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# State feedback linearization using block companion similarity transformation

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**ABSTRACT**

In this research work, a new method is proposed for linearizing a class of nonlinear multivariable system; where the number of inputs divides exactly the number of states. The idea of proposed method consists in representing the original nonlinear system into a state-dependent coefficient form and applying block similarity transformations that allow getting the linearized system in block companion form. Because the linearized system's eigenstructure can determine system performance and robustness far more directly and explicitly than other indicators, the given class multivariable system is chosen. Examples are used to illustrate the application and show the effectiveness of the given approach.



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## 1. INTRODUCTION

The feedback linearization technique is one of modern tools, which allows synthesizing a control law for smooth, continuous, non-linear systems [1-5]. The basic idea is to transform a nonlinear system into a linear one via nonlinear change of coordinates only (state linearization problem) or by nonlinear feedback and change of coordinates (feedback linearization problem) so that the linear control techniques can be applied [1]. Feedback linearization has been one of the most active research topics in recent years; linearization of affine systems [6], systems with nonsmooth nonlinearities [7], exact linearization [8]. Various methods exist to find feedback linearizable form for single input [9], and multi-input nonlinear systems [10-15].

Significant research effort has been devoted to the construction of approximate solutions to the problem of linearizing nonlinear systems by state or output feedback. Main reason for that is the limited applicability of the rigorous methods, and the complexity, sensitivity and design difficulties of the exact linearizing compensators, if any [16]. The analysis and the comparison of approximate linearization and exact linearization are proposed by [17]. If the original nonlinear system cannot be linearized exactly by state feedback, the method of approximate feedback linearization [16, 18-22] is used. This process consists of finding approximate output functions that satisfy the involutivity condition up to determined system order, [16, 19] adopted a two-step procedure to solve the approximate linearization; first a state transformation matrix is settled, so that the nonlinear system is transformed approximately into the controllable canonical form. Second, a standard nonlinear linearization method is used to transform the controllable canonical form into a stable linear system.

Kabanov [19] offered an approach that consists in representing the original nonlinear (single and multivariable) system into a state-dependent coefficient form and applying similarity transformations that allow getting the system to canonical form, that considerably simplifies the problem of feedback linearization afterwards. Such similarity transformations allow accomplishing linearization of system without considering the virtual output function $y = h(x)$ [19].This work consists in applying a block companion similarity transformation to a class of nonlinear multivariable system; where the number of inputs divides exactly the number of states. It is known that if a linear-time-invariant MIMO system described by a state space equation has a number of states divisible by the number of inputs and it can be transformed to block controller form, we can design a state feedback controller using block pole placement technique by assigning a set of desired Block poles. These may be left or right block poles [23-26]. The eigenvalues and eigenvectors (eigenstructure) of the state matrix can determine system performance and robustness more directly and explicitly than other indicators. Hence their assignment should improve feedback system performance and robustness distinctly and effectively [25].

In the present work, firstly we have started by introducing feedback linearization used in multivariable systems. As a second section a problem of transforming a multivariable system into block controller form is formulated, it is then followed by the discussion of the feedback linearization using similarity block transformation for the class of MIMO system which is the heart of this work. In a forth section examples are used to show different steps and the effectiveness of the proposed method, and finally the paper is finished by a conclusion.

## 2. PROBLEM FORMULATION

Consider a class of multivariable nonlinear system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)\, u_i$$

$$\dot{x} = f(x) + G(x)u \tag{1-a}$$

$$y_i = h_i(x), i = 1, \ldots, m \tag{1-b}$$

or $Y = H(x)$

where $f, g$and h are sufficiently smooth, $m$ is the number of inputs and outputs,$n$ is the number of states. The multivariable system (1), in the state-dependent coefficient form is given by:

$$\dot{x}(t) = A(x)x + B(x)u \tag{2}$$

where $A(x)$ is an$n \times n$ and $B(x)$ is an $n \times m$ matrices respectively; are continuous differentiable.

Consider the system (1), the feedback linearization problem consists in finding the nonsingular coordinate transformation $z = T_{FL}(x)$ and the input control law $u = u_{FL}$ such that the system (1) can be transformed into linear block canonical form. Under condition of the existence of smooth function $H(x)$, transformation $z = T_{FL}(x)$ is described by (1).

$$T_{FL}(x) = \begin{bmatrix} H(x) \\ L_f^1 H(x) \\ \vdots \\ L_f^{l-1} H(x) \end{bmatrix} \tag{3}$$

and linearizing control is given as,

$$u_{FL} = s^{-1}(x)(v - q(x)) \tag{4}$$

where $q(x) = L_f^l H(x)$ and $s(x) = L_G L_f^{l-1} H(x)$and $v$ is the new control input;$L_f^i H(x)$ is the Lie derivative of the $i^{th}$ order from the function $H(x)$ along the vector field $f$ [1, 19].

Applying feedback linearizing control $u_{FL}$ (4), in respect of the transformation $T_{FL}(x)$ (3), the system (1) has the following form,

$$\dot{z} = A_c z + B_c v \tag{5}$$

where $(A_c, B_c)$ is in block canonical form.

$$A_c = \begin{bmatrix} 0_m & I_m & \cdots & 0_m \\ 0_m & 0_m & & 0_m \\ \vdots & & \ddots & \vdots \\ 0_m & 0_m & \cdots & I_m \\ 0_m & 0_m & & 0_m \end{bmatrix}, B_c = \begin{bmatrix} 0_m \\ \vdots \\ 0_m \\ I_m \end{bmatrix} \quad (6)$$

where $0_m$ and $I_m$ are $m \times m$ zero and identity matrices, respectively.

Applicability of the normal form method for feedback linearization is ensured by the existence of diffeomorphism (3) and observance of the terms [1, 19],

$$L_G H(x) = L_G L_f^1 H(x) = \ldots = L_G L_f^{l-2} H(x) = 0_m$$
$$s(x) = L_G L_f^{l-1} H(x) \neq 0_m$$

which is not always true. In such situations, as a rule, methods of approximate feedback linearization are used [16, 19, 27]. The block canonical similarity transformation used for feedback linearizing control design is:

$$z = T(x)x, x = T^{-1}(x)z, T(x) \in \mathcal{R}^{mxn} \quad (7)$$

## 3. FEEDBACK LINEARIZATION

System (1) can be represented in the state dependent coefficient form [28]:

$$\dot{x}(t) = A(x)x + B(x)u \quad (8)$$

Let us assume that matrix $A(x)$ is $n \times n$ and vector $B(x)$ is, $n \times m$ is continuously differentiable. It is also supposed that the pair $(A(x), B(x))$ is a full block controllable, i.e. for the system (8) controllability matrix has full rank (rank of $w(x) = l$), for $t \geq t_0$.

$$w(x) = [B(x) \quad A(x)B(x) \quad \ldots \quad A^{l-1}(x)B(x)]$$

where $\frac{n}{m} = l$, $l$ (an integer) is the number of blocks.

Applying transformation (7) to (8) we get (5) for which the following relations are true.

$$\dot{z}(t) = \dot{T}(x)x + T(x)\dot{x}$$

$$\dot{z}(t) = \dot{T}(x)T^{-1}(x)z + T(x)[A(x)T^{-1}(x)z + T(x)B(x)u]$$

$$\dot{z}(t) = A_c(z)z + B_c(z)u(z) \quad (9)$$

$$A_c(z) = T(x)A(x)T^{-1}(x) + \dot{T}(x)T^{-1}(x) \quad (10)$$

where

$$A_c = \begin{bmatrix} 0_m & I_m & \ldots & 0_m \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & I_m \\ -A_0(z) & -A_1(z) & \ldots & -A_{l-1}(z) \end{bmatrix} \quad (11\text{-a})$$

$$B_c = T(x)B(x) = \begin{bmatrix} 0_m \\ \vdots \\ 0_m \\ I_m \end{bmatrix} \quad (11\text{-b})$$

where $A_i \in \Re^{mxm}, i = 1,2, \ldots, . l$. $I_m$ and $0_m$are $m\ x\ m$ identity and null matrices respectively, $l$ is an integer such that $l = \frac{n}{m}$ where $n$ is the order of the system and $m$ is the number of inputs.

The right matrix fraction description (RMFD) [24-26] of the system can be formulated directly from (11-a) as,

$$F(\lambda) = N_R(\lambda)D_R^{-1}(\lambda) \tag{12}$$

where,

$$D_R(\lambda) = I_m\lambda^l + A_1\lambda^{l-1}+\ldots+A_l \tag{13}$$

$$N_R(\lambda) = I_m\lambda^l + C_1\lambda^{l-1}+\ldots+C_l \tag{14}$$

$C_i \in \mathcal{R}^{mxm}$ are constant matrices. The desired matrix polynomial $A_D(\lambda)$if $R_i$is a right solvent is given by,

$$A_D(\lambda) = R_i^l + D_1R_i^{l-1} + \cdots + D_l$$

$$[D_{dl}\, D_{d(l-1)}\, \ldots . D_{d1}] = -[R_1^l R_2^l\ \ldots\ R_l^l]V_R^{-1}$$

where $V_R$is the right block Vandermonde matrix[24, 25]. An alternative factorization of $F(\lambda)$ is the left matrix fraction description (LMFD) defined by the following equation;

$$F(\lambda) = D_L^{-1}(\lambda)N_L(\lambda) \tag{15}$$

where $D_L(\lambda)$ is the left denominator (13) and $N_L(\lambda)$ is the left numerator (14). The desired matrix polynomial$A_D(\lambda)$ if $L_i$ is a left solvent is:

$$A_D(\lambda) = L_i^l + L_i^{l-1}D_1 + \cdots + D_l$$

the desired solvents are given as,

$$\begin{bmatrix} D_{dl} \\ D_{d(l-1)} \\ \vdots \\ D_{d1} \end{bmatrix} = -\begin{bmatrix} I_m & L_1 & \ldots & L_1^{l-1} \\ I_m & L_2 & \ldots & L_2^{l-1} \\ \vdots & \vdots & \ddots & \vdots \\ I_m & L_l & \ldots & L_l^{l-1} \end{bmatrix}^{-1} \begin{bmatrix} L_1^l \\ L_2^l \\ \vdots \\ L_l^l \end{bmatrix}$$

Linearizing control for block companion system (11) can be defined in the form of the feedback;

$$u(z) = u_{FL}(z) + v(z) = K(z)z + G_c(z)z \tag{16}$$

where matrix $G_c(z)$ can be calculated, for instance, by means of the method of the placement of the closed-loop system block poles. $K(z)$ is computed by [19, 21].

$$K(z) = -[A_{l-1}(z) \quad A_{l-2}(z) \quad \ldots \quad A_0(z)] \tag{17}$$

In terms of initial system state variables the linearizing control (16) can be written as,

$$u(x) = u_{FL}(x) + v(x) = K(x)T(x)x + G_cT \tag{18}$$

The procedure to find the coordinate of change is [24, 25]:

$$T(x) = \{T_i(x)\}\, , i = 1, \ldots, l \tag{19-a}$$

let

$$T_{c1} = [0_m \quad 0_m \quad \cdots \quad I_m]w^{-1}(x) \tag{19-b}$$

$$T(x) = \begin{bmatrix} T_{c1} \\ T_{c1}A \\ \vdots \\ T_{c1}A^{l-1} \end{bmatrix} \quad \text{(19-c)}$$

where $0_m$and $I_m$ are null and identity matrices, respectively. The transformation (7), (19) exists, if and only if:
- controllability matrix $w(x)$ has full rank $n$ at $t \geq t_0$
- $\frac{n}{m} = l, l$ is an integer

If we suppose that the block canonical form has the form (6).

$$A_c = \begin{bmatrix} 0_m & I_m & \cdots & 0_m \\ 0_m & 0_m & & 0_m \\ \vdots & & \ddots & \vdots \\ 0_m & 0_m & \cdots & I_m \\ 0_m & 0_m & & 0_m \end{bmatrix}, B_c = \begin{bmatrix} 0_m \\ \vdots \\ 0_m \\ I_m \end{bmatrix}$$

$$\text{i.e. } K(z) = [0_m \quad \dots \; 0_m] \quad (20)$$

The linearizing control is:

$$u(z) = v(z) = G_c(z)z = K_c(z)z \quad (21)$$

In the next section, numerical examples are illustrated to show the effectiveness of the proposed method.

## 4. ILLUSTRATIVE EXAMPLES

### 4.1. Example 1

Let consider the following nonlinear multivariable system, with a number of states is $n = 3$ and a number of inputs is $m = 3$ therefore a number of blocks is $l = \frac{3}{3} = 1$. The control problem is to stabilize the system (22) by means of the feedback.

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2^2 + u_1 \\ \dot{x}_2 &= -x_1 + 2x_3 + 2u_2 \\ \dot{x}_3 &= (x_1 - 1)x_2 + u_3 \end{aligned} \quad (22)$$

where,

$$f(x) = \begin{bmatrix} x_1 + x_2^2 \\ -x_1 + 2x_3 \\ (x_1 - 1)x_2 \end{bmatrix} \text{ and } g(x) = \begin{bmatrix} u_1 \\ 2u_2 \\ u_3 \end{bmatrix}$$

We write $f(x)$ and $g(x)$ in state-dependent coefficient form; $A(x)$ and $B(x)$, respectively.

$$A(x) = \begin{bmatrix} 1 & x_2 & 0 \\ -1 & 0 & -2 \\ x_2 & -1 & 0 \end{bmatrix}, B(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (23)$$

We compute the similarity transformation matrix.

$$w(x) = [B(x)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (24)$$

The rank of $w(x)$ is 3 (full rank), hence the system is controllable. Now we compute the coordinate of change $T(x)$we calculate first.

$$T_{c1} = [I_3]w^{-1}(x)$$

then,

$$T(x) = [T_{c1}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{25}$$

hence,

$$A_c(z) = T(x)A(x)T^{-1}(x)$$

$$A_c(z) = \begin{bmatrix} 1 & 2x_2 & 0 \\ -\frac{1}{2} & 2 & -1 \\ x_2 & -2 & 0 \end{bmatrix} \tag{26}$$

and $K(z) = -A_c(z)$, and we have

$$B_c(z) = T(x)B(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{27}$$

In this application, $G_c(z)$ is computed using desired block poles of the closed loop system. The eigenvalues of close-loop system matrix are $\lambda_1 = -3$, $\lambda_2 = -4$ and $\lambda_3 = -5$. The characteristic matrix polynomial of the closed-loop system is forced to equal a desired matrix polynomial of the form.

$$A_D(\lambda) = I_m\lambda^l + D_1\lambda^{l-1} + \cdots + D_l \tag{28}$$

The desired characteristic matrix polynomial $A_D(\lambda)$may be constructed from a set of desired $m \times m$ block poles or matrix roots of $A_D(\lambda)$; These block poles can be constructed under different forms either an $m \times m$ diagonal form matrix, an $m \times m$ controller canonical form matrix or an $m \times m$ observer canonical matrix. Using block poles in diagonal form gives a better response, robustness and smaller sensitivities [24], for this reason only the case of the diagonal form will be studied. Feedback gain of the closed-loop system $G_c$is computed as:

$$A - BG_c = A_D \tag{29}$$

$A_D$ is a desired closed-loop matrix in block diagonal form where each block is constructed from desired eigenvalues. The desired block pole constructed in diagonal form is given by,

$$R = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -5 \end{bmatrix} \tag{30}$$

The corresponding $3 \times 3$ desired right denominator matrix polynomial of degree 1 is:

$$D_f(s) = Is + D_{f1} \text{where } D_{f1} = -R^2V_R^{-1} \text{ i.e., } D_{f1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Now we can find $\mathrm{G_c}$

$$G_c = \begin{bmatrix} 3 & 2x_2 & 0 \\ -1/2 & 4 & -1 \\ x_2 & -2 & 5 \end{bmatrix}$$

The linearizing control.

$$u(z) = u_{FL}(z) + v(z) = K(z)z + G_c(z)z = [K(z) + G_c(z)]z = K_c(z)z$$

where

$$Kc = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -5 \end{bmatrix} \tag{31}$$

its norm $\|K_c\|_2 = 5$. In the terms of initial system state variables linearizing control (18) can be written as,

$$u(x) = K(x)T(x)x + G_c(x)T(x)x = (K(x) + G_c(x))T(x)x = K_c(x)T(x)x$$

hence,

$$u(x) = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -5 \end{bmatrix} \tag{32}$$

The results of the nonlinear system (22) with the linearizing control (32), For modeling the following initial values were taken: $x_1(0) = 0,\ x_2(0) = 1, x_3(0) = -1$. Using MATLAB software (M. file), the state variables responses are shown in this Figure 1. The results of the figure demonstrate that the system is stable.

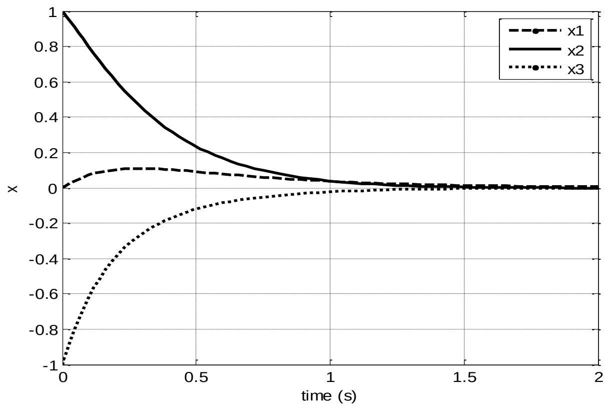


Figure 1. State variables result for the system (22)

### 4.2. Example 2

In the following nonlinear multivariable system, a number of states is $n = 4$ and a number of inputs is $m = 2$ therefore a number of blocks is $l = \frac{4}{2} = 2$. The control problem is to stabilize the system (33) by means of the feedback.

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 + x_3^2 \\ \dot{x}_2 &= -x_1 - 2x_3 + u_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= (x_1 - 1)x_2 + u_2 \end{aligned} \tag{33}$$

where,

$$f(x) = \begin{bmatrix} x_1 + x_2 + x_3^2 \\ -x_1 - 2x_3 \\ x_4 \\ (x_1 - 1)x_2 \end{bmatrix} \text{ and } g(x) = \begin{bmatrix} 0 \\ u_1 \\ 0 \\ u_2 \end{bmatrix}$$

We write $f(x)$ and $g(x)$ in state-dependent coefficient form; $A(x)$ and $B(x)$, respectively.

$$A(x) = \begin{bmatrix} 1 & 1 & x_3 & 0 \\ -1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ x_2 & -1 & 0 & 0 \end{bmatrix}, B(x) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{34}$$

We compute the block similarity transformation matrix.

$$w(x) = [B(x)\ A(x)B(x)] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \quad (35)$$

The rank of $w(x)$ is 4 (full rank), hence the system is controllable. Now we compute the coordinate of change $T(x)$, we compute first.

$$T_{c1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} w^{-1}(x)$$

then,

$$T(x) = \begin{bmatrix} T_{c1} \\ T_{c1}A \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & x_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (36)$$

hence,

$$A_c(z) = T(x)A(x)T^{-1}(x) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -2 & 1 & x_3 \\ x_2+1 & x_3 & -1 & 0 \end{bmatrix} \quad (37)$$

and we have

$$B_c(z) = T(x)B(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (38)$$

In this application, $G_c(z)$ is computed using desired block poles of the closed loop system. The eigenvalues of close-loop system matrix are $\lambda_1 = -1$, $\lambda_2 = -2$ and $\lambda_3 = -3$ and $\lambda_4 = -4$. The desired block pole constructed in diagonal form is given by,

$$R_1 = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \text{ and} R_2 = \begin{bmatrix} -3 & 0 \\ 0 & -4 \end{bmatrix} \quad (39)$$

hence, $D_f(s) = Is^2 + D_{f1}s + D_{f0}$

The linearizing control.

$$u(z) = u_{FL}(z) + v(z) = K(z)z + G_c(z)z = [K(z) + G_c(z)]z = K_c(z)z$$

where,

$$Kc = \begin{bmatrix} 2 & -2 & 5 & x_3 \\ x_2 + 1 & x_3 + 8 & -1 & 6 \end{bmatrix} \quad (40)$$

In the terms of initial system state variables the linearizing control is.

$$u(x) = \begin{bmatrix} 0.43 & 1.4 & 1.4x_3 - 2 & x_3 \\ x_2 & -1 & 0.08 & 0.6 \end{bmatrix} \quad (41)$$

The results of the nonlinear system (33) with the linearizing control (41), for modeling the following initial values were taken: $x_1(0) = 0$ , $x_2(0) = 1, x_3(0) = -1$ and $x_4(0) = -0.5$. Using MATLAB software (M. file), the state variables responses are shown in this Figure 2. The system is stable as it is indicated in the results of the figure.

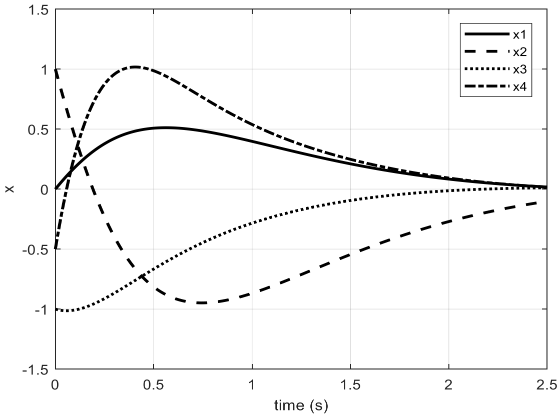


Figure 2. State variables result for the system (33)

## 5. CONCLUSION

In this work, state feedback linearization of a class of nonlinear multivariable systems; the number of states divides exactly the number of inputs, by using block canonical similarity transformation is presented. First the original nonlinear system is represented in state-dependent coefficient form then a block companion similarity transformation is applied to the system. Linearizing control for block canonical system is defined in the form of the feedback; where the feedback gain is computed by forcing the characteristic matrix polynomial of the closed-loop system to equal a desired matrix polynomial constructed from a set of desired block poles or matrix roots in diagonal form. In this approach some advantages are taken: i) Because there exists an approximate linearization method when the exact feedback linearization is not applicable, ii) Because using block controllable form, the eigenvalues and eigenvectors (eigenstructure) of the state matrix, can determine system performance and robustness, iii) Using block poles (solvents) in diagonal form gives a better time response, robustness and smaller sensitivities of the system, iv) Such block similarity transformations allow accomplishing linearization of system without considering the virtual output function $Y = H(x)$ or $y_i = h_i(x)$. In this work, similarity block transformation is developed for class of MIMO system, which seems to be a successful approach. As further work, an enhancement to the present method is proposed, the similarity block transformation will be applied to a general form of MIMO systems; for the case where $l$ (or $\frac{n}{m}$) is not an integer.

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