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# EFFECTS OF NONTHERMAL ION DISTRIBUTION AND DUST TEMPERATURE ON NONLINEAR DUST ACOUSTIC SOLITARY WAVES

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**Abstract**

An investigation of dust-acoustic solitary waves in unmagnetized dusty plasma whose constituents are inertial charged dust grains, Boltzmannian electrons and nonthermal ions has been conducted taking into account finite dust temperature. The pseudo-potential been used to study solitary solution. The existence of solitary waves of a negative potential is reported.

**Introduction**

In recent years, there has been a great deal of interest in understanding different types of collective processes in dusty plasmas, which are very common in laboratory and astrophysical environments**.** It has been found that the presence of charged dust grains modifies the existing plasma wave spectra, whereas the dust dynamics may even introduce new eigenmodes in the plasma. Indeed, Rao *et al.* **[1]**, were the first to predict theoretically the existence of extremely low-phase velocity dust acoustic waves in unmagnetized dusty plasmas whose constituents are inertial charged dust grains and Boltzmann distributed ions and electrons. These waves have been reported experimentally and their nonlinear features investigated by Barkan *et al.* **[2]**. In this work, we consider the ions non-thermal **[3]** and take into account the dust temperature and study their effect on the properties of dust acoustic waves.

**Model equations**

We consider a three-component dusty plasma with extremely massive, micron-sized, negatively charged dust grains, Boltzmannian electrons and nonthermally distributed ions. The quasi-neutrality at equilibrium is written as, $N_{i0} = Z_d N_{d0} + N_{e0}$ where, $N_{i0}$, $N_{e0}$ and $N_{d0}$ are the unperturbed ion, electron and dust densities respectively, and $Z_d$ being the number of elementary charges residing on the dust grain. The electron and nonthermal ions densities are given respectively by,

$$N_e = N_{e0} \exp(\frac{e\phi}{T_e}), \qquad \textbf{(1)}$$

$$N_i = N_{i0}\left(1+\beta\frac{e\phi}{T_i}+\beta\left(\frac{e\phi}{T_i}\right)^2\right)\exp\left\{-\frac{e\phi}{T_i}\right\}, \quad \textbf{(2)}$$

where $\beta = \dfrac{4\alpha}{1+3\alpha}$ , α being a parameter defining the population of nonthermal ions. Furthermore, the governing equations of the plasma evolution are given by,

$$\frac{\partial N_d}{\partial t}+\frac{\partial}{\partial x}\left(N_d V_d\right)=0, \quad \textbf{(3)}$$

$$\frac{\partial V_d}{\partial t}+V_d\frac{\partial V_d}{\partial x}=-\frac{Z_d e}{m_d}\frac{\partial \phi}{\partial x}, \quad \textbf{(4)}$$

$$\frac{\partial^2 \phi}{\partial x^2}=4\pi e\left(N_e - N_i + Z_d N_d\right), \quad \textbf{(5)}$$

Here, $N_d$, $V_d$ and $m_d$ are the numerical density, velocity and mass of the charged dust grains, respectively.

## ARBITRARY AMPLITUDE SOLITARY STRUCTURES

In this section we will be looking for arbitrary large amplitude solutions of the nonlinear equations system. The normalised governing equations of the plasma evolution are given by,

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}\left(n_d u_d\right)=0, \quad \textbf{(13)}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}+3\sigma\frac{\partial n_d}{\partial t}=\frac{\partial \Phi}{\partial x}, \quad \textbf{(14)}$$

$$\frac{\partial^2 \Phi}{\partial x^2}=n_d+\mu_e n_e-\mu_i n_i, \quad \textbf{(15)}$$

We have taken the equation of state as $p = n_d^{\gamma} p_0$ , where $\gamma$ is the ratio of specific heats and taking as $\gamma = 3$, $n_d$ is the dust particle number density normalized to $n_{d0}$, $u_d$ is the dust fluid velocity normalized to the dust acoustic speed $c_s = \left(Z_d T_i / m_d\right)^{1/2}$, $\Phi$ is the electrostatic potential normalized to $T_i/e$. The time and space variables are in the units of $\omega_{pd}^{-1} = \left(\sqrt{m_d/4\pi n_{d0} Z_d e^2}\right)$ the dust plasma period and the Debye length $\lambda_d = \sqrt{T_i/4\pi Z_d n_{d0} e^2}$ respectively. $\mu = \dfrac{n_{e0}}{n_{i0}}$, $\mu_i = \dfrac{1}{1-\mu}$ and $\mu_e = \dfrac{\mu}{1-\mu}$ . $\sigma_e = T_e/T_i$ , $\sigma = T_d/T_i$ , $T_i$ being the ion temperature.

We confine ourselves to investigate stationary solutions that depend on space and time in the following way, $\xi = x - Mt$ (where $M$ is the Mach number). In the stationary frame, we obtain from **Eqs.(13,14)** the density as**,**

$$n_d = \frac{\sqrt{2}M}{\sqrt{M^2 + 2\Phi + 3\sigma + \sqrt{\left(M^2 + 2\Phi + 3\sigma\right) - 12M^2\sigma}}}, \qquad \textbf{(16)}$$

where we have imposed appropriate boundary conditions for the localized disturbances, viz., $u_d \to 0$ , $n_d \to 1$, $\Phi \to 0$ at $\xi \to \infty$. Substituting for $n_d$ from **Eq.(7)** into **Eq.(3)** and multiplying both sides of the resulting equation by $d\Phi/d\xi$, integrating once, and taking into account the appropriate boundary conditions, i.e., $\Phi \to 0$ and $d\Phi/d\xi \to 0$ at $\xi \to \infty$, we obtain the energy integral equation,

$$\frac{1}{2}\left(\frac{d\Phi}{d\xi}\right)^2 + V(\Phi) = 0, \qquad \textbf{(17)}$$

where the Sagdeev potential for our purpose reads as,

$$V(\Phi) = -\mu_i\left(\left(1 + 3\beta + 3\beta\Phi + \Phi^2\right)\exp(-\Phi) - (1 + 3\beta)\right) + \frac{\mu_e}{\sigma_e}\left(1 - \exp(\sigma_e\,\Phi)\right)$$
$$+ \frac{\mathrm{M}\left(12\sigma\mathrm{M}^2\right)^{1/4}}{\sqrt{2}}\left[\exp\frac{1}{2}\cosh^{-1}\left(a/\sqrt{b}\right) + \frac{1}{3}\exp(-\frac{3}{2})\cosh^{-1}\left(a/\sqrt{b}\right)\right] \qquad \textbf{(18)}$$
$$- \frac{\mathrm{M}\left(12\sigma\mathrm{M}^2\right)^{1/4}}{\sqrt{2}}\left[\exp\frac{1}{2}\cosh^{-1}\left(c/\sqrt{b}\right) + \frac{1}{3}\exp(-\frac{3}{2})\cosh^{-1}\left(c/\sqrt{b}\right)\right]$$

**Discussion**

The solitonic solutions of **Eq.(18)** exist when the usual conditions, namely, $V(\Phi) = dV(\Phi)/d\Phi = 0$ at $\Phi = 0$ and $V(\Phi) < 0$ for $0 < |\Phi| < |\Phi_0|$, where $|\Phi_0|$ is the maximum amplitude of the solitons, are satisfied. We plotted the Sagdeev potential versus the electrostatic potential $\Phi$ for different parameters. It is clearly seen from **figure 1**, that for $\beta = 0.50$, $\mu = 0.1$, $\sigma_e$ and $\sigma = 0.2$, compressive solitons exist for $M \geq 1.25$. The dust temperature has a crucial role on the existence of solitons, **Figure 2** shows that the well potential is deeper for the cooler dust. **Figure 3** shows solitons existence is no longer possible when $\mu$ exceeds the value of $0.10$ .

**Conclusion**

In summary, let us recall that for Boltzmannian ions the soliton solutions exist for $0.95 < M < 1.52$, and that solitons are exclusively rarefactive (cavitons) (c.f. Ref.**[4]).**

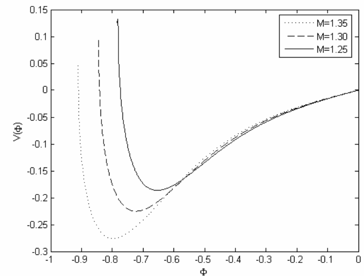


**Fig.1.** $V(\Phi)$ vs. $\Phi$ for $\beta = 0.50$, $\mu = 0.1$, $\sigma_e = 0{,}2$ and $\sigma = 0.02$ shows the existence of compressive solitons

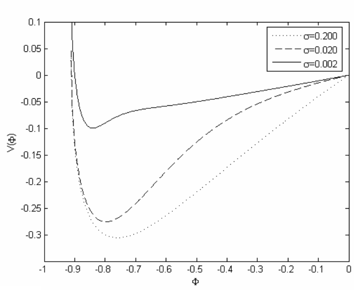


**Fig.2.** $V(\Phi)$ vs. $\Phi$ for $\beta = 0.50$, $\mu = 0.1$ and $M = 1.35$ shows the existence of compressive solitons

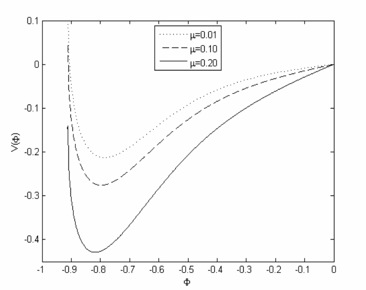


**Fig. 3.** $V(\Phi)$ vs. $\Phi$ for $\beta = 0.50$, $\sigma_e$=0.2, $\sigma = 0.02$ and $M$ Shows the existence of compressive solitons

The condition on the Mach number may be modified if ions are nonthermally distributed. Indeed, the allowed Mach numbers are given by, $1.41 < M < 1.62$ (in case $\mu = \sigma = 0.01$). Moreover, it is found that inclusion of such a distribution allows the coexistence of solitons and cavitons. Besides, it is interesting to point out that dusty plasmas can support both solitary waves of positive and negative potential corresponding to a dip (caviton) and a hump (soliton) in the dust density, for $\alpha \geq 0.178$ (and $M \geq 1.41$). It is also found that when $0.155 \leq \alpha < 0.178$, we have only rarefactive solitons **[5].** Furthermore, the effects of nonthermal ion distribution and finite dust temperature on a two component plasma (plasma consisting of dust particles and nonthermal ion) was discussed; possibility of coexistence of rarefactives as well as compressive soliton was reported **[6].** In this work, we have investigated large amplitude solitary waves with finite dust temperature incorporating the effect of nonthermal ion distribution in three component plasma. Is it found, that such plasma can support only solitary waves of negative potential.

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