# Distributed Dynamics Approximation through Lumping and Fuzzy Clustering: Reliability and Validity Issues

## Ahcene Habbi

Applied Automation Laboratory, F.H.C., University of Boumerdès, Avenue de l'indépendance, 35000 Boumerdès, Algeria

# Madjid Kidouche

Applied Automation Laboratory, F.H.C., University of Boumerdès, Avenue de l'indépendance, 35000 Boumerdès, Algeria

ABSTRACT: This paper presents a fuzzy reasoning based approach for modeling the distributed dynamics of a parallel flow heat exchanger. Distributed-parameter and lumped-parameter models of the heat exchanger are first derived. The validity of such model representations is discussed and limitations regarding their use for practical purposes are presented. The alternative modeling approach presented in this work aims at developing a moderately complex model with interpretable structure for the parallel flow heat exchanger which is the main component of a pilot thermal plant. The proposed multivariable fuzzy rule-based model achieves simultaneous prediction of the air and water temperatures. The experimental study conducted on the real plant shows the performance of the fuzzy model in capturing the key dynamical properties of the physical plant over a wide operating range and under varying dynamics.

# 1 INTRODUCTION

In process industry, heat exchangers are generally used to achieve efficient heat transfer from one fluid to another. These thermal systems can be found in particular in the chemical, petrochemical and manufacturing industries. They are usually arranged in units as a part of complex network systems. Given their extremely complex dynamics, modeling of heat exchangers is still the subject of many studies (Fink et al. 2000, Zavala-Rio and Santiesteban-Cos 2007, Arbaoui et al. 2007, Zhang et al. 2009). The complex dynamics are represented through distributed-parameter models that are derived from conservation laws. However, such model representation is in general difficult to analyze and complicated for numerical simulation.

Lumped-parameter models have been widely used to approximate the distributed dynamics of heat exchangers (Zavala-Rio and Santiesteban-Cos 2007, Astorga-Zaragoza et al. 2008, Shang et al. 2005, Astorga-Zaragoza et al. 2007). These simplified models are involved in various engineering issues such as control design, process monitoring (Peng et al. 1997, Persin and Tovornik 2005) and parameter estimation (Weyer et al. 2000). The lumping procedure consists in dividing the whole exchanger in a finite number of cells so that it becomes possible to derive a set of ordinary differential equations that describes its key dynamical properties. However, this gives rise to high order models when accurate modeling is required. On the other hand, some studies proposed a special type of low-order models that have been considered as a reliable representation of the heat exchanger dynamics (Zavala-Rio and Santiesteban-Cos 2007). Nevertheless, such models could not ensure good prediction capability for wide-range operation or under varying dynamics.

This work presents a fuzzy reasoning based approach applied to a parallel flow heat exchanger for better characterization of its distributed dynamics. The main aim is to derive an interpretable moderately complex model that should guarantee a high prediction capability over a

wide operating range and under varying operating conditions. Briefly, the paper is organized as follows. Section 2 describes the parallel flow heat exchanger and presents its fundamental model. The fuzzy modeling procedure applied to the pilot heat exchanger is presented in Section 3. Experimental results are shown in Section 4 and a conclusion is given in Section 5.

#### 2 FUNDAMENTAL MODEL OF THE DISTRIBUTED HEAT EXCHANGER

The parallel flow heat exchanger considered in this work is the main part of the pilot thermal plant depicted in Fig. 1. It consists of three subsystems: the heater, the air circuit and the water circuit. In more detail, the system is composed of an electric heater of the air (E) which generates the heating power P (kW), pipes for air and water circulation, a co-current gas-liquid exchanger (HE), two valves,  $(V_r)$  and  $(V_e)$ , to control the portion of the air flow which is recycled and the portion which is evacuated, respectively, and a variable speed pump (SP) to control the water flow  $Q_w$  (m³/s). The water entering the heat exchanger with the temperature  $T_{33}$  (°C) is heated up to the temperature  $T_{34}$  (°C) with hot air. The amount of air coming from the electric heater with temperature  $T_{14}$  (°C) enters the heat exchanger with the temperature  $T_{16}$  (°C) after flowing through the air circulation pipe, and leaves the heat exchanger with temperature  $T_{15}$  (°C). Total or partial recycling of air can be considered depending on the position of the two motor-driven valves  $V_r$  (%) and  $V_e$  (%). To derive the distributed-parameter model of the parallel flow heat exchanger, a common procedure consists in subdividing the plant in several elemental volumes with length  $d\xi$ ,  $\xi \in [0, L]$ . To this end, let us consider the following simplifying assumptions:

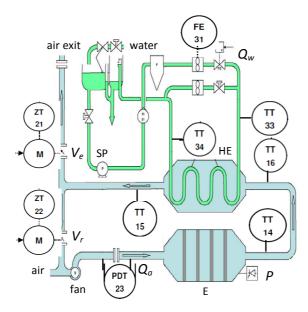


Figure 1: Schematic of the pilot heat exchanger.

- A1. The air and water temperatures and velocities are radially uniform.
- A2. The thermophysical properties of air and water are constant (in time and space).
- A3. Heat conduction along the flow axis is neglected.
- A4. The fluids are incompressible and single phase.
- A5. The heat transfer coefficient is axially uniform and time invariant.
- A6. There is no energy storage in the walls.
- A7. Inlet temperatures are constant.

Under assumptions A1-A6, energy balance applied to a differential volume of air and water leads to the following partial differential equations:

$$\frac{\partial T_w}{\partial t} = -v_w \frac{\partial T_w}{\partial \xi} + \frac{h_a d_a}{\Omega_w C_w} (T_a - T_w)$$
 (1)

$$\frac{\partial T_a}{\partial t} = -\nu_a \frac{\partial T_a}{\partial \xi} + \frac{h_a d_a}{\Omega_a C_a} (T_w - T_a) \tag{2}$$

where T: temperature (°C),  $\Omega$ : heat transfer area (m²), C: specific heat (Kcal/m³°C), d: diameter (m), v: fluid velocity (m/s), and h: heat transfer coefficient (Kcal/m²s°C). Subscripts a and w denote air and water, respectively.

As mentioned above, the distributed-parameter model (1) and (2) is difficult to manipulate for practical purposes, such as dynamics analysis, control design and performance monitoring, for instance. Approximations through lumped-parameter models are to be derived and used at least for such purposes. To this end, one should refer to the reliable representation presented in (Zavala-Rio and Santiesteban-Cos 2007) and (Astorga-Zaragoza et al. 2007), which describes the system dynamics using a 2nd-order lumped-parameter model with the logarithmic mean temperature difference (LMTD). The model is given by the following form:

$$\dot{T}_{wo} = \frac{2Q_w}{Vol_w} (T_{wi} - T_{wo}) + \frac{2h_a d_a}{\Omega_w C_w} \Delta T(T_{wo}, T_{ao})$$
(3)

$$\dot{T}_{ao} = \frac{2Q_a}{Vol_a} (T_{ai} - T_{ao}) - \frac{2h_a d_a}{\Omega_a C_a} \Delta T(T_{wo}, T_{ao})$$
(4)

where Vol denotes the total volume (m³) and  $\Delta T(T_{wo}, T_{ao})$  stands for the mean temperature difference throughout the heat exchanger, modeled through the LMTD as shown in (Shang et al. 2005). Here, the subscripts i and o are used to denote input and output temperatures. According to Fig. 1, the corresponding model variables are:  $T_{wi} = T_{33}$ ,  $T_{wo} = T_{34}$ ,  $T_{ai} = T_{16}$ , and  $T_{ao} = T_{15}$ .

Under assumptions A1-A7, the dynamic model (3) and (4) has been analytically proved to keep the main features of the dynamic behavior of the parallel flow heat exchangers. However, when considering one of the following operation requirements i.e.

- Persistent dynamic excitation in heating power,
- Variable recycling of air,
- Varying dynamics induced by water leak flows, for instance,

it seems impossible to maintain the validity of the fundamental model (3) and (4). Other alternative approaches should be investigated to deal with these wide-range operation requirements.

# 3 FUZZY CLUSTERING BASED IDENTIFICATION OF THE DITRIBUTED-PARAMETER HEAT EXCHANGER

Fuzzy identification aims at building dynamical fuzzy models in the form of IF-THEN fuzzy logical rules (De Bruin and Roffel 1996, Driankov et al. 1993, Tsekouras et al. 2005). The Takagi-Sugeno fuzzy model representation often provides an attractive solution to many process identification problems (Habbi et al. 2003) and (Fink et al. 2000). Such model structure is considered here to describe the distributed dynamics of the parallel flow heat exchanger. The water and air circuits of the pilot thermal plant depend strongly on the following physical variables: the heating power P, the air recycling valve position  $V_r$ , the air evacuation valve position  $V_e$ , the air temperatures  $T_{14}$  and  $T_{16}$ , and the water temperature at the outlet of the heat exchanger  $T_{34}$ . Total or partial air recycling may induce nonlinear effects on the plant dynamics. Simple experiments recording the time-step responses of water and air circuits show us the principal

local behavior of the heat exchanger. From these experiments we concluded that there is no evidence for higher than first-order local dynamics.

Therefore, we need to find a supervision scheme of the five measurements of  $T_{34}$ ,  $T_{14}$ , P,  $V_r$ , based on the following NARX structure (Ciftcioglu and Sariyildiz 2006):

$$Y(k+1) = \Psi(\phi(k)) \tag{5}$$

where  $Y = [T_{34} \ T_{16}]^t$  is the temperature output vector,  $\phi(k) = [T_{34} \ T_{16} \ T_{14} \ PV_r]$  the global regression vector and  $\Psi$  denotes nonlinear functions. To identify a dynamic Takagi-Sugeno fuzzy model for each circuit, real data from the pilot heat exchanger is generated in normal operation mode. During the identification experiment, the water flow rate  $Q_w$  is kept constant while the air flow rate  $Q_a$  varies according to the position of the two valves  $V_r$  and  $V_e$ . The heating power P is manipulated over its whole operating domain from 0 to 10 kW. The positions of the air recycling valve and the air evacuation valve are controlled simultaneously in the range [0-100%]. The multivariable Takagi-Sugeno fuzzy model structure used for the prediction of the process temperatures is described by a set of IF-THEN fuzzy rules where the ith rule is of the form:

Rule i:

$$IF \ P(k) is \ A_{11}^i \ and \ V_r(k) is \ A_{12}^i \ and \ T_{16}(k) is \ A_{13}^i \ and \ T_{34}(k) is \ A_{14}^i$$

$$THEN \ T_{34}(k+1) = b_1^i + a_{11}^i P(k) + a_{12}^i V_r(k) + a_{13}^i T_{16}(k) + a_{14}^i T_{34}(k)$$

$$ALSO$$

$$IF \ P(k) is \ A_{21}^i \ and \ V_r(k) is \ A_{22}^i \ and \ T_{14}(k) is \ A_{23}^i \ and \ T_{16}(k) is \ A_{24}^i$$

$$THEN \ T_{16}(k+1) = b_2^i + a_{21}^i P(k) + a_{22}^i V_r(k) + a_{23}^i T_{14}(k) + a_{24}^i T_{16}(k)$$

$$(6)$$

The fuzzy model (6) is to be derived through fuzzy clustering (De Bruin and Roffel 1996, Gomez-Skarmeta 1999, Driankov et al. 1993) using the well-established Gustafson-Kessel (GK) clustering algorithm described in (Gustafson and Kessel 1979). The number of fuzzy rules is determined by clustering the real data for different values of the parameter c which stands for the number of clusters. It was found that a fuzzy model structure with three rules (c=3) for each output is very accurate as desired and any further increase in the model complexity does not improve the model performance.

The task is now to determine the parameters of the multivariable fuzzy model structure, i.e. the fuzzy partitions of the rule premise variables and the rule-consequent parameters by means of fuzzy clustering. To this end, the data measurements collected in normal operation from the pilot heat exchanger through a PC-based data acquisition system are used. This set of training data contains 2000 samples with noise. The system was sampled every 2s.

The detected clusters are then projected onto the product space of the input-output variables of the TS fuzzy model and gaussian-type fuzzy sets are determined for each rule-premise variable with  $i=1,\ldots,3$  and  $j=1,\ldots,4$ . Thus, the whole operating domain of each rule-premise variable is partitioned into three fuzzy regions corresponding to different operating points of the parallel flow heat exchanger. The rule-consequent parameters of the fuzzy model (6) are determined using the weighted ordinary least-square method (Setnes 2000).

#### 4 EXPERIMENTAL RESULTS

To assess the performance of the proposed multivariable fuzzy model, experimental validation under varying dynamics is conducted on the real plant. Varying operating conditions are emphasized by introducing leaks with different magnitudes in the water circulation pipe of the heat exchanger. It is well known that tube and pipe leaks in heat exchangers are sources of strong changes in the process dynamics (Habbi et al. 2009) and (Sun et al. 2002). A reliable model representation should give an accurate or at least an acceptable prediction of the air and water

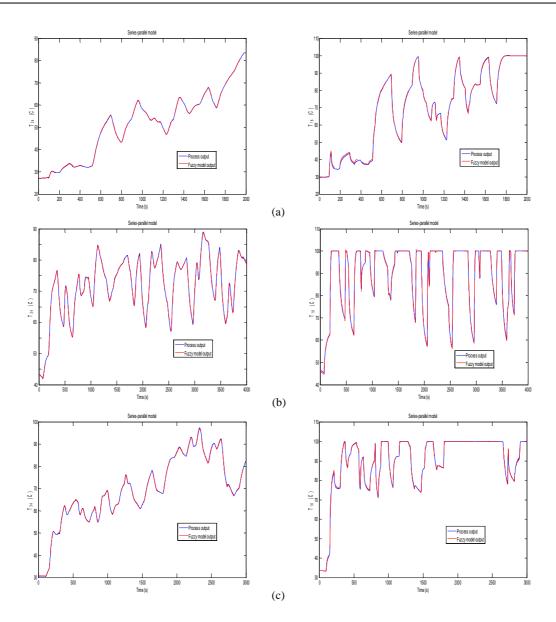


Figure 2: Experimental validation of the rule-based fuzzy model under varying dynamics: (a) 15% leak flow, (b) 25% leak flow and (c) 30% leak flow.

temperatures under this critical situation, which is frequently encountered in practice, without any prior tuning. To demonstrate the performance of the data-driven fuzzy model under varying dynamics, several experiments with different leak magnitudes are conducted. Leaks are introduced at different time instants using a bypass valve situated on the water circulation pipe of the heat exchanger. In the experimental study, single leak flows are emphasized under varying excitation signals on heating power P and air recycling and evacuation valve positions  $V_r$  and  $V_e$ . The following test cases were considered:

Case 1: 15% leak flow introduced at t = 924 s. Case 2: 25% leak flow introduced at t = 0 s. Case 3: 30% leak flow introduced at t = 1220 s.

The resulting process and fuzzy model outputs for each test case are shown in Fig. 2. These experimental results correspond to the series-parallel configuration. The good prediction capability of the developed fuzzy model is clearly visible. Indeed, it is clear that the fuzzy model shows higher performance in predicting the water temperature  $T_{34}$  and the air temperature  $T_{16}$ 

in all leak test cases. From a weak (15%) to a considerable (30%) leak flow, the fuzzy model performs considerably well with a very acceptable degree of accuracy. This important result is maintained even during the saturation of the air temperature  $T_{16}$ .

## 5 CONCLUSION

In this paper, distributed dynamics approximations through lumping procedure and fuzzy clustering are derived for a parallel flow heat exchanger. Limitations of the distributed-parameter and lumped-parameter models are presented and an alternative fuzzy rule-based model is suggested. Given the extremely complex dynamics of the thermal plant, it is of big interest to construct moderately complex models with flexible structure that can capture the key dynamical properties of the plant over a wide operating range. These models are to be used as a basement for a new way to analyze the system dynamics and to design monitoring and control systems, for instance. The experimental study conducted on the real plant demonstrates the good performance of the proposed multivariable fuzzy model in wide-range operation and under varying operating conditions. Emphasizing different operation modes of heat exchangers may help in building more reliable representations for better characterization of the complex distributed dynamics that still remain a complex issue in engineering problems.

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