Fuzzy Distributed Control of the Forced Burgers-Fisher Equation under Periodic Boundary Conditions

Sadia RAAB and Hacene HABBI
Applied Automation Laboratory
M'Hamed Bougara University of Boumerdès, Boumerdès, Algeria
Email: sadiazrl@yahoo.fr
Email: habbi@univ-boumerdes.dz

Ahmed MAIDI
Design and Control of Production Systems Laboratory, FGEI,
University of Tizi-Ouzou, Algeria
Email: ahmed.maidi@gmail.com

Karim BELHARET
Laboratoire PRISME, HEI
Ecole d'Ingénieur, Campus Centre
36000, Châteauroux, France
Email: karim.belharet@yncrea.fr

***Abstract*—This paper presents a novel control design methodology for a class of distributed parameter systems (DPSs) described by the nonlinear Burgers-Fisher equation model. The proposed approach is built upon a fuzzy Burgers-Fisher equation model of T-S type which is derived to approximate the nonlinear DPS dynamics over the spatio-temporal dimension. Relying on the geometric control theory, a complete control design procedure is developed based on the constructed fuzzy Burgers-Fisher equation model with distributed control. The resulting fuzzy distributed controller is infinite dimensional showing good stabilizing effect and performs considerably well within the specified operating conditions.**

***Keywords*—Burgers-Fisher equation, fuzzy model, distributed controller, geometric control, stabilization.**

I. INTRODUCTION

Although the application of linear geometric control tools to nonlinear distributed parameter systems (NDPSs) has shown some primary interesting results, this major research field of interest remains still less developed and many open problems need further attention [2] [3]. From practical viewpoint, it becomes actually relatively feasible to deal with distributed control problems with the growing technology of distributed actuators and sensors acting on the full spatial domain of the NDPS. Problem formulation and solving also gained interesting findings with the aid, for instance, of the concept of semi-group [1] [4] which has been used to extend the geometric control tools developed for lumped parameter systems (LPSs) to distributed parameter ones.

Among the manipulated classes of NDPSs, nonlinear Burgers-Fisher equation models [5] [6], described mathematically by nonlinear partial differential equations (PDEs), can basically be controlled by distributed actuations. For control purpose, two main approaches are employed: the early and the late lumping approaches [7] [10]. The early lumping approach is based on approximation methods [10] [11] [12] that are applied to the infinite dimensional DPS in order to derive a finite dimensional representation. The late lumping approach involves the distributed dynamics directly in the control design procedure without any prior model reduction. Obviously, this may lead to an infinite dimensional control realization with distributed nature [14] [13] [15]. Designing a late lumping controller is generally achieved by assuming an operating point [7], which helps obtaining a local linear PDE model based on which appropriate linear control tools [16] could be applied considering a given operating region. Another interesting idea consists in using the multi-modeling approach by assuming multiple operating points [18] [24] [26] [17]. On this issue, fuzzy modeling represents an effective paradigm that has shown satisfactory results in a variety of applications of model-based control design methods [19] [22] [23] [27].

In this paper, a fuzzy distributed geometric controller is developed for a NDPS represented by a Takagi-Sugeno (T-S) fuzzy Burgers-Fisher equation model with periodic boundary conditions. The design methodology is achieved from a local viewpoint assuming local dynamics described by nonlinear Burgers-Fisher equation models. To apply the geometric control tools, we considered a T-S fuzzy Burgers-Fisher equation model with distributed control based on which an infinite dimensional geometric control methodology is developed. The effectiveness of the proposed fuzzy distributed geometric controller is checked through numerical simulation.

The rest of the paper is organized as follows. Section II describes briefly the Burgers-Fisher equation model and provides a general formulation of the fuzzy control problem to solve. The detailed control design procedure of the proposed fuzzy distributed controller is given in Section III. The performance of the proposed control strategy on the evolution of the forced Burgers-Fisher equation is shown in Section IV. The paper ends with a conclusion.

II. FUZZY CONTROL PROBLEM FORMULATION

A. *The Burgers-Fisher Equation Model*

Consider the class of nonlinear distributed parameter systems described by the one-dimensional nonlinear Burges-Fisher equation with distributed control in the space domain:

$$\frac{\partial x(z,t)}{\partial t} = \nu \frac{\partial^2 x(z,t)}{\partial z^2} - x(z,t)\,\frac{\partial x(z,t)}{\partial z} + f\big(x(z,t)\big) + b(z)\,u(t) \tag{1}$$

978-1-7281-6999-6/20/$31.00 ©2020 IEEE

subject to the periodic boundary conditions:

$$x(0,t) = x(l,t), \qquad \left.\frac{\partial x(z,t)}{\partial z}\right|_{z=0} = \left.\frac{\partial x(z,t)}{\partial z}\right|_{z=l} \tag{2}$$

and the initial condition:

$$x(z,0) = x_0(z) \tag{3}$$

where $x(z,t)$ is the state variable, $z \in \Omega = [0,l]$ and $t \in [0,+\infty)$ are the space position and time variables, respectively, Ω is the spatial domain and ν is a positive constant. $u(t)$ denotes the manipulated distributed control input, $b(z)$ is a known smooth function of z that characterizes the distribution of $u(t)$ on Ω and $x_0(z)$ is the initial spatial profile. $f\big(x(z,t)\big)$ is a sufficiently smooth nonlinear function in $x(z,t)$ that satisfies $f(0) = 0$.

Defining the controlled output $y(t)$, which represents the spatial weighted mean value of the state variable $x(z,t)$ along the *z*-axis, as:

$$y(t) = \int_0^l c(z)\, x(z,t)\, dz \tag{4}$$

where $c(z)$ is a smooth shaping function of z, it follows the general control objective which consists in the design of a distributed control $u(t)$ for the DPS system (1)-(3) that should achieve perfect tracking of the desired set-point reference.

Assumption 1: The function $c(z) \in L^2(\Omega)$ is not orthogonal to $b(z)$ on Ω, that is,

$$\langle c(z), b(z)\rangle = \int_0^l c(z)\, b(z)\, dz \neq 0 \tag{5}$$

Remark 1: $L^2(z)$ being the space of square-integrable functions endowed with the inner product (5) and the norm $\|c(z)\|_{L^2(z)} = \langle c(z), c(z)\rangle^{1/2}$ [28].

B. *The fuzzy distributed control form*

The control design method described in the present contribution relies on a T-S fuzzy Burgers-Fisher equation model which is derived by means of the sector nonlinearity method [19], in order to describe exactly the dynamical spatio-temporal behavior of the nonlinear Burgers-Fisher equation (1)-(3). Applying this concept, the following fuzzy IF-THEN model representation can be obtained:

Model Rule *i*:
IF $\mu_1(\zeta,t)$ is G_{i1} and … and $\mu_p(\zeta,t)$ is G_{ip},
THEN

$$\frac{\partial x(z,t)}{\partial t} = \nu \frac{\partial^2 x(z,t)}{\partial z^2} - \gamma_i \frac{\partial x(z,t)}{\partial z} + \beta_i\, x(z,t) + b(z)\, u_i(t) \tag{6}$$

subject to the periodic boundary conditions:

$$x(0,t) = x(l,t), \qquad \left.\frac{\partial x(z,t)}{\partial z}\right|_{z=0} = \left.\frac{\partial x(z,t)}{\partial z}\right|_{z=l} \tag{7}$$

where G_{ij}, $i \in \mathbb{S} \triangleq \{1,2,\dots,r\}$, $j \in \{1,2,\dots,p\}$ are fuzzy sets, $\mu_j(\zeta,t)$ are the premise variables and r is the number of IF-THEN rules. γ_i and β_i are real known constants. The operating domain is defined as $\mathfrak{J} \triangleq \{x(z,t) \in \mathcal{L}^2([0,l])|\ \alpha_{min} \leq x(z,t) \leq \alpha_{max}\}$, where $\alpha_{min} \geq 0$ and $\alpha_{max} \geq 0$ are known parameters.

Applying fuzzy operators, the overall fuzzy Burgers-Fisher equation dynamics can be expressed as follows:

$$\frac{\partial x(z,t)}{\partial t} = \nu \frac{\partial^2 x(z,t)}{\partial z^2} + \left(\sum_{i=1}^{r} h_i\big(\mu(\zeta,t)\big)\right) \times \left(-\gamma_i \frac{\partial x(z,t)}{\partial z} + \beta_i\, x(z,t) + b(z)\, u_i(t)\right) \tag{8}$$

where $\mu(\zeta,t) = \big[\mu_1(\zeta,t) \dots \mu_p(\zeta,t)\big]^T$, and $h_i\big(\mu(\zeta,t)\big) = w_i\big(\mu(\zeta,t)\big)/\sum_{i=1}^{r} w_i\big(\mu(\zeta,t)\big)$, $i \in \mathbb{S}$, with $w_i\big(\mu(\zeta,t)\big) = \prod_{j=1}^{p} G_{ij}\big(\mu_j(\zeta,t)\big)$ such that $w_i\big(\mu(\zeta,t)\big) \geq 0, i \in \mathbb{S}$ and $\sum_{i=1}^{r} w_i\big(\mu(\zeta,t)\big) > 0$ for all $z \in [0,l]$ and $t \geq 0$. Then, we can obtain the following conditions: $h_i\big(\mu(\zeta,t)\big) \geq 0, i \in \mathbb{S}$ and $\sum_{i=1}^{r} h_i\big(\mu(\zeta,t)\big) = 1$ for all $z \in [0,l]$ and $t \geq 0$.

Based on the T-S fuzzy Burgers-Fisher equation model (6)-(7), the fuzzy distributed controller for the nonlinear Burgers-Fisher equation model (1)-(3) can be written as:

Control Rule *i*:
IF $\mu_1(\zeta,t)$ is G_{i1} and … and $\mu_p(\zeta,t)$ is G_{ip},
THEN $u_i(t) = \Psi_i(x(\zeta,t))$ (9)

where $\Psi_i(x(\zeta,t))$ stands for a local distributed control law.

Applying the same fuzzy operators to (9) yields a global fuzzy distributed controller of the form:

$$u(t) = \sum_{i=1}^{r} h_i\big(\mu(0,t)\big)\ \Psi_i(x(\zeta,t)) \tag{10}$$

III. THE FUZZY DISTRIBUTED GEOMETRIC CONTROL DESIGN PROCEDURE

With reference to the concept of characteristic index [10] manipulated in the framework of geometric control of DPSs, the calculation of the first-time derivative of the output variable $y(t)$ gives:

$$\frac{dy(t)}{dt} = \int_0^l c(z) \frac{dx(z,t)}{dt}\, dz = \int_0^l c(z)\left(\nu \frac{\partial^2 x(z,t)}{\partial z^2} - \gamma_i \frac{\partial x(z,t)}{\partial z} + \beta_i\, x(z,t) + b(z)\ u_i(t)\right) dz$$

$$= \int_0^l c(z)\left(\nu \frac{\partial^2 x(z,t)}{\partial z^2} - \gamma_i \frac{\partial x(z,t)}{\partial z} + \beta_i\, x(z,t)\right) dz + \left[\int_0^l c(z)\, b(z)\, dz\right] u_i(t) \quad (11)$$

As can be noticed, the manipulated distributed control input $u(t)$ appears explicitly in the expression of the first-time derivative of the output variable (11). Hence, the characteristic index σ of the nonlinear Burgers-Fisher equation is equal to one. Consequently, a control law that enforces, in closed-loop, a dynamic behavior of a first-order system, i.e.

$$\tau \frac{dy(t)}{dt} + y(t) = \vartheta(t) \quad (12)$$

can be easily developed. In Eq. (12), $\vartheta(t)$ is the set-point reference and τ is the controller tuning parameter.

Combining Eqs. (11) and (12), the following local distributed geometric control law follows:

$$u_i(t) = \frac{1}{\tau \int_0^l c(z) b(z) dz} \times \left(\vartheta(t) - y(t) - \tau \int_0^l c(z)\left(\nu \frac{\partial^2 x(z,t)}{\partial z^2} - \gamma_i \frac{\partial x(z,t)}{\partial z} + \beta_i\, x(z,t)\right) dz\right)$$

$$= \frac{1}{\tau \int_0^l c(z) b(z) dz}\left(\vartheta(t) - y(t) - \tau\nu \underbrace{\int_0^l c(z) \frac{\partial^2 x(z,t)}{\partial z^2} dz}_{J_1} + \tau\gamma_i \int_0^l c(z) \frac{\partial x(z,t)}{\partial z} dz - \tau\beta_i \int_0^l c(z) x(z,t) dz\right) \quad (13)$$

Integrating by parts the second integral term J_1 of equation (13), and taking into account the periodic boundary conditions (6), we get:

$$J_1 = \int_0^l c(z) \frac{\partial^2 x(z,t)}{\partial z^2} dz$$

$$= c(z) \frac{\partial x(z,t)}{\partial z}\bigg|_{z=0}^{z=l} - x(z,t) \frac{\partial c(z)}{\partial z}\bigg|_{z=0}^{z=l} + \int_0^l \ddot{c}(z) x(z,t)\, dz$$

$$= [c(l) - c(0)] \frac{\partial x(z,t)}{\partial z}\bigg|_{z=l} + [\dot{c}(0) - \dot{c}(l)] x(l,t) + \int_0^l \ddot{c}(z) x(z,t)\, dz \quad (14)$$

Substituting equation (14) into (13), it results the following local controller:

$$u_i(t) = \frac{1}{\tau \int_0^l c(z) b(z) dz} \times \left(\vartheta(t) - y(t) + \tau\gamma_i \int_0^l c(z) \frac{\partial x(z,t)}{\partial z} dz + \tau\nu\, [\dot{c}(l) - \dot{c}(0)] x(l,t) - \tau\nu\, [c(l) - c(0)] \frac{\partial x(z,t)}{\partial z}\bigg|_{z=l} - \tau \int_0^l (\nu\, \ddot{c}(z) + \beta_i\, c(z))\, x(z,t)\, dz\right) \quad (15)$$

Consequently, the overall fuzzy distributed geometric controller can be obtained as:

$$u(t) = \sum_{i=1}^{r} h_i\big(\mu(0,t)\big) \frac{1}{\tau \int_0^l c(z) b(z) dz} \times \left(\vartheta(t) - y(t) + \tau\gamma_i \int_0^l c(z) \frac{\partial x(z,t)}{\partial z} dz + \tau\nu\, [\dot{c}(l) - \dot{c}(0)] x(l,t) - \tau\nu\, [c(l) - c(0)] \frac{\partial x(z,t)}{\partial z}\bigg|_{z=l} - \tau \int_0^l (\nu\, \ddot{c}(z) + \beta_i\, c(z))\, x(z,t)\, dz\right) \quad (16)$$

III. SIMULATION RESULTS

In order to check the stabilizing performance of the designed fuzzy distributed geometric controller, let us consider the following forced Burgers-Fisher equation with distributed control [5]:

$$\frac{\partial x(z,t)}{\partial t} = \nu \frac{\partial^2 x(z,t)}{\partial z^2} - x(z,t) \frac{\partial x(z,t)}{\partial z} + f\big(x(z,t)\big) + b(z)\, u(t) \quad (17)$$

subject to the periodic boundary conditions:

$$x(0,t) = x(l,t), \qquad \frac{\partial x(z,t)}{\partial z}\bigg|_{z=0} = \frac{\partial x(z,t)}{\partial z}\bigg|_{z=l} \quad (18)$$

and the initial condition:

$$x(z,0) = x_0(z) \quad (19)$$

where $x(z,t)$ is the state variable, and $q(t)$ is the distributed control input on the spatial domain $\Omega = [0,l]$; t, z and l denote the independent time, spatial position and the length of spatial domain, respectively. The parameters of the forced Burgers-Fisher equation (17)-(19) are set as follows: $\nu = 1.5$, $l = 2\pi$, $f\big(x(z,t)\big) = \cos\big(x(z,t)\big)$ and $x_0(z) = \sin(z)$.

As mentioned above, the first task is to construct a T-S fuzzy forced Burgers-Fisher equation model for the nonlinear DPS (17)-(19) by using the sector nonlinearity method. For this purpose, let the nonlinear terms $x(z,t)\frac{\partial x(z,t)}{\partial z}$ and $\cos\big(x(z,t)\big)$ in Eq. (17) be denoted as $\mu_1(z,t)$ and $\mu_2(z,t)$, respectively. The first term $\mu_1(z,t)$ can be found in the local sector bounds $[-1{,}1]$ which is described by two lines 1 and -1, while the second term $\mu_2(z,t)$ can be found in the local sector bounds $[\cos(88), 1]$, which is represented by two lines $x(z,t)$ and $\cos(88)\, x(z,t)$.

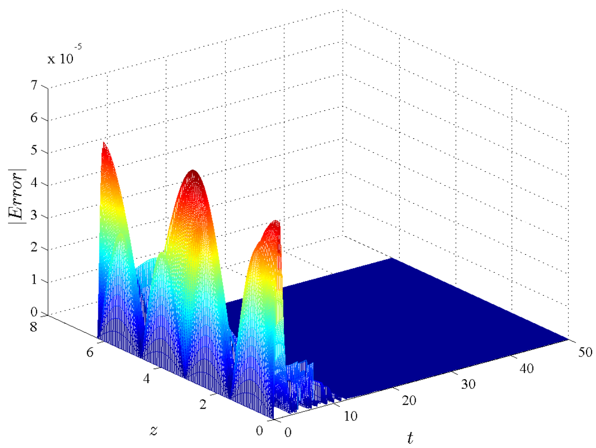

Fig. 1. Open-loop profile of the fuzzy approximation error.

The two nonlinear functions in the forced Burgers-Fisher equation (17)-(19) can be written as:

$$\mu_1(z,t) = \Big(G_{11}\big(\mu_1(z,t)\big).(1) + G_{21}\big(\mu_1(z,t)\big).(-1)\Big)\frac{\partial x(z,t)}{\partial z}$$
$$\mu_2(z,t) = \Big(G_{12}\big(\mu_2(z,t)\big).1 + G_{22}\big(\mu_2(z,t)\big).\cos(88)\Big)x(z,t)$$

Therefore, the resulting membership functions $G_{11}\big(\mu_1(z,t)\big)$ and $G_{21}\big(\mu_1(z,t)\big)$ are obtained as:

$$G_{11}\big(\mu_1(z,t)\big) = \begin{cases} \dfrac{\mu_1(z,t)+1}{2}, & x(z,t) \neq 0 \\ 1, & x(z,t) = 0 \end{cases} \quad (20)$$

$$G_{21}\big(\mu_1(z,t)\big) = 1 - G_{11}\big(\mu_1(z,t)\big) \quad (21)$$

while the membership functions $G_{12}\big(\mu_2(z,t)\big)$ and $G_{22}\big(\mu_2(z,t)\big)$ are given as:

$$G_{12}\big(\mu_2(z,t)\big) = \begin{cases} \dfrac{\mu_2(z,t) - \cos(88)\, x(z,t)}{(1-\cos(88))x(z,t)}, & x(z,t) \neq 0 \\ 1, & x(z,t) = 0 \end{cases} \quad (22)$$

$$G_{22}\big(\mu_2(z,t)\big) = 1 - G_{12}\big(\mu_2(z,t)\big) \quad (23)$$

Labeling the fuzzy sets as 'G_{11} = Big', 'G_{21} = Small', 'G_{12} = Positive Big', 'G_{22} = Negative Small', the forced Burgers-Fisher equation (17)-(19) can be approximated by a T-S fuzzy model subject to the periodic boundary conditions (18), which is composed of four fuzzy rules on $\mathfrak{I} \triangleq \{x(z,t) \in \mathcal{L}^2([0,l]) \,|\, -2 \le x(z,t) \le 2\}$:

System Rule 1:
IF $\mu_1(z,t)$ is Big and $\mu_2(z,t)$ is Positive Big,
THEN $$\frac{\partial x(z,t)}{\partial t} = \nu\frac{\partial^2 x(z,t)}{\partial z^2} - \gamma_1\frac{\partial x(z,t)}{\partial z} + \beta_1 x(z,t) + b(z)\, u_1(t)$$

System Rule 2:
IF $\mu_1(z,t)$ is Big and $\mu_2(z,t)$ is Negative Small,
THEN $$\frac{\partial x(z,t)}{\partial t} = \nu\frac{\partial^2 x(z,t)}{\partial z^2} - \gamma_1\frac{\partial x(z,t)}{\partial z} + \beta_2 x(z,t) + b(z)\, u_2(t)$$

System Rule 3:
IF $\mu_1(z,t)$ is Small and $\mu_2(z,t)$ is Positive Big,
THEN $$\frac{\partial x(z,t)}{\partial t} = \nu\frac{\partial^2 x(z,t)}{\partial z^2} - \gamma_2\frac{\partial x(z,t)}{\partial z} + \beta_1 x(z,t) + b(z)\, u_3(t)$$

System Rule 4:
IF $\mu_1(z,t)$ is Small and $\mu_2(z,t)$ is Negative Small
THEN $$\frac{\partial x(z,t)}{\partial t} = \nu\frac{\partial^2 x(z,t)}{\partial z^2} - \gamma_2\frac{\partial x(z,t)}{\partial z} + \beta_2 x(z,t) + b(z)\, u_4(t)$$

The parameters of the rule-consequents of the fuzzy forced Burgers-Fisher model are obtained as: $\gamma_1 = 1$, $\gamma_2 = -1$, $\beta_1 = 1$ and $\beta_2 = \cos(88)$.

The overall T-S fuzzy forced Burgers-Fisher equation model can be written as follows:

$$\frac{\partial\, x(z,t)}{\partial t} = \nu\frac{\partial^2 x(z,t)}{\partial z^2} + \left(\sum_{i=1}^{4} h_i\big(\mu(z,t)\big)\right) \times \left(-\gamma_i\frac{\partial x(z,t)}{\partial z} + \beta_i\, x(z,t) + b(z)\, u_i(t)\right) \quad (24)$$

where

$$w_1\big(\mu(z,t)\big) = G_{11}\big(\mu_1(z,t)\big) \times G_{12}\big(\mu_2(z,t)\big),$$
$$w_2\big(\mu(z,t)\big) = G_{11}\big(\mu_1(z,t)\big) \times G_{22}\big(\mu_2(z,t)\big),$$
$$w_3\big(\mu(z,t)\big) = G_{21}\big(\mu_1(z,t)\big) \times G_{12}\big(\mu_2(z,t)\big),$$
$$w_4\big(\mu(z,t)\big) = G_{21}\big(\mu_1(z,t)\big) \times G_{22}\big(\mu_2(z,t)\big),$$

and $h_i(\mu(z,t)) = \frac{w_i(\mu(z,t))}{\sum_{i=1}^{4} w_i(\mu(z,t))}$, $i = 1, \ldots, 4$.

The constructed T-S fuzzy model (24) represents the forced nonlinear Burgers-Fisher equation in the region $[-1, 1] \times [\cos(88), 1]$. Here, it can be easily found that:

$w_i(\mu(z,t)) \ge 0$, $i = 1, \ldots, 4$ and $\sum_{i=1}^{4} w_i(\mu(z,t)) > 0$
$h_i(\mu(z,t)) \ge 0$, $i = 1, \ldots, 4$ and $\sum_{i=1}^{4} h_i(\mu(z,t)) = 1$.

The profile of the fuzzy approximation error between the original nonlinear model (17)-(19) and the proposed fuzzy model (24), obtained for $u(t) = 0$, is depicted in Fig. 1. The global approximation error is in the scale 10^{-5} which is very acceptable. Setting the values of the control parameters of Eq.(16) as $b(z) = 1$, $c(z) = z$, $\tau = 1$, $y(t) = \int_0^{2\pi} z\, x(z,t)\, dz$ and $\vartheta(t) = 0$, we get a fuzzy distributed geometric control of the form:

$$u(t) = \sum_{i=1}^{4} h_i\big(\mu(\zeta,t)\big) \frac{1}{\int_0^{2\pi} z\, dz} \times \left(-\int_0^{2\pi} z\, x(z,t)\, dz - 1.5 \frac{\partial x(z,t)}{\partial z}\bigg|_{z=l} + \gamma_i \int_0^{2\pi} z \frac{\partial x(z,t)}{\partial z} dz - \beta_i \int_0^{2\pi} z\, x(z,t)\, dz \right) \quad (25)$$

Applying the obtained distributed control (25) to the forced Burgers-Fisher equation (17)-(19), it results the closed-loop trajectory of $\|x(.,t)\|_{L^2(z)}$ shown in Fig. 2. From the obtained results of Fig. 3 and Fig. 4, it can be observed that the stabilization of the system output around the specified set-point reference has been indeed achieved under a smooth evolution of the manipulated control variable.

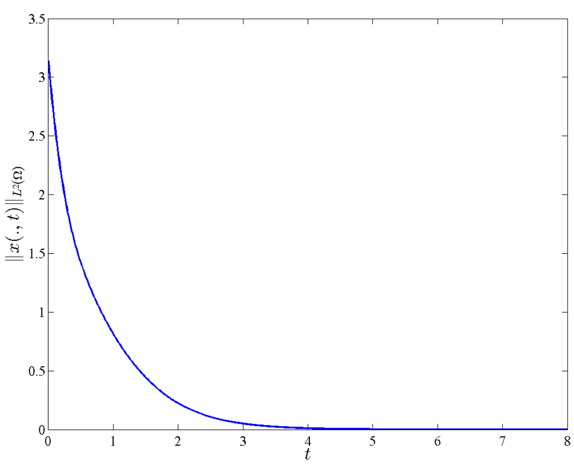

Fig. 2. Evolution of the closed-loop trajectory of $\|x(.,t)\|_{L^2(z)}$.

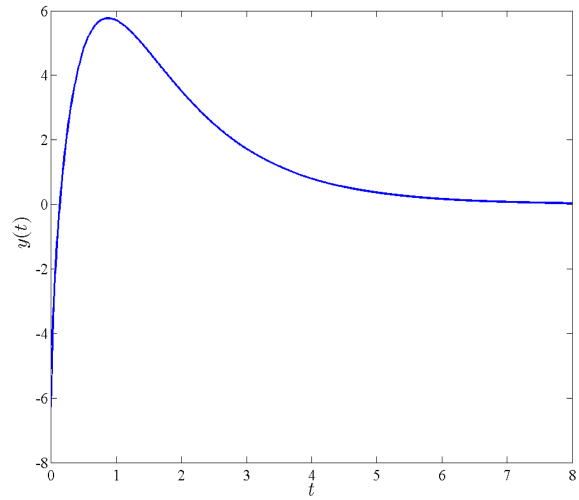

Fig.3. Evolution of the controlled output $y(t)$.

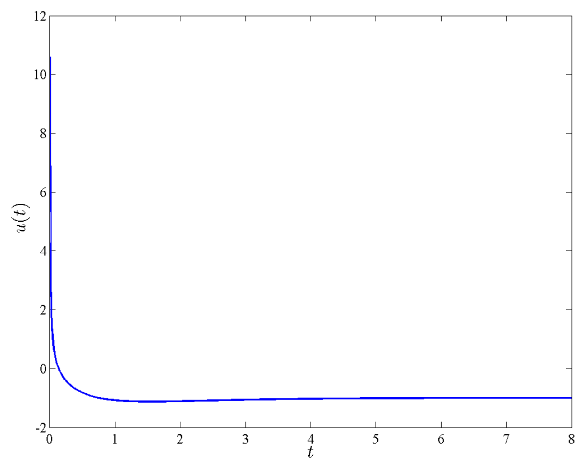

Fig. 4. Evolution of the manipulated variable $u(t)$.

In addition, the resulting closed-loop profile of evolution of the fuzzy forced Burgers-Fisher equation model is illustrated in Fig. 5. From that figure, it can be clearly seen that the proposed fuzzy distributed geometric controller (25) was able to ensure an efficient stabilizing task of the forced Burgers-Fisher equation (17)-(19) under the periodic boundary conditions.

V. Conclusion

This paper has addressed the problem of control of the nonlinear Burgers-Fisher equation model by means of fuzzy reasoning. More precisely, a fuzzy distributed geometric controller has been designed and applied to a nonlinear forced Burgers-Fisher equation model under periodic boundary conditions. To this end, a T-S fuzzy forced Burgers-Fisher equation model with distributed control has been first proposed. The constructed fuzzy model has then been used to derive an infinite dimensional fuzzy controller that stabilizes the DPS around its equilibrium state. A simulation study has been performed in order to demonstrate the effectiveness of the proposed fuzzy distributed controller. The obtained results have clearly shown the performance of the resulting closed-loop system which impacted considerably well the control of the complex dynamics of the nonlinear forced Burgers-Fisher equation model. Moreover, it is worth noting that, with regard to the described design procedure, the proposed fuzzy PDE model-based controller is easy to derive, through "fuzzy blending" of local distributed control laws, and easy to implement as well.

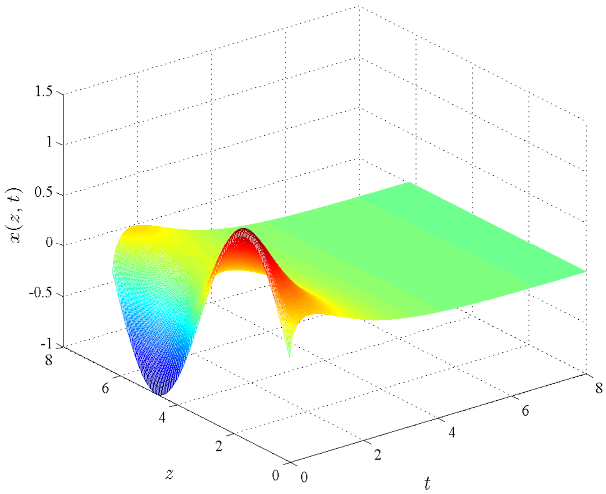

Fig. 5. 3-D profile of the controlled Burgers-Fisher equation model.

References

[1] R. Curtain and H. Zwart, "An introduction to infinite-dimensional linear systems theory," Springer, New York, 1995.

[2] P. D. Christofides, "Control of nonlinear distributed process systems: recent developments and challenges," AIChE Journal, vol. 47, no. 3, pp. 514-518, 2001.

[3] Ch.-D. Yang, J. Qiu and J.-W. Wang, "Robust H_∞ control for a class of nonlinear distributed parameter systems via proportional-spatial derivative control approach," Abstract and Applied Analysis, vol. 2014, ID 631071, 2014.

[4] Z. Emirsajłow, A. Krakowiak, A. Kowalewski and J. Sokołowsk, "Sensitivity analysis of time delay parabolic-hyperbolic optimal control problems with boundary conditions involving time delays," 18th IEEE International Conference on Methods & Models in Automation & Robotics (MMAR), pp. 514–519, 2013.

[5] N. Smaoui, "Boundary and distributed control of the viscous Burgers equation," Journal of Computational and Applied Mathematics, vol. 182, pp. 91–104, 2005.

[6] N. Smaoui, "Nonlinear boundary control of the generalized Burgers equation," Nonlinear Dynamics, vol. 37, pp. 75–86, 2004.

[7] W. H. Ray, "Advanced process control," Butterwoth, Boston, 1989.

[8] S. Raab, H. Habbi and A. Maidi, "Fuzzy boundary control control of nonlinear distributed parameter systems through an equivalent punctual control form," 2019 International Conference on Advanced Electrical Engineering (ICAEE), Algiers, pp. 1-6, 2019.

[9] S. Raab, H. Habbi and A. Maidi, "Fuzzy PDE model-based geometric control design for a diffusion-reaction process;" The 1st Conference on Electrical Engineering (CEE), Algiers, Algeria, 2019.

[10] P. D. Christofides, "Nonlinear and robust control of PDE systems: methods and applications to transport-reaction processes," Birkhauser, Boston, 2001.

[11] H.-X. Li and C. Qi, "Modeling of distributed parameter systems for applications-a synthesized review from time-space separation," Journal of Process Control, vol. 20, no. 8, pp. 891–901, 2010.

[12] A. Singh and J. Hahn, "Effect of finite-dimensional approximation on observability analysis of distributed parameter models," In the Proceedings of the 8th IFAC Symposium on Dynamics and Control of Processes Systems, pp. 199-204, June 6-8, Cancun, Mexico, 2007.

[13] P.D. Christofides and P. Daoutidis, "Nonlinear control of diffusion-convection-reaction processes," Computer and Chemical Engineering, vol. 20, no. S2, pp. S1071–S1076, 1996.

[14] P.D. Christofides and P. Daoutidis, "Feedback control of hyperbolic PDE systems," AIChE Journal, vol. 42, no. 11, pp. 3063–3086, 1996.

[15] A. Maidi and J.P. Corriou, "Boundary geometric control of a nonlinear diffusion system with time-dependent spatial domain," Asian Journal of Control, vol. 18, no. 5, pp. 1–10, 2015.

[16] A. Bátkai and S. Piazzera, "Semigroups and linear partial differential equations with delay," Journal of Mathematical Analysis and Applications, vol. 264, no. 1, pp. 1–20, 2001.

[17] C. Qi, H.-X. Li, S. Li, X. Zhao and F. Gao, "A fuzzy-based spatio-temporal multi-modeling for nonlinear distributed parameter processes," Applied Soft Computing, vol. 25, no. 12, pp. 306–321, 2009.

[18] H. Habbi, M. Kidouche, M. Kinnaert and M. Zelmat, "Fuzzy model-based fault detection and diagnosis for a pilot heat exchanger," International Journal of Systems Science, vol. 42, no. 4, pp. 587–599, 2011.

[19] K. Tanaka and H. O. Wang, "Fuzzy control systems design and analysis: a linear matrix inequality approach," Wiley, New York, 2001.

[20] K.-N. Wu and B. S. Chen, "Synchronization of partial differential systems via diffusion coupling," IEEE Transactions on Circuits and Systems |: Regular Papers, vol. 59, no. 11, pp. 2655–2668, 2012.

[21] B.-S. Chen and Y.-T. Chang, "Fuzzy state-space modeling and robust observer-based control design for nonlinear partial differential systems," IEEE Transactions on Fuzzy Systems, vol. 17, no. 5, pp. 1025–1043, 2009.

[22] X. Song, M. Wang, S. Song and J. Man, "Dissipativity-based controller design for time-delayed T-S fuzzy switched distributed parameter systems," Complexity, vol. 2018, pp. 1–11, 2018.

[23] J. W. Wang and H. N. Wu, "Exponential pointwise stabilization of semi-linear parabolic distributed parameter systems via the Takagi-Sugeno fuzzy PDE model," IEEE Transactions on Fuzzy Systems, vol. 26, no. 1, pp. 155–173, 2018.

[24] H. Habbi, M. Kidouche and M. Zelmat, "Data-driven fuzzy models for nonlinear identification of a complex heat exchanger," Applied Mathematical Modelling, vol. 35, no. 3, pp. 1470–1482, 2011.

[25] A. Maidi and J.P. Corriou, "Distributed control of nonlinear diffusion systems by input-output linearization," International Journal of Robust and Nonlinear Control, vol. 24, no. 3, pp. 389–405, 2014.

[26] O. Lamraoui, Y. Boudouaoui and H. Habbi, "Heat transfer dynamics modeling by means of clustering and swarm methods," International Journal of Intelligent Engineering Informatics, vol. 7, no. 4, pp. 346–365, 2019.

[27] O. Lamraoui and H. Habbi, "Enhanced RECCo controller with integrated removing clouds mechanism," In: Chikhi S., Amine A., Chaoui A., Saidouni D. (eds) Modelling and Implementation of Complex Systems. MISC 2018. Lecture Notes in Networks and Systems, vol. 64, pp.3-15, 2019, Springer, Cham.

[28] K. Atkinson and W. Han. Theoretical Numerical Analysis. A Functional Analysis Framework. Springer, Dordrecht, 2009.