Fractal behavior of total organic carbon in shale-gas reservoirs with an example from the Barnett Shale, Texas, USA

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Abstract

The behavior of fractal analysis using the continuous wavelet transform in shale-gas reservoirs is studied based on estimation of the so-called Hölder exponent by analyzing a total-organic-carbon (TOC) well log using the continuous wavelet transform; the Morlet is the analyzing wavelet. Application to the TOC well-log data of a horizontal well drilled in the Fort Worth Basin, Texas, USA, where the main objective is the lower Barnett Shale, clearly shows no special behavior of the Hölder exponents for known sweet spots. This process can be applied to other well-log data of shale-gas reservoirs to compare results and generalize a rule about the fractal behavior in shale-gas reservoirs.

Introduction

Fractal analysis has become a popular tool in geophysics. Ouadfeul and Aliouane (2012) apply wavelet-based fractal analysis to amplitude-variation-with-offset (AVO) seismic data and obtain results showing the power of fractal analysis in the identification of heterogeneities from seismic data. Ouadfeul et al. (2012b) show the ability of generalized fractal dimensions to identify heterogeneities in the case of low signal-to-noise ratio. Ouadfeul and Aliouane (2011) establish a method of lithofacies segmentation from well-log data using the wavelet transform modulus maxima lines (WTMM).

In this article, we will explore the fractal behavior of the total-organic-carbon log in the presence of sweet spots. Data from a horizontal well drilled in the lower Barnett Shale are used. The sweet spots are discriminated using conventional well-log data analysis. Details of the determination of TOC can be found in Aliouane and Ouadfeul (2014).

We begin by introducing wavelet-based fractal analysis. Then a TOC log is analyzed by continuous wavelet transform (CWT). We end with interpretation of results and give a conclusion.

Wavelet-based fractal analysis

Here we review some of the important properties of wavelets, without any attempt at being complete. What makes wavelet transform special is that the set of basis functions, known as wavelets, are chosen to be well localized (have compact support) both in space and frequency (Arnéodo et al., 1988; Arnéodo et Bacry, 1995). Thus, one has some kind of dual localization of the wavelets. This contrasts with the Fourier transform, in which one has only monolocalization, meaning that localization in position and frequency simultaneously is not possible.

Grossmann and Morlet (1985) give the CWT of a function $s(z)$ as

$$C_s(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} s(z)\psi^*(z)dz. \quad (1)$$

Each family test function is derived from a single function defined by the analyzing wavelet according to (Torréasiani, 1995)

$$\psi_{a,b}(z) = \psi(\frac{z-b}{a}), \quad (2)$$

where $a \in R^+$ is a scale parameter, $b \in R$ is the translation, and ψ^* is the complex conjugate of ψ. The analyzing function, $\psi(z)$, generally is chosen to be well localized in space (or time) and wavenumber. Usually, $\psi(z)$ is required only to be of zero mean, but for the particular purpose of multiscale analysis, $\psi(z)$ also is required to be orthogonal to some low-order polynomials, up to the degree $n-1$ (Aliouane et al., 2012), i.e., to have n vanishing moments:

$$\int_{-\infty}^{+\infty} z^n\psi(z)dz = 0 \text{ for } 0 \le n \le p-1. \quad (3)$$

According to equation 3, the p order moment of the wavelet coefficients at scale a reproduces the scaling properties of the processes. Thus, while filtering out the trends, the wavelet transform reveals the local characteristics of a signal and, more precisely, its singularities.

It can be shown that the wavelet transform can reveal the local characteristics of s at a point z_0. More precisely, we have the following power-law relation (Ouadfeul and Aliouane, 2011):

$$\left|C_s(a,z_0)\right| \approx a^{h(z_0)}, \text{ when } a \to 0^+, \quad (4)$$

where h is the Hölder exponent (or singularity strength). The Hölder exponent can be understood as a global indicator of the local differentiability of a function s.

The scaling parameter (the so-called Hurst exponent) estimated when analyzing a process by using the Fourier transform (Ouadfeul and Aliouane, 2011) is a global measure of a self-affine process, whereas the singularity strength h can be considered a local version (i.e., it describes "local similarities") of the Hurst exponent. In the case of monofractal signals, which are characterized by the same singularity strength everywhere ($h(z)$ = constant), the Hurst exponent equals h. Depending on the value of h, the input signal could be long-range correlated ($h > 0.5$), uncorrelated ($h = 0.5$), or anticorrelated ($h < 0.5$) (Aliouane et al., 2012).

Application to real data

The Hölder exponent has been used widely as a tool for conventional hydrocarbon reservoir characterization. Ouadfeul et al. (2012a) suggest the use of the Hölder exponent estimated using the wavelet transform as a tool for lithofacies segmentation from

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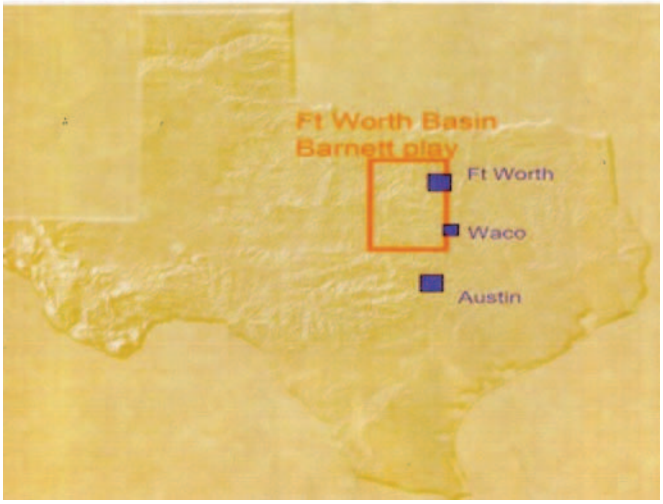

Figure 1. Geographic location of the Fort Worth Basin in Texas.

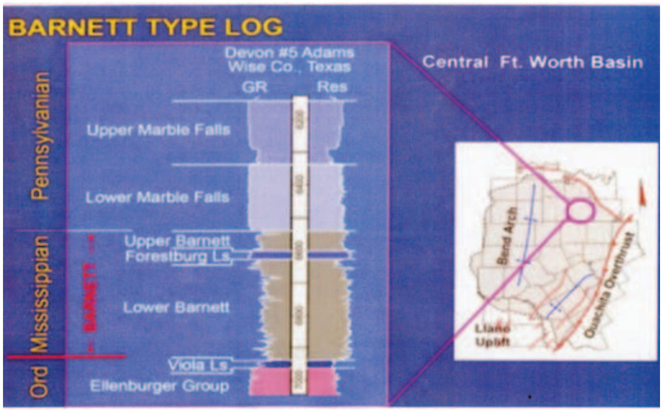

Figure 2. Stratigraphic column of Mississippian and Pennsylvanian ages as related to the Forth Worth Basin (Browning and Martin, 1980).

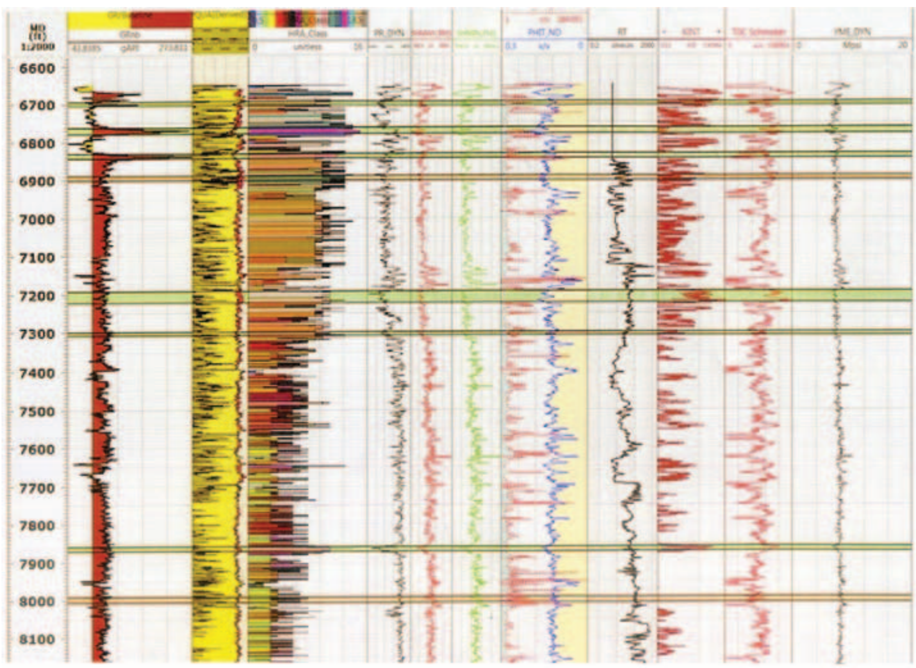

Figure 3. Well-log data and their associated sweet spots for horizontal well 01H. After Aliouane and Ouadfeul (2014), Figure 5.

well-log data. Results clearly show the usefulness of this exponent. Ouadfeul and Aliouane (2014) demonstrate the ability of the Hölder exponent to detect the oil-water contact by analyzing the AVO seismic attributes using the wavelet transform.

Fedi (2003) shows the ability of the Hölder exponent derived from the susceptibility log recorded in the Kontinentales Tiefbohrprogramm der Bundesrepublik Deutschland (KTB) main borehole to obtain evidence for several rather homogeneous zones. These results are well correlated to the lithologic units in the stratigraphic column.

The objective of this work is to examine the utility of the Hölder exponent in shale-gas reservoir characterization and particularly the fractal behavior of total organic carbon in sweet-spot intervals. To illustrate this, data from a horizontal well drilled in the Barnett Shale are analyzed. Let us start by describing the geologic context of the Barnett Shale.

Geologic setting. The Barnett Shale was deposited over present north-central Texas during the late Mississippian age at a time when marine transgression was caused by the closing of the lapetus Ocean basin (Figure 1). By the end of the Pennsylvanian, the Ouachita Thrust Belt began to encroach into the present north Texas area. The thrust belt owes its existence to the subduction of the South American Plate under the North American Plate. The emergence of the Ouachita Thrust Belt created the foreland basin along the front of the thrust. Early studies of the basin attributed thermal maturation of the Barnett Shale to burial history and the thermal regimes associated with depth of burial (Aliouane and Ouadfeul, 2014).

Explorationists began to doubt this hypothesis as more data became available, however. Kent Bowker, formerly of Mitchell Energy/Devon, proposed a different model (Bowker, 2003), suggesting that the maturation process was driven by displacement of hot fluids, from east to west, associated with the Ouachita Thrust (Aliouane and Ouadfeul, 2014).

Figure 2 shows the stratigraphic column of Mississippian and Pennsylvanian ages. Our shale-gas reservoir target is the lower Barnett Shale (Aliouane and Ouadfeul, 2014). The top of the reservoir is at 6650 ft (Givens and Zhao, 2014).

Data processing. To check the fractal behavior of the total-organic-carbon log in sweet-spot areas, we have used the results of Aliouane and Ouadfeul (2014). Figure 3 shows the sweet-spot intervals associated with all well logs. Sweet-spot areas are characterized by good geochemical, petrophysical, and geomechanical parameters. Analysis of Figure 3 shows the existence of eight sweet spots with the depth intervals mentioned in Table 1.

The TOC log is processed using wavelet-based fractal analysis. First, the modulus of the continuous wavelet

Table 1. Sweet-spot depth intervals extracted from Figure 3.

Sweet spot *n*°	Depth interval
1	6690 to 6703 ft
2	6760 to 6770 ft
3	6840 to 6850 ft
4	6895 to 6905 ft
5	7190 to 7220 ft
6	7300 to 7310 ft
7	7870 to 7880 ft
8	7990 to 8010 ft

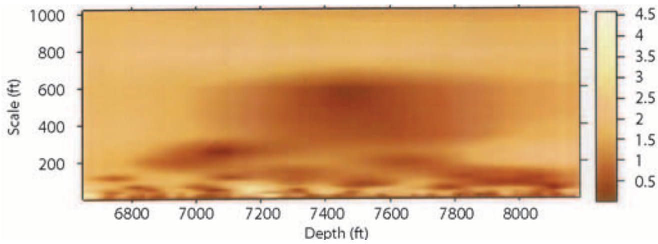

Figure 4. Modulus of continuous wavelet transform of the TOC log. The analyzing wavelet is the complex Morlet.

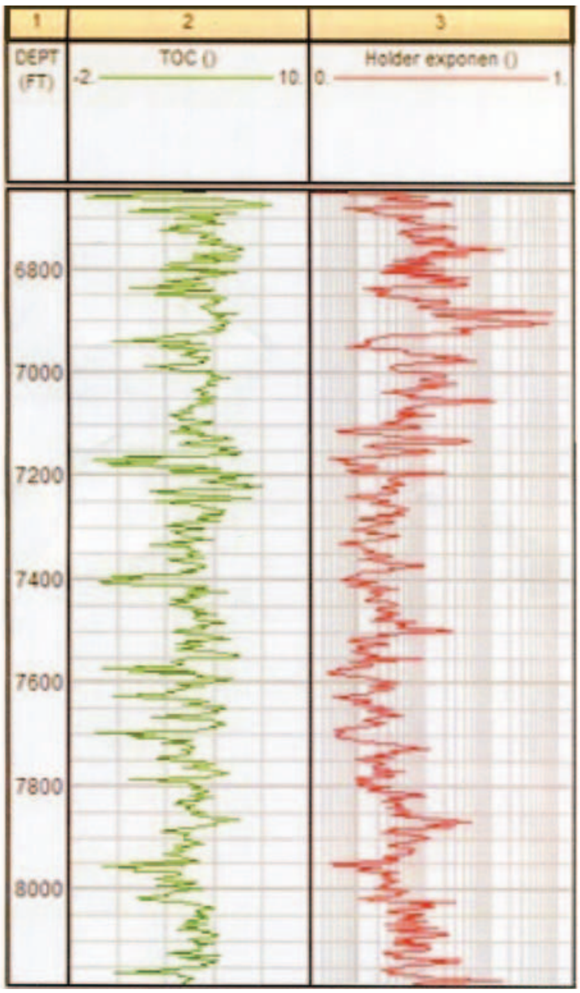

Figure 5. Total-organic-carbon log compared with the Hölder exponents estimated by using the continuous wavelet transform.

is calculated using the complex Morlet as an analyzing wavelet. Figure 4 shows this modulus versus the depth and scale. Hölder exponents then are calculated using the linear regression of the logarithm of the modulus of the continuous wavelet transform versus logarithm of the scale. Figure 5 presents the results with the TOC log.

Interpretation of results

Analysis of Figure 5 clearly shows that the Hölder exponents exhibit a special behavior for high and low values of TOC content. For example, the depth interval of 6845 ft through 6855 ft shows high values of Hölder exponents without moderate values of the TOC.

We can observe that the sweet spots defined in Table 1 are not characterized by any high values of Hölder exponents except in some depth intervals.

Conclusion

By consequence, the Hölder exponents cannot be used as a sweet-spot indicator and as a tool for shale-gas reservoir characterization. The results are important for unconventional hydrocarbon resource characterization because they indicate that care is needed in the use of the fractal analysis tool in exploration of such huge hydrocarbon resources.

We recommend the fractal analysis of other logs such as Poisson's ratio and Young's modulus to examine the behavior of the Hölder exponents of these logs in sweet-spot intervals. TLE

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Suggestion for further reading

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