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Title:

Altitude Backstepping control of quadcopter

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Praise is only to Allah the Almighty, whose blessing and guidance have helped me to accomplish this work first.

I would like to share my immense happiness which could never be achieved without the supervision and the diligence of my supervisor ***Dr. R.Boushaki*** who contributed by all means all along the fulfillment of this thesis.

My most thanks to all IGEE family (students, teachers, workers) who were present since day one of entry till the last day of leaving.

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Dedication

To my beloved parents, to my precious grandmother Fatima Zohra GHERBI and to all other genuine family members.

To my friends Amine BOUDERHEM, Abderrahmane OULDSLIMANE, Idris DERRASHOUK, Ahmed Seddik Bouhamidi and to all those who will be happy for me.

Abstract

This work deals with the study of the stabilization process of a nonlinear control system taking a certain model and derive state space equations through implementation of kinetic and dynamic equations where we present a challenging tool known as backstepping controllers based on Energy functions concept as presented by the famous Russian mathematician Lyaponuv. Simulation of the obtained results is done with *Simulink*.

Parameter	Meaning
ξ	The absolute linear position
η	Euler angles
θ	Pitch angle “y”
φ	Roll angle.
ψ	Yaw angle
R	Rotation matrix
V_B	Linear velocity
v	Angular velocities
R	Rotation Matrix
F_i	Thrust force
W_i	Angular velocity of rotor
α_i	Collective pitch
β_i	Blade orientation
$C\alpha$	Lift Coefficients
K_i	Lift coefficient
K_d	Drag constant
D_i	Drag force
I_x, I_y, I_z	The moment of inertia
u	The control
B_δ	Spherical radius
B_ϵ	Radius

INTRODUCTION

In the last few decades UAV's use has become considerably famous in its applications along the technical and other domains. Because of the maneuverability and simple hovering ability that comes optimally costless many researchers and engineers in control and automatic field put much efforts and cast more focus in order to achieve advances and more usage comfort.

The general topic of this thesis is to study the stability process of the entire dynamics of the quadcopter while applying nonlinear controllers.

The work presented in this project starts by introducing some generalities on quadcopters technology and history, while in the second chapter working and flying principles are to be concisely explained in terms of degrees of freedom and system actuation summing up by a simple model known as DROUIN which will be highlighted in more details in chapter three where BACKSTEPPING command is to take place. By the end we will show simulation results of both regulation and tracking of the altitude control.

CHAPTER ONE

1.1 Introduction

A Quadcopter also named quadrotor is avertical take-off and landing vehicle, classified as rotorcraft as it requires four rotors to provide lift throughout the flight; quadcopters are usually mounted in a cross symmetrical configuration, they have several advantages over the fixed wing aircrafts, can move in any direction and are capable of hovering and flying at low speeds.

Each rotor of a quadrotor plays a specific role for controlling the system, either directional or lift control, rotors are also responsible for a certain amount of thrust and torque about their center of rotation, as well as for a drag force opposite to the rotorcraft’s direction of flight. The quadrotor’s propellers are divided in two pairs, two pusher and two puller blades that rotate in opposite direction. As a consequence, the resulting net torque can be null if all propellers turn with the same angular velocity, thus allowing the aircraft to remain still around its center of gravity.

Nowadays, quadcopters are more and more used in various environments;surveillance, search and rescue, construction inspections,agricultural surveying, post natural disaster analysis, amusements and several other applications.

1.2 Objectives and Motivation

This work will focus on the modeling and the control of an Unmanned Aerial Vehicle “UAV” type quadrotor. The reason of choosing the quadrotor is due to its advantages that will be addressed later,in addition to that; most of studies done on quadrotors use linear flight controllers, these controllers can only perform when the quadrotor is flying around an operating point, they suffer from a huge performance degradation whenever the latter leaves the nominal conditions or performs aggressive maneuvers, the purpose is to go beyond these limits and control the system as it is.

Quadrotor is an under actuated system, it has six Degrees Of Freedom (DOF) and only four actuators (motors). The research field is still facing challenges in controlling quadrotors as being highly nonlinear multivariable systems, they are very difficult to control due to the nonlinear coupling between the actuators and the degrees of freedom.

1.3 Unmanned Aerial Vehicles

UAVs are small aircrafts that are flown without a pilot. They can either be remotely operated by a human or be autonomous; autonomous vehicles are controlled by an onboard computer which can be preprogrammed to perform different or a specific task; while in other literatures, UAVs may refer to powered or unpowered, tethered or untethered aerial vehicles.

1.3.1 History of UAV's

UAVs were first manufactured by Lawrence and Sperry (USA) in 1916. Its name was the Aviation Torpedo as shown in Figure 1.1.It could be flown for a distance of 30 miles. It was reported that Lawrence and Sperry used a gyroscope to balance the body [1].

Figure 1.1: Lawrence and Sperry UAV

1.3.1.1 Military History

A great interest was shown by the USA to develop UAVs to be used in the World War I (WWI) and two projects where funded. The first was by Elmer Sperry to develop the "Flying Bomb" UAV and the second project was the "Kettering Bug" manufactured by General Motors. Both projects were cancelled and the funding stopped as they proved unsuccessful. This is due to the fact of the absence of the required technological advances in the fields of guidance systems and engines. Development of UAVs started increasing tremendously by the end of the 1950s; the USA deployed them during the Vietnam War to decrease the casualties in pilots when flying over hostile territories. After their success, the USA and Israel decided to invest more to build smaller and cheaper UAVs, they used small motors like those found in motorcycles to result in smaller sized and lighter UAVs.

In addition, a video camera was added on the UAVs to transmit images to the ground operator. In 1991, the USA used UAVs extensively in the Gulf War, and the most famous model was the Predator shown in Figure 1.2. UAVs were intensively used by the USA in many conflicts and wars in the late 1990's and early 2000's and later on, UAVs were used abundantly in the war against Iraq.

Figure 1.2: Predator Military UAV

1.3.1.2 Civil History

The use of UAVs was not only confined to military use; in 1969, NASA grew a concern to automatically control an aircraft, the first trials was the PA-30 program. The program was successful but they had a pilot onboard to take over the control of the aircraft in case anything went wrong. Other research programs followed the success of the PA-30 program like: Drones for Aerodynamic and Structural Testing (DAST) and Highly Maneuverable Aircraft Technology (HiMAT) programs [8]. Following that era, in the 1990's NASA then partnered with industrial companies to develop a nine-year long research project called the Environmental Research Aircraft and Sensor Technology Project (ERAST). They developed several UAVs models that were able to fly for altitudes up to 30 Km and endured flights up to 6 months. The resulting UAV models included the: Pathfinder, Helios, Atlus and Perseus B. The developed UAVs carried several sensors to carry out environmental measurements, the onboard sensors included a camera, a digital Array Scanned Interferometer (DASI) and an active detect, see and avoid (DSA) system.

1.3.2 Application of UAVs

In addition to the military use, UAVs can be used in many civil or commercial applications that are too dull, too dirty or too dangerous for manned aircrafts. These uses include but not limited to:

Earth Science observations from UAVs can be used side-to-side with that acquired from satellites. Such missions include:

(a) Measuring deformations in the Earth's crust that may be indications to natural disasters like earthquakes, landslides or volcanoes.

(b) Cloud and Aerosol Measurements.

(c) Troposphere pollution and air quality measurements to determine the pollution sources and how plumes of pollution are transported from one place to another

(d) Ice sheet thickness and surface deformation for studying global warming.

(e) Gravitational acceleration measurements, since the gravitational acceleration varies near Earth, UAVs are used to accurately measure gravitational acceleration at multiple places to define correct references.

(f) River discharge is measured from the volume of water flowing in a river at multiple points. This will help in global and regional water balance studies.

Search and rescue UAVs equipped with cameras are used to search for survivors after natural disasters like earthquakes and hurricanes or survivors from shipwrecks and aircraft crashes.

Wild fire suppression UAVs equipped with infrared sensors are sent to fly over forests prone to fires in order to detect it in time and send a warning back to the ground station with the exact location of the fire before it spreads.

Law enforcement UAVs are used as a cost efficient replacement of the traditional manned police helicopters.

Border surveillance UAVs are used to patrol borders for any intruders, illegal immigrants or drug and weapon smuggling.

Research UAVs are also used in research conducted in universities to proof certain theories. Also, UAVs equipped with appropriate sensors are used by environmental research institutions to monitor certain environmental phenomena like pollution over large cities.

Industrial applications UAVs are used in various industrial applications such as pipeline inspection or surveillance and nuclear factories surveillance.

Agriculture development UAVs also have agriculture uses such as crops spraying [2].

1.3.3 Classification of UAVs

There are different ways to classify UAVs, either according to their range of action, aerodynamic configuration, size and payload or according to their levels of autonomy.

1.3.3.1 Range of Action Classification

UAVs can be classified into 7 different categories based on their maximum altitude and endurance as follows [4]:

(a) High-Altitude Long-Endurance (HALE): they can fly over 15000 m high with an endurance of more than 24 hr. They are mainly used for long-range surveillance missions.
(b) Medium-Altitude Long-Endurance (MALE): they can fly between 5000- 15000 m of altitude for a maximum of 24 hr. MALE UAVs are also used for surveillance missions.
(c) Medium-Range or Tactical UAV (TUAV): They can fly between 100 and 300 km of altitude. They are smaller and operated with simpler systems that their HALE and MALE counterparts.
(d) Close Range UAV: They have an operation range of 100 km. They are mainly used in the civil application such as powerline inspection, crop-spraying, traffic monitoring, homeland security, etc.
(e) Mini UAV (MUAV): They have a weight of about 20 kg and an operating range of about 30 km.
(f) Micro UAV (MAV): They have a maximum wingspan of 150 mm. They are mainly used indoors where they are required to fly slowly and hover.

(g)Nano Air Vehicles (NAV): They have a small size of about 10 mm. they are mainly used in swarms for applications such as radar confusion. They are also used for short range surveillance if equipped with an equally small camera [3].

1.3.3.2 Aerodynamic Configuration Classification

UAVs can be classified into two main categories based on their aerodynamic configuration as follows:

(a) Fixed-wing UAVs: require a run-way to take-off and land. They can fly for a long time and at high cruising speeds. They are mainly used in scientific applications such as meteorological reconnaissance and environmental monitoring, shown in Figure 1.3

Figure 1.3: Fixed Wings UAVs

(b) Rotary-wing UAVs: they can take off and land vertically. They can also hover and fly with high maneuverability. The Rotary-wing UAVs can be further classified into four groups.

(a) Single Rotor

(b) Coaxial

(c) Quadrotor

(d) Multi-Rotor

Figure 1.4: Rotary Wings UAVs

(i) Single-rotor: they have a main rotor on top and another rotor at the tail for stability, same like the helicopter configuration. Shown in Figure 1.4 (a).

(ii) Coaxial: they have two rotors rotating in opposite directions mounted to the same shaft. Shown in Figure 1.4(b).

(iii) Quadrotor: they have four rotors fitted in a cross-like configuration. Shown in Figure 1.4(c).

(iv) Multi-rotor: UAVs with six or eight rotors. They are agile type and fly even when a motor fails, as there is redundancy due to the large number of rotors. Shown in Figure 1.4(d).

Increasing the number of rotors in turn increases the payload and maximum altitude of the UAVs but it comes at the cost of increasing the size and power consumption.

1.4 Quadrotors

The quadrotor concept for aerial vehicles was developed a long time ago. It was reported that the Breguet-Richet quadrotor built in 1907 had actually flown. A quadrotor is considered to be a rotary-wing UAV due to its configuration that will be discussed later.

1.4.1 The Quadrotor Concept

A quadrotor consists of four rotors, each fitted in one end of a cross-like structure as shown in Figure 1.5. Each rotor consists of a propeller fitted to a separately powered DC motor. Propellers 1 and 3 rotate in the same direction while propellers 2 and 4 rotate in an opposite direction leading to balancing the total system torque and cancelling the gyroscopic and aerodynamics torques in stationary flights.

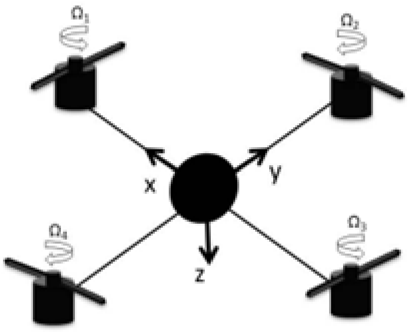

Figure 1.5: Quadrotor Configuration

The quadrotor is a 6 DOF object, thus 6 variables are used to express its position in space (x, y, z, φ, θ and ψ). x, y and z represent the distances of the quadrotor's center of mass along the x,y and z axes respectively from an Earth fixed inertial frame. Φ, θ and ψ are the three Euler angles representing the orientation of the quadrotor. φ is called the roll angle Which is the angle about the x-axis, θ is the pitch angle about the y-axis, while ψ is the yaw angle about the z-axis. Figure 1.6 clearly explains the Euler Angles. The roll and pitch angles are usually called the attitude of the quadrotor, while the yaw angle is referred to as the heading of the quadrotor. For the linear motion, the distance from the ground is referred to as the altitude and the x and y position in space is often called the position of the quadrotor.

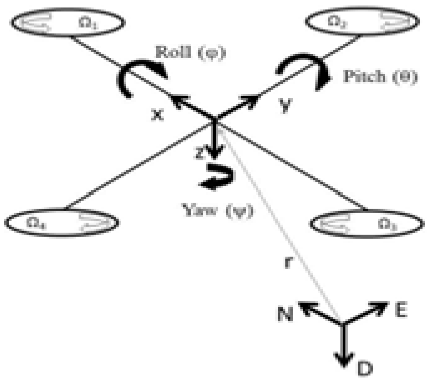

Figure 1.6: Euler Angles for a Quadrotor UAV

To generate vertical upwards motion, the speed of the four propellers is increased together whereas the speed is decreased to generate vertical downwards motion. To produce roll rotation coupled with motion along the y-axis, the second and fourth propellers speeds are changed while for the pitch rotation coupled with motion along the x-axis, it is the first and third propellers speeds that need to be changed. One problem with the quadrotor configuration is that to produce yaw rotation, one need to have a difference in the opposite torque produced by each propeller pair. For instance, for a positive yaw rotation, the speed of the two clockwise turning rotors need to be increased while the speed of the two counterclockwise turning rotors need to be decreased. Figure 1.7 shows how different movements can be produced, note that a thicker arrow means a higher propeller speed.

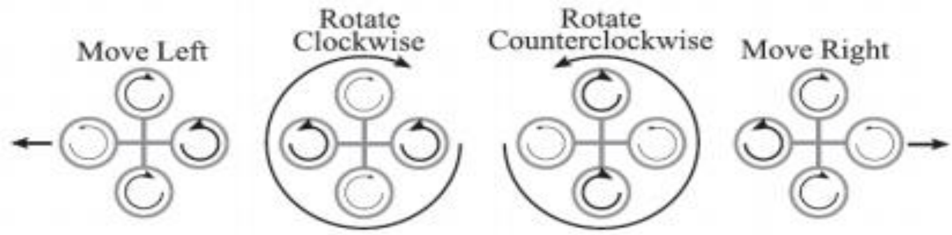

Figure 1.7: Generated Motion of the Quadrotor

1.4.2 Advantages and Drawbacks of Quadrotors

Some advantages of the quadrotor over helicopters is that the rotor mechanics are simplified as it depends on four fixed pitch rotors unlike the variable pitch rotor in the helicopter, thus leading to easier manufacturing and maintenance. Moreover, due to the symmetry in the configuration, the gyroscopic effects are reduced leading to simpler control. Stationary hovering can be more stable in quadrotors than in helicopters due to the presence of four propellers providing four thrust forces shifted a fixed distance from the center of gravity instead of only one propeller centered in the middle as in the helicopters structure. More advantages are the vertical take-off and landing capabilities, better maneuverability and smaller size due to the absence of a tail; these capabilities make quadrotors useful in small area monitoring and buildings exploration. Moreover, quadrotors have higher payload capacities due to the presence of four motors thus providing higher thrust. On the other hand, quadrotors consume a lot of energy due to the presence of four separate propellers. Also, they have a large size and heavier than some of their counterparts again to the fact that there is four separate propellers.

1.4.3 Hardware Components of Quadrotor

Quadcopter hardware componentsvary and are application dependent. Standard components are: microcontrollers, sensors, motors, Global Positioning System (GPS) power supply and telemetry devices. The arms and center plate of the quadcopter frame is in most cases made of carbon fiber. Connections between the center plates and arms, as well as the motor mounts can be made of Aluminum. The modular integration of the frame allows components to be replaced easily if necessary. The propulsion system is mounted directly onto this frame. Another important part is the propulsion unit. The propulsion unit for the quadrotor consists of four brushless DC motors and four electronic speed controllers. The power source for the system can be cell lithium polymer battery. Propellers mounted on the motors must be several cm lengths and have a fixed pitch angle. This propulsion configuration allows safe operations of the frame and ensures excellent lift and thrust performance for all of the flight.

Addressing other sensors like accelerometer and barometer that measure linear acceleration and altitude from the ground respectively is essentially important.

1.5 Future of the industry

Technological improvements will make UAVs faster, stronger and safer. Recent innovations such as hydrogen fuel cells promise to keep them flying for hours. But the real innovations will not come from the aircraft itself, but from its equipment, the analysis of the data gathered and the algorithms that make UAVs react to the external environment. A team of scientists has demonstrated that UAVs were able to build a rope bridge, assemble items to create a structure, or detect and catch an object in the air. These improvements in capabilities are still at an experimental stage but they open up great perspectives for applications in the engineering and construction industries in the coming decade.

1.6 Conclusion

Quadcopter is a special kind of vehicle, which can be implemented in different applications. Quadcopters could be used for a variety of new policing functions. They could be used for safety inspections, perimeter patrols around prisons and thermal imaging to check for cannabis being grown in roof lofts and other not easy to access locations. The police could use them to capture number plates of speeding drivers, for detecting theft from cash machines, railway monitoring, combat fly-posting, fly tipping, abandoned vehicles, waste management. Future research will be in field of search and rescue. In future an effort will be directed to development of a system for defining evacuation/safe path in case of natural disasters and accidents and many other fields.

CHAPTER TWO

2.1 Introduction

In control system, the model is necessary for the analysis and the design; it is essentially a mathematical description which consists of differential equations that can be derived using mathematical and physical laws. This model must be as simple as possible and provides sufficient information about the system behavior.

In the following chapter we will mainly describe all the nonlinear dynamics of a quadcopter, as well as present different models and end up by showing the model used.

2.2 Rotations and Angular Velocities Representation.

The quadcopter structure is presented in Figure 2.1 including the corresponding angular velocities, torques and forces created by the four rotors (numbered from 1 to 4)

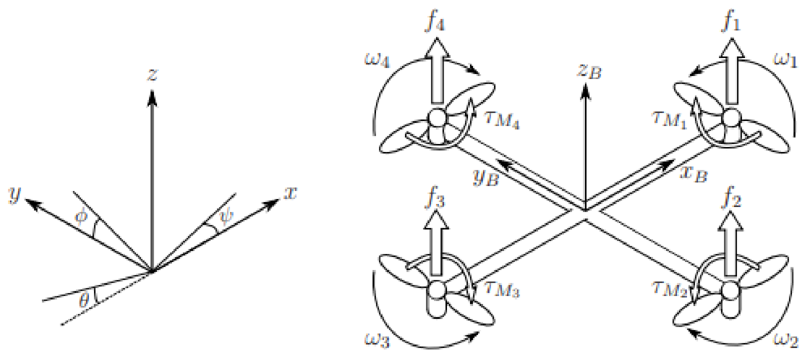

Figure 2.1 The inertial and body fixed frame of Quadcopter.

The absolute linear position of the quadcopter is defined in the inertial frame x,y, and z axes with ξ. The attitude (the angular position) is defined in the inertial frame with three Euler angles η. Pitch angle θ determines the rotation of the quadcopter around the y-axis. Roll angle φ determines the rotation around the x-axis and yaw angle ψ around the z-axis. Vector q contains the linear and angular position vectors.

$$\xi = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \ \eta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, q = \begin{bmatrix} \xi \\ \eta \end{bmatrix}. \tag{2.1}$$

The origin of the body frame is in the center of mass of the quadcopter. In the body frame, the linear velocities are determined by V_B and the angular velocities by Va.

$$V_B = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad Va = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

The rotation matrix from the body frame to the inertial frame is:

$$R = \begin{bmatrix} c\varphi c\theta & c\varphi s\theta s\psi - s\varphi c\psi & c\varphi s\theta c\psi + s\varphi s\psi \\ s\varphi c\theta & s\varphi s\theta s\psi + c\varphi c\psi & s\varphi s\theta c\psi - c\varphi s\psi \\ -s\theta & c\theta s\psi & c\theta c\psi \end{bmatrix} \tag{2.2}$$

In which $S_x = \sin(x)$ and $C_x = \cos(x)$. The rotation matrix R is orthogonal thus $R^{-1} = R^T$ which is the rotation matrix from the inertial frame to the body frame[5].

Instantaneous angular velocity: With 3 angles, we can obtain an instantaneous angular velocity with three components$\dot{\varphi}, \dot{\theta}, \dot{\psi}$. The reference being linked to the center of gravity, the vector v is expressed as follows:

$$Va = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\varphi} \\ 0 \\ 0 \end{bmatrix} + R(x,\varphi)^{-1} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + (R(y,\theta)R(x,\varphi))^{-1} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$$

We can write:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\varphi} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ C_\varphi \dot{\theta} \\ -S_\varphi \dot{\theta} \end{bmatrix} + \begin{bmatrix} -S_\theta \dot{\psi} \\ C_\theta S_\varphi \dot{\psi} \\ C_\theta C_\varphi \dot{\psi} \end{bmatrix}$$

So:

$$P = \dot{\varphi} - S_\theta \dot{\psi} \tag{2.3}$$

$$q = C_\varphi \dot{\theta} + C_\theta S_\varphi \dot{\psi} \tag{2.4}$$

$$r = -S_\varphi \dot{\theta} + C_\theta C_\varphi \dot{\psi} \tag{2.5}$$

We inversely deduce:

$$\dot{\varphi} = p + \tan(\theta)\sin(\varphi)\, q + \tan(\theta)\cos(\varphi)\, r \tag{2.6}$$

$$\dot{\theta} = \cos(\varphi)\, q - \sin(\varphi)\, r \tag{2.7}$$

$$\dot{\psi} = \frac{\sin(\varphi)}{\cos(\theta)}\, q + \frac{\cos(\varphi)}{\cos(\theta)}\, r \tag{2.8}$$

2.3 Forces and Moments

2.3.1 Thrust Force

The thrust force is the force that enables the quadcopter to lift up and fly at some height.
The quadcopter is assumed to have symmetric structure with the four arms aligned with the body x- and y-axes.

The angular velocity of rotor i, denoted with ω_i, creates a force F_i in the direction of the rotor axis.

In General, the thrust force is written as $F_i = C_\alpha \omega_i^2 (\alpha_i - \beta_i)$ proportional to the difference between the collective pitch α_i and the blade orientation β_i, the lift coefficient C_α and the square of the angular speed.

Most of authors reduce the calculation of the lift force to $F_i = k_i\, \omega_i^2$ where k_i is the lift constant.

It should be noted that B_i= Cst when dealing with quadcopters since there is no adjustment of the collective pitch.

The lift coefficient C_α is still present, however, rarely used, we simply integrated into the lift constant k_i.

The total lift force is expressed as follows:

$$F = \sum_{i=1}^{4} F_i = \sum_{i=1}^{4} k_i\, \omega_i^2 \qquad (2.9)$$

2.3.2 Drag Moment

The drag moment is a force acting opposite to the relative motion of a quadcopter moving with respect to a surrounding gas/air.
Clearly, the aerodynamic forces and moments depend on the geometry of the propeller and the air density. Since for the case of quadrotors, the maximum altitude is usually limited, thus the air density can be considered constant.

The drag force can be written as

$$D_i = k_d \omega_i^2 \qquad (2.10)$$

where k_d is the drag constant; which can be determined experimentally for each propeller type.

2.3.3 Gyroscopic effect

It is necessary to underline the importance of this effect as far as the rotors have a significant rotation. The propellers of quadcopters are very weak so if one needs to deliver a precise input signal that is flawless of this effect then taking the weight of the motor rotors is of great importance

2.3.4 Ground effect

Besides the lift force and the drag moment, which are the predominant aerodynamic forces and moments created by a rotor, there exist another external aerodynamic influences which acts on a propeller,it is called ground effect. This refers to the variation of the lift coefficient when the rotor is in close proximity to the ground. Due to its complexity, this effect is only considered for single rotor helicopters.

2.4 Six DOF equations of motion

The equations that determine a 6 DOF body are:
Forces:

$$m\,[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X \qquad (2.13)$$

$$m\,[\dot{v} - wr + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})] = Y \qquad (2.14)$$

$$m\,[\dot{w} - uq + vp - Z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z \qquad (2.15)$$

Moments:

$$I_x\dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K \qquad (2.16)$$

$$I_y\dot{q} + (I_x - I_z)rp - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{zx} + (qp - \dot{r})I_{yz} + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = M \qquad (2.17)$$

$$I_z\dot{r} + (I_y - I_x)pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{zx} + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N \qquad (2.18)$$

X_G, Y_G, Z_G will disappear if we consider the calculations at the center of gravity:

$$m\,[\dot{u} - vr + wq\,] = X \qquad (2.19)$$

$$m\,[\dot{v} - wp + ur\,] = Y \qquad (2.20)$$

$$m\,[\dot{w} - uq + vp\,] = Z \qquad (2.21)$$

$$I_x\dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} = K \qquad (2.23)$$

$$I_y\dot{q} + (I_x - I_z)rp - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{zx} + (qp - \dot{r})I_{yz} = M \quad (2.24)$$

$$I_z\dot{r} + \left(I_y - I_x\right)pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{zx} = N \quad (2.25)$$

Considering that the inertia $I_{xy} = I_{xz} = I_{yz} = 0$, the model will be simplified:

$$m\,[\dot{u} - vr + wq\,] = X \quad (2.26)$$

$$m\,[\dot{v} - wp + ur\,] = Y \quad (2.27)$$

$$m\,[\dot{w} - uq + vp\,] = Z \quad (2.28)$$

$$I_x\dot{p} + \left(I_z - I_y\right)qr = K \quad (2.29)$$

$$I_y\dot{q} + (I_x - I_z)rp = M \quad (2.30)$$

$$I_z\dot{r} + \left(I_y - I_x\right)pq = N \quad (2.31)$$

The quadcopter is assumed to have symmetric structure $I_x = I_y$, and the model is further reduced to:

$$m\,[\dot{u} - vr + wq\,] = X \quad (2.32)$$

$$m\,[\dot{v} - wp + ur\,] = Y \quad (2.33)$$

$$m\,[\dot{w} - uq + vp\,] = Z \quad (2.34)$$

$$I_x\dot{p} + \left(I_z - I_y\right)qr = K \quad (2.35)$$

$$I_y\dot{q} + (I_x - I_z)rp = M \quad (2.36)$$

$$I_z\dot{r} = N \quad (2.37)$$

Speed terms such as *vr*, *wp*,…are of the 2nd order and often over looked, even they are at the origin of couplings. We obtain then a system of equations very simplified

$$m\,\dot{u} = X \quad (2.38)$$

$$m\,\dot{v} = Y \quad (2.39)$$

$$m\,\dot{w} = Z \quad (2.40)$$

$$I_x\dot{p} = K \quad (2.41)$$

$$I_y\dot{q} = M \quad (2.42)$$

$$I_z\dot{r} = N \quad (2.43)$$

2.5 Drouin Model

Drouin uses a model that neglects the gyroscopic effect, ground effect as well as a negligible air density effects .

$$k_2 = \frac{(I_z - I_y)}{I_x} \qquad (2.44)$$

$$k_4 = \frac{(I_x - I_z)}{I_x} \qquad (2.45)$$

I_x, I_y and I_z being the moments of inertia in body-axis and m the total mass of the rotorcraft. The moment equations can be written as [6]:

$$\dot{p} = \left(\frac{l}{I_x}\right)(F_4 - F_2) + k_2\, q\, r \qquad (2.46)$$

$$\dot{q} = \left(\frac{l}{I_y}\right)(F_1 - F_3) + k_4\, p\, r \qquad (2.47)$$

$$\dot{r} = \left(\frac{k}{I_z}\right)(F_2 - F_1 + F_4 - F_3\,) \qquad (2.48)$$

The Euler equations are given by:

$$\dot{\phi} = p + \tan(\theta)\sin(\phi)\, q + \tan(\theta)\cos(\phi)\, r \qquad (2.49)$$

$$\dot{\theta} = \cos(\phi)\, q - \sin(\phi)\, r \qquad (2.50)$$

$$\dot{\psi} = \left(\frac{\sin(\phi)}{\cos(\theta)}\right) q + \left(\frac{\cos(\phi)}{\cos(\theta)}\right) r \qquad (2.51)$$

Where θ, φ, and ψ are respectively the pitch, roll and yaw angles.

The acceleration equations written directly in the local Earth reference system are such as:

$$\ddot{x} = \frac{1}{m}(\cos(\psi)\sin(\theta)\cos(\phi) + \sin(\psi)\sin(\phi))F \qquad (2.52)$$

$$\ddot{y} = \frac{1}{m}(\sin(\psi)\sin(\theta)\cos(\phi) - \cos(\psi)\sin(\phi))F \qquad (2.53)$$

$$\ddot{z} = -g + \frac{1}{m}\cos(\theta)\cos(\phi)\, F \qquad (2.54)$$

Where x, y and z are the center of gravity coordinates
Where:

$$F = F_1 + F_2 + F_3 + F_4 \qquad (2.55)$$

With the constraints:

$$0 \le F_i \le F_{max} \qquad i \in \{1,2,3,4\}$$

2.6 DC Motors

The DC-motor is an actuator which converts electrical energy into mechanical energy and vice versa. It is composed of two interactive electromagnetic circuits.The first one called rotor, is free to rotate around the second one that is called stator which is fixed instead.

2.6.1 Brushed DC Motors

In the rotor, several groups of copper windings are connected in series and are externally accessible through a device called commutator. In the stator, two or more permanent magnets impose a magnetic field which affects the rotor. By applying a DC-current flow into the windings, the rotor turns because of the force generated by the electrical and magnetic interaction.The rotor and commutator geometries, keeps the motorturning while supplied by a DC-voltage on its terminals.

The circuit of the DC-motor is controlled by a real voltage generator $v[V]$which gives the control input. In theory, another resistor should be added in series of the voltage generator representing the driver losses. However, in a good project, the generator losses are kept low therefore it is possible to neglect the min the model. The basic electrical circuit which describes the steady state behavior of the DC motor is shown in figure 2.2

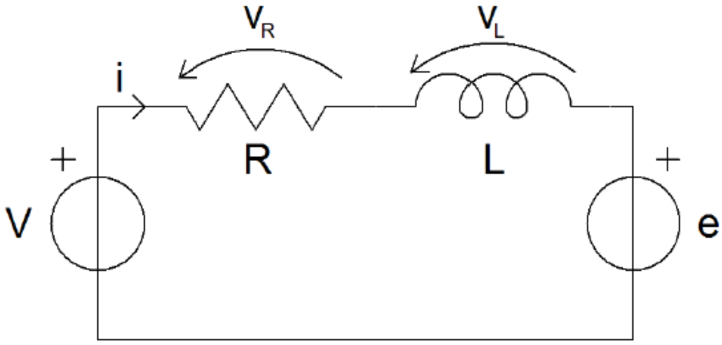

Figure 2.2 Simple Example of a Motor Circuit

2.6.2 Brushless DC Motors

Instead of brushes, the BLDC motor accomplishes commutation electronically using rotor position feedback to determine when to switch the current. Feedback usually entails an attached Hall sensor or a rotary encoder. The stator windings work in conjunction with permanent magnets on the rotor to generate a nearly uniform flux density in the air gap. This permits the stator coils to be driven by a constant DC voltage (hence the name brushless DC).

2.6.3 Use of DC Motors in Quadcopters

Most of Quadcopters use BLDC motors as they offer several advantages over brushed DC motors which include more torque per weight, reduced noise, increased reliability, longer life time and increased efficiency.

The motors should be selected in such a way that it follows thrust to weight relationship.

$$\text{Ratio} = \text{Thrust/Weight} = ma/mg = a/g \qquad (2.56)$$

Thus, vertical take-off and vertical landing (VTOL) is possible only when, (a/g) > 1 or in other words, the total thrust to total weight ratio should be greater than 1 so that the quadcopter can accelerate in the upward direction [7]

$$\text{Total Thrust} = 2*(\text{Total weight of Quadcopter}) \qquad (2.57)$$

Propellers size however is also very important and it is controlled by its two dimensions length and pitch, large propellers are efficient and energy consuming. They can fly to higher altitudes but the increase and the decrease of speed is different in time than the small propellers as opposed.

Quadcopters aerodynamics is majorly affected by the thrust drag. And these forces are constantly impacted by the angular velocity at which the motor blades turn and the two respective coefficients that go with.

$$\mathrm{D_i} = \mathrm{k_d}\omega_\mathrm{i}^2 F_i = k_i\,\omega_i^2 \qquad (2.58)$$

2.7 Sensors

Mainly we have mainly two types of sensors, Accelerometers that determine the position and the direction of the flight and the gyroscope that senses the orientation of the aircraft and allow a good measuring unit for attitude control, in our work however the simulation afforded an easy master over all the dynamics of the system where we showed a simple access to state manipulation and control.

2.7.1 Accelerometer

Accelerometers measure the **lateral acceleration** of the sensor, in a quadcopter they are called a *Microelectromechanical systems* (MEMS)

2.7.2 Gyroscope

This sensor however does a slightly different job since it measures the attitude angles taking the angular velocity as a data and develop a basic integrator block to find the angular position.

2.8 Conclusion

Mathematical models are inherently nonlinear and making use of this, systematically require the application of linear control by linearizing these models. In this quadcopter however we are going to deal with the nonlinear system in the most natural way as Drouin stated. The grounding effect and gyroscopic effect may present other terms of nonlinearities that if a designer set nonlinear controllers these nonlinearities are for sure going to be compensated and dealt with for granted. Unfortunately for simplicity reasons this model pays attention to the moment and acceleration equations that we will tend to control in the next chapter as our main focus will be to control the altitude of the quadcopter while pitch, roll and yaw angles will be null and allow our model to hover and attempt trajectory tracking along one axis.

CHAPTER THREE

3.1 – Introduction to Nonlinear Systems

Physical systems are inherently nonlinear, and the study of matters where nonlinearities are to be handled carefully and widening the operating range has recently showed a strong interest in areas like Aircraft and Spacecraft control, robotics, and biomedical engineering.

In the past, applications of nonlinear control have been limited due to the lack of analytical computations faced, however with advances in computer technology a great enthusiasm has born for researchers and engineers and came to reveal many uncompensated nonlinearities problems by designing their suitable nonlinear controllers.

We distinguish two types of nonlinearities, inherent or natural nonlinearities that come with the system's hardware and motion and which have undesirable effects, the intentional nonlinearities which are artificially introduced by the designer

Nonlinearities can be classified mathematically as continuous and discontinuous nonlinearities which cannot be locally approximated by linear functions and that are known by hard nonlinearities. We have many types of hard nonlinearities (such as, e.g. dead zone, hysteresis or on off nonlinearities)

The subject of nonlinear control is of a great interest to automatic and control applications; it helps a control engineer to get acquainted with practical applications while presenting more significant ways and tools for analysis and systems control.

3.1.1 - Nonlinear Systems Representation

A nonlinear dynamic system is usually represented by a set of nonlinear differential equation

$$\dot{x} = f(x, t) \quad (3.1)$$

Where f is a (n x 1) nonlinear vector function, and x is the (n x l) state vector. The number of states n is called the order of the system. A solution x (t) of the equations (3.1) corresponds to a curve in state space as t varies from zero to infinity, this curve is referred as the phase plan as seen and discussed before for the case where n = 2.

The latter can represent the dynamic system where no control signal is involved which is the case for a swinging pendulum. And it is directly applicable to feedback control systems if it is to be representing the closed-loop dynamic of a control system, with the input being a control of state x and time t which disappears if the plant dynamic is

$$\dot{x} = f(x, u, t)$$

With some control of

$$u = g(x, t)$$

The closed loop dynamics turn into

$$\dot{x} = f(x, g(x, t)\ , t)$$

Which can be written of the form of equation (3.1)

3.2 – Fundamentals of Lyaponuv Theory

Given a control system, stability is the most crucial property one might question himself about, because an unstable system literarily makes no use and a stable system is the one that if starting it somewhere near its operating point implies that it will stay in the neighborhood of this point. So namely the trajectory that a designer wants his control system whether linear or nonlinear to track is characterized by the operating point in the absence of disturbance, so at a given moment if the quadcopter is disturbed by air gust and it deviates from the assigned trajectory and never comes back we say the system is unstable.

The general problem of motion stability was introduced in late 19th century by the great Russian mathematician AlexandrMikhailovichLyapunov which extends for two essential works (Linearization and Direct method). The linearization is restricted for local motion as it draws conclusions about the stability of the system as linearly approximated. However the Direct method is a powerful tool for design purpose mainly and which consists of forming and making restrictions for stability boundaries by analyzing the Energy like functions.

3.2.1 - Stability Concepts

Stability is a fundamental issue in control system analysis and design. Essentially we have three types of stability, stability in the Lyaponuv sense or marginal stability, asymptotic stability and exponential stability.

Given the nonlinear system $\dot{x} = f\{x(t)\}$and the equilibrium point $x = 0$

A simplifying formulation of the stability concept is to illustrate the systems stability by a simple spherical region where the state is initially excited within a certain operating range of the inner BallB_{ϵ}, known for local excitation. (Ball) isdefined by $\|x\| = \delta$ is B_{δ}

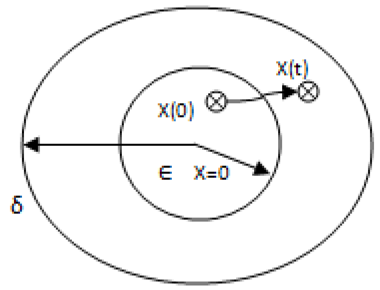

Figure 3.1 Spherical representation of the concept of stability

Definition 1: the equilibrium point x=0 is stable in the Lyaponuv sense (marginal stability) if $\forall \in > 0, \exists \delta > 0$ such that $\|x(0)\| < \in \;=> \|x(t)\| < \delta$ for all $t \geq 0$ [8]

So if the initial state starts somewhere not far from the operating range, as time goes by if the state at $t \geq 0$ remains in the neighborhood of the operating point $x = 0$ then we say the operating point $x = 0$ is marginally stable.

In some applications marginal stability is limited and is not of a great interest. From a control engineering optic if we consider our quadrotor to track a certain reference formulated by the designer in case intentional nonlinearities take places and attempt to perturb the system and we want to maintain the altitude with the respect to the fixed inertial frame x, y the state may deviate from its trajectory, respecting the inner stability process but not knowing how to come back to the origin $x = 0$.

Asymptotic stability will therefore guarantee the return of the aforementioned and enable the nonlinear controller to rapidly compensate the process of current deviation.

Definition 2: Equilibrium point 0 is asymptotically stable if it is stable, and if in Addition there exists some $r > 0$ such that $\|x(0)\| < r$ implies that $x(t) \to 0$ as $t \to \infty$

one may question the need of stability requirements, as viewed in asymptotic stability which brings the concept of trajectory convergence toward 0. In some applications the trajectory convergence speed is needed to be estimated and carefully studied and in for this concept we introduce the so-called exponential stability.

Definition 3: Equilibrium point 0 is exponentially stable if there exists two strictly positive numbers α, λ such that,

$$\forall t > 0, \|x(t)\| \leq \alpha\|x(0)\|e^{-\lambda t}$$

In some ball B_ϵ around the origin, so the above definitions are formulated to characterize the local behavior of the system, however if asymptotic or exponential stability holds for any initial state then we say that the system is globally asymptotically or exponentially stable.

3.2.2 – Lyaponuv Direct Method

Central idea of Lyaponuv's theory is when system is at rest its energy is zero and small perturbation might provide the system with a certain amount of energy and observing the latter yield to three different choices. Energy remains constant as time goes by, energy dissipates and the state returns to the equilibrium point $= 0$, energy increases which eventually yield to an unstable system.

3.2.2.1 – Local Stability

Theorem: If, in a ball B_ϵ centered at $x = 0$ we can find a function $V(x)$ with continuous first partial derivative such that [10]

i) $V(x)$ is locally positive definite in Be

ii) $\dot{V}(x)$is negative semi definite in Be

Then the equilibrium point 0 is stable in the Lyaponuv sense. If, actually, the derivative of energy like function $\dot{V}(x)$ is locally negative definite inB_ϵ, then the system is asymptotically stable.

3.2.2.2 – Global Stability

Theorem: Assume there exists a function $V(x)$ with continuous first partial derivative such that,

i) $V(x)$ is globally positive definite

ii) $\dot{V}(x)$is globally negative definite

iii) $\lim_{\|x\| \to \infty} V(x) = \infty$

Then the equilibrium point 0 is stable is globally asymptotically stable.

Along the same lines, it is important to realize that the theorems in Lyapunov analysis are all sufficiency theorems. If for a particular choice of Lyapunov function candidate V, the conditions on V are not met, one cannot draw any conclusions on the stability or instability of the system - the only conclusion one should draw is that a differentLyapunov function candidate should be tried. [9]

3.2.3 – Control Design Based on Lyapunov's Direct Method

In the previous sections we were only dealing with the stability analysis on the basis of a real lyaponuv candidate function, now we move to the design where someone needs to use a nonlinear controller, hypothesize the Lyaponuv function and guarantee its usefulness by setting a control law to justify the use and make the lyaponuv function a real lyaponuv candidate to asymptotically stabilize the system and yield to practical nonlinear feedback control application.

Many feedback control techniques are based on the idea of designing the feedback control in such a way that a Lyapunov function, or more specifically the derivative of a Lyapunov function, has certain properties that guarantee boundededness of trajectories and convergence to an equilibrium point or an equilibrium set. Backstepping is a recursive technique that normally breaks the design problem into subsystems of lower order, by exploiting the flexibility of lower order and even scalar system

3.3– Backstepping Command

Backstepping is an iterative algorithm that relax the stabilization of a nonlinear control system by exciting it with the appropriate control law, while verifying Lyaponuv energy like functions and nominating them as a real candidate and that's by meeting asymptotic stability conditions.

We consider this basic second order approach

$$\dot{x}_1 = x_2 (3.2) \ \ \dot{x}_2 = u$$

Step 1: We desire x_1 to track our desired state x_{1d} , we consider x_2 as the first input to the system and we denote it by α, we shall now write the first error equation [11]

$$\epsilon_1 = \mathrm{x}_{1\mathrm{d}} - \mathrm{x}_1 \qquad (3.3)$$

Let $V_1(x)$ be a function of x, $V_1(x)$ is said to be lyaponuv function if

i) $V_1(x)$ is positive definite

ii) $\dot{V}_1(x)$ is locally negative definite

We define the first lyaponuvfunction and its first derivative

$$V_1(x) = \frac{1}{2}\epsilon_1^2 \qquad (3.4)$$

$$\dot{V}_1(x) = \epsilon_1\dot{\epsilon}_1 \qquad (3.5)$$

By deriving the error and using relation (3.2) we get

$$\dot{\epsilon}_1 = \dot{x}_{1d} - x_2 \qquad (3.6)$$

By plugging (3.6) in (3.5) and replacing $x_2$by α we get

$$\dot{V}_1 = \epsilon_1(\dot{x}_{1d} - \alpha) \qquad (3.7)$$

So that the system converges and Lyaponuv condition is valid for asymptotic stability these two conditions suffice

$$V_1(x) > 0\ \forall x$$

$$\dot{V}_1(x) < 0\ \forall x$$

To guarantee $\dot{V}_1(x) < 0$ (Convergence condition) the following condition must be verified.

$$-\epsilon_1 K_1 = (\dot{x}_{1d} - \alpha) \qquad (3.8)$$

if$K_1 > 0$ is positive then the condition is fulfilled and we get the law forOur pseudo-entry.

$$\alpha = (\dot{x}_{1d}+\epsilon_1 K_1)$$

Which is from equation (3.3) equals

$\alpha = (\dot{x}_{1d} + K_1\ (x_{1d} - x_1))$ (3.9)

So classically if we wish that x_1 to track x_{1d} and remain at this value then, $\dot{x}_{1d} = 0$, therefore $\dot{x}_1 = 0$,

$+K_1\ x_{1d} - K_1\ x_1 = 0$ then $x_1 = x_{1d}$

Step 2 : First step implies that if $\alpha = (\dot{x}_{1d} + \epsilon_1 K_1)$ then this yield to this equality $x_1 = x_{1d}$

Meanwhile there exists another possibility for x_2 to deviate from α and form a second error ϵ_2

$\epsilon_2 = \alpha - x_2$

Take the derivative of the second error,

$\dot{\epsilon}_2 = \dot{\alpha} - \dot{x}_2$ (3.10)

We have from the derivative of the first error

$$\dot{\epsilon}_1 = \dot{x}_{1d} - x_2$$

$\dot{\epsilon}_1 = \dot{x}_{1d} + \epsilon_2 - \alpha$ (3.11)

And also from state space equation (3.2) we have

$$\dot{\epsilon}_2 = \dot{\alpha} - u$$

$\dot{\epsilon}_2 = \ddot{x}_{1d} + \dot{\epsilon}_1 K_1 - u$ (3.12)

Now we exert another Lyaponuv function bearing in mind the two errors

$V_2(x) = \frac{1}{2}\epsilon_1^2 + \frac{1}{2}\epsilon_2^2$ (3.13)

We obtain the first derivative with respect to time

$\dot{V}_2(x) = \epsilon_1\dot{\epsilon}_1 + \epsilon_2\dot{\epsilon}_2$ (3.14)

Putting (3.11) and (3.12) and by substitution equation $\dot{V}_2(x)$ becomes,

$$\dot{V}_2(x) = -K_1\epsilon_1^2 + \epsilon_2(\epsilon_1 + K_1\dot{\epsilon}_1 + \ddot{x}_{1d} - u)$$

If wecould have

$$(\epsilon_1 + K_1\dot{\epsilon}_1 + \ddot{x}_{1d} - u) = -K_2\epsilon_2$$

Second Lyaponuv function becomes

$$\dot{V}_2(x) = -K_1\epsilon_1^2 - K_2\epsilon_2^2 < 0$$

And the command law is of the form:

$$\boldsymbol{u = \epsilon_1 + K_1\dot{\epsilon}_1 + \ddot{x}_{1d} + K_2\epsilon_2} \qquad \textbf{(3.15)}$$

3.3.1 – Quadcopter reviewed dynamics

Used Model is called the Drouin model as described precisely in chapter two where the gyroscopic effect is not taken into account and the latter will alleviate much complexity on the conducted study of the following technique.

$$k_2 = \frac{(I_z - I_y)}{I_x}$$

$$k_4 = \frac{(I_x - I_z)}{I_x}$$

For the adopted vector state, state variables are of this approach

$$\dot{X} = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, z, \dot{z}, y, \dot{y}, x, \dot{x}]^T]$$

With a system of differential equation as the following, the acceleration equations written directly in the local Earth reference system [6]

$$\dot{x} = u$$

$$\dot{y} = v$$

$$\dot{z} = w$$

$$\dot{u} = \frac{1}{m}(\cos(\psi)\sin(\theta)\cos(\phi) + \sin(\psi)\sin(\phi))F$$

$$\dot{v} = \frac{1}{m}(\sin(\psi)\sin(\theta)\cos(\phi) - \cos(\psi)\sin(\phi))F$$

$$\dot{w} = -g + \frac{1}{m}\cos(\theta)\cos(\phi)\,F$$

I_x, I_y and I_z being the moments of inertia in body-axis and m the total mass of the quadcopter.

The moment equations can be written as:

$$\dot{\phi} = p$$
$$\dot{\theta} = q$$
$$\dot{\psi} = r$$

From Chapter two,

$$\dot{p} = \left(\frac{l}{I_x}\right)(F_4 - F_2) + k_2 qr$$
$$\dot{q} = \left(\frac{l}{I_y}\right)(F_1 - F_3) + k_4 pr$$
$$\dot{r} = \left(\frac{k}{I_z}\right)(F_2 - F_1 + F_4 - F_3)$$

3.3.2 - Control Law Design

Now we can attempt to apply the backstepping mentioned above in both Euler angles not which yield mainly to nonlinearities compensation within the system.

Our main mission is to find the total control input denoted by$U = [U1\ U2\ U3\ U4]$, bearing in mind that our used model is the model described both in chapter 2 and chapter 3 in the above section which takes the drouin model for simplicity reasons.

Recall the state vector is$\dot{X} = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, z, \dot{z}, y, \dot{y}, x, \dot{x}]^T$

Now we shall present the four second order subsystems

$$\dot{\phi} = p$$
$$\dot{p} = \left(\frac{l}{I_x}\right)(F_4 - F_2) + k_2 qr \qquad (3.16)$$

$U1$=$(F_4 - F_2)$

$$\dot{\theta} = q$$
$$\dot{q} = \left(\frac{l}{I_y}\right)(F_1 - F_3) + k_4 pr \qquad (3.17)$$

$U2$=$(F_1 - F_3)$

$$\dot{\psi} = r$$

$$\dot{r} = \left(\frac{k}{I_z}\right)(F_2 - F_1 + F_4 - F_3) \qquad (3.18)$$

$U3 = (F_2 - F_1 + F_4 - F_3)$

And last the 4th subsystem which attempts to regulate the altitude of this quadcopter along the z Axis.

$$\dot{z} = w$$

$$\dot{w} = -g + \frac{1}{m} cos(\theta) cos(\phi) F \qquad (3.19)$$

$U4 = (F_1 + F_2 + F_3 + F_4) = F$

Now we shall start presenting the backstepping gains assigned for this work, in which I used two gains for Z altitude control denoted by L_3 and L_4

For the Euler angles I used two main gains for the entire process, thereby for the three remaining subsystem we will have L_1 and L_2

<u>Control command of the Roll angle.</u>

$$\dot{\phi} = p$$

$$\dot{p} = \left(\frac{l}{I_x}\right)(F_4 - F_2) + k_2 qr \qquad (3.16)$$

<u>Step1:</u>

We define the first error that represents the deviation between the actual and the desired value which is zero.

$\epsilon_1 = \phi_d - \phi$ (3.17)

We introduce now α to be the next pseudo input for the subsystem, by taking the derivative and subsitututing the first derivative of the error becomes

$\dot{\epsilon_1} = \dot{\phi}_d - \alpha$(3.18)

We define the first Lyaponuvfunction and its derivative:

$$V_1(x) = \frac{1}{2}\epsilon_1^2$$

$$\dot{V}_1(x) = \epsilon_1 \dot{\epsilon}_1$$

Now to meet lyaponuv conditions as discussed in the previous section we have to verify this equation where $L_1 > 0$

$$\dot{\epsilon}_1 = -L_1 \epsilon_1$$

$$\dot{\phi}_d - \alpha = -L_1 \epsilon_1$$

Which after substitution yields to a pseudo input α of the form

$\alpha = \dot{\phi}_d + L_1 \epsilon_1$(3.19)

Step 2:

Now we form a second error

$$\epsilon_2 = \alpha - p \qquad (3.20)$$

By taking the derivative of the second error

$$\dot{\epsilon}_2 = \dot{\alpha} - \dot{p} \qquad (3.21)$$

From equation (3.18) and (3.20) we have

$$\dot{\epsilon}_1 = \dot{\phi}_d - p$$

$$\dot{\epsilon}_1 = \dot{\phi}_d + \epsilon_2 - \alpha \qquad (3.22)$$

And also from state space equation (3.16) we have

$$\dot{\epsilon}_2 = \dot{\alpha} - \left(\frac{l}{I_x}\right)(F_4 - F_2) - k_2 q r$$

$$\dot{\epsilon}_2 = \ddot{\phi}_d + L_1 \dot{\epsilon}_1 - \left(\frac{l}{I_x}\right)(F_4 - F_2) - k_2\, q\, r \qquad (3.23)$$

Now we exert another Lyaponuv function bearing in mind the two errors

$$V_2(x) = \frac{1}{2}\epsilon_1^2 + \frac{1}{2}\epsilon_2^2$$

We obtain the first derivative with respect to time

$$\dot{V}_2(x) = \epsilon_1 \dot{\epsilon}_1 + \epsilon_2 \dot{\epsilon}_2$$

Putting 3.22 and 3.23 and by substitution, equation $\dot{V}_2(x)$ becomes,

$$\dot{V}_2(x) = \epsilon_1(\dot{\phi}_d + \epsilon_2 - \alpha) + \epsilon_2(\ddot{\phi}_d + L_1\dot{\epsilon}_1 - \left(\frac{l}{I_x}\right)(F_4 - F_2) - k_2qr)$$

$$\dot{V}_2(x) = \epsilon_1(\dot{\phi}_d + \epsilon_2 - \dot{\phi}_d - L_1\epsilon_1) + \epsilon_2(\ddot{\phi}_d + L_1\dot{\epsilon}_1 - \left(\frac{l}{I_x}\right)(F_4 - F_2) - k_2qr)$$

$$\dot{V}_2(x) = -L_1\epsilon_1^2 + \epsilon_2(\epsilon_1 + \ddot{\phi}_d + L_1\dot{\epsilon}_1 - \left(\frac{l}{I_x}\right)(F_4 - F_2) - k_2\, q\, r)$$

If we could have

$$\left(\epsilon_1 + \ddot{\phi}_d + L_1\dot{\epsilon}_1 - \left(\frac{l}{I_x}\right)(F_4 - F_2) - k_2qr\right) = -L_2\epsilon_2$$

Second Lyaponuv function becomes

$$\dot{V}_2(x) = -L_1\epsilon_1^2 - L_2\epsilon_2^2 < 0$$

And the command law will achieve asymptotic stability of the subsystem

$$\boldsymbol{U1} = (\boldsymbol{F_4} - \boldsymbol{F_2}) = \frac{I_x}{l}(\boldsymbol{\epsilon_1} + \ddot{\boldsymbol{\phi}}_d + \boldsymbol{L_1}\dot{\boldsymbol{\epsilon}}_1 + \boldsymbol{L_2}\boldsymbol{\epsilon_2} - \boldsymbol{k_2}\boldsymbol{qr})$$

Control command of the Pitch angle.

$$\dot{\theta} = q$$

$$\dot{q} = \left(\frac{l}{I_y}\right)(F_1 - F_3) + k_4pr \qquad (3.17)$$

Step1:

We define the first error that represents the deviation between the actual and the desired value which is zero.

$$\epsilon_1 = \theta_d - \theta \qquad (3.24)$$

we introduce now $\alpha$to be the next pseudo input for the subsystem, by taking the derivative and subsitututing the first derivative of the error becomes

$$\dot{\epsilon}_1 = \dot{\theta}_d - \alpha \qquad (3.25)$$

We define the first lyaponuv function and its derivative:

$$V_1(x) = \frac{1}{2}\epsilon_1^2$$

$$\dot{V}_1(x) = \epsilon_1\dot{\epsilon}_1$$

Now to meet lyaponuv conditions as discussed in the previous section we have to verify this equation where $L_1 > 0$

$$\dot{\epsilon}_1 = -L_{1}\epsilon_1$$

$$\dot{\theta}_d - \alpha = -L_{1}\epsilon_1$$

Which after substitution yields to a pseudo input α of the form

$\alpha = \dot{\theta}_d + L_1\epsilon_1$ *(3.26)*

Step 2:

Now we form a second error

$\epsilon_2 = \alpha - q$(3.27)

by taking the derivative of the second error

$\dot{\epsilon}_2 = \dot{\alpha} - \dot{q}$(3.28)

From equation (25) and (27) we have

$$\dot{\epsilon}_1 = \dot{\theta}_d - q$$

$$\dot{\epsilon}_1 = \dot{\theta}_d + \epsilon_2 - \alpha$$

$\dot{\epsilon}_1 = \epsilon_2 - L_1\epsilon_1$(3.29)

And also from state space equation (3.17) we have

$$\dot{\epsilon}_2 = \dot{\alpha} - \left(\frac{l}{I_y}\right)(F_1 - F_3) - k_4 pr$$

$\dot{\epsilon}_2 = \ddot{\theta}_d + L_1\dot{\epsilon}_1 - \left(\frac{l}{I_y}\right)(F_1 - F_3) - k_4 pr$(3.30)

Now we exert another Lyaponuv function bearing in mind the two errors

$$V_2(x) = \frac{1}{2}\epsilon_1^2 + \frac{1}{2}\epsilon_2^2$$

We obtain the first derivative with respect to time

$$\dot{V}_2(x) = \epsilon_1\dot{\epsilon}_1 + \epsilon_2\dot{\epsilon}_2$$

Putting 3.29 and 3.30 and by substitution equation $\dot{V}_2(x)$ becomes,

$$\dot{V}_2(x) = \epsilon_1(\epsilon_2 - L_{1}\epsilon_1) + \epsilon_2(\ddot{\theta}_d + L_{1}\dot{\epsilon}_1 - \left(\frac{l}{I_y}\right)(F_1 - F_3) - k_4\, p\, r)$$

$$\dot{V}_2(x) = -L_{1}\epsilon_1^2 + \epsilon_2(\epsilon_1 + \ddot{\theta}_d + L_{1}\dot{\epsilon}_1 - \left(\frac{l}{I_y}\right)(F_1 - F_3) - k_4\, p\, r)$$

If wecould have

$$\left(\epsilon_1 + \ddot{\theta}_d + L_1\dot{\epsilon}_1 - \left(\frac{l}{I_y}\right)(F_1 - F_3) - k_4\, p\, r\right) = -L_2\epsilon_2$$

Second Lyaponuv function becomes

$$\dot{V}_2(x) = -L_1\epsilon_1^2 - L_2\epsilon_2^2 < 0$$

And the command law will achieve asymptotic stability of the subsystem

U2=$\mathbf{(F_1 - F_3) = \frac{I_y}{l}(\epsilon_1 + \ddot{\theta}_d + L_1\dot{\epsilon}_1 + L_2\epsilon_2 - k_4\, p\, r)}$

<u>Control command of the Yaw angle</u>.

$$\dot{\psi} = r$$

$\dot{r} = \left(\frac{k}{I_z}\right)(F_2 - F_1 + F_4 - F_3)$ *(3.18)*

<u>Step1:</u>

We define the first error that represents the deviation between the actual and the desired value which is zero.

$\epsilon_1 = \psi_d - \psi$ (3.31)

we introduce now α to be the next pseudo input for the subsystem, by taking the derivative and subsitututing the first derivative of the error becomes

$\dot{\epsilon}_1 = \dot{\psi}_d - \alpha$(3.32)

We define the first Lyaponuv function and its derivative:

$$V_1(x) = \frac{1}{2}\epsilon_1^2$$

$$\dot{V}_1(x) = \epsilon_1\dot{\epsilon}_1$$

Now to meet lyaponuv conditions as discussed in the previous section we have to verify this equation where $L_1 > 0$

$$\dot{\epsilon}_1 = -L_1\epsilon_1$$

$$\dot{\psi}_d - \alpha = -L_1\epsilon_1$$

Which after substitution yields to a pseudo input α of the fo*rm*

$$\alpha = \dot{\psi}_d + L_1\epsilon_1 \qquad (3.33)$$

<u>Step 2:</u>

Now we form a second error

$$\epsilon_2 = \alpha - r \qquad (3.34)$$

by taking the derivative of the second error

$$\dot{\epsilon}_2 = \dot{\alpha} - \dot{r} \qquad (3.35)$$

From equation (3.32) and (3.34) we have

$$\dot{\epsilon_1} = \dot{\psi}_d - r$$

$$\dot{\epsilon_1} = \dot{\psi}_d + \epsilon_2 - \alpha$$

$$\dot{\epsilon_1} = \epsilon_2 - L_1\epsilon_1 \ (3.36)$$

And also from state space equation (3.18) we have

$$\dot{\epsilon}_2 = \dot{\alpha} - \left(\frac{k}{I_z}\right)(F_2 - F_1 + F_4 - F_3)$$

$$\dot{\epsilon}_2 = \ddot{\psi}_d + L_1\dot{\epsilon}_1 - \left(\frac{k}{I_z}\right)(F_2 - F_1 + F_4 - F_3) \quad (3.37)$$

Now we exert another Lyaponuv function bearing in mind the two errors

$$V_2(x) = \frac{1}{2}\epsilon_1^2 + \frac{1}{2}\epsilon_2^2$$

We obtain the first derivative with respect to time

$$\dot{V}_2(x) = \epsilon_1\dot{\epsilon}_1 + \epsilon_2\dot{\epsilon}_2$$

Putting 3.36 and 3.37 and by substitution equation $\dot{V}_2(x)$ becomes,

$$\dot{V}_2(x) = \epsilon_1(\epsilon_2 - L_1\epsilon_1) + \epsilon_2\left(\ddot{\psi}_d + L_1\dot{\epsilon}_1 - \left(\frac{k}{I_z}\right)(F_2 - F_1 + F_4 - F_3)\right)$$

$$\dot{V}_2(x) = -L_1\epsilon_1^2 + \epsilon_2\left(\epsilon_1 + \ddot{\psi}_d + L_1\dot{\epsilon}_1 - \left(\frac{k}{I_z}\right)(F_2 - F_1 + F_4 - F_3)\right)$$

If wecould have

$$\left(\epsilon_1 + \ddot{\psi}_d + L_1\dot{\epsilon}_1 - \left(\frac{k}{I_z}\right)(F_2 - F_1 + F_4 - F_3)\right) = -L_2\epsilon_2$$

Second Lyaponuv function becomes

$$\dot{V}_2(x) = -L_1\epsilon_1^2 - L_2\epsilon_2^2 < 0$$

And the command law will achieve asymptotic stability of the subsystem if

U3=$\mathbf{(F_2 - F_1 + F_4 - F_3) = \frac{I_z}{k}(\epsilon_1 + \ddot{\psi}_d + L_1\dot{\epsilon}_1 + L_2\epsilon_2)}$

<u>Control command of the Z position.</u>

$$\dot{z} = w$$

$\dot{w} = -g + \frac{1}{m} cos(\theta)\, cos(\phi)\, F$ *(3.19)*

<u>Step1:</u>

We define the first error that represents the altitude deviation between the actual and the desired value

$$\epsilon_1 = z_d - z$$

we introduce now α to be the next pseudo input for the subsystem, by taking the derivative and subsitututing the first derivative of the error becomes

$\dot{\epsilon_1} = \dot{z}_d - \alpha$ (3.36)

We define the first lyaponuv function and its derivative:

$$V_1(x) = \frac{1}{2}\epsilon_1^2$$

$$\dot{V}_1(x) = \epsilon_1\dot{\epsilon}_1$$

Now to meet lyaponuv conditions as discussed in the previous section we have to verify this equation where $L_3 > 0$

$$\dot{\epsilon_1} = -L_3\epsilon_1$$

$$\dot{\psi}_d - \alpha = -L_3\epsilon_1$$

Which after substitution yields to a pseudo input α of the form

$\alpha = \dot{z}_d + L_3\epsilon_1$ (3.37)

<u>Step 2:</u>

Now we form a second error

$\epsilon_2 = \alpha - W$(3.38)

by taking the derivative of the second error

$\dot{\epsilon}_2 = \dot{\alpha} - \dot{W}$ (3.39)

From equation (3.36) and (3.38) we have

$$\dot{\epsilon_1} = \dot{z}_d - W$$

$$\dot{\epsilon_1} = \dot{\psi}_d + \epsilon_2 - \alpha$$

$\dot{\epsilon_1} = \epsilon_2 - L_3\epsilon_1$ (3.40)

And also from state space equation (3.19) we have

$$\dot{\epsilon}_2 = \dot{\alpha} + g - \frac{1}{m}\cos(\theta)\cos(\phi)\,F$$

$$\dot{\epsilon}_2 = \ddot{Z}_d + L_3\dot{\epsilon}_1 + g - \frac{1}{m}\cos(\theta)\cos(\phi)\,F \qquad (3.41)$$

Now we exert another Lyaponuv function bearing in mind the two errors

$$V_2(x) = \frac{1}{2}\epsilon_1^2 + \frac{1}{2}\epsilon_2^2$$

We obtain the first derivative with respect to time

$$\dot{V}_2(x) = \epsilon_1\dot{\epsilon}_1 + \epsilon_2\dot{\epsilon}_2$$

Putting 3.40 and 3.41 and by substitution equation $\dot{V}_2(x)$ becomes,

$$\dot{V}_2(x) = \epsilon_1(\epsilon_2 - L_3\epsilon_1) + \epsilon_2(\ddot{Z}_d + L_3\dot{\epsilon}_1 + g - \frac{1}{m}\cos(\theta)\cos(\phi)\,F)$$

$$\dot{V}_2(x) = -L_3\epsilon_1^2 + \epsilon_2(\epsilon_1 + \ddot{Z}_d + L_3\dot{\epsilon}_1 + g - \frac{1}{m}\cos(\theta)\cos(\phi)\,F)$$

If wecould have

$$\left(\epsilon_1 + \ddot{Z}_d + L_3\dot{\epsilon}_1 + g - \frac{1}{m}\cos(\theta)\cos(\phi)\,F\right) = -L_4\epsilon_2$$

Second Lyaponuv function becomes

$$\dot{V}_2(x) = -L_1\epsilon_1^2 - L_2\epsilon_2^2 < 0$$

And the command law will achieve asymptotic stability of the subsystem if

$$\mathbf{U4 = F = m/\cos(\theta)\cos(\phi)\,(\epsilon_1 + \ddot{Z}_d + L_3\dot{\epsilon}_1 + L_4\epsilon_2 + g)}$$

3.4 Conclusion

The Backstepping is an effective tool for stabilizing nonlinear system as it deals with the fact of stabilizing the system iteratively starting from upper order and meet lower order result.

Now by the end we can say that we have obtained the four input forces, and what comes next practically is simply the operation of exciting the dynamics of our model in a systematic way and study the behavior of the latter and observe further altitude control as this can deviate if not setting the right gain that nevertheless applies to respect the Lyaponuv direct method but also to good trajectory tracking as will be seen in the next chapter.

CHAPTER FOUR

4.1. Introduction

In the previous chapter, using Backstepping command I have designed four input control for each subsystem to be controlled and so that its actual and desired states will be alike.

In this chapter we shall start seeing the response of the nonlinear control system as excited by two sort of inputs. The first one will be a simple step function that I have programmed in which the quadcopter's velocity and acceleration is initially at rest and as time goes by the hovering process will start and reach a fixed value and remains there.

The second input is slightly complementary as the system is not only required to rich a certain position but to land the quadcopter in a very rigorous manner and we shall see the variation of the speed and acceleration that corresponds to this command.

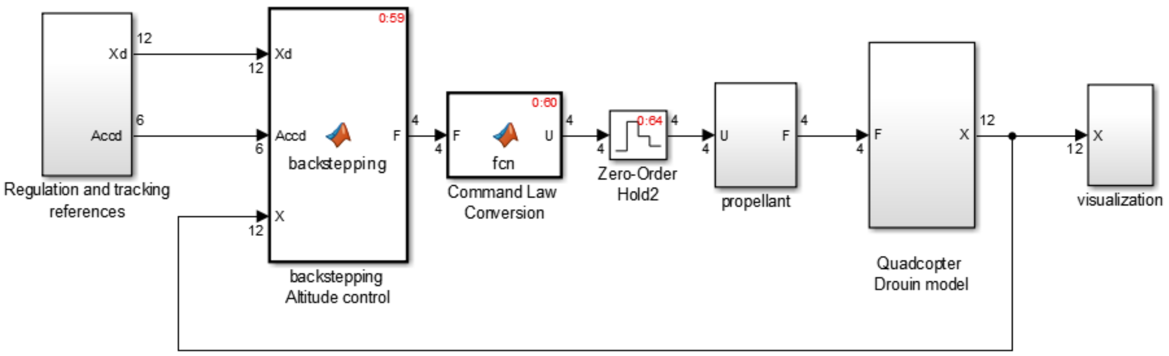

Figure 4.1 general diagram representation of the nonlinear system

4.2. Angular Design Approach

As we all know for an excellent command of a quadcopter altitude the Euler angles with respect to both body fixed frame and inertial frame must be zeroed out. This not only ensures the visionary stability of the quadcopter with respect to the horizontal access but will also guarantee no variation along the X and Y direction.

If the nonlinear controller comes to achieve a desired angles which in this case is Zero then controlling the altitude would be much easier to focus on.

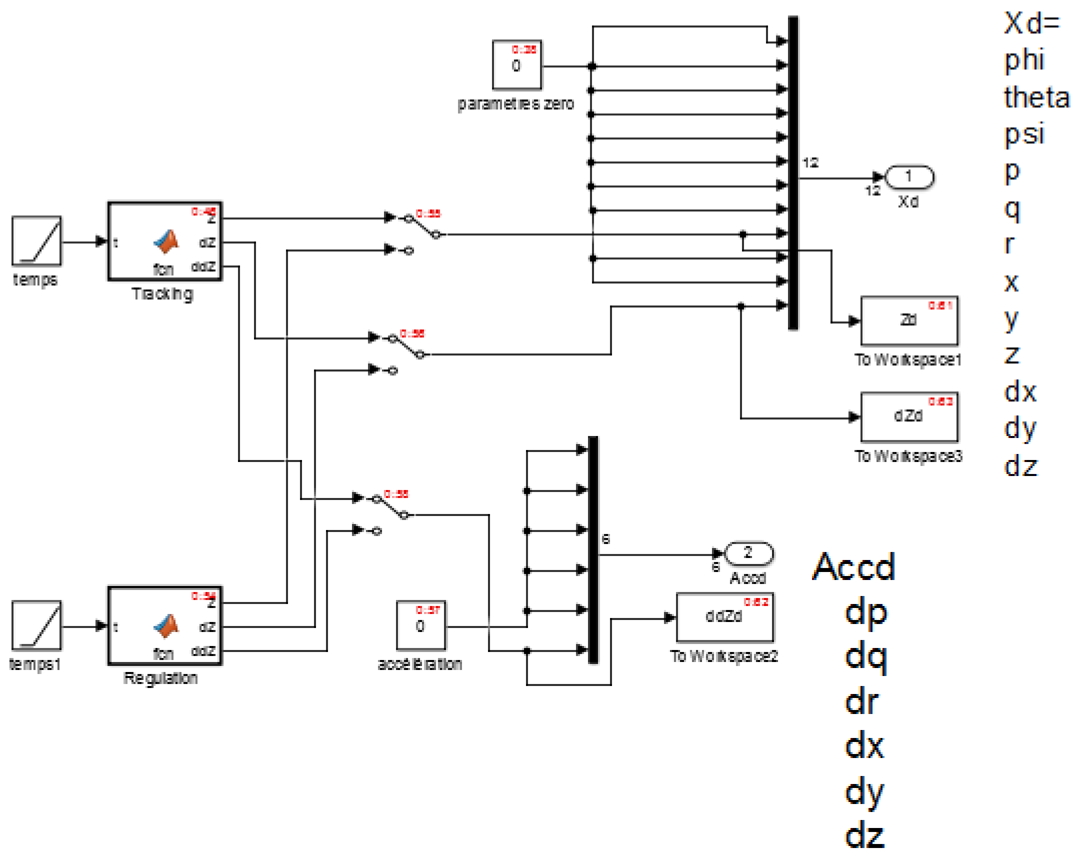

Figure 4.2 Reference generation functions

As discussed earlier it can clearly be seen that the Euler angles are set to be equal to zero so that the latter will avoid angle variation and position along other Cartesian axis which will lack and oppose in other word the altitude control command and the system deviate from desired trajectory and becomes unstable since no control is done on the X and Y direction.

If the Euler angles are all set to zero the backstepping command U1, U2, U3 will compensate for any variation and get them equal to the desired value in this way if the angular angles are equal to zero therefore both Angulat velocity and acceleration will also be zero**.**

4.3. Trajectory planning

4.3.1 Regulation

As it is seen in figure 4.2 the two first blocks are to design the function to be tracked by the system the first function follows a step form for which the quadcopter is desired to reach a height of 2 meters and remains fixed.

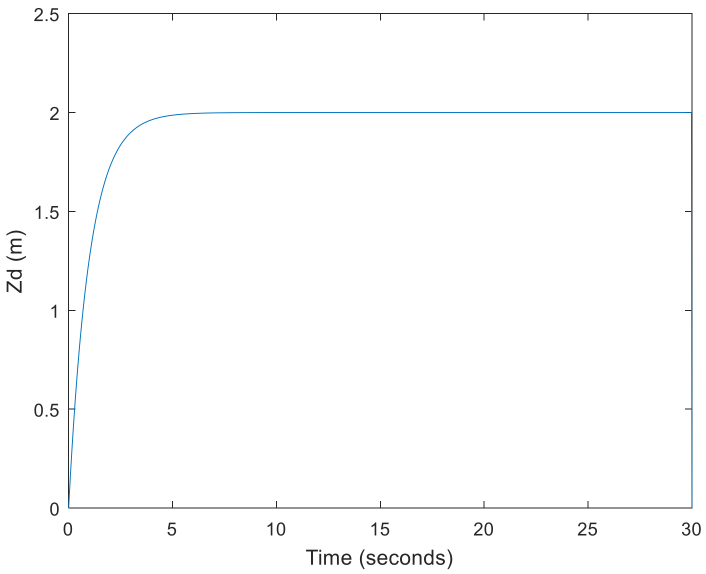

Figure 4.3 Step input function (Regulation)

4.3.2 Tracking

The second function is a polynomial of 5th degree where it is smoothly designed. The quadcopter will initially be at rest and then hover for an altitude of 10 meters this is during the first 10 seconds. The quadcopter then is set to remain fixed for 5 seconds at this height and land smoothly after.

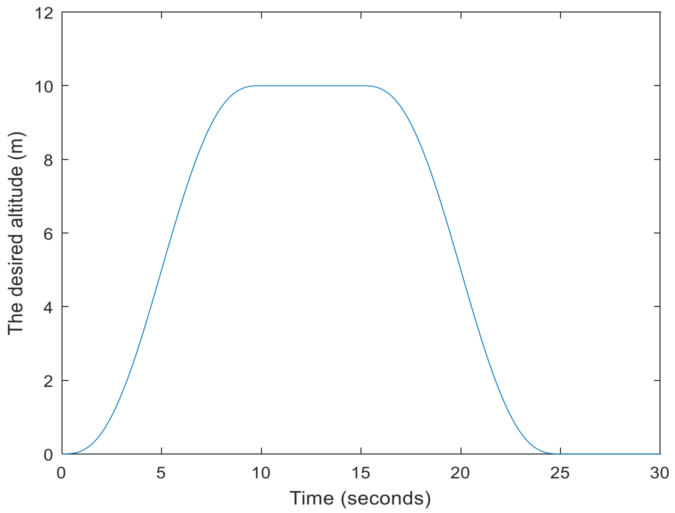

Figure 4.4 Fifth order polynomial function (Tracking)

4.4 Simulation result

4.4.1 Backstepping Gains

After Simulation process the systems altitude Z and its linear first and second derivative were dependent on the BACKSTEPPING gain.

If we recall from chapter 3 the closed loop nonlinear control dynamics were based on the Lyaponuv approach when we set the backstepping gain L_1 to be positive to achieve asymptotic stability, a pseudo input from relation (3.19) was then injected in the second part of the subsystem where we presented a second lyaponuv function.

And we said the origin can be asymptotically stable if we could set a second gain L_2 to be positive and therefore to obtain the overall input command for Roll angle control. We did the same thing for Pitch and yaw angles only in which the overall subsystem commands U2 and U3 were originally based on different moment equations for sure.

For the 4th input control U4= F1+F2+F3+F4, we had two different gains L_3 and L_4 which followed the same procedure but did a different job which is to control the altitude instead of the attitude.

4.4.2 Backstepping Gains Tuning

Our control design followed a basic theoretical approach; however simulation and practice may do slightly different than what we think. Setting L_2 and L_1 to be positive was very straight forward; however it wasn't the case for L_3 and L_4.

Choosing arbitrarily was the key point but not everything since I had to fix one gain $L_4 = 8$, and I varied L_3. The following result reveals the complexity of this approach.

The desired Zd versus the actual altitude Z for $L_3 = 36$:

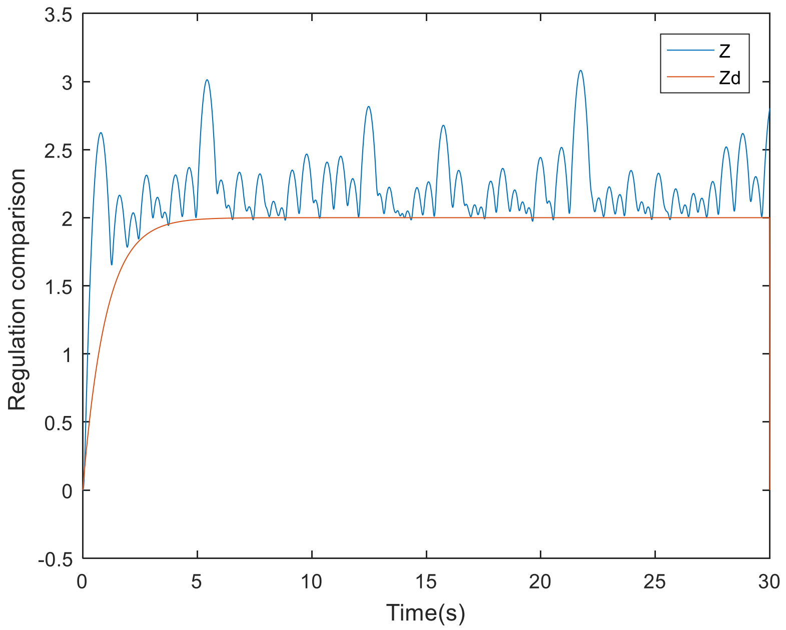

Figure 4.5 Altitude Regulation comparison

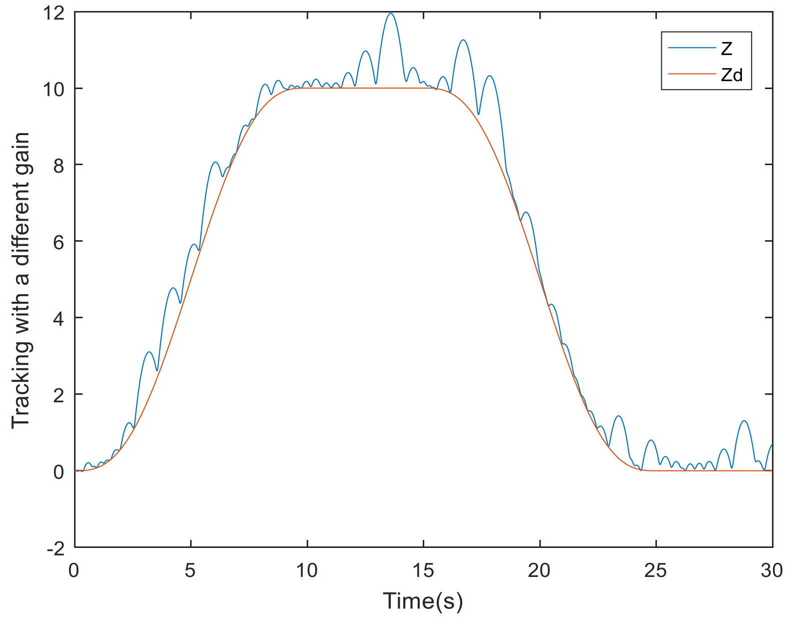

Figure 4.6 Tracking Comparison

The desired Zd versus the actual altitude Z for $L_3 = 25$:

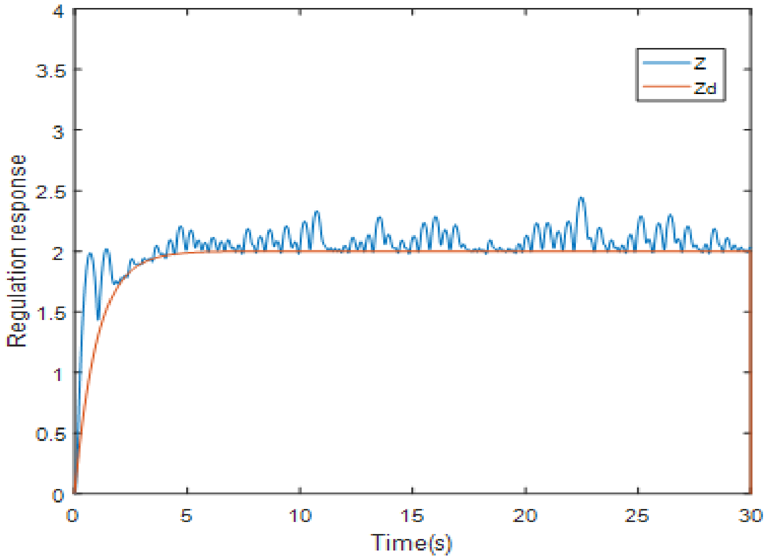

Figure 4.7 Altitude Regulation comparison

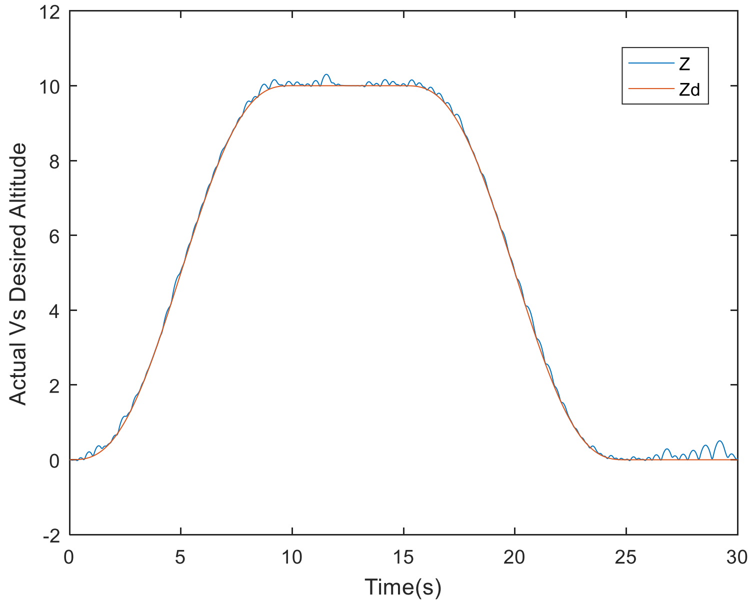

Figure 4.8 Tracking with a different gain

We notice here as we decrease the backstepping gain the origin seems to be more asymptotically stable and trajectory is reached in this situation we can choose lower gain and observe the behavior of the actual altitude.

This happens mainly due to the saturation of the actuators. As I have changed the thrust coefficient and made a bit larger in a previous experiment with the same backstepping gain system seemed to converge toward the desired altitude.

NB: this was only done to check the efficiency of the controller under a variety of condition not to change the motors parameters.

The desired Zd versus the actual altitude Z for $14 \leq L_3 \leq 1$

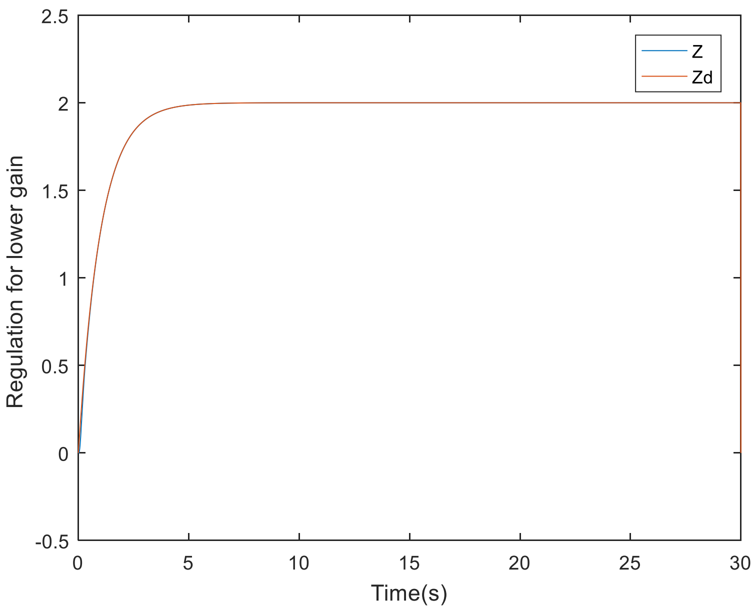

Figure 4.9 Regulation with a lower backstepping gain

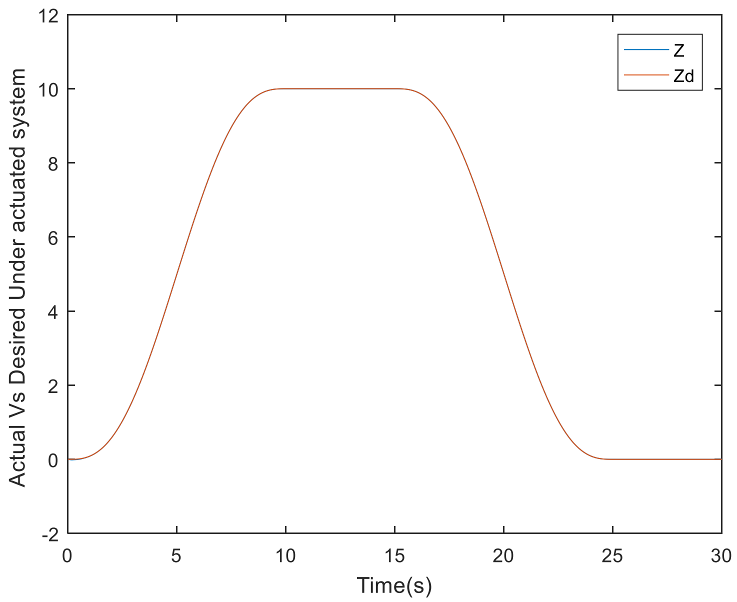

Figure 4.10 Tracking with a lower Backstepping gain

The desired linear velocity dZd versus the actual velocity dZ (along the Z axis)

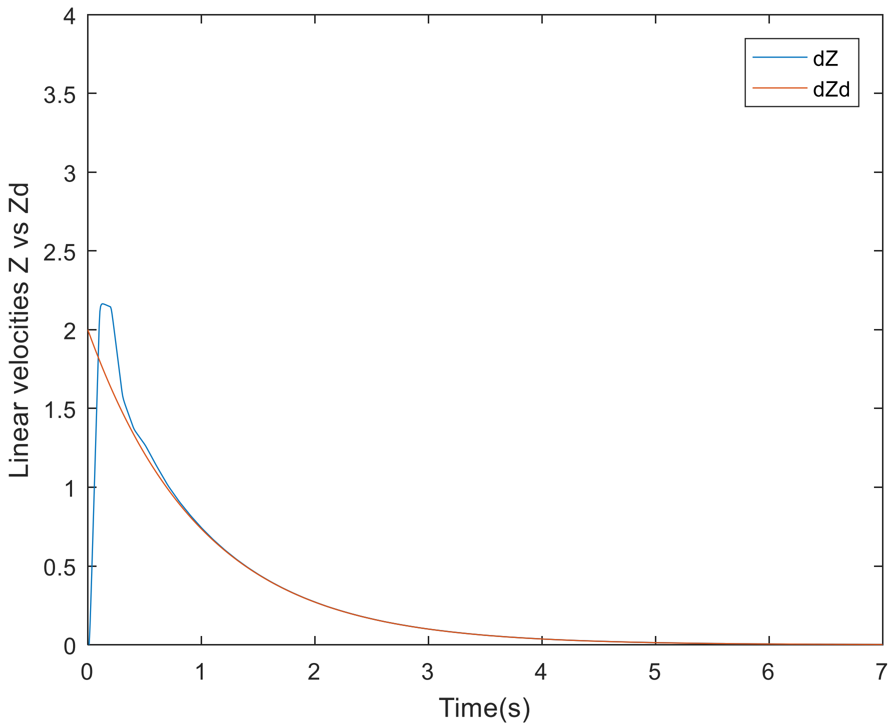

Figure 4.11 The linear velocity Regulation.

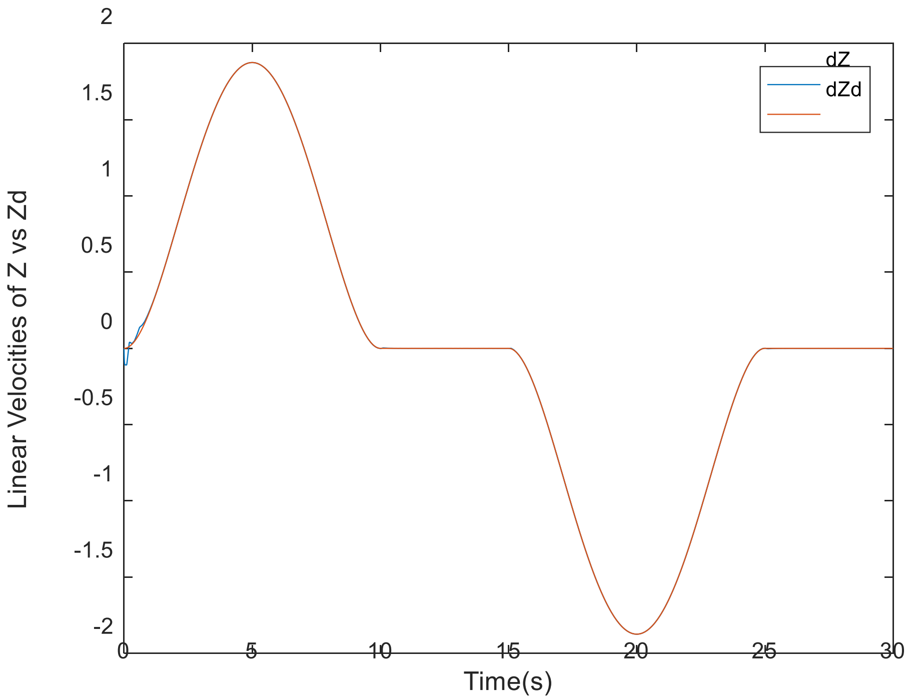

Figure 4.12 The linear velocity Tracking.

The desired linear Acceleration ddZd versus the actual acceleration ddZ.

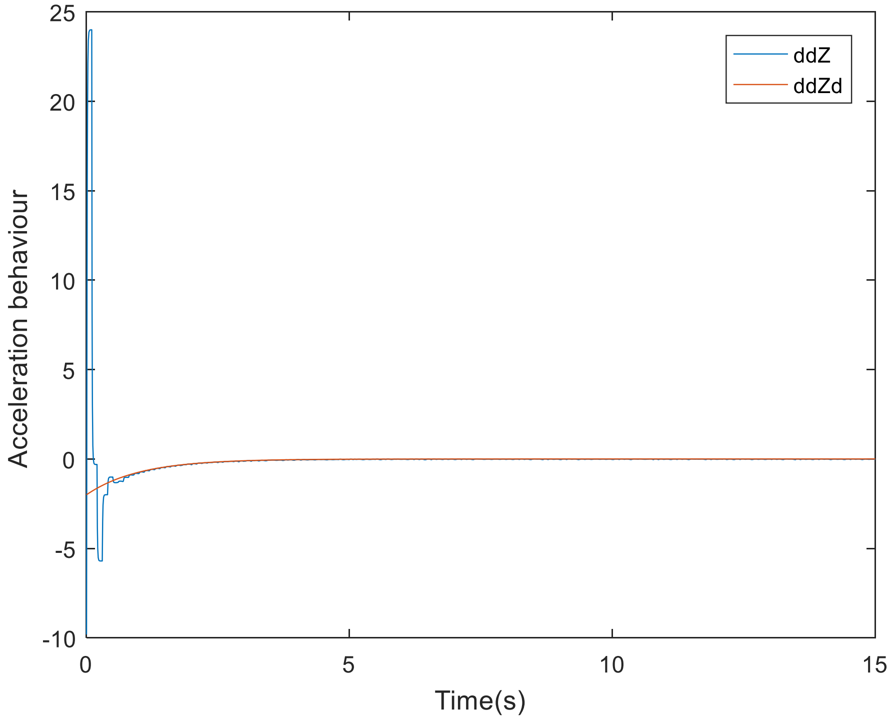

Figure 4.13 The linear acceleration Regulation.

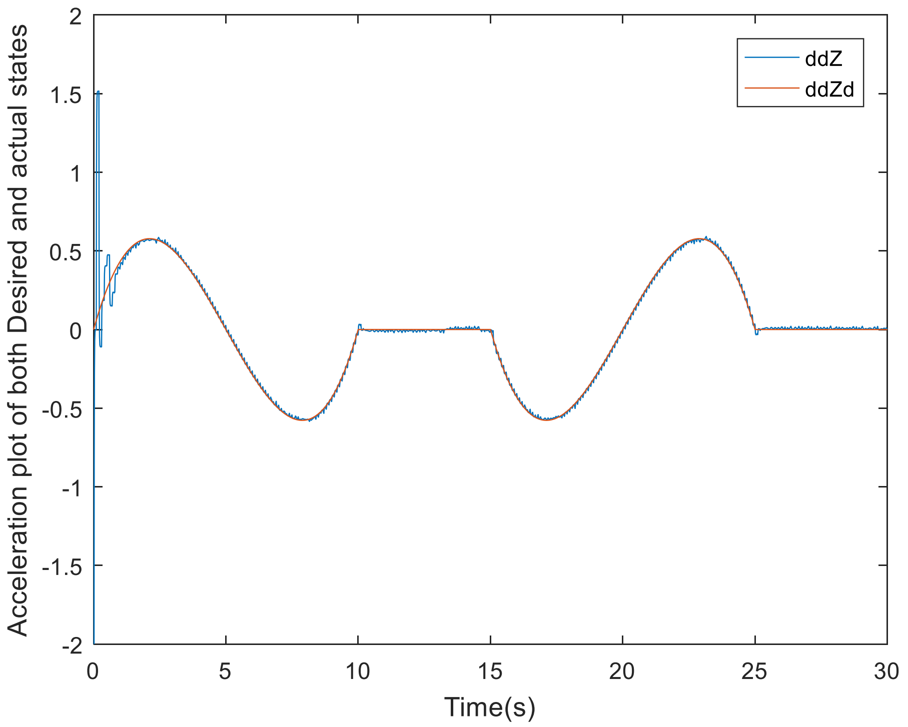

Figure 4.14 The linear acceleration Tracking.

4.4.3 Euler angles

As we have mentioned earlier Roll, yaw and pitch angles are set to be zero therefore no variation and displacement exists along the X and Y axis. It is very straight forward to show that the total force required to lift the quadcopter would be an equal force fed to all rotors therefore the angular velocity of the rotors must be the same.

If the controller achieves this, there will be no chance left for the Euler angle to appear. The figures below will illustrate both tracking and regulation process influence on the X and Y direction and Vice versa. And will show the resulting angles.

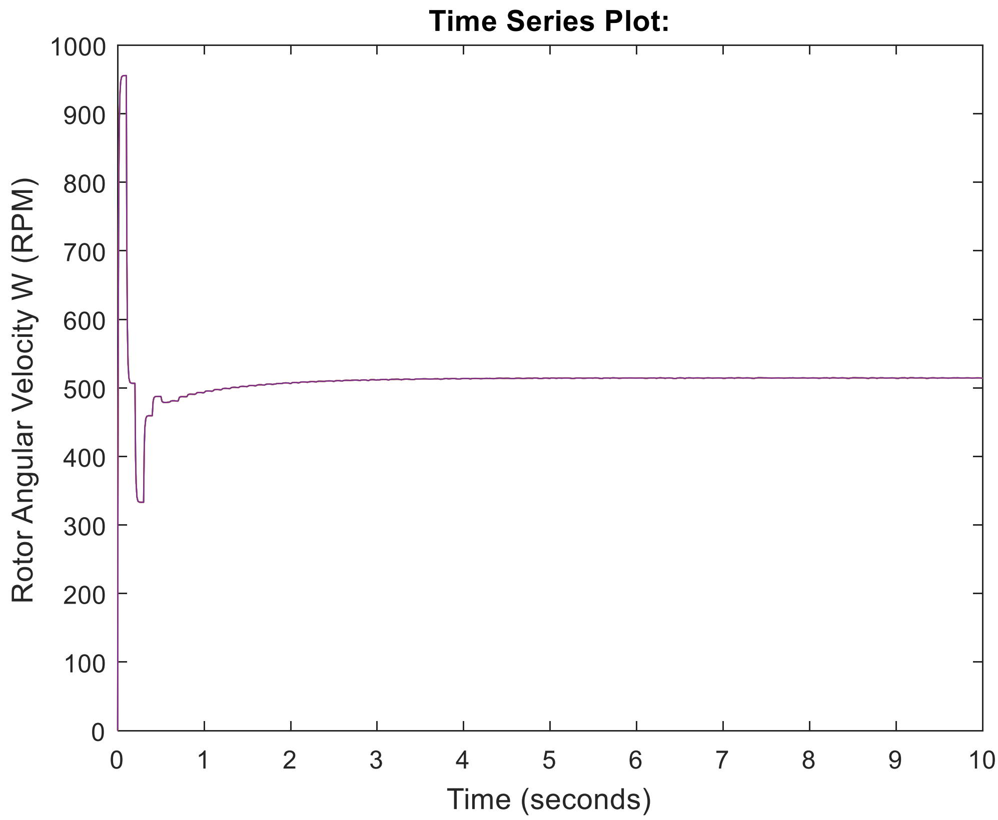

Figure 4.15 The angular velocity of the 4 rotors (RPM)

As seen in **Figure 4.15** the angular velocity of the four rotors is the same and is of a constant value in Regulation.

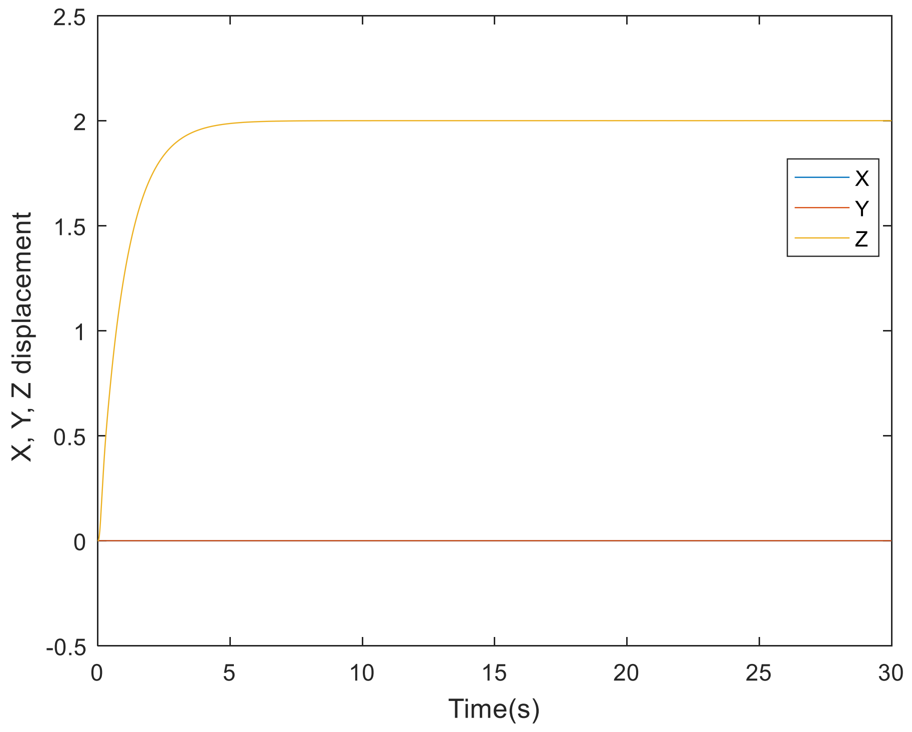

Figure 4.16 Regulation influence on X and Y direction

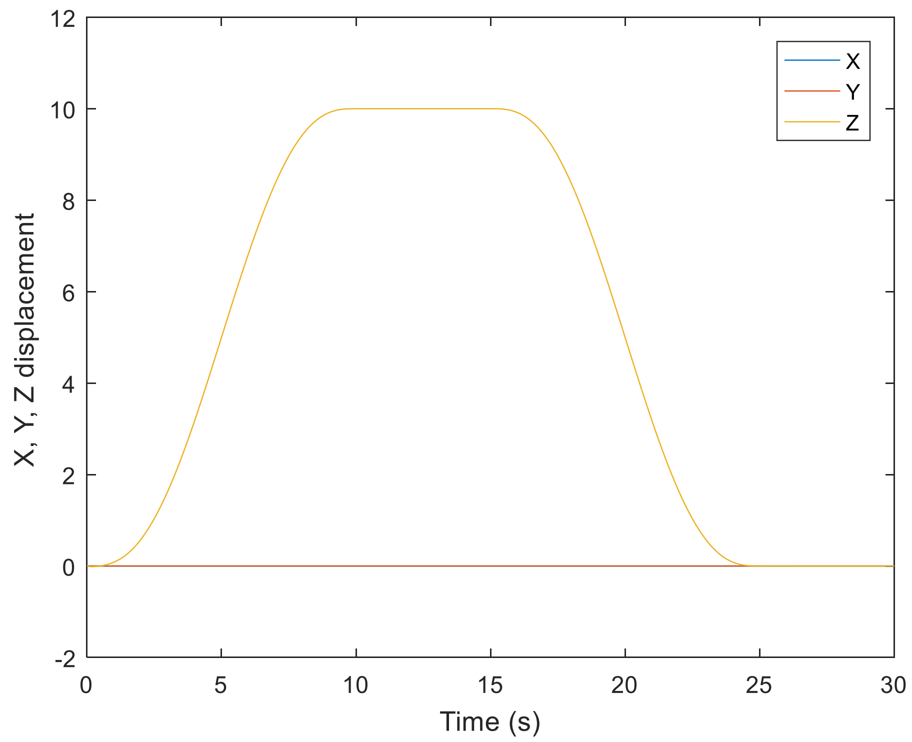

Figure 4.17 Tracking influence on X and Y direction

4.5 Conclusion

From our quadrotor model we realized a backstepping command based on Lyapunov's theorem. We managed to generate a trajectory of the quadrotor, so that the system convergences toward the desired values (Stability).

We have seen that this nonlinear controller is valid for systems stabilization under some conditions which were shown above such as backstepping gain tuning which depends on the model and the actuators you're feeding. So the backstepping responses loses tracking when system reaches saturation.

Therefore Backstepping has a high tracking performance.

CONCLUSION

General Conclusion

In this work, I reviewed the nonlinear control design and I wanted to check its efficiency in term of stability maintaining since the linear counterpart techniques are preferred due to the fact that they have been proven to work for a large class of control and automatic problems. The nonlinear technique explained in this project is efficiently reliable for nonlinear system stabilization.

When analyzing the results, backstepping has positive and negative marks, it is true that we have achieved the stability objectives and prior tracking but for hard nonlinearities encountered by the quadcopter the system may show no resistance and compensation for the last.

To sum up I would suggest designing a more vulnerable Lyaponuv function for more error regulation, because based upon the lyaponuv theory there may exist many types of lyaponuv functions that will globally stabilize the system and yet achieving the best one is of great interest and consequence to the system.

The last thing that I can add for future work is adding an integral control which means adding another state that helps in steady state tracking performance and rejecting constant disturbances that is known by n’th order integral backstepping.

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