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SIMPLIFIED NUMERICAL SIMULATION OF TRANSIENTS IN GAS NETWORKS

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A set of equations governing an isothermal compressible fluid flow is resolved numerically for two practical cases. The first case concerns the fast fluid flow in short gas pipelines where the equations, written in conservative form, are resolved by a predictor-corrector scheme for the interior mesh points: an improved Lax-Friedricks scheme as a predictor and a leapfrog scheme as a corrector. Characteristics and upwind methods are used for the boundary conditions. The second case is concerned with massic slow fluid flow in relatively long gas pipelines. The equations, written in non conservative form, are resolved by a simple explicit finite difference scheme. The boundary conditions are considered by using the characteristic form of the equations including an inertial multiplier (Yow model) and resolved by a Newton-Raphson method. The obtained results agree with those of other methods. These numerical experiments permit the user to gain more computational time and simplicity in comparison with methods.

Keywords: gas transients; gas pipeline; characteristics method

INTRODUCTION

The mathematical model of transient gas flow in a pipe can be, as has been demonstrated in theory, one partial differential equation or a system of equations. The form of these equations varies with the assumptions made with regards to the operating conditions of the pipeline. The equations may be linear or, generally non linear. They may be parabolic or hyperbolic of the first or second order.

In this study, relatively simplified models will be presented. It is required that on the one hand the description of the phenomena is accurate, and on the other hand, that it is reasonable to use simple computational calculations to solve these models. As a rule, simple models are an alternative which presents a reasonable compromise between the description accuracy and the cost of solution. The simplified models are obtained by neglecting some terms in the basic set of equations, as a result of a quantitative estimation of the particular elements of the equations for given operating conditions of the pipeline. This means that the models of gas transient flow used for simulation should fit the given conditions of operation of the pipe. A necessary condition for proper selection of the model can, therefore, be the previous analysis of these conditions.

The methods for solving partial differential equations can generally be classified as analytical and numerical. The analytical methods are very laborious compared to numerical computations, and are unsuitable for solving problems of this kind. The finite difference methods, most frequently used to solve transient partial differential equations in the case of dynamic simulation of gas networks, are implicit and explicit methods both for parabolic and hyperbolic forms of the equations.

The most currently accepted numerical procedures include the method of characteristics, Wylie and Streeter[1], the explicit and the implicit finite difference methods, Wylie and Al[2] and Streeter and Wylie[3], and the variational methods. Racheford and Dupond[4]. With any of these methods, the numerical errors, especially the truncation error in the solution, may be appreciable if a large reach length (Δx) is used.

The characteristic method and the explicit method have obvious advantages by requiring relatively little computer storage compared to the implicit methods. But the ratio of the time increment to reach length ($\Delta t/\Delta x$) is limited by certain stability criteria so that the time increment is very small. The computational time becomes excessive for long duration transients.

Under isothermal conditions, the continuity and momentum equations, together with the state equations, constitute the governing equations describing transient flows in natural gas pipelines. From these equations, the aim of this study is to consider two types of gas flow in pipelines: fast and slow fluid flows. Taking into account the physical nature of these flows, different numerical methods are proposed for the resolution of the respective equation sets. The assumptions usually made include isothermal flow, applicability of steady-state friction and negligible wall expansion or contraction under pressure loads.

DEVELOPMENT OF THE MATHEMATICAL MODEL

Consider a pipeline with constant cross-sectional area, the one dimensional continuity equation for the gas is:

$$\frac{\partial \rho_g}{\partial t} + \frac{\partial}{\partial x}(\rho_g V) = 0 \tag{1}$$

The one dimensional momentum equation for gas flux in pipelines with spatially constant temperature distribution along the pipeline is given by:

$$\frac{\partial}{\partial t}(\rho_g V) + \frac{\partial}{\partial x}(\rho_g V^2) = -\frac{\partial P}{\partial x} - \frac{f_g \rho_g V|V|}{2D} - \rho_g g \frac{\partial z}{\partial x} \quad (2)$$

Writing an equation of state for natural gas as:

$$P = \rho_g \frac{ZRT}{\mu_g} \quad (3)$$

and taking into account the isothermal conditions, the acoustic wave speed becomes:

$$C = \left(\frac{ZRT}{\mu_g}\right)^{0.5} \quad (4)$$

The transient flows in pipelines may be divided as a function of the gas speed in fast and slow fluid transients. These cases have been studied by many authors in gas dynamics, Leveque and Yee[5], Wylie and Al[6], Racheford and Dupond[7] and Harten[8]. The aim here is to reformulate the above equations for practical applications. Setting $m = \rho_g V_g$ and putting equation (4) into (3), equations (1) and (2) may be rearranged in the following conservative form:

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{F}}{\partial x} = \vec{A} \quad (5)$$

Where

$$\vec{F} = \vec{F}(\vec{U}) \quad \text{and} \quad \vec{A} = \vec{A}(\vec{U}) \quad (6)$$

And

$$\vec{U} = \begin{pmatrix} \rho_g \\ m \end{pmatrix}; \quad \vec{F} = \begin{pmatrix} m \\ \dfrac{m^2}{\rho_g} + C^2 \rho_g \end{pmatrix}$$
$$\vec{A} = \begin{pmatrix} 0 \\ -\dfrac{f_g m|m|}{2D\rho_g} \end{pmatrix} \quad (7)$$

The above equation set is one dimensional, first order, non-linear and hyperbolic.

In the absence of field data, steady state variable distribution constitutes the initial conditions. These steady state initial conditions are obtained using an appropriate analytical equation written as in Zhou and Adewumi[9]:

$$\bar{\rho} = \frac{f_g m_1^2}{DC^2 \rho_{g_i} 2}\left(\frac{D}{f_g} \ln \bar{\rho} - \Delta l\right) + 1 \quad (8)$$

Where

$$\bar{\rho} = \left(\frac{\rho_g}{\rho_{g_i}}\right)^2$$

This equation, which is implicit in $\bar{\rho}$, is well suited for the iterative method to determine density or pressure distribution.

THE NUMERICAL SCHEMES

Fast Fluid Flow

Several classical numerical schemes were tested to integrate equation (5). These include the Lax-Wendroff, the MacCormack and the Godunov schemes, (respectively Leveque and Yee[5], Sod[10] and Warren[11]). These second order schemes, used for the above constitutive equations, have the advantage that shock wave problems and other discontinuities can be treated with relatively good accuracy.

This paper considers two practical cases. The first case concerns fast fluid flow in short gas pipelines, where the equations, written in conservative form, are resolved by a two time step predictor-corrector scheme for the interior mesh points: an improved Lax-Friedricks scheme as a predictor and a leapfrog scheme as a corrector. Characteristics and upwind methods are used for the boundary conditions.

Then the conservative form of equation sets (1) and (2) can be written as:

$$\frac{\partial q_{1i}}{\partial t} + \frac{\partial q_{2i}}{\partial x} = q_{3i} \quad (9)$$

Where q_{ij} (for $j = 1, 2, 3$ and $i = 1, 2$) are function of the density (or pressure) and the velocity (or mass rate).

The Lax-Friedricks scheme applied to equation (9), yields:

$$\begin{aligned} q_{1i}(x, t+\Delta t) = {} & 0.5 \times \lfloor q_{1i}(x+\Delta x, t) + q_{1i}(x-\Delta x, t\rfloor \\ & - [q_{2i}(x+\Delta x, t) - q_{2i}(x-\Delta x, t)] \\ & \times \Delta t/2/\Delta x + [q_{3i}(x+\Delta x, t) \\ & + q_{3i}(x-\Delta x, t)] \times \Delta t/2 \end{aligned} \quad (10)$$

This scheme is obtained by using a stabilizing procedure which consists of replacing $q_{li}(x, t)$ of the original scheme by $[q_{li}(x+\Delta x, t) + q_{li}(x-\Delta x, t)]/2$. This corresponds, in fact, to the addition of a dissipative term proportional to the second derivative of $q_{li}(x, t)$, Hirch[12].

The numerical viscosity is then introduced by the first term on the right hand side of equation (10). Unfortunately, this scheme causes considerable damping of the waves, owing to its first-order accuracy. This would lead to too low values for the maximum pressures. Second-order accuracy can be obtained by adding an adapted second step (leapfrog scheme) to equation (10):

$$\begin{aligned} q_{1i}(x, t+\Delta t) = {} & q_{1i}(x, t) - \begin{bmatrix} q_{2i}(x+\Delta x, t+\Delta t) - \\ q_{2i}(x-\Delta x, t+\Delta t) \end{bmatrix} \\ & \times \Delta t/\Delta x + 0.5 \times \Delta t \times \begin{bmatrix} q_{3i}(x+\Delta x, t+\Delta t) + \\ q_{3i}(x-\Delta x, t+\Delta t) \end{bmatrix} \end{aligned} \quad (11)$$

q_{3i} corresponds to the source terms in equations (1) and (2).

Equations (10) and (11) together constitute a predictor-corrector two-step, and also one among the Lax-Wendroff family schemes. The numerical damping by this scheme is appreciable, provided a sufficient number of mesh points is chosen. This scheme can be shown to be consistent with diffusional equations (9), and to be linearly stable if:

$$\frac{\Delta t}{\Delta x} \leq \frac{1}{C + |V|}$$

Slow Fluid Flow

The second case is concerned with slow fluid flow in relatively long gas pipelines. The non-conservative equations are resolved by a simple explicit finite difference scheme. The boundary conditions are considered by using the characteristic form of the equations including an inertial multiplier (Yow[13] model), and resolved by a Newton-Raphson method.

Substituting for ρ_g from equation (3) and writing the mass rate as $M = m \times A$, the relations (1) and (2) yield:

$$\frac{C^2}{A}\frac{\partial M}{\partial x} + \frac{\partial P}{\partial t} = 0 \tag{12}$$

$$\frac{\partial P}{\partial x} + \rho_g\left(V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t}\right) + \rho_g g\frac{\partial z}{\partial x} + f_g\rho_g\frac{V|V|}{2D} = 0 \tag{13}$$

Where A is the cross-sectional area of the flow.

Substituting for ρ_g from (3) and (4) yields:

$$v = \frac{MC^2}{AP}$$

Then the equation of motion (13) becomes:

$$\left[1 - \frac{M^2C^2}{A^2P^2}\right]\frac{\partial P}{\partial x} + \frac{2MB^2}{A^2P}\frac{\partial M}{\partial x} + \frac{1}{A}\frac{\partial M}{\partial t} + \frac{Pg}{C^2}\frac{\partial z}{\partial x} + f_g C^2\frac{M|M|}{2DA^2g^2P} = 0 \tag{14}$$

In this equation, the second term in the coefficient of $\partial P/\partial x$ is negligible compared to unity. Also, the term $\partial M/\partial x$ is negligible compared to the other terms. With these simplifications and multiplying equation (14) by P, the equation of motion for transient gas flow yields:

$$\frac{1}{2}\frac{\partial P^2}{\partial x} + \frac{P}{A}\frac{\partial M}{\partial t} + \frac{P^2g}{C^2}\frac{\partial z}{\partial x} + f_g C^2\frac{M|M|}{2DA^2} = 0 \tag{15}$$

Equations (12) and (15), describing the slow transient flow of gas in a pipeline, constitute a hyperbolic system of two non-linear partial differential equations which cannot be solved analytically. Any analytical solution must incorporate some simplifications, or assume some specific set of initial and boundary conditions.

Generally, the analytical solution thus generated reduces computational expense, but is only applicable to analysis of a subproblem, or a very simplified problem. Thus, these equations must be solved numerically.

Several types of numerical methods for solving the above system for gas transient flow have been reported in the literature: explicit and implicit finite difference methods, method of characteristics and variational methods. All these methods proceed in steps, computing the required parameter values (pressure and flow rate) at various points along the pipeline at the instant $t + \Delta t$, on the basis of the known distribution of these parameters along the pipeline at time t.

The partial differential equations (12) and (15) can be discretized along the space x and time t in three ways: backward, centred and forward difference schemes. Let Δx be the length of each pipe segment; solutions are computed successively at the value of time, increasing in time steps of size Δt. Then, using the explicit forward finite difference method, the partial differentials (12) and (15) can be expanded for any grid point i at time $t + \Delta t$ as follows:

$$\frac{\partial P}{\partial t} = \frac{P_i^t + \Delta t - P_i^t}{\Delta t}$$

$$\frac{\partial P^2}{\partial x} = \frac{P_{i+1}^t - P_i^t}{\Delta x}$$

$$\frac{\partial M}{\partial t} = \frac{M_i^t + \Delta t - M_i^t}{\Delta x}$$

$$\frac{\partial M}{\partial x} = \frac{M_{i+1}^t - M_i^t}{\Delta x}$$

Where the subscript i indicates the grid along the x direction, and superscript t indicates the parameter value at the previous time step. The pressure P and the mass flow M are discretized as follows:

$$P = \frac{P_i^t + P_{i+1}^t}{2} \quad \text{and} \quad M = \frac{M_i^t + M_{i+1}^t}{2}$$

Then the transient flow equations (12) and (15), can be linearized using the above finite difference equations.

Solving for $P_i^{t+\Delta t}$ and $M_i^{t+\Delta t}$ yields:

$$P_i^{t+\Delta t} = P_i^t - \frac{\Delta t}{\Delta x}\frac{C^2}{A}[M_{i+1}^t - M_i^t] \tag{16}$$

$$M_i^{t+\Delta t} = M_i^t - \frac{\Delta t}{\Delta x}\left(\frac{A}{P_i^t + P_{i+1}^t}\right)x((P^t)_{i+1}^2 - (P^t)_i^2) - \frac{Ag\,\Delta t}{2(P_i^t + P_{i+1}^t)C^2}\sin\alpha - f_g\frac{C^2\Delta t}{4DA(P_i^t + P_{i+1}^t)}x \times [(M_i^t + M_{i+1}^t)|M_i^t + M_{i+1}^t|] \tag{17}$$

The first previous methods are faster because they require lesser computation time than implicit methods, but they are subject to instability and a restricted time step size. For these reasons, and also because of the inaccuracies in the computations that generally result from the use of this method, it is proposed in the following section, to use it in conjunction with the method of characteristics.

In this study, the choice of an explicit method for the mesh interior points and characteristics form of the equations for the boundaries, responds mainly to the requirement to minimize the computational time of the program.

INITIAL AND BOUNDARY CONDITIONS

Initial conditions, as well as boundary conditions to the previous equation sets, must be specified in order to obtain an appreciable solution for the differential equations (1), (2) and (12), (15). The initial conditions of these systems are required to resolve initial density and velocity for the first case, and pressure and mass rate for the second case, as a function of the position x along the pipeline. Boundary conditions must also be specified to obtain a unique solution. In this study, the initial conditions are given by the relations (8) and (13) for fast and slow gas flow, respectively.

Boundary Conditions for the Fast Fluid Flow

Physical boundary conditions are imposed to allow the consideration of a wide variety of operating situations. The practical example used considers the following case: a constant mass flux (or a known function of time) at the inlet when the outlet mass flux is a known function of time (or a constant). Densities or pressures at the boundaries constitute the unknowns. It is proposed in this study to treat numerically the two boundary conditions by the characteristics method and by an upwind finite difference method.

The first method converts the initial partial differential equation sets (1) and (2) into ordinary differential equations. The physical interpretation is that the waves travel with speed C which is given by the relation (4), propagating then the effect of the initial boundary conditions. This theory is generally used in different ways to follow wave development. Then the transformation of equations (1) and (2) into ordinary differential form yields:

$$dP \pm \rho_g C dV = \pm \rho_g C \frac{f_g |V| V}{2D} dt \mp \rho_g g C \sin\theta dt \qquad (18a)$$

A computational procedure to obtain P or V is necessary with incremental Δt, and equal space Δx. Values of the fluid properties at the previous time are interpolated linearly for subsonic flow from mesh $i-1$, i and $i+1$.

The second method is based on upwind differentiation of the relation (1) at the boundaries. The relations used by Zhou and Awedumi[14] are considered as:

$$(\rho_g)_1^{t+\Delta t} = (\rho_g)_1^t + \frac{\Delta t}{\Delta x}(m_1^t - m_2^t)\frac{1 - \dfrac{m_1^t}{C(\rho_g)_1^n}}{1 + \dfrac{m_1^t}{C(\rho_g)_1^t}} - \frac{\Delta t}{\Delta x}(m_1^t - m_2^n)\left(\frac{m_1^t}{(\rho_g)_1^n} - C\right) + \frac{m_1^{t+\Delta t} - m_1^t}{C + \dfrac{m_1^t}{(\rho_g)_1^t}} + \frac{f_g m_1^t |m_1^t| \Delta t}{2D(\rho_g)_1^t \left(C + \dfrac{m_1^t}{(\rho_g)_1^t}\right)} \qquad (18b)$$

$$(\rho_g)_{n+1}^{t+\Delta t} = (\rho_g)_{n+1}^t \frac{\Delta t}{\Delta x}(m_{n+1}^t - m_n^t)\frac{1 + \dfrac{m_{n+1}^t}{C(\rho_g)_{n+1}^t}}{1 - \dfrac{m_{n+1}^t}{C(\rho_g)_{n+1}^t}} - \frac{\Delta t}{\Delta x}(m_{n+1}^t - m_n^t)\left(\frac{m_{n+1}^t}{(\rho_g)_{n+1}^t} + C\right) - \frac{m_{n+1}^{t+\Delta t} - m_{n+1}^t}{C + \dfrac{m_{n+1}^t}{(\rho_g)_{n+1}^t}} - \frac{f_g m_{n+1}^1 |m_{n+1}^t| \Delta t}{2D(\rho_g)_{n+1}^t \left(C - \dfrac{m_{n+1}^t}{(\rho_g)_{n+1}^t}\right)} \qquad (18c)$$

The results obtained are in relatively good agreement for the first times of the transient, i.e. the times corresponding to the Figures 1, 2, 3 and 4. Unfortunately, instabilities were observed for the time corresponding to Figure 5.

Boundary Conditions for the Slow Fluid Flow

For the reasons involved in the section on slow fluid flow, this explicit scheme is rarely used. Nevertheless, it is possible to avoid the previous undesirable instabilities by using the characteristics method to resolve equations (12) and (13) for the boundaries. Rewriting these equations without taking into account the gas inertia, the corresponding compatibility equations obtained by the classical mathematical method (Wylie and Streeter[1]) are:

$$\alpha \frac{C}{A}(M_i^{t+\Delta t} - M_{i\pm 1}^t) \pm P_i^{t+\Delta t} \mp P_{i\mp 1}^t + f_g \frac{C_{\Delta x}^2}{DA(P_i^{t+\Delta t} + P_{i\pm 1}^t)} X \frac{e^S - 1}{s} \times \left. \frac{M_i^{t+\Delta t} + M_{i\pm 1}^t}{2} \left| \frac{M_i^{t+\Delta t} + M_{i\pm 1}^t}{2} \right| \right) + \frac{(P_i^{t+\Delta t})^2}{P_i^{t+\Delta t} + P_{i\pm 1}^t}(e^S - 1) = 0 \qquad (19)$$

Where:

$$s = (2g\Delta x \sin\theta)/C^2$$

The parameter α is the so-called 'inertial multiplier,' introduced by Yow[15] in the equation of gas motion. He made an extensive analysis of the value of α to error, based on gas flow in a single horizontal pipe with sine wave variation of the input boundary condition. His procedures also permitted a study of this discretization on error. Then, in equation (19) a second order evaluation of the friction term is used by averaging linearly the mass flow. An iterative Newton-Raphson method permits discovery of the unknown functions P and M in the boundaries for the time $t + \Delta t$. The signs + and − correspond respectively to outlet and inlet condition. In the current application, the inertial multiplier α is calculated by a more suitable engineering method (Wylie and Streeter[1]).

RESULTS AND DISCUSSION

Two practical examples, corresponding to fast and slow fluid flows, are simulated using both previously combined methods. The boundary conditions for these two cases are treated by characteristics methods with changing the primitive variables of the corresponding equation set of the fast fluid flow.

The first example, which concerns the transport of a fast gas transient in a short pipeline with an impulse supply of gas mass flux at the inlet of the line, has been simulated using the previous predictor-corrector scheme. This example is taken from Wylie and Al[2], Racheford and Dupond[4] and Zhou and Adewumi[14] in which the solutions were obtained respectively using the method of characteristics (MOC), variational methods, and first-order three-point explicit Godunov scheme, and the second-order five-point

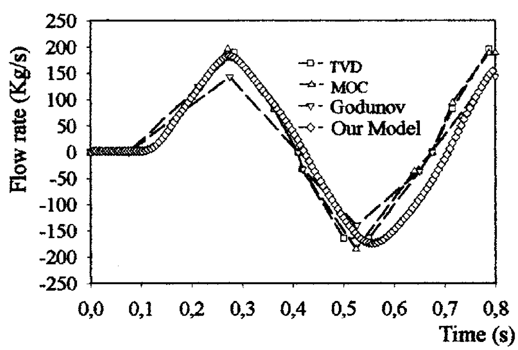

Figure 1. Fast fluid flow: Flow rate version at the midpoint of the pipeline.

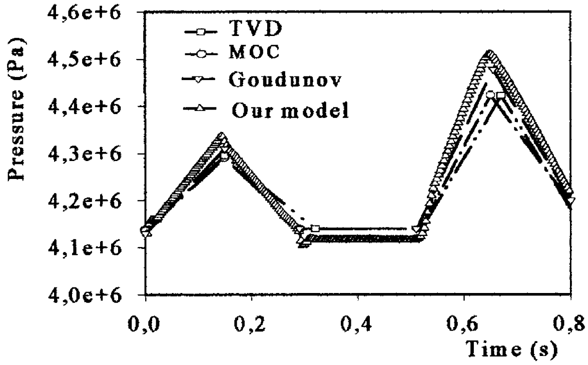

Figure 2. Fast fluid flow: Pressure at the inlet of the pipe.

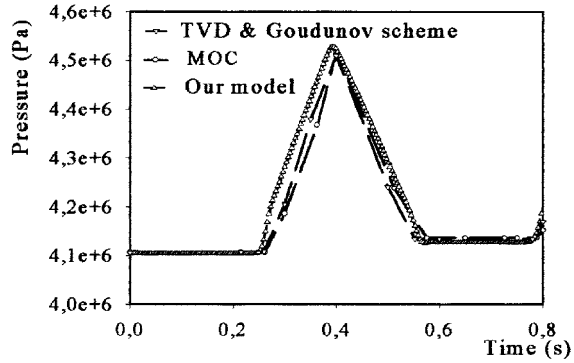

Figure 4. Fast fluid flow: Pressure at the outlet of the pipe.

TVD scheme for the source free to solve the full set of the governing equations.

A pipeline 91.44 m long, 0.609 m interior diameter and having initially $41.368 \times 10^5\ \mathrm{N\,m^{-2}}$ with a shut downstream extremity. At time zero, the upstream inflow begins to increase linearly and reaches $41.368 \times 10^5\ \mathrm{N\,m^{-2}}$ at 0.145 seconds, decreases linearly to zero again at 0.29 seconds, and then remains constant at zero. The downstream end is closed. For the simulation of the above fast transient problem, the predictor-corrector scheme adopts with the characteristics method the same Δt which is imposed by the stability criteria of Courant, Friedricks and Lewy (Wylie and Streeter[1]).

Using the predictor corrector scheme (10) and (11) to resolve system (9), Figure 1 shows mass rate time evolution, rate of the gas at midpoint of the gasline ($x = 0.5 \times \mathrm{L}$), where these results are compared with those obtained by the authors referenced[2,14]. A good agreement between them is observed. The gas flow rate fronts are completely solved within the first 0.8 seconds. The behaviour of the gas flow rate evolution at the midpoint of the pipeline is the result of the reflected pressure impulse at the upstream end of the pipe.

Figure 2 shows the comparison of the predicted pressure (by the present model) at the inlet of the line ($x/L = 0$) with the reported data. Again, a relatively good agreement between the predicted results and the reported data is obtained. It can be seen that the duration of the pressure pulse of the peak is the same. The pressure wave is maintained and captured during the first 0.80 seconds.

In Figure 3, the pressure wave front is reproduced over 2.4 seconds without significant loss of accuracy. However, the pressure amplitude seems close to that obtained by the characteristics method. The slight differences with the curves which correspond to the other methods may be due to some simplifications introduced by Zhou and Adewumi[14] (i.e. the value of the coefficient of the friction losses).

Figure 4 shows the comparison, as regards the pressure at the outlet point of the pipeline ($x/L = 1$), between the obtained numerical results and the reported data. Again, a relatively good agreement can be noted. At the outlet of the pipeline, the pressure wave fronts are completely resolved within the first 0.8 seconds.

In order to check the numerical method described herein, the pressure evolution was calculated for the times close to the end of the transient phenomenon. Figure 5 shows that the agreement is satisfactory, in comparison with those of other authors. This indicates that the method described in the previous sections is reliable. Also, the computed results show that the introduced numerical damping has not produced any undesirable effect.

The second example is the propagation of slow transient, for 30 minutes, in a 1.930×10^3 m long, 0.365 m interior diameter transmission pipeline, with a specified sinusoidal boundary condition at the downstream (Figure 6). The procedure to determine Δx and α is illustrated in the original study of Yow[13], where the solution obtained with the standard characteristics method is compared to an accurate solution. The sinusoidal boundary condition takes the following form:

$$M = M_o + \Delta M \sin(\omega t) \tag{20}$$

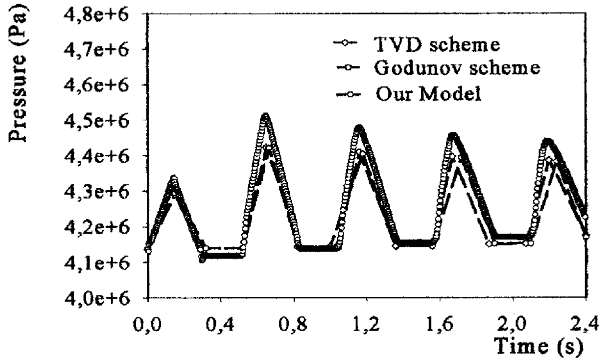

Figure 3. Fast fluid flow: Pressure at the inlet of the pipeline.

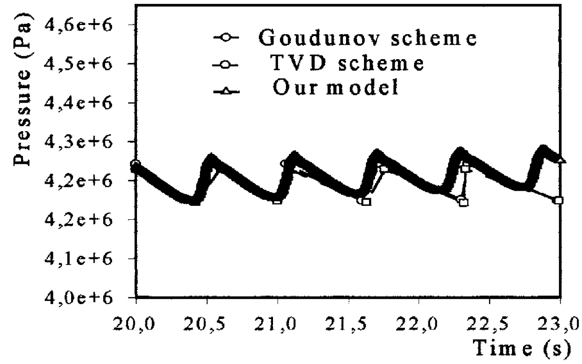

Figure 5. Pressure response at the inlet of the pipe.

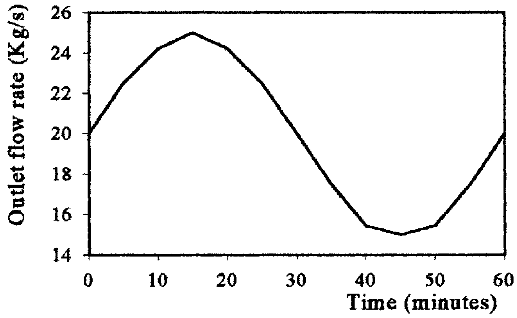

Figure 6. Mass rate variation at the outlet of the pipeline.

where the mass initial rate $M_0 = 20\,\mathrm{kg\,s^{-1}}$, t is in minutes and ω the frequency in rad/sec.

For this example, Yow[13] compares the obtained accurate (analytical) solution for an equation set equivalent to (12) and (14) (where $\Delta x = L/10$ and $\alpha = 1$), with those resulting from using the equivalent relations to (19). In this last case two values of α (3 and 8) are tested.

Figure 7 shows the predicted gas mass rate curves at the inlet of the pipeline by using the relations (16) and (17) in the grid interior points, and (19) for the boundaries. In comparison with the previous author results[13], the agreement is very good. The present model reproduces the propagation of the imposed outlet signal very well, without any attenuation.

The pressure curves at the outlet of the pipeline are drawn in Figure 8. In this figure, the obtained gas pressure evolution at the inlet is closer to the measured data than that predicted by the previous author[13]. Hence the agreement between the obtained results and the measured and reported data is good, and it appears to be fairly in agreement with the accurate solution.

Thus, it is considered that the numerical models presented in the previous section, describing slow fluid flow for gas pipelines, without neglecting any important term in the momentum equation represents faster programming and a more economical solution.

CONCLUSION

The numerical methods presented in this study allow calculation of the propagation of pressure or mass rate perturbations from one boundary of a gas pipeline. It is shown that the coupling of two different types of finite difference schemes, can reduce the computational time and also avoid the instabilities, errors and difficulties existing during the use of sophisticated finite difference schemes, especially for more complex gas networks distribution.

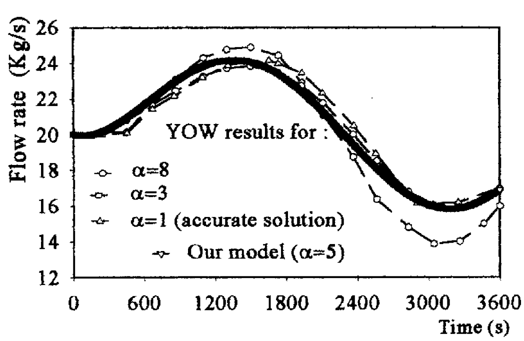

Figure 7. Slow fluid flow: Flow rate at the inlet of the pipeline.

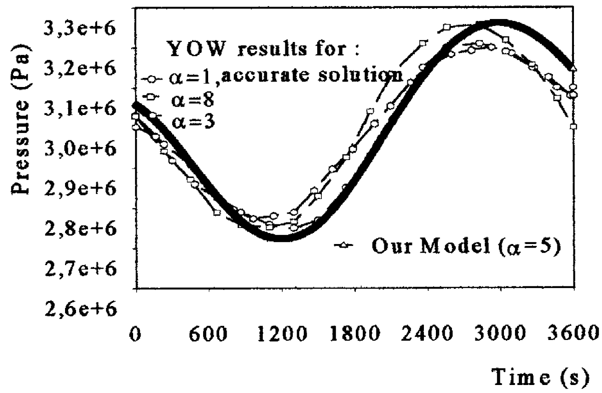

Figure 8. Transient pressure evolution at the outlet of the pipeline.

NOMENCLATURE

A	cross-sectional area of pipeline, vector
C	isothermal speed of sound
d	pipeline diameter
fg	gas friction factor
F	flux vector in equation (5)
g	gravitational acceleration
m	gas mass flux
M	gas mass rate
p	pressure
R	universal gas constant
t	time
T	absolute gas temperature
V	gas velocity
X	axial coordinate, L
z	compressibility factor

Greek letters

ω	pulsation
ρ_g	gas density
ρ_i	inlet gas density
μ_g	molecular gas weight
Δt	uniform time step
Δx	uniform grid size
θ	angle of the pipe makes with the horizontal

Subscripts

g	gas
i	inlet, upstream, node

REFERENCES

1. Wylie, E. B. and Streeter, V. L., 1969, *Fluid Transients*, (McGraw Hill, New York) pp 271–280.
2. Wylie, E. B., Stoner, M. A. and Streeter, V. L., 1971, Network system transient calculation by implicit methods, *SPEJ*, December.
3. Streeter, V. L. and Wylie, E. B., 1969, Natural gas pipeline transients, *SPEJ*, December pp 357–364.
4. Racheford, H. H. and Dupond, T., 1974, A fast high, accurate means of modeling transient flow in gas pipeline systems by variational methods, *SPEJ*, April.
5. Leveque, R. J. and Yee, H. C., 1990, A study of numerical methods for hyperbolic conservation laws with stiff source terms, *J Comp Phys*, 86: 187–210.
6. Wylie, E. B., Streeter, V. L. and Stiner, M. A., 1974, Unsteady state natural gas calculations in complex piping systems, *SPEJ*, February.
7. Racheford, H. H. and Dupond, T., 1974, Some application of transient

flow simulation to promote understanding the performance gas pipeline systems, *SPEJ*, April.
8. Harten, A., 1983, High resolution schemes for hyperbolic conservation laws, *J Comp Phys*, 49.
9. Zhou, Junyang and Adewumi, M. A., 1995, The development and testing of a new flow equations, *Proc 1995 pipeline Simulation Interest Group 27th Annual Meeting, Albuquerque, NM, Oct.*, 19–20.
10. Sod, G. A., 1978, A survey of several finite difference methods systems of nonlinear conservation laws, *J Comp Phys*, 27: 1–35.
11. Warren, M. D., 1984, Open-end outflow boundary approximations for the solution of the equations of unsteady one-dimensional gas flow by the Lax-Wendroff method, *Int J Heat & Fluid Flow*, 5(1).
12. Hirch, C., 1997, *Numerical Computation of Internal and External Flows*, Vol 2. (John Wiley & Sons) pp 224–240.
13. Yow, W., 1972, Numerical error in natural gas transient calculations, *ASME Trans J of Fluids Eng*, 94(2): 422–428.
14. Juanyang Zhou and Adewumi, M. A., 1996, Simulation of transients in natural gas pipelines, *SPEJ*, November, 202–208.

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