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ION-ACOUSTIC SOLITONS IN ELECTRON POSITRON NONTHERMAL PLASMA

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Abstract

Electron-positron plasmas are believed to exist in the early universe, in active galactic nuclei, and in pulsar magnetospheres. Most of the astrophysical plasmas usually contain ions as well in addition to electrons and positrons. In this work, the ion-acoustic solitons are investigated in three-component plasmas, whose constituents are inertial ions, nonthermal electrons, and Boltzmannian positrons. The properties of stationary structures are studied by pseudo-potential approach, which is valid for large amplitude. It is found that the conditions which support the existence of solitons are altered.

Introduction

Nonlinear propagation of intense electrostatic waves in electron-positron plasmas has received large amount of theoretical interest mainly because such plasma are naturally produced under certain astrophysical conditions. Since electron-positron are thought to have been present in the early Universe, plasma processes are expected to have played an important role in the early history, as well as the evolution of the Universe. In contrast to the usual plasma consisting of electrons and positive ions, it has been observed that nonlinear waves in plasmas having an additional component of positrons be- have differently. Electron-positron-ion plasmas appear in the early universe, intergalactic jets and in the pulsar magnetosphere, and also in the solar atmosphere and in fact most of the astrophysical plasmas usually consist of ions, in addition to electrons and positrons, and it is pertinent to study the behaviour of nonlinear wave motions in an electron-positron-ion (e-p-i) plasma. When positrons are introduced in the plasma, the response of the plasma to disturbances changes drastically **[1,2]**. Recently, there has been a great deal of interest in studying linear as well as nonlinear wave motions in electron-positron. The nonlinear studies have been focused on the nonlinear self-consistent structures, such as envelope solitons, vortices, etc. Therefore, it seems important to study linear and nonlinear wave propagation in plasma.

Formulation

We consider three component plasma consisting of singly charge positive ions, nonthermal distribution electrons and Boltzmannian positrons. The quasi-neutrality at equilibrium is written as, $N_{e0} = N_{i0} + N_{p0}$ where, N_{e0}, N_{i0} and N_{p0} are the unperturbed electron, ion and positron densities respectively. The positrons and nonthermal electrons densities are given respectively by,

$$N_p = N_{p0} \exp\left(-\frac{e\Phi}{T_p}\right) \qquad (1)$$

$$N_e = N_{e0}\left[1 - \beta\frac{e\Phi}{T_e} + \beta\left(\frac{e\Phi}{T_e}\right)^2\right]\exp\left(\frac{e\Phi}{T_e}\right) \qquad (2)$$

where $\beta = \frac{4\alpha}{1+3\alpha}$, α being a parameter defining the population of nonthermal electrons **[2]**.

Finite amplitude linear ion-acoustic waves

We first linearize **Eqs. (1-5)** and assume that the first order quantities of N_p, N_e, N_i, V_i and ϕ are proportional to $\exp[i(kx - \omega t)]$, to get the dispersion relation of the ion-acoustic waves in an electron-positron-ion plasma,

$$\omega^2 = \frac{k^2 C_s^2}{1 + k^2\lambda_D^2}\,\frac{1-p}{1-\beta+p\,T_e/T_p} \qquad (3)$$

where, $p = \frac{n_{p0}}{n_{e0}}$, $C_s = \left(\frac{T_e}{m_i}\right)^{1/2}$, $\lambda_D^2 = \left(\frac{4\pi e^2 N_0}{T_0}\right)^{-1}$ and $\frac{N_0}{T_0} = \frac{N_{p0}}{T_p} + (1-\beta)\frac{N_{e0}}{T_e}$, ω the frequency, k the wave number C_s the ion-sound velocity, λ_D^2 the effective Debye length, N_0 the effective number density and T_0 the effective temperature.

Arbitrary amplitude solitary structures

In this section we will be looking for arbitrary large solutions of the nonlinear equations system. The normalised governing equations of the plasma evolution are given by,

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i u_i}{\partial x} = 0 \qquad (4)$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial x} = -\frac{\partial \Phi}{\partial x} \qquad (5)$$

$$\frac{\partial^2 \Phi}{\partial x^2} = (1 - \beta\Phi + \beta\Phi^2)\exp(\Phi) - p\exp(-\sigma\,\Phi) - n_i \qquad (6)$$

where, n_i is the ion number density normalized to N_{e0}, u_i is the ion fluid velocity normalized to the ion acoustic speed $C_s = (T_e/m_i)^{\frac{1}{2}}$, Φ is the electrostatic potential normalized to T_e/e, the time and space variables are written in units of the plasma period $\omega_p^{-1} = (4\pi e^2 N_0/m_i)^{1/2}$ and the effective Debye length $\lambda_D = (T_0/4\pi e^2 N_0)^{1/2}$. We confine ourselves to investigate stationary equations that depend on space and time in the following way, $\xi = x - Mt$ where M is the Mach number. In the stationary frame, we obtain from **Eqs. (4-6)** the density as,

$$n_i = \frac{Mn_{i0}}{\sqrt{M^2 - 2\Phi}}, \qquad (7)$$

where we have imposed appropriate boundary conditions for the localized disturbances, viz., $u_i \longrightarrow 0$, $n_i \rightarrow n_{i0}$ at $\xi \rightarrow \pm\infty$, which can then be substituted into **Eq(6)**, and multiplying both sides of the resulting equation by $\partial\Phi/\partial\xi$, integrating once, and taking into account the appropriate boundary conditions, i.e., $\Phi \rightarrow 0$ and $d\Phi/\partial\Phi \rightarrow 0$ at $\xi \rightarrow \pm\infty$, we obtain the energy integral equation,

$$\frac{1}{2}\left(\frac{d\Phi}{d\xi}\right)^2 + V(\Phi) = 0 \qquad (8)$$

where the Sagdeev potential for our purpose reads as,

$$V(\Phi) = 1 + 3\beta + \frac{p}{\sigma} - (1 + 3\beta - 3\beta\Phi + \beta\Phi^2)\exp(\Phi) - \frac{p}{\sigma}\exp(-\sigma\Phi) + (1-p)M\left(M - \sqrt{M^2 - 2\Phi}\right) \qquad (9)$$

where, $\sigma = T_e/T_p$. The solitonic solution of **Eq. (8)** exist when the usual conditions, namely, $V(\Phi) = dV(\Phi)/d\Phi = 0$ at $\Phi = 0$ and $V(\Phi) < 0$ for $0 < |\Phi| < |\Phi_0|$, where $|\Phi_0|$ is the maximum amplitude of the solitons, are satisfied. The condition on the Mach number can be obtained by expanding the Sagdeev potential$V(\Phi)$, given by **Eq. (8)**, around the origin. Therefore, the critical Mach number, at which the second derivative sign (the lower value of M), can be found as,

$$M_l = \sqrt{1 - p/(1 + \sigma p - \beta)} \qquad (10)$$

At this critical value of$M$, the third derivative of Sagdeev potential is negative, in the case we have $\alpha > \alpha_{cr}$ (α_{cr} being a critical value to be determined). Indeed, the condition$V''' < 0$, yields, $\frac{1-p}{(1+\sigma\, p-\beta)^2} - \frac{3}{1-\sigma^2 p} < 0$ which leads to $\beta \sim 0.5$. Furthermore, at $\Phi_{\mathrm{m}} = -\mathrm{M}^2/2$ the Sagdeev potential is required at least too vanish, i.e.,

$$V(\Phi_{\mathrm{m}}) = 1 + 3\beta - \left(1 + 3\beta + \frac{3\beta M}{2} + \frac{\beta \mathrm{M}^4}{4}\right)\exp\left(-\frac{\mathrm{M}}{2}\right) + \frac{p}{\sigma}\left(1 - \exp\left(\frac{\sigma M^2}{2}\right)\right) + (1 - p)M^2\left(1 - \sqrt{2}\right) \geq 0 \qquad (11)$$

Eqs. (9) and **(10)** determine the lower and upper limits for M, respectively. The variation of M_l versus σ is giving by **Fig 1** which shows that the lower limit of Mach number increase with increasing β and decreasingσ. Indeed, it is shown that for$p = 0{,}01$, $\sigma = 0{,}01$ and $1{,}35 \leq M \leq 1{,}57$ solitons exist. **Fig 2-4** show the function $V(\Phi)$ determined by **Eq. (11)** for different values of the Mach number, p, β andσ.

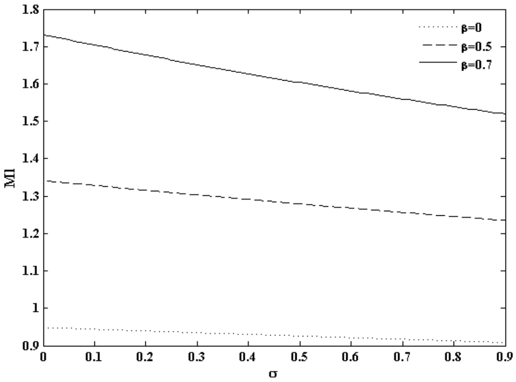

FIG.1 Variation of $\boldsymbol{M_l}$ versus $\boldsymbol{\sigma}$

$p = 0{,}01$, $\sigma = 0{,}01$ and different values of β

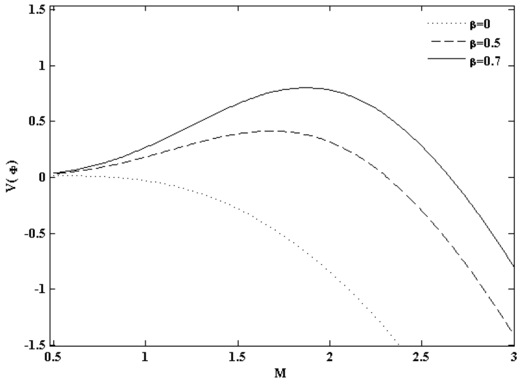

FIG.2 Variation of $\boldsymbol{V(\Phi_m)}$ versus $\boldsymbol{M}$ for

$p = 0{,}01$, $\sigma = 0{,}01$ and different values of β

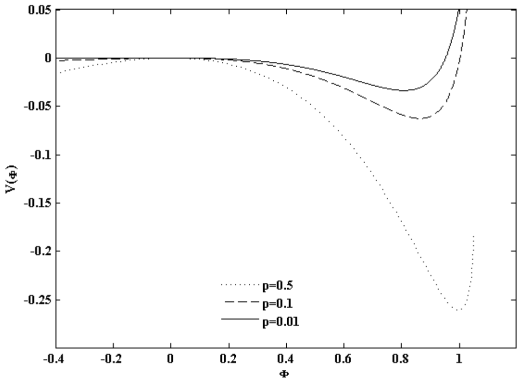

FIG.3 Variation of $\boldsymbol{V(\Phi)}$ =versus $\boldsymbol{\Phi}$

$M = 1{,}45$, $\sigma = 0{,}01$ and different values of p

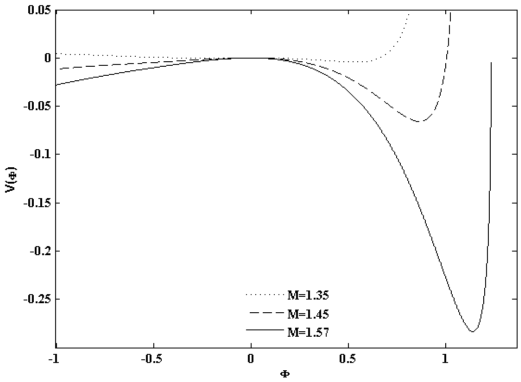

FIG.4 Variation of $\boldsymbol{V(\Phi)}$ =versus $\boldsymbol{\Phi}$

$p = 0{,}01$, $\sigma = 0{,}01$ and different values of M

Summary

Electrostatic ion-acoustic structures have been investigated by pseudopotential approach in unmagnetized three-component plasma consisting of inertial ions, nonthermal electrons and Boltzmannian positrons. It emerges that in electron-positron ion plasmas, the ion acoustic solitons would have an electrostatic potential hump. Moreover, the speed of solitons of given amplitude decreases as the fractional number $\boldsymbol{p}$ increases. We think that the present investigation will be helpful in understanding the nonlinear propagation of electrostatic perturbation associated with positrons which may occur in space.

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