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Systematic study of the isovector pairing effect on the moment of inertia of proton-rich nuclei in the region $30 \leq Z \leq 40$

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Abstract. A systematic study of the isovector neutron-proton (np) pairing effect on the moment of inertia is performed at zero temperature. This study is based on a recently established expression obtained using the framework of the quantum perturbation theory and the Inglis cranking method, at the limit when the temperature is nil.

We considered even–even proton-rich nuclei such as $30 \leq Z \leq 40$ and $N - Z = 0, 2, 4$ using the single-particle energies and eigen-states of a deformed Woods-Saxon mean-field. The obtained results are compared to their homologues of the conventional BCS theory (i. e. when only the pairing between like-particles is considered).

1. Introduction

The nuclear moment of inertia plays a very important role in the description of rotating nuclei. Indeed, this observable being sensitive to the deformation, its measurement indirectly provides significant information about the shape of the nuclei. The study of the moment of inertia has been the subject of several works taking into account the pairing correlations between like-particles at zero and finite temperature [1, 2, 3, 4]. It has been also calculated at zero and finite temperature with inclusion of isovector pairing correlations [5, 6, 7, 8, 9].
The aim of the present work is to carry out a systematic study of the isovector pairing effect on the moment of inertia of proton-rich nuclei in the region $30 \leq Z \leq 40$.

2. Formalism

Let us consider a heated nucleus constituted by N neutrons and Z protons. The symmetry axis being Oz, the system is cranked around the Ox axis of a rotating frame. The grand-partition function is given by [8, 9]:

$$Z = Tr\left\{\exp(-\beta[H - \sum_t \lambda_t N_t - \hbar\omega J_x])\right\} \tag{1}$$

where β is the inverse of the temperature T and H is the Hamiltonian of the system given, in the second quantization and isospin formalism, in the isovector pairing case, by [10]:

$$H = \sum_{\nu>0,t} \varepsilon_{\nu t} \left(a^\dagger_{\nu t} a_{\nu t} + a^\dagger_{\tilde{\nu} t} a_{\tilde{\nu} t} \right) - \sum_{tt'} G_{tt'} \sum_{\nu\mu>0} \left(a^\dagger_{\nu t} a^\dagger_{\tilde{\nu} t'} a_{\tilde{\mu} t'} a_{\mu t} + a^\dagger_{\nu t} a^\dagger_{\tilde{\nu} t'} a_{\tilde{\mu} t} a_{\mu t'} \right) \tag{2}$$

The subscript t corresponds to the isospin component $(t = n, p)$, $a^\dagger_{\nu t}$ and $a_{\nu t}$ respectively represent the creation and annihilation operators of the particle in the state $|\nu t\rangle$, of energy $\varepsilon_{\nu t}$; $|\widetilde{\nu} t\rangle$ is the time-reverse of $|\nu t\rangle$, $G_{tt'}$ characterizes the pairing-strength. The neutron and proton are supposed to occupy the same energy levels. In all that follows, it is assumed that the single-particle energies are independent from the temperature [11]. $\lambda_t (t = n, p)$ are the Fermi level energies and N_t are the particle-number operators given by:

$$N_t = \sum_{\nu>0} \left(a^\dagger_{\nu t} a_{\nu t} + a^\dagger_{\tilde{\nu} t} a_{\tilde{\nu} t} \right) \qquad t = n, p \tag{3}$$

ω is the rotation frequency and J_x is the projection over the Ox axis of the angular momentum. The usual Inglis [12] expression of the energy may be easily generalized in order to include the temperature effects as well as the np pairing correlations. The energy E is then given by [4]:

$$E = \left(-\frac{\partial \ln Z}{\partial \beta} \right)_{\lambda_t \beta = cte} \tag{4}$$

The expansion to the second order in ω of expression (4) is given by:

$$E \simeq E_0 - \omega^2 \hbar^2 \int_0^\beta \langle J_x (\beta) J_x (\chi) \rangle_0 \, d\chi \tag{5}$$

where

$$J_x (\kappa) = e^{\kappa H'} J_x e^{-\kappa H'} \; and \quad \langle J_x (\beta) J_x (\chi) \rangle = \frac{Tr e^{-\beta H'} J_x (\beta) J_x (\chi)}{Tr e^{-\beta H'}} \qquad \kappa = \beta, \chi \tag{6}$$

$J_x(\kappa)$ is the Heisenberg transform of J_x and the thermal average in Eq. (5) is evaluated using the grand-canonical ensemble associated to the Hamiltonian without rotation H' given by:

$$H' = H - \sum_{t=n,p} \lambda_t N_t \tag{7}$$

This thermal average value may easily be determined using the quasiparticle representation. In the latter, the auxiliary Hamiltonian has been approximately diagonalized by means of the Feynman path integral technique and using the Hubbard-Stratonovich transformation [10]. In fact, the diagonal form of the Hamiltonian H' becomes:

$$H' = \sum_{\nu>0,\ \tau=1,2} E_{\nu\tau} \left(\alpha^\dagger_{\nu\tau} \alpha_{\nu\tau} - \alpha_{\tilde{\nu}\tau} \alpha^\dagger_{\tilde{\nu}\tau} \right) + \sum_{\nu>0,t} \widetilde{\varepsilon}_{\nu t} \tag{8}$$

where we set

$$\widetilde{\varepsilon}_{\nu t} = (\varepsilon_{\nu t} - \lambda_t) - (G_{tt} + G_{np}) / 2 \qquad t = p, n \tag{9}$$

$\alpha^{\dagger}_{\nu\tau}$ (respectively $\alpha_{\nu\tau}$) is the creation (respectively annihilation) operator of a quasiparticle of $\tau\,(\tau=1,2)$ type, given by the generalized Bogoliubov-Valatin transformation [10]:

$$\alpha^{\dagger}_{\nu\tau} = \sum_{\nu>0,\ t} \left(u_{\nu\tau t} a^{\dagger}_{\nu t} + v_{\nu\tau t} a_{\tilde{\nu} t} \right) \qquad \tau = 1,2 \tag{10}$$

The corresponding energies $E_{\nu\tau}$ are defined in Ref. [10]. The perpendicular moment of inertia, is then defined by [8, 9]:

$$\Im^{np}_{\perp} = 2\hbar^2 \int_0^{\beta} \langle J_x(\beta)\, J_x(\chi) \rangle_0 \, d\chi \tag{11}$$

One has, after some algebra,

$$\begin{aligned} \Im^{np}_{\perp} &= \hbar^2 \sum_{\nu\mu\tau\tau' t t'} \langle \nu t \left| J_x \right| \mu t \rangle \langle \mu t' \left| J_x \right| \nu t' \rangle \\ &\left\{ \left[v_{\nu\tau t} u_{\mu\tau' t} - v_{\mu\tau' t} u_{\nu\tau t} \right] \left[v_{\nu\tau t'} u_{\mu\tau' t'} - v_{\mu\tau' t'} u_{\nu\tau t'} \right] \left(\frac{\tanh\left(\frac{\beta E_{\nu\tau}}{2}\right) + \tanh\left(\frac{\beta E_{\mu\tau'}}{2}\right)}{E_{\nu\tau} + E_{\mu\tau'}} \right) \right. \\ &\left. + \left[u_{\nu\tau t} u_{\mu\tau' t} + v_{\nu\tau t} v_{\mu\tau' t} \right] \left[u_{\nu\tau t'} u_{\mu\tau' t'} + v_{\nu\tau t'} v_{\mu\tau' t'} \right] \left(\frac{\tanh\left(\frac{\beta E_{\mu\tau'}}{2}\right) - \tanh\left(\frac{\beta E_{\nu\tau}}{2}\right)}{E_{\mu\tau'} - E_{\nu\tau}} \right) \right\} \end{aligned} \tag{12}$$

If the np pairing effects vanish, expression (12) reduces to the sum of the moments of inertia of the neutron and proton systems considered separately in the framework of the pairing between like-particles approach [4, 13, 14].

3. Numerical results-Discussion

The previously described formalism has been numerically applied using the single-particle energies and eigen-states of a deformed Woods-Saxon mean-field [15]. We used the parameters of Ref. [16] with a maximum number of shells $N_{max} = 12$. The ground state deformations are those of the Möller table [17]. We considered even-even nuclei such as $N - Z = 0, 2, 4$. Indeed, in such nuclei, it has been shown that np pairing effects are not negligible [18, 19]. In the present work, we considered only nuclei with a deformed ground-state such as $30 \leq Z \leq 40$. The pairing-strength values G_{pp}, G_{nn} and G_{np} have been deduced from the Δ_{pp}, Δ_{nn} and Δ_{np} values at zero temperature. The latter are obtained using the odd-even mass differences [20].
It appears that the inclusion of the np pairing effect clearly modifies the values of the moment of inertia obtained within the framework of usual BCS theory. Indeed, the relative discrepancy between the previsions of the present model and those of the BCS one varies from one nucleus to another and is about 15.57% on average for all the studied nuclei. In addition, the relative discrepancy between the theoretical values and the experimental data for $N = Z$ nuclei is on average 28.60% and 23.15% respectively with and without inclusion of the np pairing. With regard to other nuclei, this discrepancy is, with and without inclusion of the np pairing, respectively 27.11% and 37.73% for nuclei such as $(N - Z) = 2$ and 39.57% and 45.56% for nuclei such as $(N - Z) = 4$.
It is thus necessary to take into account this type of correlations in the evaluation of the moment of inertia of proton rich nuclei like those considered in this work.

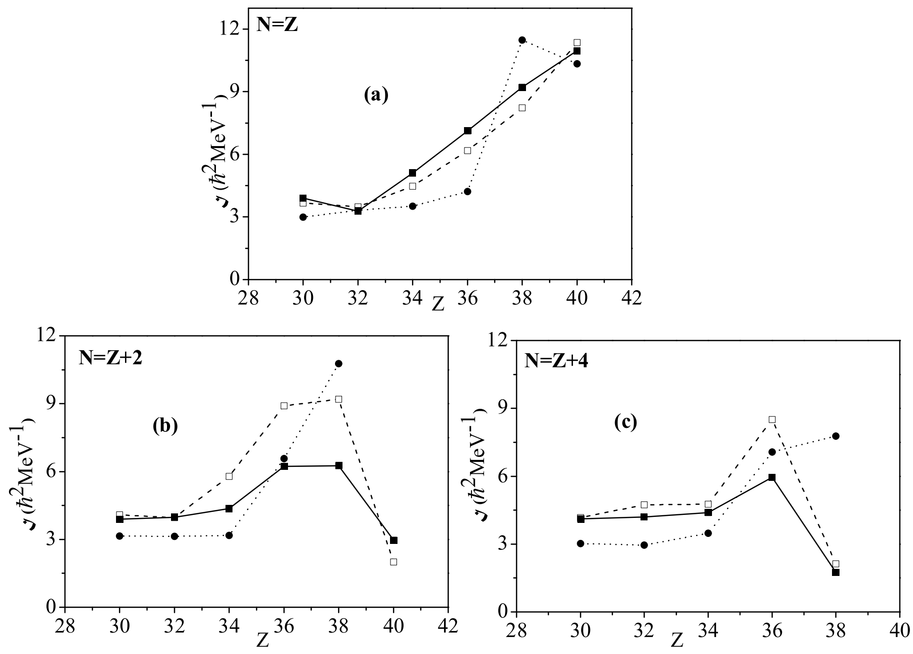

Figure 1. Variation of the moment of inertia as a function of the proton number Z, for nuclei such as $N = Z$ **(a)**, $N - Z = 2$ **(b)** and $N - Z = 4$ **(c)** and whose experimental values are known; with (−■−) and without (−□−) inclusion of np pairing. (− • −) represent the experimental values [21, 22].

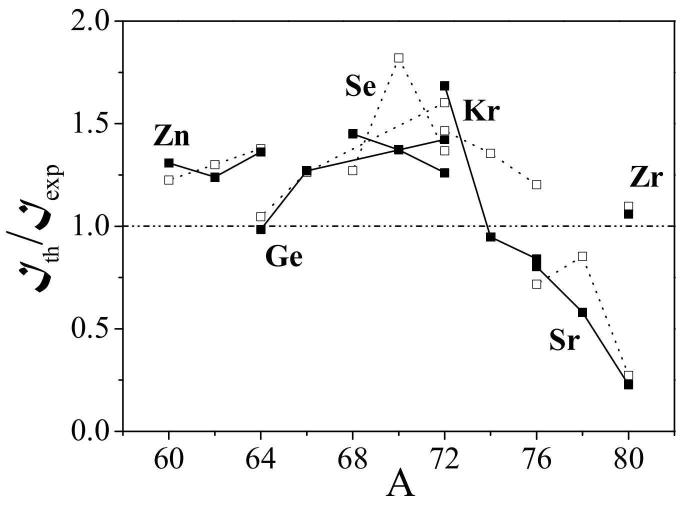

Figure 2. Points (−■−) are our results and (−□−) those of the BCS approximation .

Table 1. Moment of inertia of even-even proton-rich nuclei evaluated without (column 2) and with (column 3) inclusion of np pairing. The last column deals with the relative discrepancy.

Nucleus	$\Im_{FTBCS}(\hbar^2 MeV^{-1})$	$\Im_{\perp}^{np}(\hbar^2 MeV^{-1})$	$\frac{\Im_{\perp}^{np}-\Im_{FTBCS}}{\Im_{FTBCS}}$ %
^{60}Zn	3.66	3.91	6.80
^{62}Zn	4.09	3.90	4.78
^{64}Zn	4.17	4.12	1.14
^{64}Ge	3.48	3.27	5.96
^{66}Ge	3.96	3.98	0.71
^{68}Ge	4.73	4.20	11.23
^{68}Se	4.47	5.10	13.93
^{70}Se	5.78	4.36	24.52
^{72}Se	4.76	4.39	7.68
^{72}Kr	6.19	7.12	15.15
^{74}Kr	8.91	6.23	30.06
^{76}Kr	8.51	5.95	30.09
^{76}Sr	8.23	9.21	12.03
^{78}Sr	9.19	6.25	32.00
^{80}Sr	2.12	1.76	16.60
^{80}Zr	11.35	10.96	3.48
^{82}Zr	2.00	2.97	48.50

[1] Allal N. H and Fellah M, 1991 *Phys. Rev.* C **43** 2648.
[2] Hasegawa M and Tazaki S, 1993 *Phys. Rev.* C **47** 188.
[3] Zeng J. Y, Jin T. H and Zhao Z. J, 1994 *Phys. Rev.* C **50** 1388.
[4] Allal N. H, Fellah M, Benhamouda N and Oudih M. R, 2008 *Phys.Rev.* C **77** 054310.
[5] Mokhtari D, Ami I, Fellah M and Allal N.H, 2008 *Int. J. Mod. Phys.* E **17** 655.
[6] Gercekliočlu M and Calik A. E, 2005 *Acta Phys. Slo.* **55** 197.
[7] Calik E, Deniz C and Gercelioglu M, 2009 *Pramana-J. Phys.* **73** 847.
[8] Ami I, Fellah M, Benhamouda N, Allal N. H and Belabbas M, 2012 *J. Phys.: Conf. Series* **338** 012019.
[9] Ami I, Fellah M, Allal N.H, Benhamouda N, Belabbas M and Oudih M.R, 2011 *Int. J. Mod. Phys.* E **20** 1947.
[10] Fellah M, Allal N.H, Belabbas M, Oudih M.R and Benhamouda N, 2007 *Phys. Rev.* C **76** 047306.
[11] Brack M and Quentin P, 1981 *Nucl. Phys.* A **361** 35.
[12] Inglis D. R, 1954 *Phys. Rev.* **96** 1059; 1955 *Phys. Rev.***97** 701.
[13] Brack M et al., 1972 *Rev. Mod. Phys.* **44** 320.
[14] Brack M et al., 1974 *Nucl. Phys.* A **234** 185.
[15] Woods R. D and Saxon D. S, 1954 *Phys. Rev.* **95** 577.
[16] Allal N.H and Fellah M, 1994 *Phys. Rev.* C **50** 1404.
[17] Moller P, Nix J. R, Myers W. D and Swiatecki W. J, 1995 *Atomic Data and Nuclear Data Tables* **59** 185.
[18] Civitarese O, Reboiro O, 1997 *Phys. Rev.* C **56** 1179.
[19] Civitarese O, Reboiro M and Vogel P, 1997 *Phys. Rev.* C **56** 1840.
[20] Simkovic F, Moustakidis Ch. C, Pacearescu L and Faessler A, 2003 *Phys. Rev.* C **68** 054319.
[21] Firestone R. B and Shirley V. S, *Table of isotopes* (John and Wiley & Sons, New York,1999).
[22] Fischer S.M, Lister C. J and Balamuth D. P, 2003 *Phys. Rev.* C **67** 064318.