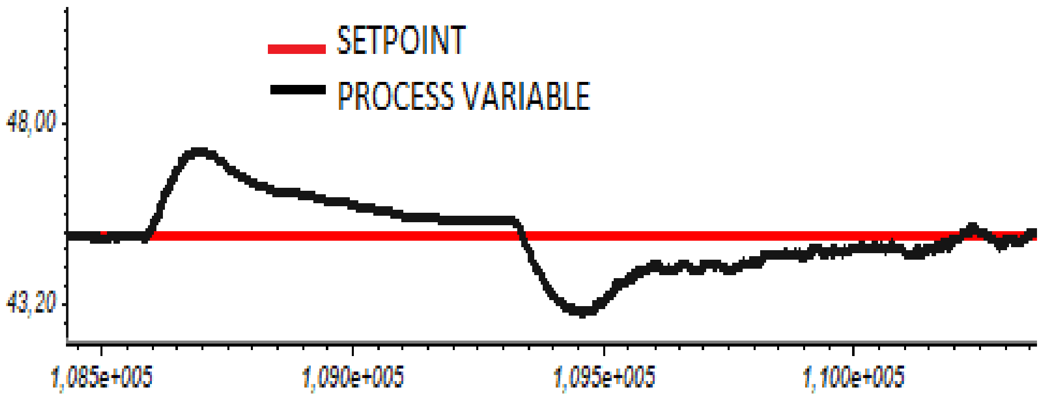



**Figure IV - 21 : TIC-100 response to a 10% disturbance with PID parameters**

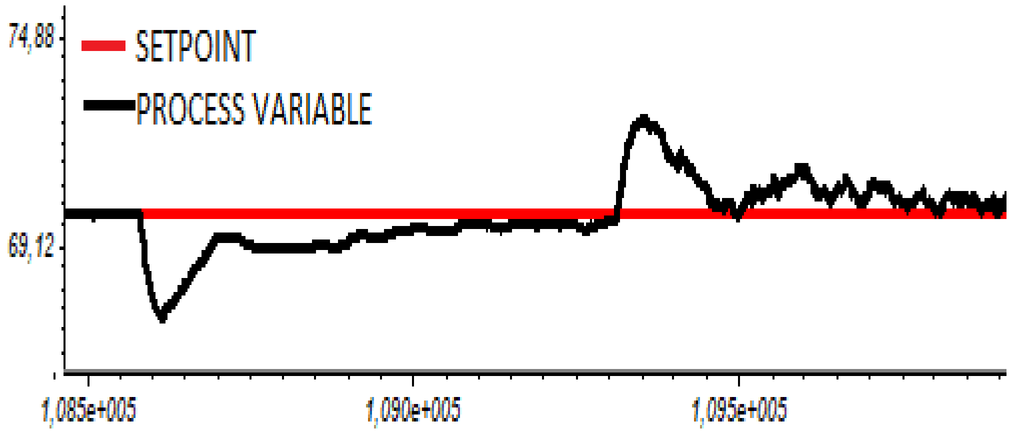



**Figure IV - 22 : TIC-101 response to a 10% disturbance with PID parameters**

The performance of the two controllers is shown in Table IV-9:

**Table IV - 9: Performance analysis of the controllers with PID parameters**

| PID Controller | $T_s$(min) | $T_r$(min) | Disturbance rejection | SP change in TIC-100 | SP change In TIC-101 |
|---|---|---|---|---|---|
| TIC-100 | 221 | 183 | 5.5% | No overshoot | 4.4% |
| TIC-101 | 235 | 148 | 3.37% | 3.17% | 0.64% |

According to the results obtained in Table IV-9, the settling time is decreased by 32.42% for TIC-100.

It was observed that temperature fluctuations at tray 5 influenced the temperature at tray 25 by 0.7% less than PI controller. As for the changes in temperature at tray 25 temperature, it caused an overshooting of 0.64% at TIC-101, but reduced its impact on tray 5 by 0.14%.

The derivative action did not have a great impact on the disturbance rejection; whereas it can be noticed a 0.51% increase in TIC-101, and a 0.5% decrease on TIC-100.Overall, it can be stated that the derivative action helped in slightly smoothening the response of the controllers, but it did not solve interaction & disturbance rejection problems.

The forward decouplers will be used next in attempt to remove the already stated problems.

## IV.4.4 Adding Decouplers

We have observed that interaction occurred between two control loops. The degree of interaction can be evaluated by calculating the Relative Gain Array (RGA) matrix; by applying Equation (II-14), we get the gain matrix:

$$k = \begin{bmatrix} -2.18 & 3.12 \\ -2.17 & 3.25 \end{bmatrix}$$

The corresponding RGA by applying Equation (II-15) is:

$$\Lambda = RGA = \begin{bmatrix} 22.5207 & -21.5207 \\ -21.5207 & 22.5207 \end{bmatrix}$$

This matrix shows that the interaction is very strong, thus the use of decouplers is essential to eliminate or to reduce the effect of loop interaction. Decoupler elements are calculated as follows:

$$D_{C1} = -\frac{G_{12}}{G_{11}} = 1.05$$

$$D_{C2} = -\frac{G_{21}}{G_{22}} = 0.67$$

Implementing the forward decouplers and testing the performances again.

### Setpoint change in TIC-100

By applying a setpoint change on the TIC-100 controller from 45°C to 50°C, the reflux valve close thus the temperature of tray 5 will increase up to the value required. The response is shown in Figure IV-23:

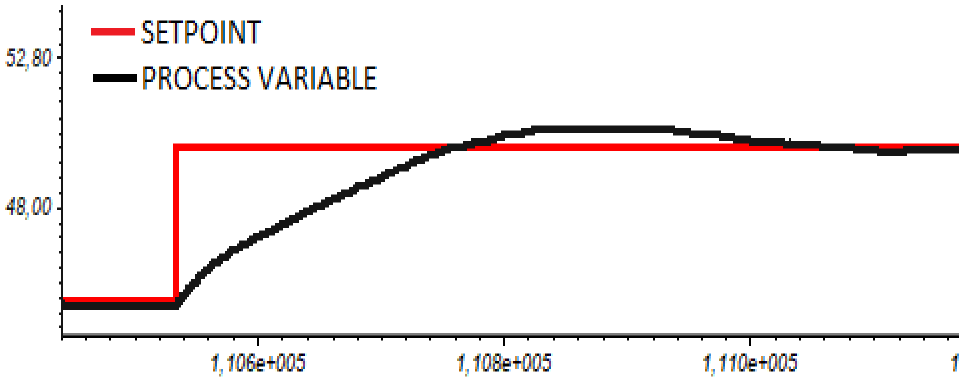



**Figure IV - 23 : Setpoint change in TIC-100 with a decoupler**

The 25th tray temperature response shows an overshoot of 0.84°C (1.2%) responding the 5°C increment in the setpoint of TIC-100. As shown in Figure IV-24:

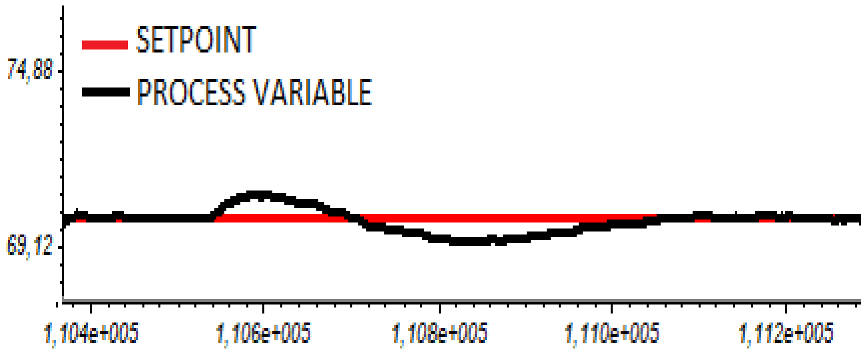



**Figure IV - 24 : TIC-101 response to setpoint change in TIC-100 with a decoupler**

## Setpoint change in TIC-101

By applying a setpoint change on the TIC-101 controller from70°C to 75°C, the reboiler valve opens thus the temperature of tray 25 will increase up to the SP value required.

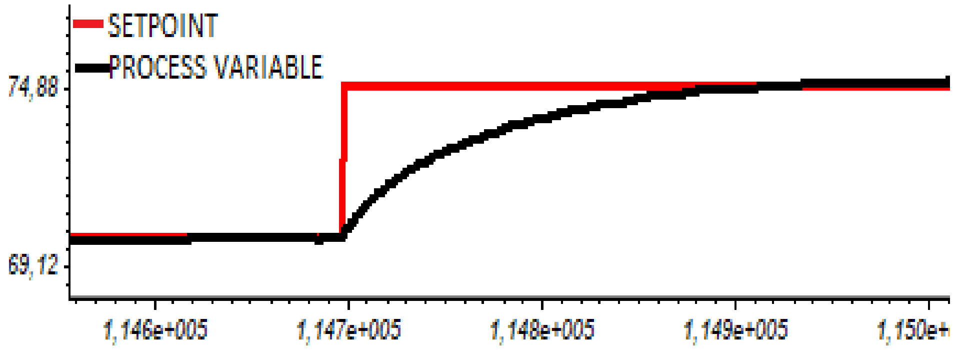



**Figure IV - 25 : Setpoint change in TIC-101 with a decoupler**

The 5-tray temperature response shows an overshoot of 0.64°C (1.42%) to the 5°C increment in the setpoint of TIC-101.

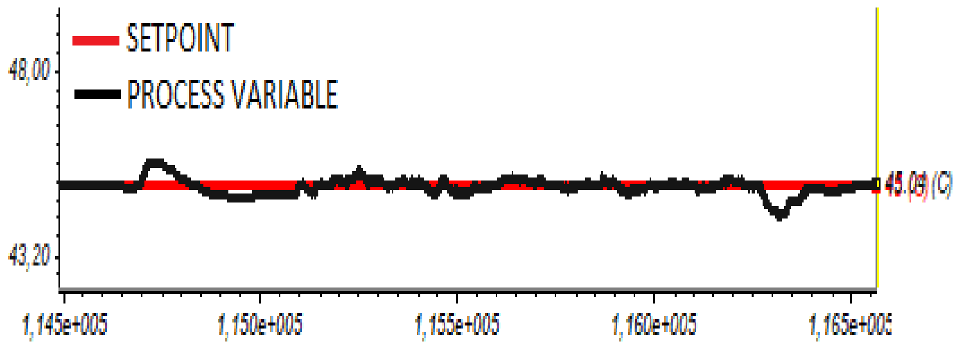



**Figure IV - 26 : TIC-100 response to setpoint change in TIC-101 with a decoupler**

## Disturbance rejection

Opening 10% on the feed valve (increasing from 50% to 60%) leads to an overshoot of 2.55°C (5.66%) in the response of TIC-100 as shown in Figure Ⅳ-27:

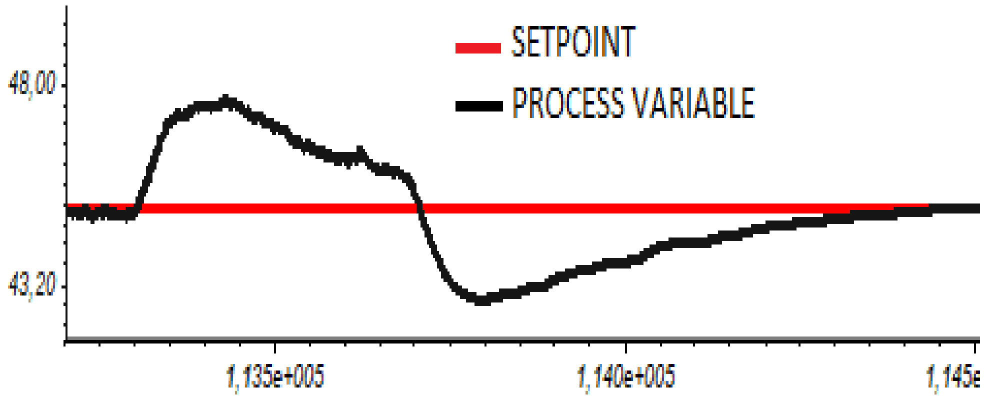



**Figure IV - 27 : TIC-100 response to 10% disturbance with a decoupler**

In the same manner, we apply a 10% disturbance on TIC-101.This will lead to an overshoot of 2.3°C (3.3%) as shown in Figure Ⅳ-28:

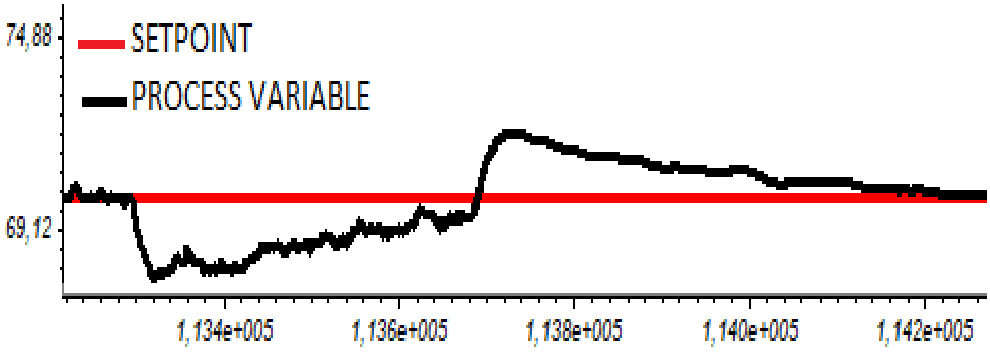



**Figure IV - 28 : TIC-101 response to 10% disturbance with a decoupler**

Table Ⅳ-10 illustrates the results obtained:

**Table IV - 10: Performance analysis of the controllers when using forward decoupling**

| PID With decouplers | $T_s$(min) | $T_r$(min) | Disturbance rejection | SP change in TIC-100 | SP change In TIC-101 |
|---|---|---|---|---|---|
| TIC-100 | 232 | 194 | 5.66% | 1.16% | 1.42% |
| TIC-101 | 226 | 149 | 3.3% | 1.2% | No overshoot |

Upon comparing the dynamic model analysis before and after decoupling, significant improvements are evident. The overshoot in tray 25 temperature, caused by setpoint change in tray 5, decreased notably from 2.22°C (3.17%) to 0.84°C (1.2%), demonstrating the effectiveness of decouplers in mitigating the influence of tray 5 temperature variations. Furthermore, the overshoot in tray 5 temperature, caused by change in tray 25, decreased by 3.02% (from 2°C to 0.64°C) indicating that the variation in tray 5 temperature does not have a huge impact on tray 25 anymore.Since, decouplers do not directly handle disturbances, their primary focus on reducing interactions; let us try a further minimization to these problems by using an MPC controller.

## IV.5 Applying MPC Controller

Model Predictive Controller (MPC) is used instead of two PID controllers for managing the temperatures of Tray 5 and Tray 25.

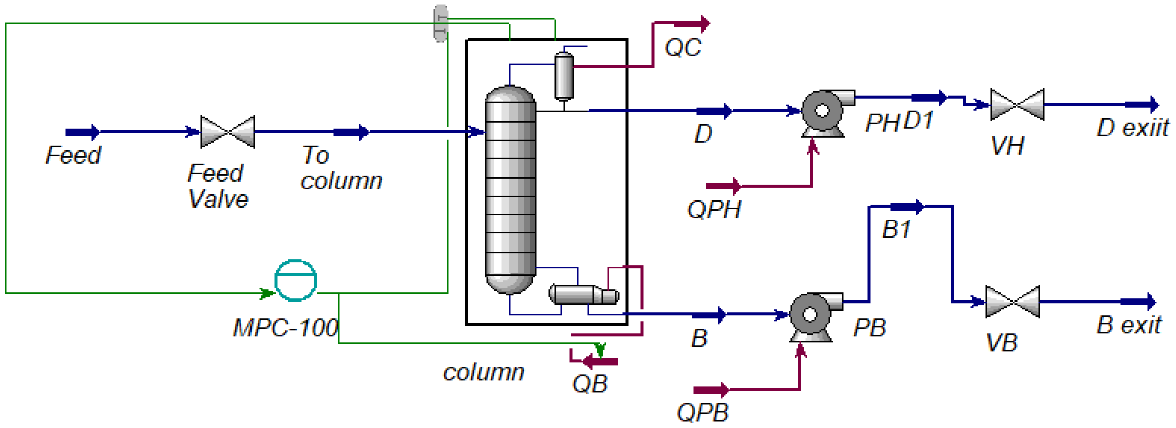



**Figure IV - 29 : Distillation column control with MPC**

We provide the transfer matrix G(s), obtained earlier, to the MPC controller:

$$G(s) = \begin{bmatrix} \frac{-2.18}{1+127s} & \frac{3.12}{1+193s} \\ \frac{-2.17}{1+95s} & \frac{3.25}{1+111s} \end{bmatrix}$$

In a multi-variable system, the slowest loop needs to be considered in calculating the MPC parameters; in other words, the calculations are based on the dynamics of the loop having the largest time constant and delay.

The initial parameters of the MPC, calculated according to equations of section Ⅲ.4, are shown on the next Table Ⅳ-11:

**Table IV - 11: MPC calculated parameters**

| $T_s$ | N | P | M | Gamma-U | Gamma-Y |
|---|---|---|---|---|---|
| 19.3 | 75 | 52 | 15 | 1 | 1 |

An initial comparative response using MPC controller & PID shown next:

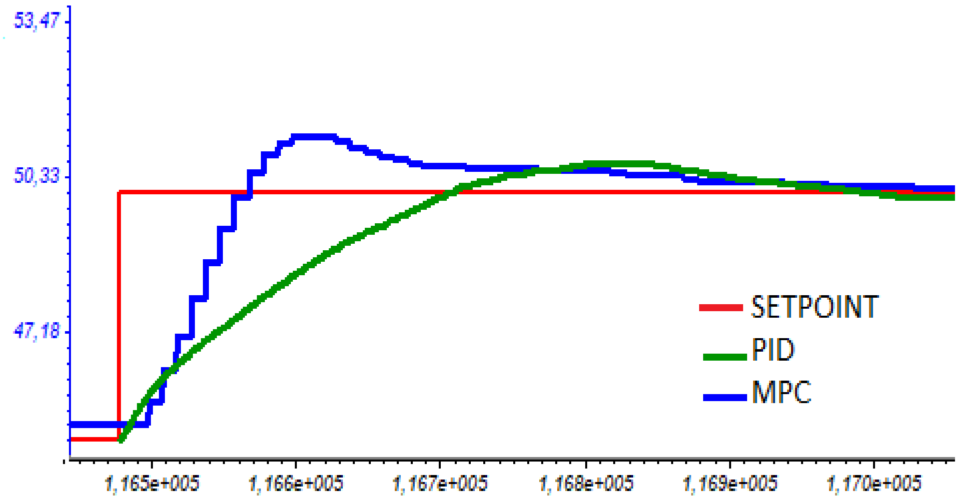



**Figure IV - 30 : Response to step change in tray 5 using MPC and PID-Decoupler**

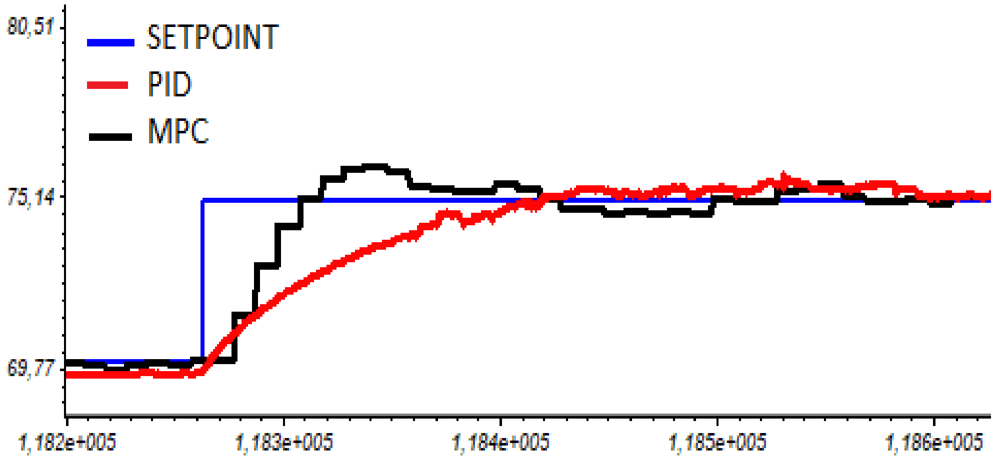



**Figure IV - 31 : Response to step change in tray 25 using MPC and PID-Decoupler**

Based on responses illustrated in Figure Ⅳ-30 and Figure Ⅳ-31, MPC controller respond faster to step change in comparison to PID with a decoupler.

MPC works well when adding disturbance forbid the massive overshoot as well, as shown on Figure Ⅳ-32, and Figure Ⅳ-33:

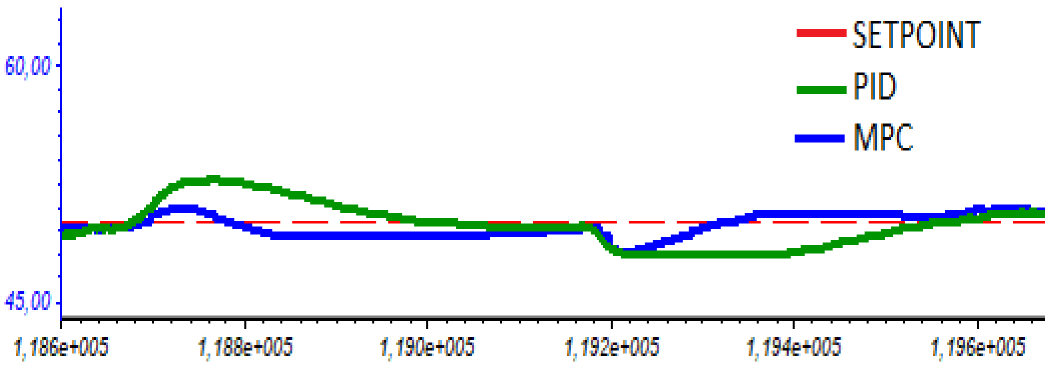



**Figure IV - 32: Response to disturbance in tray 5 with MPC and PID-Decoupler**

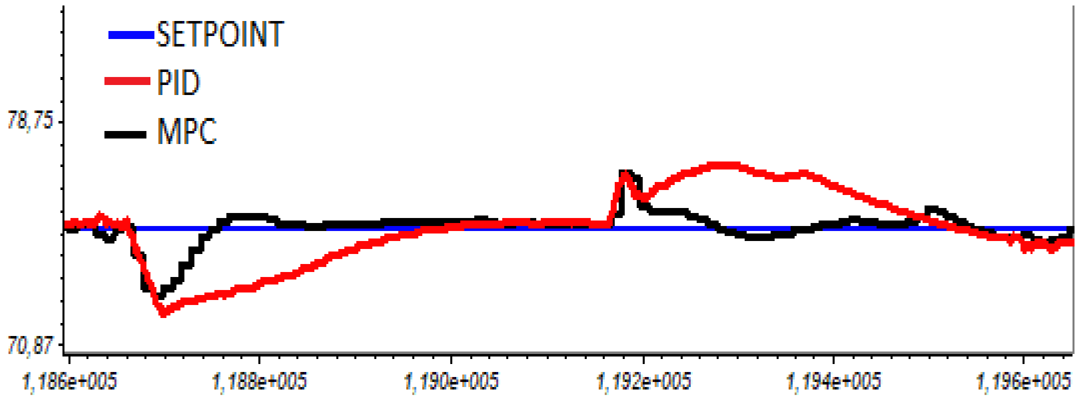



**Figure IV - 33 : Response to disturbance in tray 25 with MPC and PID-Decoupler**

The desired performance would be met by the fine-tuning procedure, which is the disturbance rejection, the loop interaction problems, and the settling time of the loops.

## IV.6 Tuning MPC Controller

After multiple trial and error experiment based on the ground rules recommended at the end of chapter-Ⅲ, the final parameters are shown on Table Ⅳ-12:

**Table IV - 12: MPC tuned parameters**

| $T_s$ | N | P | M | Gamma-U | Gamma-Y |
|---|---|---|---|---|---|
| 9.3 | 116 | 97 | 11 | 0.7 | 0.5 |

### Setpoint change in Tray-5

By applying a setpoint change on the Tray-5 from 45°C to 50°C, the reflux valve close thus the temperature of tray 5 will increase up to the setpoint. This latest results in decreasing the settling time of Tray-5 temperature up to 79min (1h19min) with an overshooting of 1°C (2.2%).

As for the effect on Tray-25, it is completely eliminated (no overshooting). The results are shown on Figure Ⅳ-34:

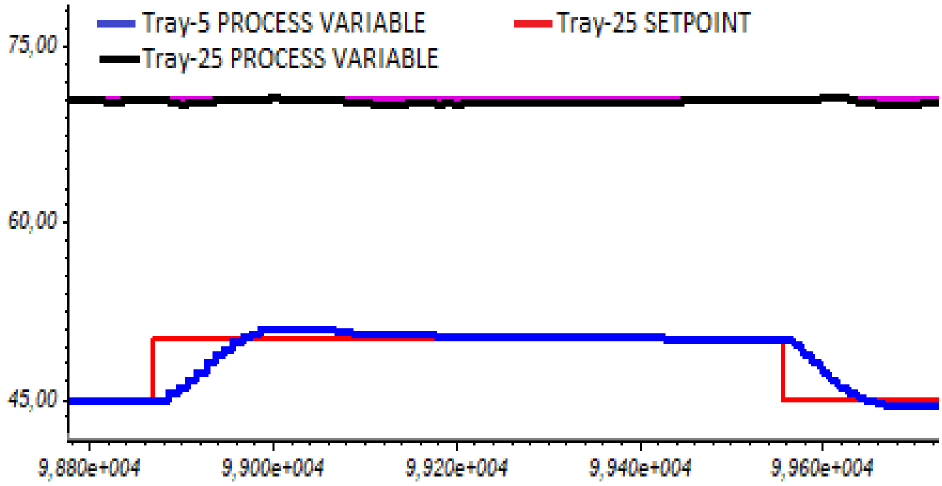



**Figure IV - 34 : Response of temperature at tray-5 and 25 to a setpoint change at Tray-5 with MPC**

## Setpoint change in Tray-25

By applying a setpoint change on Tray-25 controller from 70°C to 75°C, the reboiler valve opens thus the temperature of tray 25 will increase up to the SP value required. This results in decreasing the settling time up to 48 min making it the faster loop, and an overshoot of 1.33°C (1.9%).

While Tray-5 respond by an overshooting of 1.19°C (2.64%). The results are demonstrated in Figure IV -35:

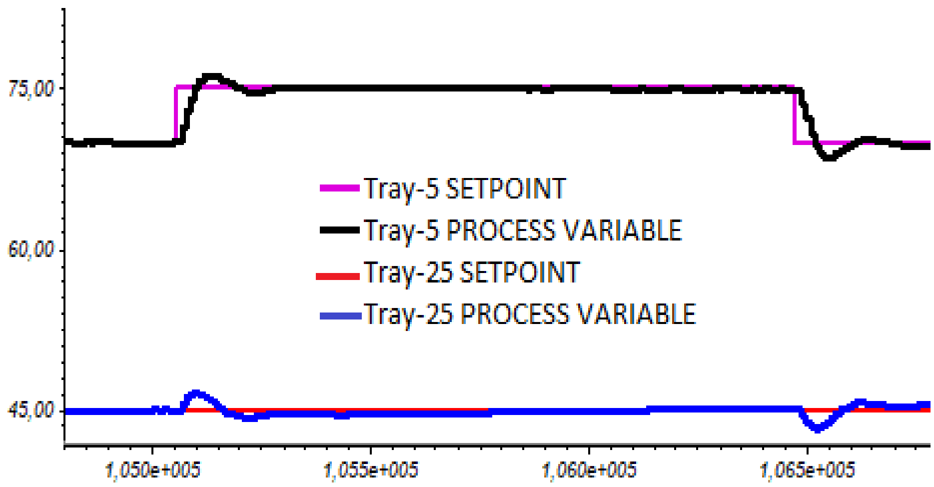



**Figure IV - 36 : Response of temperature at tray-5 and 25 to a setpoint change at Tray-25 with MPC**

## Disturbance rejection

Opening 10% on the feed valve (increasing from 50% to 60%) leads to an overshoot of 1.6°C (3.5%) in the response of Tray-5, and a 2.04°C (2.8%) for Tray-25 as in Figure Ⅳ-36:

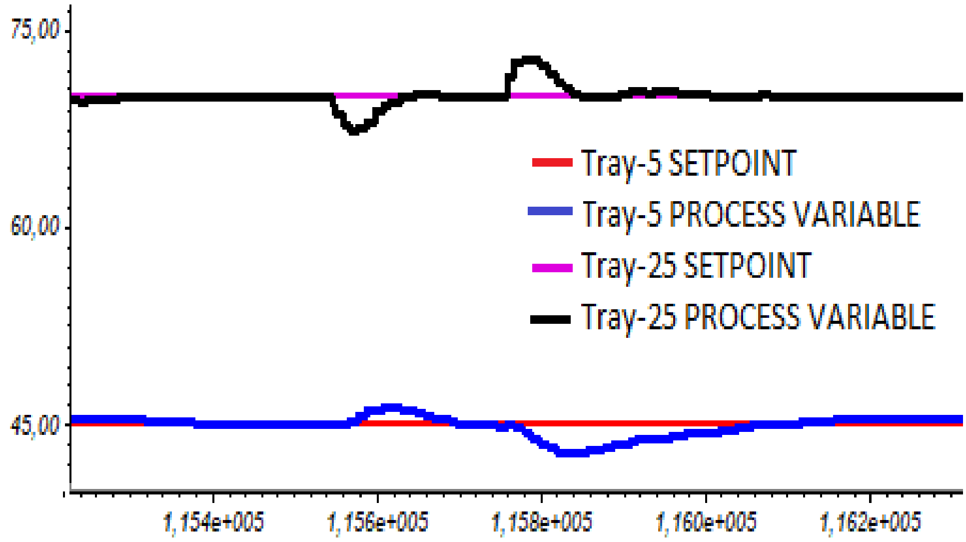



**Figure IV - 37 : Response of temperature at tray-5 and 25 to a 10% disturbance with MPC**

The results are recorded in Table Ⅳ-13:

**Table IV - 13: Performance analysis of Trays 5 and 25 using MPC controller**

| MPC | $T_s$(min) | $T_r$(min) | Disturbance rejection | SP change in TIC-100 | SP change In TIC-101 |
|---|---|---|---|---|---|
| Tray-5 | 79 | 68 | 3.5% | 2% | 2.64% |
| Tray-25 | 48 | 47 | 2.8% | No overshoot | 1.77% |

## Results:

Comparing the dynamic model analysis before and after using MPC, remarkable performance improvements can be noticed. The settling time of both loops has decreased by almost three times (for Tray-5) and four times (for Tray-25) in comparison with the decouplers; allowing the fast process stability when a setpoint change occurs.

Disturbance rejection has been reduced as well; a 2% reduction in the case of Tray-5, and 0.8% in Tray-25.

The MPC controller offers the best minimization on the loop interaction problems and disturbance rejection, as well as reducing settling time for a faster response. However, in attempt to decrease the settling time and loop interactions, a small overshooting resulted when changing the setpoint. This problem, in addition to disturbance rejection, can be solved by DMC (dynamic matrix control) provided by ASPEN-TECH or RMPC (robust model predictive control) provided by HONEYWELL company.

Unfortunately, these technologies are licensed and are not available for use.

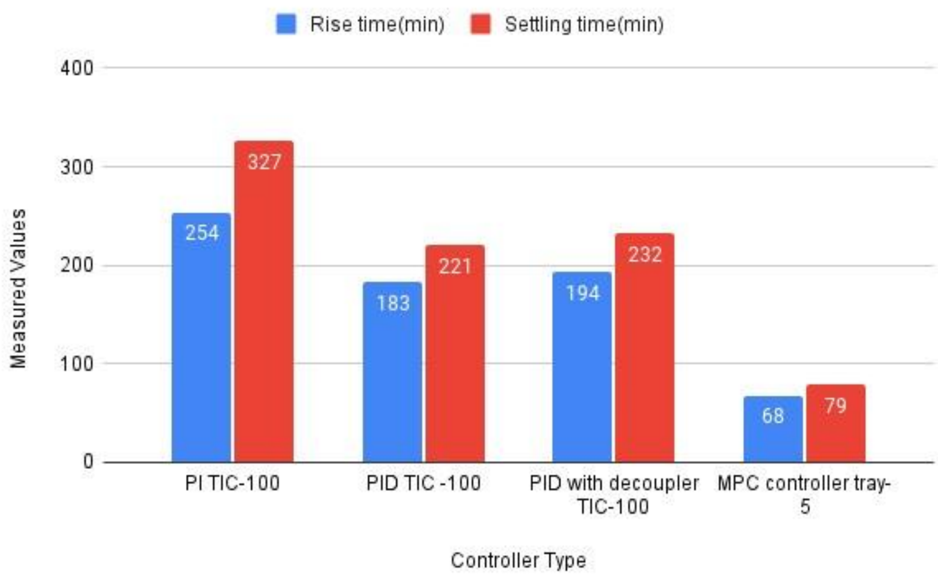
A comparison between the controller’s performances analysis is shown next:

**Table IV - 14: Comparison between Tray-5 controllers based on the performance analysis**

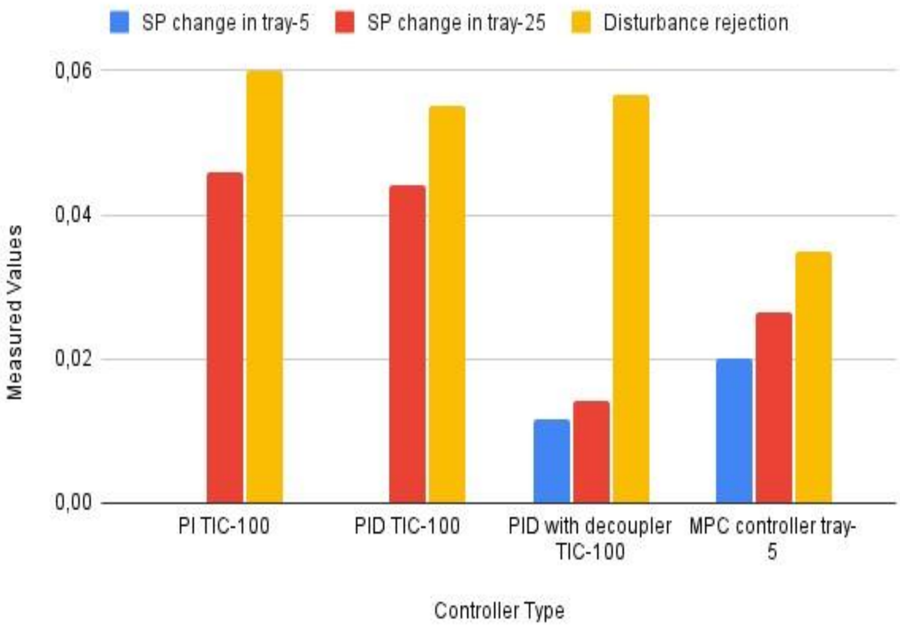
| Controllers/ /Index | PI TIC-100 | PID TIC-100 | PID with decoupler TIC-100 | MPC controller Tray-5 |
|---|---|---|---|---|
| $T_r$(min) | 254 | 183 | 194 | 68 |
| $T_s$(min) | 327 | 221 | 232 | 79 |
| SP change in Tray-5 | No overshoot | No overshoot | 1.16% | 2% |
| SP change In Tray-25 | 4.58% | 4.4% | 1.42% | 2.64% |
| Disturbance rejection | 6% | 5.5% | 5.66% | 3.5% |

**Table IV - 15: Comparison between Tray-25 controllers based on the performance analysis**

| Controllers/ /Index | PI TIC-101 | PID TIC-101 | PID with decoupler TIC-101 | MPC controller Tray-25 |
|---|---|---|---|---|
| $T_r$(min) | 150 | 148 | 149 | 47 |
| $T_s$(min) | 236 | 235 | 226 | 48 |
| SP change in Tray-5 | 3.87% | 3.17% | 1.2% | No overshoot |
| SP change In Tray-25 | No overshoot | 0.64% | No overshoot | 1.77% |
| Disturbance rejection | 2.86% | 3.37% | 3.3% | 2.8% |

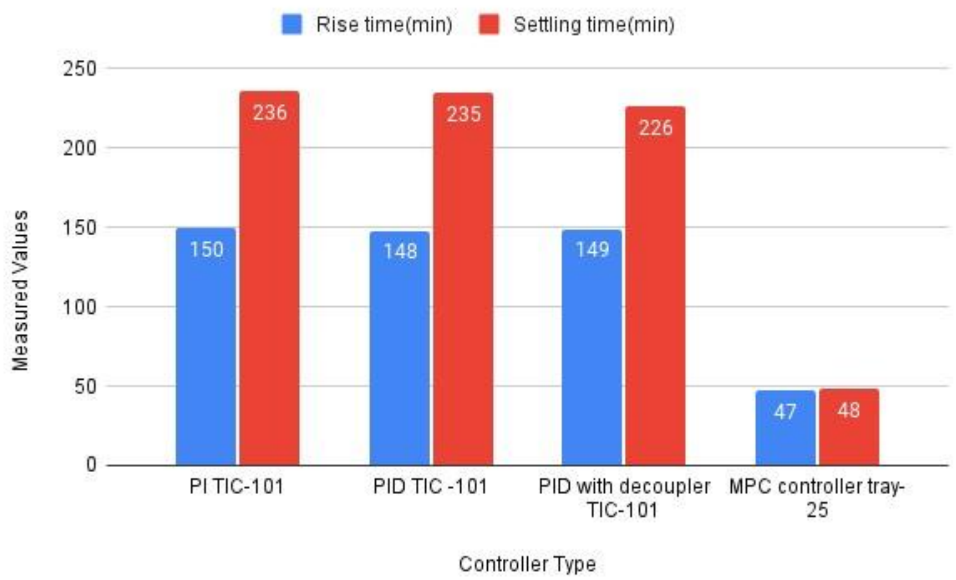



**(a)**

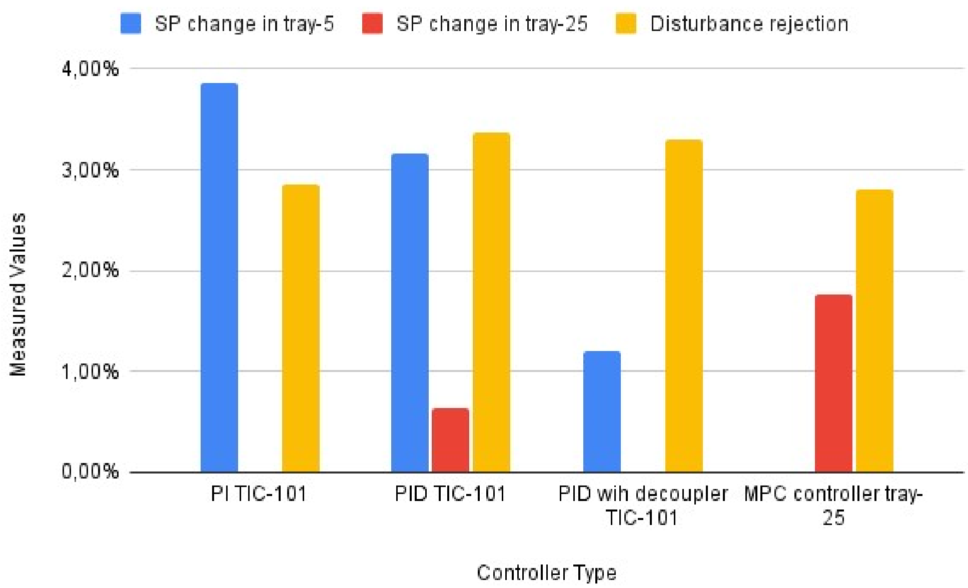



**(b)**

**Figure IV - 38 : Final Comparison between Tray-5 controllers' performance: (a) according to settling time & rise time –(b) according to overshooting**




**(a)**




**(b)**

**Figure IV - 39 : Final Comparison between Tray-25 controllers' performance: (a) according to settling time & rise time –(b) according to overshooting**

Based on Table Ⅳ-14, Table Ⅳ-15, Figure Ⅳ-38, Figure Ⅳ-39, and previous discussions, it is safer to state that MPC controller is the best solution in handling the complexity of multiple variables system, and dealing with interactions and disturbance rejection by offering the optimal results.

## Difficulties & Challenges:

Model Predictive Control can handle MIMO complex systems with interactions between variables, effectively reject disturbances and maintain control performance, and can be applied to a wide range of processes, including nonlinear and time-varying systems. However, set of drawbacks can be mentioned:

- The performance of MPC heavily relies on the accuracy of the process model. Inaccurate models can lead to unstable control; a model tuning is required first.
- The parameters tuning of MPC to set an optimal control is highly exhausting and time consuming due to the multiple trials and errors tests.
- It requires expertise in both control theory and the specific process being controlled.

Despite these drawbacks, MPC remains a powerful control strategy in many industries, Especially where high performance and constraint handling are needed.

## Rules of Thumb:

After the study and simulation of MPC applied on a depropanizer distillation column, and the previous ground rules suggested at chapter-III; a set of recommendations to be followed when dealing with a column having the same specifications.

<u>Column Specifications:</u>

**Table IV - 16: Specifications of the tuned distillation column (Depropanizer)**

| Tower specifications | |
|---|---|
| Number of trays/Stage numbering | 30/Top-down |
| Type of Trays | Sieve |
| Tray diameter (m) | 10.94 |
| Tray volume ($m^3$) | 51.75 |
| Tray space (m) | 0.55 |
| Condenser | |
| Condenser Type | Total |
| Condenser Diameter (m) | 1.193 |
| Condenser Height (m) | 1.789 |
| Condenser Volume ($m^3$) | 2 |
| Condenser Pressure (KPa) | 1350 |
| Condenser Temperature (°C) | 41.46 |
| Reboiler | |

| | |
|---|---|
| Reboiler Type | Regular |
| Reboiler Diameter (m) | 1.193 |
| Reboiler Height (m) | 1.789 |
| Reboiler Volume ($m^3$) | 2 |
| Reboiler Pressure (KPa) | 1369 |
| Reboiler Temperature (°C) | 81.33 |
| Feed specifications | |
| Composition | 40% Propane - 60% I-Butane |
| Temperature (°C) | 48.85 |
| Pressure (at) | 20 |
| Flow (kgmole/s) | 1 |

MPC Tuning Procedure:

Based on the ground rules of chapter-III, Although Some changes occurred due to changing the type of the process and dealing with a more complex MIMO system; here is a set of recommendations to be followed when tuning such a depropanizer having the specifications mentioned on Table Ⅳ-16.

The slowest loop needs to be considered (the process having the greatest lag time & dead time).

**Process Model:**

- Accuracy of the system is crucial for a good control.
- The process identification region must match the operating region; modeling the process needs to be done in the same region that we are controlling it, to avoid model mismatch.
- For a dead time, dominant process (when the delay is much larger than the process lag), the execution interval should generally be at least half of the plant time delay ($\boldsymbol{T_s}$=0.5 $\boldsymbol{\theta_p}$). However, if unmeasured load disturbances are significant and frequent (disturbances that affect the process but are not measured by the system), it's crucial to improve the model accuracy and reduce the execution interval to one tenth of the plant time delay ($\boldsymbol{T_s}$=0.1 $\boldsymbol{\tau_p}$) [27].

**Response horizon:** its value must be large enough such that the last step response model parameter is equal to the steady state gain.

**Control horizon:**

- First decrease the value of control horizon by four steps; and check whether the performance obtained is better comparing to the initial one. Check in the same way for four steps increase.
- Compare your results then choose the optimal value for the control horizon according to the desired performance.

**Prediction horizon:**

- Check the performance obtained for two steps increase and two steps down. Then pick the optimal value.

Once these steps are made, and the performance is improved, move to the optimization procedure.

**Weighting functions:**

- Start by decreasing the Gamma_U by 0.2 while keeping the Gamma_Y at 1, then check the performance obtained.
- Repeat the previous step again with Gamma_Y.
- The effect of each function will be clear to you by this step; then continue on changing their values according to the desired performance; if the goal is decreasing the settling time and fastening the process and minimizing the overshooting, then the Gamma_Y is the one that should be considered. A value that is less than 0.5 would result in an aggressive response and a massive overshoot.
- As for minimizing oscillatory response for the system, then Gamma_U should be considered in this case. A value that is less than 0.7 would lead the system to oscillatory response.
- A balance should be made, between these two functions, during the tuning procedure so that the desired performance is met.

# General conclusion

This thesis aimed to investigate the application of Model Predictive Control (MPC) on abinary distillation column to enhance process efficiency and product quality.

The research demonstrated that MPC significantly improves setpoint tracking and disturbance rejection compared to traditional PID control methods. Key findings include thesuccessful mitigation of interaction issues through forward decouplers and the superior performance of MPC under varying operating conditions.

These results highlight MPC's potential in handling the complexities of multivariable processes, offering a more robust control strategy for industrial applications. The findings align with existing literature on advanced process control but provide new insights into thespecific challenges and solutions for binary distillation columns.

This thesis contributes to the field by providing a detailed comparison of MPC and PID control methods, establishing a set of guidelines for MPC tuning, and demonstrating thepractical benefits of MPC in industrial distillation processes.

One limitation of this study was the dependency on simulation models, which may not fully capture all real-world process dynamics. Additionally, the complexity of MPC tuning requirescareful consideration to avoid suboptimal performance.

Future research could explore the application of MPC to more complex distillation processes and investigate the integration of real-time adaptive control mechanisms. Further studies should also consider the impact of varying feed compositions and more sophisticated disturbance models.

# References


[1] K. M. V. R. B. J. ,. M. Drgoňa Ján, «Model identification and predictive control of a laboratory binary distillation column,» chez *2015 20th International Conference on Process Control (PC),* 2015.

[2] A. a. F. N. a. B. H. Al-Shammari, «Model predictive control of a binary distillation column,» *Advances in Engineering Research,* vol. 8, pp. 255-270, 2014.

[3] O. GUPTA, Elements of Petroleum Refinery Engineering, Khanna Publishing House, 2015.

[4] J. G. Speight, Dewatering,Desalting,and Distillation in Petroleum Refining, CRC Press, 2022.

[5] J.-C. CICILE, «Distillation-Absorption.Etude pratique,» *Techniques de l'ingénieur.Génie des procédés,* vol. 2, n° %1J2610, pp. J2610.1-j2610.20, 2001.

[6] C. L. Smith, Distillation control: An engineering perspective, John Wiley Sons, 2012.

[7] T. R. Kuphaldt, Lessons in industrial instrumentation, Creative Commons Attribution/PAControl. com, 2008.

[8] K. Nitin, Plantwide Control of Integrated Chemical Processes, Kanpur,India: National Program on Technology Enhanced Learnning (NPTEL), 2014.

[9] L. M. J. Y. Fikar M, «CVP-based optimal control of an industrial depropanizer column,» *Computers \& Chemical Engineering,* vol. 24, n° %12-7, pp. 909-915, 2000.

[10] H. L.Wade, Basic and advanced regulatory control: system design and application, ISA, 2004.

[11] D. C. K. K. D. Abdullah Bin Gani, Information,communication and computing technology, New Delhi India: Springer Nature Singapore, 2019.

[12] S. C. L, Practical Process Control:Tuning and Troubleshooting, John Wiley & Sons, 2009.

[13] K. T. a. H. ,. J. L. Erickson, Plant-wide process control, John Wiley & Sons, 1999.

[14] «International Society of Automation,» [En ligne]. Available: https://www.isa.org. [Accès le 15 05 2024].

[15] C. L.Smith, Advanced Process Control : Bryond Single-Loop Control, John Wiley & Sons, 2010.

[16] S. a. P. Skogestad, Multivariable feedback control : analysis and design, John Wiley & Sons, 2005.

[17] J. Uhlmann, «On the relative gain array(RGA) with singular and rectangular matrices,» *Applied Mathematics Letters,* vol. 93, pp. 52-57, 2019.

[18] L. A. H. ,. ,. A. Casaverde, «A simulation study of the performance of GPC and PID applied to a binary distillation column,» IEEE, 22 03 2021. [En ligne]. Available: https://ieeexplore.ieee.org/stampPDF/getPDF.jsp?arnumber=9465198. [Accès le 21 04 2024].

[19] E. F. a. B. C. a. C. E. F. a. B. C. Camacho, Constrained model predictive control, Springer, 2007.

[20] A. a. W. A. Ahmad, «Application of model predictive control (MPC) tuning strategy in multivariable control of distillation column,» *Reaktor,* vol. 11, n° %12, pp. 66--70}, 2007.

[21] L. A. a. R. R. H. a. P. D. A. Casaverde, «A simulation study of the performance of GPC and PID applied to a binary distillation column,» chez *2021 IEEE International Conference on Automation/XXIV Congress of the Chilean Association of Automatic Control (ICA-ACCA),* 2021.

[22] H. a. B. H. A. Mansoor, «Comparitive study of linear controllers for binary distillation column».

[23] S. a. M. R. Manna, «Comparative performance analysis of LQR and MPC for active suspension system,» chez *2020 IEEE 5th International Conference on Computing Communication and Automation (ICCCA),* 2020.

[24] J. A. Rossiter, Model-based predictive control: a practical approach, CRC press, 2017.

[25] R. a. C. D. J. Shridhar, «A tuning strategy for unconstrained SISO model predictive control,» *Industrial \& Engineering Chemistry Research,* vol. 36, n° %13, pp. 729--746, 1997.

[26] E. J. a. S. M. E. a. S. C. A. Iglesias, «Tuning equation ford dynamic matrix control in siso loops,» *Ingenier{\'\i}a y desarrollo,* n° %119, pp. 88--100, 2006.

[27] G. K. a. C. R. A. McMillan, Models Unleashed: Virtual Plant and Model Predictive Control Applications, Isa, 2004.

[28] W. L. Luyben, Distillation Design And Control Using ASPEN Simulation, John Wiley & Sons, 2013.

[29] W. SÖLKEN, «Distillation Column Internals,» Obtenido de Explore the World of Piping , 2008. [En ligne]. Available: https://www.wermac.org/equipment/distillation_part2.html. [Accès le 24 4 2024].

# Appendix A

## 1. Introduction to Aspen HYSYS

HYSYS is a powerful engineering simulation tool, uniquely designed in terms of its architecture, interface, and interactive operation. Its integrated steady state and dynamic modeling capabilities allow the same model to be evaluated from either perspective, with full sharing of process information, representing a significant advancement in engineering software.

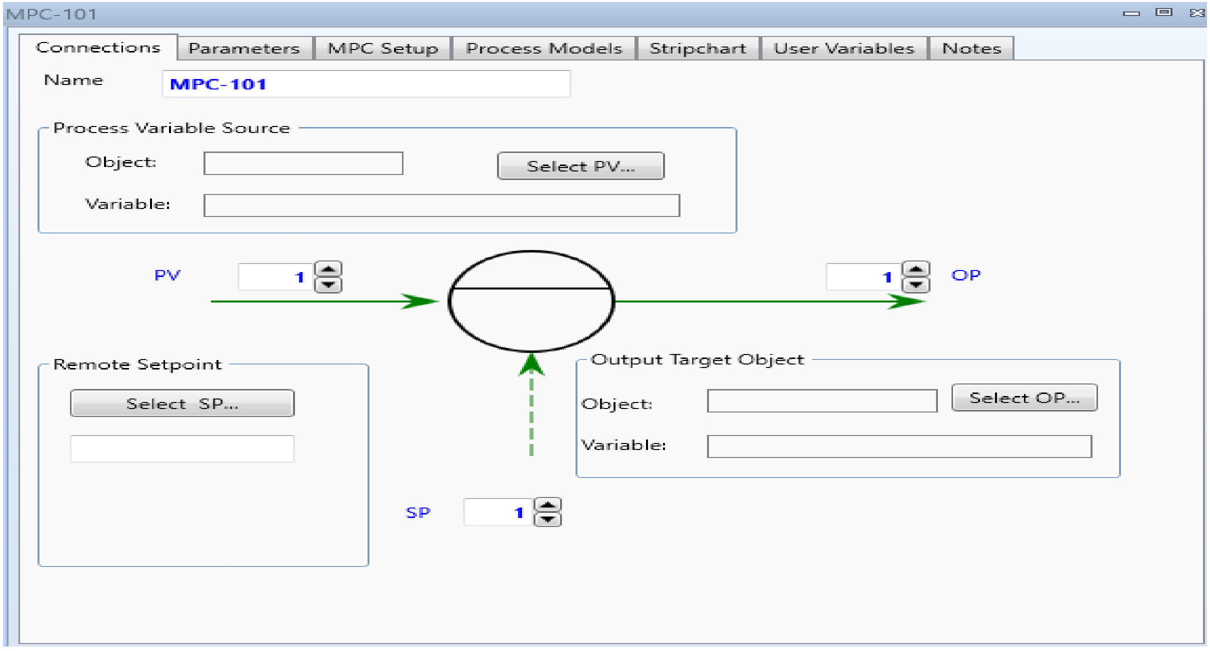
The various components of HYSYS offer a robust approach to steady state modeling. With a comprehensive selection of operations and property methods, users can confidently model a wide range of processes. The design of HYSYS enhances process understanding, maximizing the return on simulation time.

The inherent flexibility, unparalleled accuracy, and robustness of its property package calculations result in more realistic models. Widely used in chemical engineering, HYSYS supports research, development, modeling, and design across various industries, from upstream processes, gas processing, and cryogenic facilities to refining and chemical processes.

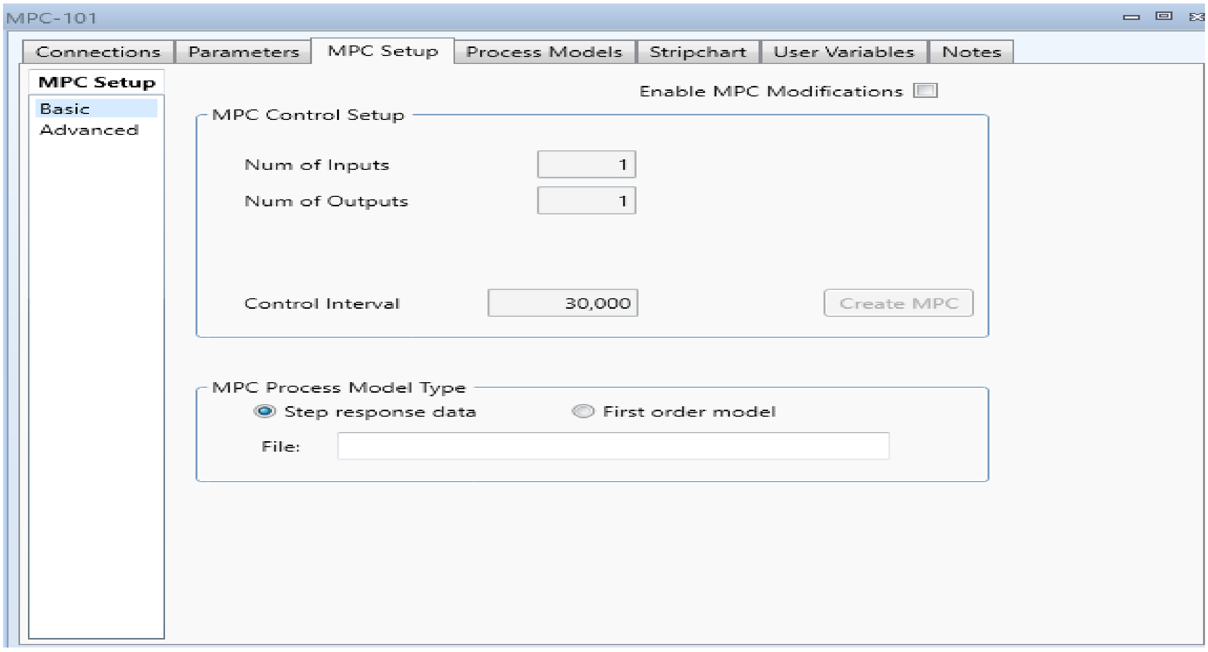
## 2. Implementation of MPC controller in HYSYS

After implementing the steady state model of the process we need to convert into a dynamic system by sizing of volume equipment and piping (pumps, compressors, valves) such that the program can calculate the pressure differences and thus the flow rates. After passing to dynamic model controllers can be added. The steps to follow in order to add an MPC controller to the system are as follows:

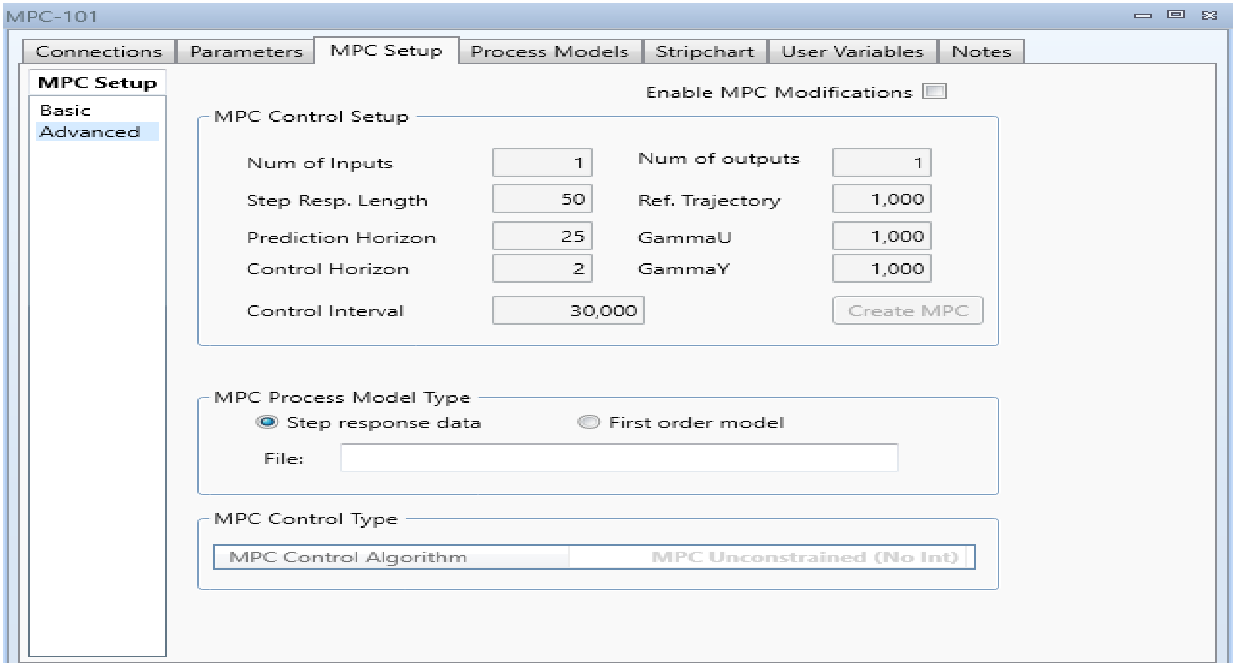
- Bring the MPC controller from the pallet and click on it to have a window as shown in the figure below then connect the MPC controller to its appropriate “process variable source” and “output target object”:




➢ move to" MPC Setup" then "Basic" and choose the Process Model Type, along with the control interval that specify the sampling interval:




➢ move to" Advanced" then enter the MPC control setup that are pre-calculated:




➢ Now, specify the process Model (process gain $K_p$, process time constant $T_p$, and delay), then click « Update Step Response » button to calculate the step response data for the process models.