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Title:

**On The Effect of Similarity Transformation On  
Robustness of State Feedback Control Design**

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# Abstract

It is well known, in the control theory, that the state feedback represents one of the most popular techniques in designing the dynamics of any system. In multivariable “MIMO” systems, the state feedback gain matrix allowing the assignment of the desired set of closed loop poles (eigenvalues) is not unique.

The aim from this project is to investigate the effect of the controller form similarity transformation on the robustness of the state feedback control design applied to an airplane case study.

In this thesis, the F-16 Fighter in the lateral model is taken as a study case. The simulation of a large number of possible state feedback gains is done using MATLAB software; and the magnitude of the gain matrices, performance and robustness measures are computed.

These results are then compared to select the best feedback gain having the smallest magnitude and best robustness performances.

# Dedication

*I want to dedicate this work to:*

*First and foremost, the sake of Allah, my Creator and my Master.*

*To my beloved parents for their unconditional love and prayers, and for their endless support and encouragement throughout my life.*

*To my brothers: Amine & Tarek and my sister in law: Chahra and to the rest of my family, I would like to express my wholehearted thanks.*

*To my fiancé: Khaled for always encouraging me and believing in me.*

*To my best friend and sister Rym for cheering me up in hard times and being there for me all the time*

*To all my friends who have always been a constant source of support and encouragement during the challenges of my whole college life. I will always appreciate what they have done, especially, Nadia for giving me wise advices whenever I needed them, Radja & Zohra for tolerating me during my bad mood and being such good friends, love you girls.*

*Finally, I would like to dedicate this thesis to every single person who had given me a hand to achieve my goal. Special thanks for you all.*

**Tinhinene**

# Dedication

*This thesis is dedicated to:*

*The sake of Allah, my Creator and my Master.*

*My lovely parents, who never stop giving of themselves in countless ways, my mother for all her sacrifice and unconditional love for us, my father for all his endless support and encouragement in every step*

*My beloved sister Soulef, who is always by my side.*

*The best friend I can ever have: my zoo for encouraging and supporting me.*

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*To the soul of my grandfather to be still here even after his death.*

*To everyone I shared a smile with.*

**Radja Ouafa**

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# Nomenclature

$x$	Vector of states
$\dot{x}$	Derivative of the state $x$
$u$	Vector of inputs
$y$	Vector of Outputs
$A$	State matrix
$B$	Control matrix
$C$	Output matrix
$D$	Feed forward matrix
$\mathcal{R}(A, B)$	Reachability matrix
$T_c$	Controller similarity transformation matrix
$\{A_c, B_c, C_c\}$	General controller state-space system
$\lambda_i$	$i^{\text{th}}$ eigen value of the system
$K$	Feedback gain matrix
$\Delta(\lambda)$	Characteristic matrix polynomial
$\Delta_d(\lambda)$	Desired characteristic matrix polynomial
$(.)^{-1}$	Inverse
$(.)^T$	Transpose
$R_i/L_i$	Right/Left general eigenvector matrix
$\mathbb{R}^n$	Set of real numbers including 0
$\mathbb{R}^{n+}$	Set of real positive numbers including 0
MIMO	Multi-Input Multi-Output

# Introduction

The aim of a control system is to force a given set of process variables to behave in some desired and prescribed way by either fulfilling some requirements of the time or frequency domain or achieving the best robustness.

Many systems, particularly in technologically advanced areas such as Aircraft, are represented by models with several inputs and several outputs signals that are highly cross coupled which makes the use of single-input single-output systems (SISO) control technique inefficient.

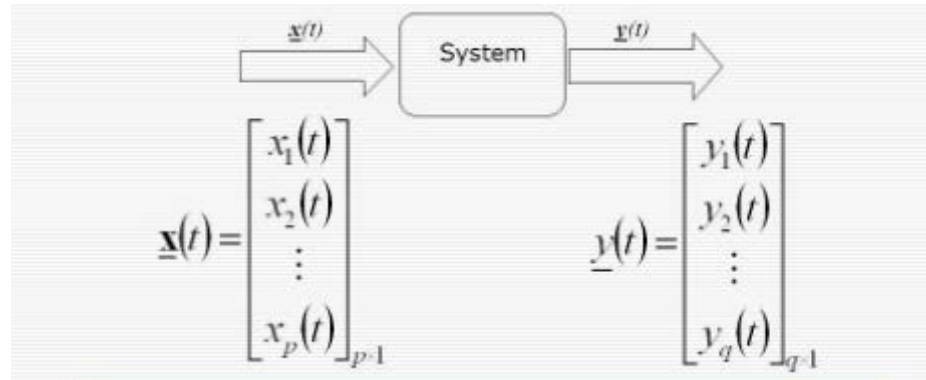
One of the well-known and widely used techniques in Multi-Input Multi-Outputs (MIMO) control design is the state feedback method, in which the state vector value is fed back to the input through a constant matrix  $K$  called the state feedback gain. This constant gain matrix permits the assignment of closed-loop poles in the desired locations under the assumption of complete reachability. In MIMO control systems, this state feedback gain is not unique, which permits to meet specifications beyond relocating closed-loop system eigenvalues.

The aim of this present report is to use general controller canonical form and to take advantage of the non-unicity to choose the suitable feedback gain matrix that will meet specifications beyond assigning the closed loop poles, in the airplane case.

## 1- Introduction

Systems with more than one input and/or more than one output are known as Multi-Input Multi-Output systems and frequently abbreviated as MIMO systems.

Figure 1.1 illustrates the representation of a MIMO system.



**Figure 1.1:** MIMO system representation.

## 2- Mathematical Models

A linear time invariant MIMO system can be described by two different mathematical models:

- The state space model.
- The transfer function model.

In this thesis, the system under study will be described using a state-space description.

## 3- State-Space Description [1]

The state space model represents the internal dynamics of a multivariable system and consists of a set of input, output and state variables related by first order differential equations or difference equations.

A state space model is of the form:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) & (1.1.a) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) & (1.1.b) \end{cases} \quad (1.1)$$

Where:

$A \in \mathbb{R}^{n \times n}$  is the state matrix.

$B \in \mathbb{R}^{n \times m}$  is the input matrix.

$C \in \mathbb{R}^{p \times n}$  is the output matrix.

$D \in \mathbb{R}^{p \times m}$  is the feed forward (direct) matrix.

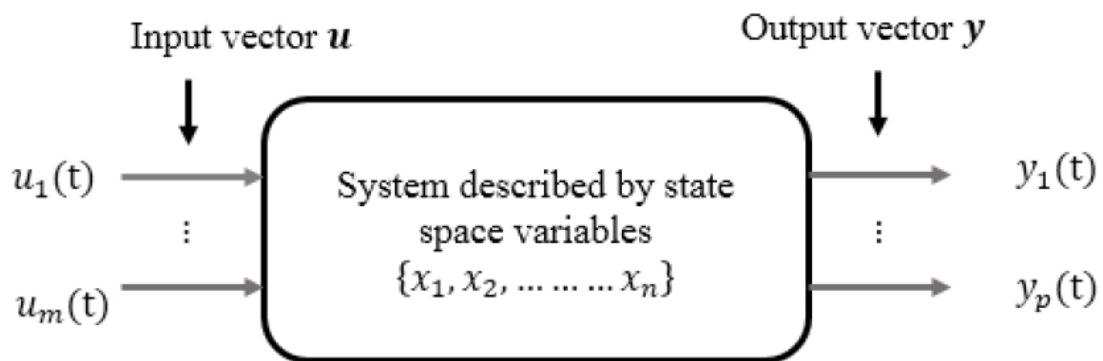
The control input  $\mathbf{u}(t)$  is a vector of dimension  $m$ .

The state variable  $\mathbf{x}(t)$  is a vector of dimension  $n$ .

The output  $\mathbf{y}(t)$  is a vector of dimension  $p$ .

In the above model, equation (1.1.a) is called the dynamic equation which describes the dynamic part of the system; and equation (1.1.b) describes how the system state variables  $\mathbf{x}(t)$  and system inputs  $\mathbf{u}(t)$  will instantly determine the system outputs  $\mathbf{y}(t)$ .

Figure 1.2 illustrates the inputs and the outputs of a MIMO system.



**Figure 1.2:** System inputs and outputs.

**Remark:**

The feed forward matrix  $D$  allows the system input to affect the system output directly. If  $D$  is the null matrix, the term  $D\mathbf{u}(t)$  has no effect on the dynamics of the system. Any similarity transformation on the system given in (1.1) will not affect the feed forward matrix  $D$ .



## 4- Reachability [2]

### 4.1- Definition 1.1

Consider the system (1.1).

- a)  $(A, B)$  is reachable if for every final state  $x_f \in \mathbb{R}^n$  there exists a finite input  $u(t)$  transferring the state of the system from any initial state to  $x_f$  in a finite time.
- b)  $(A, B) \geq 0$  is positively reachable if for every final state  $x_f \in \mathbb{R}^{n+}$  there exists a finite nonnegative input transferring the state of the system from any initial state to  $x_f$  in a finite time.

### 4.2- Theorem 1.1

The system described in (1.1) is said to be fully reachable if and only if the reachability matrix  $\mathcal{R}(A, B) = [B \ A \ B \ A^2 \ B \ \dots \ A^{n-1} \ B]$  (1.2) has a full rank.

**Proof:** [1]

#### Remarks:

- The system is completely (totally, fully) reachable i.e.  $\text{rank}(\mathcal{R}(A, B)) = n$ , means that all the eigenvalues of the system can be relocated by state feedback using the input.
- If  $\text{rank}(\mathcal{R}(A, B)) < n$  the system is said to be partially reachable and so only some eigenvalues can be relocated.

Let  $B = [b_1 \ b_2 \ \dots \ b_m]$ , where  $b_i$  is a column vector of dimension  $n \times 1$ .

Then the reachability matrix can also be written as:

$$\mathcal{R}(A, B) = [b_1 \ b_2 \ \dots \ b_m ; A b_1 \ A b_2 \ \dots \ A b_m ; \dots \dots \ A^{n-1} b_1 \ A^{n-1} b_2 \ \dots \ A^{n-1} b_m]$$

(1.3)

If  $\text{rank}(B) = \bar{m} < m$ , then  $\text{rank}[\mathcal{R}(A, B)] = \text{rank}[B \ A \ B \ A^2 \ B \ \dots \ A^{n-\bar{m}} \ B]$

### 5- General Controller Canonical Form

Consider the linear time-invariant MIMO system represented by equation (1.1). The system can be transformed into general controller canonical form if it is completely reachable .i.e. condition (1.2) is satisfied.

In order to construct a similarity transformation (change of basis),  $n$  linearly independent vectors in  $\mathcal{R}(A, B)$  should be found.

Let  $\mathcal{K}_i$  be the number of linearly independent vectors of  $\mathcal{R}(A, B)$  associated with  $b_i$  ;  $\mathcal{K}_i$  is called a reachability index or a Kronecher index.

To find the indices we have got to fill the young diagram which is represented by table 1.1:

**Tables 1.1:** Young diagram for reachability indices

	$b_1$	$b_2$	$b_3$	...	$b_m$
$A^0$					
$A^1$					
$A^2$					
$\vdots$					
$A^{n-\bar{m}}$					
$\mathcal{K}_i$					

To fill the young diagram we must test the linear independence going from left to right and from up to down, once we find a vector linearly independent with the previous set of checked vectors, we cross the corresponding cell until all the vectors are checked, finding  $\sum_{i=1}^m \mathcal{K}_i = n$  leads to  $n$  linearly independent vectors from the columns of  $\mathcal{R}(A, B)$ .

Now suppose we have determined the reachability indices  $\mathcal{K}_i, i=1,2,\dots,m$  . We can then construct the reachability base matrix  $\mathcal{R}_B(A, B)$ .

$$\mathcal{R}_B(A, B) = [b_1, A b_1, A^2 b_1, \dots, A^{\mathcal{K}_1-1} b_1, b_2, A b_2, A^2 b_2, \dots, A^{\mathcal{K}_2-1} b_2, \dots, b_m, A b_m, A^2 b_m, \dots, A^{\mathcal{K}_m-1} b_m]$$

(1.4)

Using this reachability base matrix we can construct a similarity transformation  $T_C$  such that  $x_C = T_C x$ .

Where:

$$T_C = [p_1 \quad p_1 A \quad \dots \quad p_1 A^{\mathcal{K}_1-1} \quad p_2 \quad p_2 A \quad \dots \quad p_2 A^{\mathcal{K}_2-1} \quad \dots \quad p_m \quad \dots \quad p_m A^{\mathcal{K}_m-1}]^T \quad (1.5)$$

and  $p_i = \sigma_i^{\text{th}}$  row of  $\mathcal{R}_B^{-1}(A, B)$ ,  $i=1,2,\dots,m$

$$\sigma_i = \sum_{j=1}^i \mathcal{K}_j. \quad (1.6)$$

Considering again the MIMO system (1.1) and its reachability matrix  $\mathcal{R}(A, B)$  given by (1.4), we define the input selection sequence with respect to the columns of matrix  $B = [b_1 \ b_2 \ \dots \ b_m]$ , as:

$$S = [s_1 s_2 \ \dots \ s_m] \quad (1.7)$$

The general controller canonical form of the matrices A and B is as follows:

$$A_C = T_C A T_C^{-1} = \left[ \begin{array}{cccc|ccc|cccc} 0 & 1 & \dots & 0 & & & & & & & & & \\ \vdots & \vdots & & \vdots & \dots & (0) & & & (0) & & & & \\ 0 & 0 & \dots & 1 & & & & & & & & & \\ \hline x & x & \dots & x & & x & \dots & x & & x & \dots & x & \\ & & \vdots & & \ddots & & & & & & \vdots & & \\ & & \vdots & & & \ddots & & & & & \vdots & & \\ \hline & & & & & & & & & & 0 & 1 & \dots & 0 \\ & & & & & & & & & & \vdots & \vdots & & \vdots \\ (0) & & & & \dots & & (0) & & & & 0 & 0 & \dots & 0 \\ \hline x & x & \dots & x & & x & \dots & x & & & x & x & x & x \end{array} \right] \quad (1.8)$$

$A_C$  is a matrix of companion blocks each of size  $\mathcal{K}_i$  on the diagonal and non-trivial elements at the  $\sigma_i^{\text{th}}$  rows and zeros everywhere else.

$$B_C = T_C B = \left[ \begin{array}{cccc} 0 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 1 & x & \dots & x \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 1 & \dots & x \\ & \vdots & \vdots & \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 1 \end{array} \right]$$

(1.9)

Where the  $\sigma_i^{th}$  rows are non-trivial with 1's as leading coefficients at the  $j^{th}$  column where:

$$j = \{1, 2, 3, \dots, m\} .$$

$\mathbf{C}_c$  has no particular form and  $\mathbf{D}_c = \mathbf{D}$ .

**Remark:**

The companion form takes the following structure:

$$\begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{pmatrix} \quad (1.10)$$

The characteristic polynomial of this matrix is given by the coefficients of its last row:

$$\Delta(\lambda) = \lambda^n + a_{n-1} \lambda^{n-1} + \dots + a_1 \lambda + a_0 \quad (1.11)$$

Due to the form of  $\mathbf{B}_c$ , all rows of  $\mathbf{A}_c$ , except the nontrivial row denoted by the element  $x$ , are not affected by the state feedback. Hence, the elements of those rows of the matrix  $(\mathbf{A}_c - \mathbf{B}_c \mathbf{K}_c)$  can be arbitrarily assigned.

Since the nontrivial terms of  $\mathbf{B}_c$  lead to a linear combination of  $k_{ij}$ 's in  $(\mathbf{A}_c - \mathbf{B}_c \mathbf{K}_c)$ , we multiply the matrix  $\mathbf{B}_c$  by an elementary matrix  $\mathbf{D}_{rh}$ , such that all the nontrivial terms in the  $\{k_{1th}, (k_1 + k_2)_{th}, (k_1 + k_2 + k_3)_{th} + \dots\}$  rows of the matrix  $\mathbf{B}_c$  will be zeroed. We get the following matrix:

$$E_{bc} = B_c D_{rh} = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix} \quad (1.12)$$

We define also: —

$$K_c = D_{rh}^{-1} K_c \quad (1.13)$$

As to have:

$$B_c K_c = B_c D_{rh} \overline{K_c} = E_{bc} \overline{K_c} \quad (1.14)$$

This implies:

$$A_c - B_c K_c = A_c - E_{bc} \overline{K_c} \quad (1.15)$$

## 6- Conclusion

The approach of using the similarity transformation  $T_c$  to get the feedback control design is summarized in four steps:

- Transform the given system into general controllable form.
- Compute  $\overline{K_c}$  such that  $A_c - E_{bc} \overline{K_c}$  has a set of desired eigenvalues.
- Compute  $K_c = D_{rh} \overline{K_c}$  where  $D_{rh}$  is such that  $B_c = E_{bc} D_{rh}^{-1}$ .
- Compute  $K$  from  $K_c$ , such that  $K = K_c T_c$ .

## 1- Introduction

In order to do our design some criteria have been selected to evaluate the performance of the system. This chapter presents these criteria.

## 2- Norms [3]

### 2.1- Definition 2.1

The norm is a real number, denoted as  $\|e\|$ , which satisfies the following properties:

1. Non-negative :  $\|e\| \geq 0$ .
2. Positive :  $\|e\| = 0$  iff  $e = 0$ .
3. Homogenous :  $\|\alpha \cdot e\| = \alpha \cdot \|e\|$ .
4. Triangle inequality :  $\|e_1 + e_2\| \leq \|e_1\| + \|e_2\|$ .

Where:  $e$  is a vector, and  $\alpha$  is a scalar.

In this thesis we will consider only matrix norms.

### 2.2- Definition 2.2

A norm  $\|A\|$  of a matrix  $A$  is a matrix norm which, in addition to the four norm properties given earlier in definition 2.1, satisfies the multiplicative property (also called the consistency condition) :  $\|A \cdot B\| \leq \|A\| \cdot \|B\|$ .

### 2.3- Most Common Matrix Norm Types

- i. The Matrix 1-norm

It is the maximum absolute column sum.

$$\|A\|_1 = \max_{1 \leq j \leq n} \left( \sum_{i=1}^m |a_{ij}| \right) \quad (2.1)$$

- ii. The Matrix  $\infty$ -norm

It is the maximum absolute row sum.

$$\|A\|_\infty = \max_{1 \leq i \leq n} \left( \sum_{j=1}^n |a_{ij}| \right) \quad (2.2)$$

- iii. The Matrix Euclidian norm (also called the 2-norm)

It is the square root of the largest eigen value of  $\mathbf{A}^T \cdot \mathbf{A}$  or the largest singular value of A

$$\|\mathbf{A}\|_2 = \max \lambda(\mathbf{A}^T \cdot \mathbf{A})^{1/2} \quad (2.3)$$

vi. The Matrix Frobenius-norm

$$\|\mathbf{A}\|_F = \left( \sum_{i,j} |a_{ij}|^2 \right)^{1/2} = [\text{Trace } \mathbf{A}^T \cdot \mathbf{A}]^{1/2} \quad (2.4)$$

### Remark:

The concept of norm of a matrix helps in this work to determine the amount of control effort required when state feedback is used by measuring the norm of the state feedback gain matrix.

## 3- Condition Number

### 3.1- Definition 2.3

The condition number of an invertible matrix A is defined as  $\chi(\mathbf{A}) = \|\mathbf{A}\| \|\mathbf{A}^{-1}\|$ .

This quantity enables to know how close is the matrix A to singularity. This affects the accuracy of computations based on the matrix A. It can also be seen as a function to a perturbed input argument.

Note that the condition number of a matrix is always greater or equal to 1.

### 3.2- Condition Number and Conditioning

- If  $\chi(\mathbf{A})$  is large , A is called ill-conditioned (with respect to inversion).
- If  $\chi(\mathbf{A})$  is small , A is called well-conditioned (with respect to inversion).

## 4- Robustness & Sensitivity

### 4.1- Definition 2.4

The ability of a controller to perform in a satisfactory way under any working conditions and in the presence of uncertainties is called « ROBUSTNESS ».

The aim of a robust controller is to tackle the problems created by those uncertainties that may lead to performance degradation or even to the instability of the system.

## 4.2- Types of Robustness

### I. Performance Robustness

The performance or time behavior of any control system is determined to a large extent by its eigenvalues. There are some suitable measures performance robustness.

#### I.1- Eigen Value Sensitivity

It is used to measure how much the system's eigenvalues are sensitive to the model uncertainties, it includes:

i. Individual Eigen Value Sensitivity:

The sensitivity of the  $i^{\text{th}}$  eigen value of a matrix A to perturbations in some or all of its elements is given by the following expression:

$$s(\lambda_i) = \frac{\|L_i\|_2 \cdot \|R_i\|_2}{|L_i^T R_i|} \quad (2.5)$$

Where  $L_i$  and  $R_i$  are the left and right eigenvectors corresponding to eigen value  $\lambda_i$ , respectively.

ii. Overall Eigen Value Sensitivity:

The overall eigen value sensitivity of the matrix A, which is the condition number of the modal matrix, is defined as:

$$S(V) = \|R\|_2 \cdot \|R^{-1}\|_2 \quad (2.6)$$

Where R is the right eigen vector matrix of the matrix A.

#### I.2- Relative Change:

It measures the relative change in eigen value  $\lambda_i$  following a perturbation of the system matrix A.

$$RC = |\lambda_i - \lambda_i'| / |\lambda_i| \quad (2.7)$$



Where:

$\lambda_i$  is the original eigen value.

$\lambda_i'$  is the new eigen value following the perturbation.

## II. Stability Robustness

Stability is the most important property in control design; the sensitivity to such a property is called stability robustness. Basically stability means that if every dynamic matrix eigen value has a negative real part; hence the sensitivity of these eigenvalues with respect to model uncertainties is a direct way to measure the sensitivity of the whole system stability.

Some stability robustness measures have been developed in the control literature; among these, we have the so-called  $M_1$ ,  $M_2$  and  $M_3$  measures. [6] [9] [10]

### II.1- The Robust Stability Measure $M_1$ :

It is defined as:  $M_1 = \min \{ \sigma (A - j\omega I) \}$  (2.8)

For  $0 \leq \omega \leq \infty$

Where:  $\sigma$  is the real part of the complex pair.

$M_1$  measures the smallest possible norm of matrix variation for the dynamic matrix  $A$  to have an unstable or a pure imaginary eigen value  $j\omega$ .

### II.2- The Robust Stability Measure $M_2$ :

It is defined as:  $M_2 = s(V)^{-1} |Re\{\lambda_n\}|$  where  $(|Re\{\lambda_n\}| \leq \dots \leq |Re\{\lambda_1\}|)$  (2.9)

$|Re\{\lambda_n\}|$  is the shortest distance between the unstable region and the eigenvalue  $\lambda_i$ .  $M_2$  equals this distance divided (or weighted) by the sensitivity of all the eigenvalues of the matrix. As the sensitivity goes up,  $M_2$  goes down.

### II.3- The Robust Stability Measure $M_3$ :

It is defined as:  $M_3 = \min_{1 \leq i \leq n} \{ s(\lambda_i)^{-1} |Re(\lambda_i)| \}$  (2.10)

$M_3$  measures the likelihood margin for every eigenvalue to become unstable. It is equal to  $|Re(\lambda_i)|$  divided by its corresponding sensitivity  $s(\lambda_i), i=1, \dots, n$ .

## 5- Time Domain Criteria

Some time domain criteria are often used to describe the performance of control systems. Although developed for second order systems, they can be valuable for higher order systems.

### 5.1- Maximum Overshoot

The maximum overshoot is related to the maximum peak value of the response with respect to the final value.

$$M_p = y(T_p) - y(\infty) \quad (2.11)$$

$$\text{Percent maximum overshoot} = \frac{\text{Maximum overshoot}}{y(\infty)} \times 100\% \quad (2.12)$$

### 5.2- Peak Time $T_p$

It is the time needed for the response to reach the first break of overshoot (i.e.: the peak value).

### 5.3- Settling Time $T_s$

It is the time required for the response curve to reach and stay within a range about the final value of a size specified by an absolute percentage of the final value (usually 2% or 5%).

### 5.4- Rise Time $T_r$

It is defined as the time required for the step response to reach- 10 to 90 percent of the final value.

## 6- Conclusion

In our case, we were considered only with the norms of the feedback gain, and the robustness and sensitivity of the eigenvalues.

## 1- Introduction

Any aircraft has its own characteristics and specifications for desired vehicle performances, so this should be taken into account before analyzing any flight control problem.

Accordingly, this chapter deals with the definition of the airplane dynamics, which is our study case. Following that, a brief analysis of the lateral model, which will be used throughout the thesis, will be presented.

## 2- Airplane Definition

The airplane is a powered, fixed-wing aircraft that is propelled forward by thrust from a jet engine or propeller. Airplanes come in many different shapes and sizes depending on the mission of the aircraft.

An airplane is constructed of many parts divided into four major sections: fuselage, power plant and undercarriage, wings and tail. [11]

**2.1- Fuselage:** containing two parts; the fuselage which holds the structure together and accommodates passengers and/or cargo, and the cockpit which holds the command and control section of an airplane.

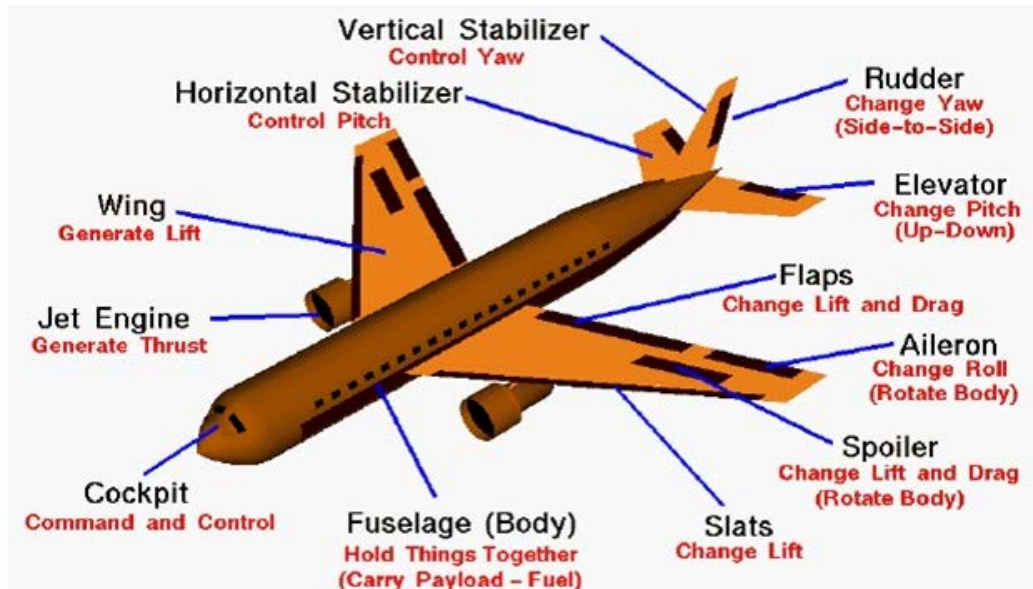
**2.2- Power Plant & Undercarriage:** containing two parts; the power plant (engines) which generate thrust and provide hydraulic and electric power, and the undercarriage (landing gear) which provides a platform for the aircraft to stand as well as plays an important obvious role in landing and take-off.

**2.3- Wings:** containing five parts; the wing which generate lift and control the airflow while flying, the slats which adjust the angle of attack of the wings, increasing lift, the flaps which adjust the camber of the wings, increasing lift, the spoilers which adjust the camber of sections of the wings, decreasing lift, and the ailerons which increase or decrease lift asymmetrically, in order to change roll and, thus, move the aircraft left or right while flying.

**2.4- Tail:** containing four parts; the horizontal stabilizer which helps maintain an airplane's equilibrium and stability in flight, the elevator which increase or decrease lift on the horizontal stabilizer symmetrically in order to control the pitch motion of

an airplane, the vertical stabilizer which prevents lateral movements of the airplane, and the rudder which controls the yaw motion of an airplane.

Figure 3.1 illustrates the different parts of an airplane.



**Figure 3.1:** The parts of an airplane

### 3- Axes of Movement

An airplane is capable of flying in any direction by rotating about in three dimensions called axes of movement which are: the normal axis, the lateral axis and the longitudinal axis.

**3.1- Normal Axis:** drawn from top to bottom, assigned the Z letter. The rotation about this axis is called “Yaw”, which is the movement of the nose of the airplane from side to side.

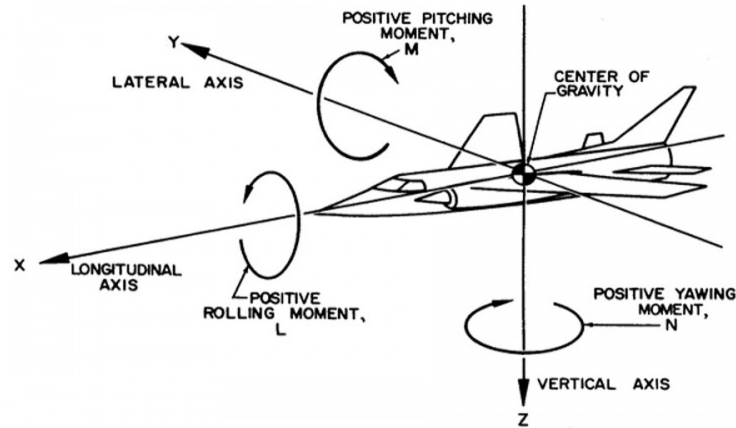
**3.2- Lateral Axis:** drawn parallel to the wings, assigned the Y letter. The rotation about this axis is called “Pitch”, which is the up or down movement of the nose of the airplane.

**3.3- Longitudinal Axis:** drawn from tail to nose, assigned the X letter. The rotation about this axis is called “Roll”, which is the up or down movement of the airplane wings.

Each axis passes through the center of gravity of the airplane and is perpendicular to the other two axes.

Three possible moments can occur about the three axes; the rolling moment denoted by  $L$ , the pitching moment denoted by  $M$  and the yawing moment denoted by  $N$ .

The figure 3.2 illustrates the axes of movement of an airplane.

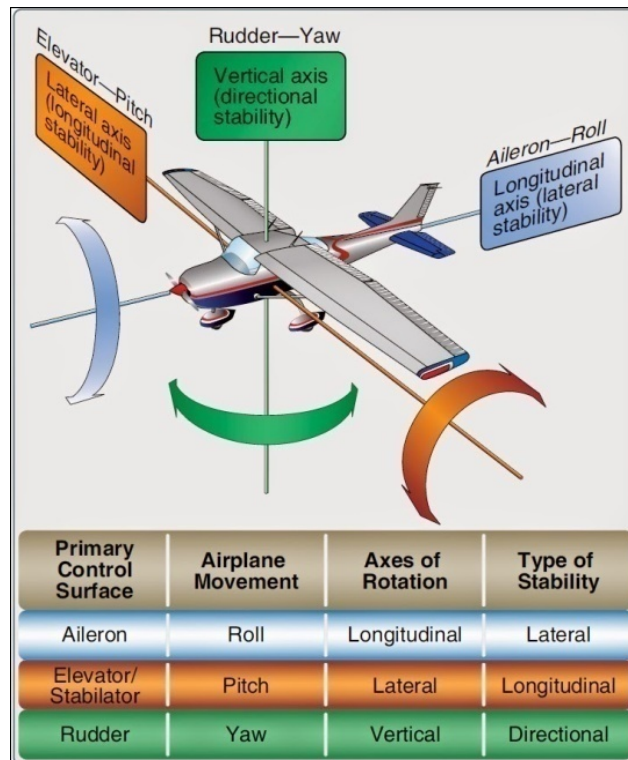


**Figure 3.2:** The axes of movement of an airplane

#### 4- Airplane Primary Flight Controls

Airplane flight controls are the means by which a pilot controls the direction and attitude of an airplane in flight. Primary flight controls are required to safely control an airplane during flight and consist of ailerons, elevators and rudder. Movement of any of the primary flight controls causes the airplane to rotate around the axis of rotation associated with the control surface. The ailerons are attached to the trailing edge of both wings and when moved, rotate the airplane around the longitudinal axis. The elevator is attached to the trailing edge of the horizontal stabilizer. When it is moved, it alters airplane pitch, which is the attitude about the horizontal or lateral axis. The rudder is hinged to the trailing edge of the vertical stabilizer. When the rudder changes position, the airplane rotates about the vertical axis.

Figure 3.3 shows the primary flight controls of a light airplane and the movement they create relative to the three axes of flight.



**Figure 3.3:** Flight control surfaces move the airplane around the three axes of flight

## 5- Forces Acting On The Airplane

During flight, there are four forces acting on an airplane. These forces are lift, weight, thrust, and drag. [12]

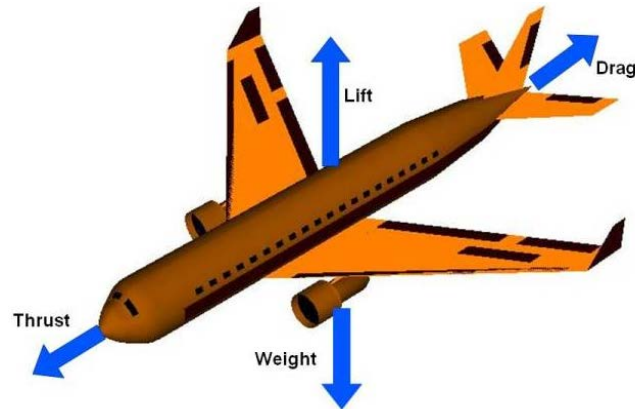
**5.1- Lift:** is the component of the aerodynamic force perpendicular to the relative wind (the flight direction).

**5.2- Drag:** is the component of the aerodynamic force parallel and opposed to the flight direction.

**5.3- Weight:** is a force that is always directed toward the center of the earth. Its magnitude depends on the mass of all the airplane parts, plus the amount of fuel, plus any payload on board.

**5.4- Thrust:** is the force produced by the engine. It is directed forward along the axis of the engine.

Figure 3.4 illustrates the different forces acting on the airplane.



**Figure 3.4:** The forces on airplane

The motion of the airplane through the air depends on the relative strength and direction of the forces shown above. If the forces are balanced, the airplane cruises at constant velocity. If the forces are unbalanced, the airplane accelerates in the direction of the largest force.

In an ideal situation, the forces acting on an airplane in flight can produce no net external force. In this situation the lift is equal to the weight, and the thrust is equal to the drag.

Figure 3.5 illustrates the resulting motion of the airplane when the forces become unbalanced.

Flight Condition	Effect
Lift > Weight	Plane Rises
Weight > Lift	Plane Falls
Drag > Thrust	Plane Slows
Thrust > Drag	Plane Accelerates

**Figure 3.5:** The motion of the airplane for unbalanced forces.

## 6- Stability of An Airplane [13] [14]

Stability of an airplane is its tendency to return to its original trimmed position.



We can distinguish three types of stability in an airplane: the longitudinal, the lateral and the directional stability.

- The longitudinal stability refers to the behavior of the airplane in pitch that is the movement of the longitudinal axis when it is rotated about the lateral axis.
- The lateral stability refers to the behavior of the airplane in roll that is the movement of the lateral axis when it is rotated about the longitudinal axis.
- The directional stability refers to the behavior of the airplane in yaw that is the movement of the longitudinal axis when it is rotated about the vertical axis.

These three stabilities can be divided into two main parts:

**6.1- Static Stability:** it deals with the direction and magnitude of the forces and moments acting on the airplane in its deflected position. It is the natural stability that tends to restore the original airplane position due to any change in pitch, roll or yaw.

The airplane is statically stable if the produced moments are in such a direction to restore the original position.

The airplane is statically unstable if the produced moments are in such a direction to increase the displacement and move the airplane further from its trimmed position.

**6.2- Dynamic Stability:** it deals with the behavior of the airplane over a period of time.

The airplane is dynamically stable, if it has been disturbed from its equilibrium position, if the maximum displacement decreases with time.

The airplane is dynamically unstable if the maximum displacement increases with time.

## 7- Motion of An Aircraft

The form of the equations of motion of an airplane depends on the axis system chosen. Generally a body fixed axis system is used, where its origin is located exactly on the airplane's center of gravity (CG).

In addition, many assumptions are made in deriving the equations of motion:

- i. The distances between any points on the airplane do not change in flight, i.e. the airplane is a rigid-body.
- ii. The inertial frame of reference, the earth, is fixed in space, i.e. no acceleration.

The airplane, which is considered as a rigid-body in space, is a dynamic system with six degrees of freedom (DOF). Its motion in space is defined by six components of velocity along and about the airplane axis system.

The six velocity components are the linear velocities  $u$ ,  $v$ ,  $w$  along the system axes and the angular velocities  $p$ ,  $q$ ,  $r$  about the system axes.

For an aircraft it is common to assume that the longitudinal modes (DOFs 1, 3 and 5) are decoupled from the lateral modes (DOFs 2, 4 and 6). The key assumption is that the fuselage is slender; that is, the length is much larger than the width and the height of the aircraft. It is also assumed that the longitudinal velocity is much larger than the vertical and transversal velocities.

For more details, the reader may refer to [15].

During this thesis, the lateral model of the F-16 Fighter will be used.

The state vectors are:

$$\mathbf{x}_{\text{lat}} = \begin{bmatrix} \beta \text{ (ft/s)} \\ \phi \text{ (ft/s)} \\ p \text{ (rad/s)} \\ r \text{ (rad)} \\ \delta_A \text{ (rad)} \\ \delta_R \text{ (rad)} \\ r_w \text{ (rad)} \end{bmatrix}, \mathbf{u}_{\text{lat}} = \begin{bmatrix} u_A \text{ (rad)} \\ u_R \text{ (rad)} \end{bmatrix}, \mathbf{y}_{\text{lat}} = \begin{bmatrix} r_w \text{ (deg)} \\ p \text{ (deg/s)} \\ \beta \text{ (deg)} \\ \phi \text{ (deg)} \end{bmatrix}$$

Where:

- $\beta$  is the sideslip angle.
- $\phi$  is the roll angle.
- $p$  is the roll rate perturbation.
- $r$  is the yaw rate perturbation.
- $\delta_A$  is the aileron deflection.
- $\delta_R$  is the rudder deflection.

The nonlinear lateral model of the F-16 Fighter was linearized around the equilibrium point:

Speed  $V_T = 502 \text{ ft/s} = 552 \text{ km/h}$

Mach-number  $M = 0.45$

The resulting system matrices a and b are shown below:

$$A = \begin{bmatrix} -0.3220 & 0.0640 & 0.0364 & -0.9917 & 0.0003 & 0.0008 & 0 \\ 0 & 0 & 1 & -0.0037 & 0 & 0 & 0 \\ -30.6492 & 0 & -3.6784 & 0.6646 & -0.7333 & 0.1315 & 0 \\ 8.5396 & 0 & -0.0254 & -0.4764 & -0.0319 & -0.0620 & 0 \\ 0 & 0 & 0 & 0 & -20.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -20.2 & 0 \\ 0 & 0 & 0 & 57.2958 & 0 & 0 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20.2 & 0 \\ 0 & 20.2 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 57.2958 & 0 & 0 & -1 \\ 0 & 0 & 57.2958 & 0 & 0 & 0 & 0 \\ 57.2958 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 57.2958 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = 0$$

And the actual eigen values are given by:

lam = [ -1.0000  
-0.4224 + 3.0633i  
-0.4224 - 3.0633i  
-0.0167  
-3.6152  
-20.2000  
-20.2000 ]

## 1- Introduction

In order to select the best feedback control design, different cases are simulated using the data of the F-16 aircraft in the lateral model presented in the precedent chapter. For each case a number of parameters are calculated then compared to get the best choice that meet the requirements.

## 2- General Controller Canonical Form

As explained earlier in chapter 1, the feedback gain matrix  $\mathbf{K}$  is computed using the controller similarity transformation  $T_c$  which, in our case, comes in two different results due to the choice of two input sequence selection (permutation):

- First permutation:  $S_1 = [b_1, b_2]$ .
- Second permutation:  $S_2 = [b_2, b_1]$ .

The pair (A,B) is fully reachable since the reachability matrix  $\mathcal{R}(A, B)$  has a full rank  $r = 7$ .

The reachability (Kronecher) indices are:

- First permutation:  $\mathcal{K}_1 = 4, \mathcal{K}_2 = 3$ .
- Second permutation:  $\mathcal{K}_1 = 3, \mathcal{K}_2 = 4$ .

In order to transform the system into general controller canonical form, the similarity transformation  $T_c$  is constructed and the matrices  $A_C, B_C, C_C$  are calculated.

### 2.1- First Permutation

The resulting  $T_c$  for the input sequence  $S_1$  and the corresponding system in general controller canonical form are shown below.

$$T_c = \begin{bmatrix} -3.0523 & 0.1116 & 0.0004 & -0.0385 & 0 & 0 & -0.0530 \\ 0.6412 & -0.1953 & -0.0001 & 0.0081 & 0 & 0 & 0.0530 \\ -0.1347 & 0.0410 & -0.1719 & 2.3980 & 0.0000 & -0.0000 & -0.0530 \\ 25.7896 & -0.0086 & 0.6075 & -4.1603 & 0.0495 & -0.1714 & 0.0530 \\ 2.1517 & -0.0469 & -0.0003 & 0.0271 & 0 & 0 & 0.0246 \\ -0.4520 & 0.1377 & 0.0319 & -0.7367 & 0 & 0.0000 & -0.0246 \\ -7.1223 & -0.0289 & 0.0228 & -0.5902 & -0.0000 & 0.0495 & 0.0246 \end{bmatrix}$$

$A_c =$

-0.0000	1.0000	0.0000	-0.0000	0	0.0000	-0.0000
-0.0000	-0.0000	1.0000	-0.0000	-0.0000	-0.0000	0.0000
0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
542.4936	-126.6049	-132.0551	-26.3613	989.8241	-235.6129	-14.0898
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
-0.0000	0	0.0000	0.0000	-0.0000	0.0000	1.0000
-152.8917	-15.3476	-0.3851	-0.0000	-278.5280	0.0381	-19.5155

$B_c =$

0	0
0	0
0.0000	-0.0000
1.0000	-3.4620
0	0
0	0.0000
-0.0000	1.0000

## 2.2- Second Permutation

The resulting  $T_c$  for the input sequence  $S_2$  and the corresponding system in general controller canonical form are shown below.

$T_c =$

-0.3827	-0.0478	0.0001	-0.0048	0	0	-0.0089
0.0804	-0.0245	-0.0618	-0.1301	0.0000	0	0.0089
0.7580	0.0051	0.2091	0.4533	0.0495	0.0000	-0.0089
0.8816	-0.0322	-0.0001	0.0111	0	0	0.0153
-0.1852	0.0564	0.0000	-0.0023	0.0000	0.0000	-0.0153
0.0389	-0.0119	0.0496	-0.6927	-0.0000	0.0000	0.0153
-7.4493	0.0025	-0.1755	1.2017	-0.0143	0.0495	-0.0153

$A_c =$

-0.0000	1.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.0000
0.0000	-0.0000	1.0000	0.0000	0.0000	-0.0000	-0.0000
7.3807	-68.0181	-23.5853	25.3084	-5.8738	-0.3528	0.0000
-0.0000	-0.0000	-0.0000	-0.0000	1.0000	0.0000	0.0000
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	1.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	1.0000
-82.5841	19.6579	1.1756	-251.1786	-223.5917	-52.7012	-22.2915

$B_c =$

0	0
0.0000	0
1.0000	0.0000
0	0
0.0000	0.0000
-0.0000	0.0000
-0.2888	1.0000

In the simulation process, we choose 12 different sets of desired closed loop poles. For each set we calculate the corresponding feedback gain, and then we do a performance analysis: feedback gain magnitude, robust performance and stability.

**Table 4.1:** The different chosen sets of desired eigenvalues

The desired eigenvalues			
case 1	case 2	case 3	case 4
-0.05 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	-0.1 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	-0.2 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	-0.0167 -1 -0.8-i3.0633 -0.8+i3.0633 -3.6152 -20.2 -20.2
case 5	case 6	case 7	case 8
-0.0167 -1 -0.4224-2.5 -0.4224+2.5 -3.6152 -20.2 -20.2	-0.0167 -1 -1-i -1+i -3.6152 -20.2 -20.2	-0.05 -1 -0.4224-2.5 -0.4224+2.5 -3.6152 -20.2 -20.2	-0.1 -1 -0.4224-2.5 -0.4224+2.5 -3.6152 -20.2 -20.2
case 9	case 10	case 11	case 12
-0.2 -1 -0.4224-2.5 -0.4224+2.5 -3.6152 -20.2 -20.2	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	-0.1 -1 -1-i -1+i -3.6152 -20.2 -20.2	-0.2 -1 -1-i -1+i -3.6152 -20.2 -20.2

The comparison is done according to:

- Number of blocks.
- Input sequence (permutation).
- Distribution of desired closed loop poles.

To distinguish between the different cases and to make the comparison easier, we introduce the following table.

**Table 4.2:** Numbering of the different cases

Cases	Meaning
case: 1.1.1 --- 12.1.1	Set from 1to 12, 1 block, first permutation
case: 1.1.2 --- 12.1.2	Set from 1to 12, 2 blocks (dominant & non-dominant distribution) , first permutation
case: 1.1.3 --- 12.1.3	Set from 1to 12, 2 blocks (random distribution), first permutation
case: 1.2.1 --- 12.2.1	Set from 1to 12, 1 block, second permutation
case: 1.2.2 --- 12.2.2	Set from 1to 12, 2 blocks (dominant & non-dominant distribution) , second permutation
case: 1.2.3 --- 12.2.3	Set from 1to 12, 2 blocks (random distribution), second permutation

### 3- Performance of The Actual System

**Table 4.3:** Performance analysis of the actual system

Eigenvalues	S(V)	S( $\lambda_i$ )	M <sub>2</sub>	M <sub>3</sub>
-0.0167	132.6212	5.1230	1.2626 * 10 <sup>-4</sup>	0.0033
-1		54.6941		
-0.4224-i3.0633		27.9476		
-0.4224+i3.0633		27.9476		
-3.6152		5.9276		
-20.2		1.0010		
-20.2		1.0010		

### 4- Performance of The Different Desired Simulated Cases

The resultant feedback gains and their corresponding performance analysis, for each case, are summarized in the following tables.



## 4.1- First Permutation

Table 4.4: Performance analysis corresponding to 1 block

Cases	Eigenvalues	Norm1	Norm2	Norm $\infty$	S(V)	S( $\lambda_i$ )	M <sub>2</sub>	M <sub>3</sub>
Case 1.1.1	-0.05 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	1.0892 *10 <sup>6</sup>	8.8085 *10 <sup>5</sup>	9.1016 *10 <sup>5</sup>	7.9862 *10 <sup>7</sup>	803.8715 4.9301*10 <sup>3</sup> 6.9168*10 <sup>4</sup> 6.9168*10 <sup>4</sup> 7.9844*10 <sup>4</sup> 2.8407*10 <sup>7</sup> 1.3157*10 <sup>6</sup>	6.3891 *10 <sup>-10</sup>	6.9417 *10 <sup>-7</sup>
Case 2.1.1	-0.1 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	1.0996 *10 <sup>6</sup>	8.8936 *10 <sup>5</sup>	9.2030 *10 <sup>5</sup>	8.0073 *10 <sup>7</sup>	865.9350 5.1152*10 <sup>3</sup> 6.9643*10 <sup>4</sup> 6.9643*10 <sup>4</sup> 8.149*10 <sup>4</sup> 2.8383*10 <sup>7</sup> 1.3109*10 <sup>6</sup>	1.2626 *10 <sup>-9</sup>	6.9451 *10 <sup>-7</sup>
Case 3.1.1	-0.2 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	1.1205 *10 <sup>6</sup>	9.0638 *10 <sup>5</sup>	9.4057 *10 <sup>5</sup>	8.0511 *10 <sup>7</sup>	1.0192*10 <sup>3</sup> 5.5554*10 <sup>3</sup> 7.056*10 <sup>4</sup> 7.056*10 <sup>4</sup> 8.4972*10 <sup>4</sup> 2.834*10 <sup>7</sup> 1.3015*10 <sup>6</sup>	2.5001 *10 <sup>-9</sup>	6.951 *10 <sup>-7</sup>
Case 4.1.1	-0.0167 -1 -0.8-i3.0633 -0.8+i3.0633 -3.6152 -20.2 -20.2	1.2963 *10 <sup>6</sup>	1.0476 *10 <sup>6</sup>	1.0650 *10 <sup>6</sup>	8.5922 *10 <sup>7</sup>	908.5925 5.7751*10 <sup>3</sup> 8.5785*10 <sup>4</sup> 8.5785*10 <sup>4</sup> 1.1536*10 <sup>5</sup> 3.0672*10 <sup>7</sup> 1.26*10 <sup>6</sup>	2.0699 *10 <sup>-10</sup>	6.3996 *10 <sup>-7</sup>
Case 5.1.1	-0.0167 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	8.5627 *10 <sup>5</sup>	6.9179 *10 <sup>5</sup>	6.9799 *10 <sup>5</sup>	7.3571 *10 <sup>7</sup>	1.2852*10 <sup>3</sup> 6.4469*10 <sup>3</sup> 4.5242*10 <sup>4</sup> 4.5242*10 <sup>4</sup> 8.1031*10 <sup>4</sup> 2.6244*10 <sup>7</sup> 1.3787*10 <sup>6</sup>	2.4585 *10 <sup>-10</sup>	7.5425 *10 <sup>-7</sup>

Case 6.1.1	-0.0167 -1 -1-i -1+i -3.6152 -20.2 -20.2	8.1359 *10 <sup>5</sup>	6.5848 *10 <sup>5</sup>	6.8877 *10 <sup>5</sup>	7.9202 *10 <sup>7</sup>	6.4775*10 <sup>3</sup> 4.2272*10 <sup>4</sup> 3.7062*10 <sup>4</sup> 3.7062*10 <sup>4</sup> 2.034*10 <sup>5</sup> 2.6852*10 <sup>7</sup> 1.3543*10 <sup>6</sup>	2.7856 *10 <sup>-10</sup>	7.3619 *10 <sup>-7</sup>
Case 7.1.1	-0.05 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	8.6421 *10 <sup>5</sup>	6.9822 *10 <sup>5</sup>	7.0502 *10 <sup>5</sup>	7.3860 *10 <sup>7</sup>	1.3493*10 <sup>3</sup> 6.6339*10 <sup>3</sup> 4.553*10 <sup>4</sup> 4.553*10 <sup>4</sup> 8.244*10 <sup>4</sup> 2.6268*10 <sup>7</sup> 1.3735*10 <sup>6</sup>	6.9666 *10 <sup>-10</sup>	7.5339 *10 <sup>-7</sup>
Case 8.1.1	-0.1 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	8.7613 *10 <sup>5</sup>	7.0787 *10 <sup>5</sup>	7.1558 *10 <sup>5</sup>	7.4321 *10 <sup>7</sup>	1.4563*10 <sup>3</sup> 6.9407*10 <sup>3</sup> 4.5953*10 <sup>4</sup> 4.5953*10 <sup>4</sup> 8.4608*10 <sup>4</sup> 2.6316*10 <sup>7</sup> 1.3669*10 <sup>6</sup>	1.3666 *10 <sup>-9</sup>	7.5176 *10 <sup>-7</sup>
Case 9.1.1	-0.2 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	8.9999 *10 <sup>5</sup>	7.2718 *10 <sup>5</sup>	7.3669 *10 <sup>5</sup>	7.5219 *10 <sup>7</sup>	1.7196*10 <sup>3</sup> 7.6710*10 <sup>3</sup> 4.676*10 <sup>4</sup> 4.676*10 <sup>4</sup> 8.9133*10 <sup>4</sup> 2.6417*10 <sup>7</sup> 1.3547*10 <sup>6</sup>	2.6833 *10 <sup>-9</sup>	7.4835 *10 <sup>-7</sup>
Case 10.1.1	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	8.2571 *10 <sup>5</sup>	6.6822 *10 <sup>5</sup>	6.9819 *10 <sup>5</sup>	7.9744 *10 <sup>7</sup>	7.0174*10 <sup>3</sup> 4.3819*10 <sup>4</sup> 3.7848*10 <sup>4</sup> 3.7848*10 <sup>4</sup> 2.074*10 <sup>5</sup> 2.6984*10 <sup>7</sup> 1.3493*10 <sup>6</sup>	6.9971 *10 <sup>-10</sup>	7.3237 *10 <sup>-7</sup>
Case 11.1.1	-0.1 -1 -1-i -1+i -3.6152 -20.2 -20.2	8.4390 *10 <sup>5</sup>	6.8287 *10 <sup>5</sup>	7.1234 *10 <sup>5</sup>	7.0559 *10 <sup>7</sup>	7.9476*10 <sup>3</sup> 4.6359*10 <sup>4</sup> 3.9053*10 <sup>4</sup> 3.9053*10 <sup>4</sup> 2.1352*10 <sup>5</sup> 2.7191*10 <sup>7</sup> 1.3429*10 <sup>6</sup>	1.3225 *10 <sup>-9</sup>	7.2650 *10 <sup>-7</sup>

Case 12.1.1	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	8.8028 *10 <sup>5</sup>	7.1215 *10 <sup>5</sup>	7.4063 *10 <sup>5</sup>	7.3579 *10 <sup>7</sup>	1.0387*10 <sup>4</sup> 5.2410*10 <sup>4</sup> 4.1548*10 <sup>4</sup> 4.1548*10 <sup>4</sup> 2.2623*10 <sup>5</sup> 2.4741*10 <sup>7</sup> 1.0966*10 <sup>6</sup>	2.8350 *10 <sup>-9</sup>	7.9632 *10 <sup>-7</sup>
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**Table 4.5:** Performance analysis corresponding to 2 blocks dominant and non-dominant distribution

Cases	Eigenvalues	Norm1	Norm2	Norm $\infty$	S(V)	S( $\lambda_i$ )	M <sub>2</sub>	M <sub>3</sub>
Case 1.1.2	-0.05 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	1.1621 *10 <sup>4</sup>	9.4868 *10 <sup>3</sup>	1.0521 *10 <sup>4</sup>	1.6783 *10 <sup>7</sup>	76.0099 681.0467 495.2866 495.2866 2.0224*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.9602*10 <sup>6</sup>	0.3009 *10 <sup>-8</sup>	0.3385 *10 <sup>-5</sup>
Case 2.1.2	-0.1 -1 -0.4224-i3.0633 - 0.4224+i3.0633 -3.6152 -20.2 -20.2	1.1621 *10 <sup>4</sup>	9.4869 *10 <sup>3</sup>	1.0521 *10 <sup>4</sup>	1.6842 *10 <sup>7</sup>	80.3083 684.839 496.0199 496.0199 2.0224*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.9602*10 <sup>6</sup>	0.5968 *10 <sup>-8</sup>	0.3385 *10 <sup>-5</sup>
Case 3.1.2	-0.2 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	1.1621 *10 <sup>4</sup>	9.4872 *10 <sup>3</sup>	1.0521 *10 <sup>4</sup>	1.6963 *10 <sup>7</sup>	90.6474 694.0623 496.266 496.266 2.0224*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.9602*10 <sup>6</sup>	0.1182 *10 <sup>-7</sup>	0.3385 *10 <sup>-5</sup>
Case 4.1.2	-0.0167 -1 -0.8-i3.0633 -0.8+i3.0633 -3.6152 -20.2 -20.2	1.164 *10 <sup>4</sup>	9.5053 *10 <sup>3</sup>	1.0541 *10 <sup>4</sup>	1.6744 *10 <sup>7</sup>	6.84612 702.4089 453.6747 453.6747 2.0224*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.960*10 <sup>6</sup>	0.1025 *10 <sup>-8</sup>	0.3385 *10 <sup>-5</sup>

Case 5.1.2	-0.0167 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.1619 *10 <sup>4</sup>	9.4864 *10 <sup>3</sup>	1.0528 *10 <sup>4</sup>	1.6777 *10 <sup>7</sup>	364.7539 702.4089 605.8303 605.8303 2.0224*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.960*10 <sup>6</sup>	0.1038 *10 <sup>-8</sup>	0.3385 *10 <sup>-5</sup>
Case 6.1.2	-0.0167 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.1647 *10 <sup>4</sup>	9.5144 *10 <sup>3</sup>	1.057 *10 <sup>4</sup>	1.7733 *10 <sup>7</sup>	2.4918*10 <sup>3</sup> 5.6523*10 <sup>3</sup> 2.3242*10 <sup>3</sup> 2.3242*10 <sup>3</sup> 2.022*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.960*10 <sup>6</sup>	1.0734 *10 <sup>-9</sup>	0.3355 *10 <sup>-6</sup>
Case 7.1.2	-0.05 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.1620 *10 <sup>4</sup>	9.4867 *10 <sup>3</sup>	1.0529 *10 <sup>4</sup>	1.6826 *10 <sup>7</sup>	378.063 954.5496 606.4325 606.4325 2.0224*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.9602*10 <sup>6</sup>	3.0153 *10 <sup>-9</sup>	0.3355 *10 <sup>-6</sup>
Case 8.1.2	-0.1 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.1620 *10 <sup>4</sup>	9.4822 *10 <sup>3</sup>	1.0529 *10 <sup>4</sup>	1.6899 *10 <sup>7</sup>	413.38 973.9613 607.2312 607.2312 2.0224*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.9602*10 <sup>6</sup>	5.9634 *10 <sup>-8</sup>	3.3855 *10 <sup>-6</sup>
Case 9.1.2	-0.2 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.1621 *10 <sup>4</sup>	9.4882 *10 <sup>3</sup>	1.053 *10 <sup>4</sup>	1.7037 *10 <sup>7</sup>	452.8884 1.0203*10 <sup>3</sup> 608.3987 608.3987 2.0224*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.9602*10 <sup>6</sup>	1.1791 *10 <sup>-8</sup>	3.3855 *10 <sup>-6</sup>
Case 10.1.2	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.1648 *10 <sup>4</sup>	9.5151 *10 <sup>3</sup>	1.0571 *10 <sup>4</sup>	1.7774 *10 <sup>7</sup>	2.6614*10 <sup>3</sup> 5.8183*10 <sup>3</sup> 2.358*10 <sup>3</sup> 2.358*10 <sup>3</sup> 2.0224*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.960*10 <sup>6</sup>	2.9532 *10 <sup>-9</sup>	3.3855 *10 <sup>-6</sup>

Case 11.1.2	-0.1 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.1649 *10 <sup>4</sup>	9.5161 *10 <sup>3</sup>	1.0572 *10 <sup>4</sup>	1.783 *10 <sup>7</sup>	2.3482*10 <sup>3</sup> 6.0916*10 <sup>3</sup> 2.4056*10 <sup>3</sup> 2.4056*10 <sup>3</sup> 2.0224*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.960*10 <sup>6</sup>	5.7628 *10 <sup>-9</sup>	3.3855 *10 <sup>-6</sup>
Case 12.1.2	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.1648 *10 <sup>4</sup>	9.5151 *10 <sup>3</sup>	1.0571 *10 <sup>4</sup>	1.7774 *10 <sup>7</sup>	2.6614*10 <sup>3</sup> 5.8183*10 <sup>3</sup> 2.358*10 <sup>3</sup> 2.358*10 <sup>3</sup> 2.0223*10 <sup>3</sup> 5.9614*10 <sup>6</sup> 5.960*10 <sup>6</sup>	2.9532 *10 <sup>-9</sup>	3.3855 *10 <sup>-6</sup>

**Table 4.6:** Performance analysis corresponding to 2 blocks random distribution

Cases	Eigenvalues	Norm1	Norm2	Norm $\infty$	S(V)	S( $\lambda_i$ )	M <sub>2</sub>	M <sub>3</sub>
Case 1.1.3	-0.05 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	1.0670 *10 <sup>4</sup>	1.0005 *10 <sup>4</sup>	1.1951 *10 <sup>4</sup>	1.4274 *10 <sup>7</sup>	429.6364 1.5378*10 <sup>3</sup> 1.6581*10 <sup>4</sup> 1.6581*10 <sup>4</sup> 532.5384 4.7596*10 <sup>6</sup> 4.7596*10 <sup>6</sup>	3.5029 *10 <sup>-9</sup>	4.2440 *10 <sup>-6</sup>
Case 2.1.3	-0.1 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	1.0670 *10 <sup>4</sup>	1.0665 *10 <sup>4</sup>	1.1951 *10 <sup>4</sup>	1.4321 *10 <sup>7</sup>	462.6911 1.564*10 <sup>3</sup> 1.6581*10 <sup>3</sup> 1.6581*10 <sup>3</sup> 534.646 4.7596*10 <sup>6</sup> 4.7596*10 <sup>6</sup>	6.9817 *10 <sup>-9</sup>	4.2440 *10 <sup>-6</sup>
Case 3.1.3	-0.2 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	1.0671 *10 <sup>4</sup>	1.0666 *10 <sup>4</sup>	1.1950 *10 <sup>4</sup>	1.4411 *10 <sup>7</sup>	543.8557 1.629*10 <sup>3</sup> 1.6581*10 <sup>3</sup> 1.6581*10 <sup>3</sup> 539.1303 4.7596*10 <sup>6</sup> 4.7596*10 <sup>6</sup>	1.3876 *10 <sup>-8</sup>	4.2440 *10 <sup>-6</sup>
Case 4.1.3	-0.0167 -1 -0.8-i3.0633 -0.8+i3.0633 -3.6152 -20.2 -20.2	1.1020 *10 <sup>4</sup>	1.1018 *10 <sup>4</sup>	1.2326 *10 <sup>4</sup>	1.6811 *10 <sup>7</sup>	409.8848 1.5219*10 <sup>3</sup> 1.6018*10 <sup>3</sup> 1.6018*10 <sup>3</sup> 531.1774 5.618*10 <sup>6</sup> 5.618*10 <sup>6</sup>	9.9311 *10 <sup>-10</sup>	3.5956 *10 <sup>-6</sup>

Case 5.1.3	-0.0167 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	6.8474 *10 <sup>3</sup>	6.8604 *10 <sup>3</sup>	7.9907 *10 <sup>3</sup>	1.1417 *10 <sup>7</sup>	409.8842 1.5219*10 <sup>3</sup> 1.1406*10 <sup>3</sup> 1.1406*10 <sup>3</sup> 539.1303 3.7880*10 <sup>6</sup> 3.7880*10 <sup>6</sup>	1.4623 *10 <sup>-9</sup>	5.3327 *10 <sup>-6</sup>
Case 6.1.3	-0.0167 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.1204 *10 <sup>3</sup>	1.4551 *10 <sup>3</sup>	2.1995 *10 <sup>3</sup>	2.6805 *10 <sup>7</sup>	409.8858 1.5219*10 <sup>3</sup> 516.6466 516.6466 531.1715 8.5864*10 <sup>5</sup> 8.5864*10 <sup>5</sup>	6.2285 *10 <sup>-9</sup>	2.3526 *10 <sup>-5</sup>
Case 7.1.3	-0.05 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	6.8476 *10 <sup>3</sup>	6.8605 *10 <sup>3</sup>	7.9905 *10 <sup>3</sup>	1.1443 *10 <sup>7</sup>	429.6362 1.5378*10 <sup>3</sup> 1.1406*10 <sup>3</sup> 1.1406*10 <sup>3</sup> 532.5368 3.788*10 <sup>6</sup> 3.788*10 <sup>6</sup>	4.3693 *10 <sup>-9</sup>	5.3327 *10 <sup>-6</sup>
Case 8.1.3	-0.1 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	6.8479 *10 <sup>3</sup>	6.8607 *10 <sup>3</sup>	7.9901 *10 <sup>3</sup>	1.1482 *10 <sup>7</sup>	462.6909 1.5643*10 <sup>3</sup> 1.1406*10 <sup>3</sup> 1.1406*10 <sup>3</sup> 534.6444 3.7888*10 <sup>6</sup> 3.7888*10 <sup>6</sup>	8.7075 *10 <sup>-9</sup>	5.3327 *10 <sup>-6</sup>
Case 9.1.3	-0.2 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	6.8485 *10 <sup>3</sup>	6.8610 *10 <sup>3</sup>	7.9893 *10 <sup>3</sup>	1.1557 *10 <sup>7</sup>	543.8554 1.6294*10 <sup>3</sup> 1.1406*10 <sup>3</sup> 1.1406*10 <sup>3</sup> 539.1286 3.788*10 <sup>6</sup> 3.788*10 <sup>6</sup>	1.7304 *10 <sup>-8</sup>	5.3327 *10 <sup>-6</sup>
Case 10.1.3	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.1206 *10 <sup>3</sup>	14549 *10 <sup>3</sup>	2.1992 *10 <sup>3</sup>	2.7227 *10 <sup>6</sup>	429.6378 1.5378*10 <sup>3</sup> 516.6466 516.6466 532.5336 8.7038*10 <sup>5</sup> 8.7038*10 <sup>5</sup>	1.8363 *10 <sup>-8</sup>	2.3208 *10 <sup>-5</sup>

Case 11.1.3	-0.1 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.1209 *10 <sup>3</sup>	1.4546 *10 <sup>3</sup>	2.1986 *10 <sup>3</sup>	2.7307 *10 <sup>6</sup>	462.6924 1.5643*10 <sup>3</sup> 516.6465 516.6465 535.6412 8.7038*10 <sup>5</sup> 8.7038*10 <sup>5</sup>	3.6614 *10 <sup>-8</sup>	2.3208 *10 <sup>-5</sup>
Case 12.1.3	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.1215 *10 <sup>3</sup>	1.4541 *10 <sup>3</sup>	2.1976 *10 <sup>3</sup>	2.7448 *10 <sup>6</sup>	543.8569 1.6294*10 <sup>3</sup> 516.6463 516.6463 539.1253 8.7038*10 <sup>5</sup> 8.7038*10 <sup>5</sup>	7.2856 *10 <sup>-8</sup>	2.3208 *10 <sup>-5</sup>

4.2- Second Permutation

Table 4.7: Performance analysis corresponding to 1 block

Cases	Eigenvalues	Norm1	Norm2	Norm $\infty$	S(V)	S( $\lambda_i$ )	M <sub>2</sub>	M <sub>3</sub>
Case 1.2.1	-0.05 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	2.4073 *10 <sup>4</sup>	2.5419 *10 <sup>4</sup>	3.6381 *10 <sup>4</sup>	9.7204 *10 <sup>6</sup>	44.40443 89.8437 526.0910 526.0910 4.6649*10 <sup>3</sup> 3.2436*10 <sup>6</sup> 3.2436*10 <sup>6</sup>	5.0978 *10 <sup>-9</sup>	6.2253 *10 <sup>-6</sup>
Case 2.2.1	-0.1 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	2.4268 *10 <sup>4</sup>	2.5673 *10 <sup>4</sup>	3.6911 *10 <sup>4</sup>	2.3407 *10 <sup>6</sup>	57.714 90.9248 530.3288 530.3288 4.7522*10 <sup>3</sup> 7.879*10 <sup>6</sup> 7.879*10 <sup>6</sup>	4.6304 *10 <sup>-9</sup>	2.5627 *10 <sup>-6</sup>
Case 3.2.1	-0.2 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	2.4979 *10 <sup>4</sup>	2.6306 *10 <sup>4</sup>	3.7657 *10 <sup>4</sup>	6.9749 *10 <sup>6</sup>	45.6368 119.0271 543.7617 543.761 4.9033*10 <sup>3</sup> 2.2965*10 <sup>6</sup> 2.2965 *10 <sup>6</sup>	2.6077 *10 <sup>-8</sup>	8.7931 *10 <sup>-6</sup>

Case 4.2.1	-0.0167 -1 -0.8-i3.0633 -0.8+i3.0633 -3.6152 -20.2 -20.2	2.7036 *10 <sup>4</sup>	2.8510 *10 <sup>4</sup>	4.0616 *10 <sup>4</sup>	2.3105 *10 <sup>6</sup>	52.5566 107.8491 768.9888 768.9888 5.7265*10 <sup>3</sup> 7.5085*10 <sup>4</sup> 7.5085*10 <sup>4</sup>	7.0925 *10 <sup>-8</sup>	2.6416 *10 <sup>-4</sup>
Case 5.2.1	-0.0167 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.7067 *10 <sup>4</sup>	1.7950 *10 <sup>4</sup>	2.5447 *10 <sup>4</sup>	8.9177 *10 <sup>6</sup>	58.8838 100.5411 549.6773 549.6773 4.0018*10 <sup>3</sup> 2.6359*10 <sup>6</sup> 2.6359*10 <sup>6</sup>	1.8165 *10 <sup>-6</sup>	7.6605 *10 <sup>-6</sup>
Case 6.2.1	-0.0167 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.0691 *10 <sup>4</sup>	1.1057 *10 <sup>4</sup>	1.4954 *10 <sup>4</sup>	8.0801 *10 <sup>6</sup>	266.309 584.6525 1.0166*10 <sup>3</sup> 1.0166*10 <sup>3</sup> 5.7986*10 <sup>3</sup> 2.3721*10 <sup>6</sup> 2.3721*10 <sup>6</sup>	1.8665 *10 <sup>-9</sup>	8.5120 *10 <sup>-6</sup>
Case 7.2.1	-0.05 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.7240 *10 <sup>4</sup>	1.8130 *10 <sup>4</sup>	3.6381 *10 <sup>4</sup>	2.5722 *10 <sup>6</sup>	62.1157 103.5171 554.6566 554.6566 4.0564*10 <sup>3</sup> 2.6524*10 <sup>6</sup> 2.6524*10 <sup>6</sup>	5.4983 *10 <sup>-9</sup>	7.6129 *10 <sup>-6</sup>
Case 8.2.1	-0.1 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.7501 *10 <sup>4</sup>	1.8401 *10 <sup>4</sup>	2.6136 *10 <sup>4</sup>	9.0063 *10 <sup>6</sup>	67.4020 108.3963 562.0659 562.0659 4.1403*10 <sup>3</sup> 2.664*10 <sup>6</sup> 2.664*10 <sup>6</sup>	1.0997 *10 <sup>-8</sup>	7.5797 *10 <sup>-6</sup>
Case 9.2.1	-0.2 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.8023 *10 <sup>4</sup>	1.8944 *10 <sup>4</sup>	2.6964 *10 <sup>4</sup>	9.1535 *10 <sup>6</sup>	79.5218 119.9641 576.5928 576.5928 4.315*10 <sup>3</sup> 2.7114*10 <sup>6</sup> 2.7114*10 <sup>6</sup>	2.1672 *10 <sup>-8</sup>	7.4471 *10 <sup>-6</sup>



Case 10.2.1	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.0935 *10 <sup>4</sup>	1.1313 *10 <sup>4</sup>	1.5324 *10 <sup>4</sup>	8.1606 *10 <sup>6</sup>	288.3824 610.4138 1.0461*10 <sup>3</sup> 1.0461*10 <sup>3</sup> 5.9566*10 <sup>6</sup> 2.3963*10 <sup>6</sup> 2.3963*10 <sup>6</sup>	5.8536 *10 <sup>-9</sup>	8.4263 *10 <sup>-6</sup>
Case 11.2.1	-0.1 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.1301 *10 <sup>4</sup>	1.1697 *10 <sup>4</sup>	1.5881 *10 <sup>4</sup>	8.2127 *10 <sup>6</sup>	325.4034 652.3381 1.0916*10 <sup>3</sup> 1.0916*10 <sup>3</sup> 46.1993* 10 <sup>3</sup> 2.4126*10 <sup>6</sup> 2.4126*10 <sup>6</sup>	1.1774 *10 <sup>-8</sup>	8.3693 *10 <sup>-6</sup>
Case 12.2.1	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	1.2033 *10 <sup>4</sup>	1.2467 *10 <sup>4</sup>	1.6993 *10 <sup>4</sup>	8.4612 *10 <sup>6</sup>	414.4044 750.1652 1.1861*10 <sup>3</sup> 1.1861*10 <sup>3</sup> 6.7050*10 <sup>3</sup> 2.4889*10 <sup>6</sup> 2.4889*10 <sup>6</sup>	2.2919 *10 <sup>-8</sup>	8.1126 *10 <sup>-6</sup>

**Table 4.8:** Performance analysis corresponding to 2 blocks dominant and non-dominant distribution

Cases	Eigenvalues	Norm1	Norm2	Norm $\infty$	S(V)	S( $\lambda_i$ )	M <sub>2</sub>	M <sub>3</sub>
Case 1.2.2	-0.05 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	627.82 24	520.58 73	669.131 5	6.0956 *10 <sup>5</sup>	24.0858 67.2119 29.8121 29.8121 105.2778 2.5765*10 <sup>5</sup> 2.576*10 <sup>5</sup>	8.1802 *10 <sup>-8</sup>	7.8314 *10 <sup>-5</sup>
Case 2.2.2	-0.1 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	627.86 62	520.59 83	669.131 5	6.0861 *10 <sup>5</sup>	24.3694 67.5865 29.8202 29.8202 105.2779 2.5765*10 <sup>5</sup> 2.5765*10 <sup>5</sup>	1.6407 *10 <sup>-7</sup>	7.8314 *10 <sup>-5</sup>
Case 3.2.2	-0.2 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	627.95 36	520.62 05	669.131 5	6.0645 *10 <sup>5</sup>	15.0491 68.4758 29.8315 29.8315 105.2779 2.5765*10 <sup>5</sup> 2.5765*10 <sup>5</sup>	3.295 *10 <sup>-7</sup>	7.8314 *10 <sup>-5</sup>

Case 4.2.2	-0.0167 -1 -0.8-i3.0633 -0.8+i3.0633 -3.6152 -20.2 -20.2	633.45 36	522.11 08	669.131 5	6.5666 $*10^5$	22.3087 69.3253 35.7961 35.7961 105.2781 $2.5765*10^5$ $2.5760*10^5$	2.5243 $*10^{-8}$	7.8314 $*10^{-5}$
Case 5.2.2	-0.0167 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	627.37 13	520.40 54	669.131 5	7.2732 $*10^5$	118.8897 92.9487 58.5536 58.5536 105.2783 $2.5765*10^5$ $2.576*10^5$	2.2684 $*10^{-8}$	7.8314 $*10^{-5}$
Case 6.2.2	-0.0167 -1 -1-i -1+i -3.6152 -20.2 -20.2	635.33 82	522.46 19	669.131 5	7.6125 $*10^5$	804.6529 550.7445 548.1290 548.1290 105.2819 $2.5765*10^5$ $2.5760*10^5$	2.1112 $*10^{-8}$	1.9973 $*10^{-5}$
Case 7.2.2	-0.05 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	627.47 31	520.43 21	669.131 5	7.2692 $*10^5$	119.9073 94.1135 58.6113 58.6113 105.2783 $2.5765*10^5$ $2.5760*10^5$	6.85 $*10^{-8}$	7.8314 $*10^{-5}$
Case 8.2.2	-0.1 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	627.62 59	520.47 24	669.131 5	7.2626 $*10^5$	121.5404 96.0189 58.6889 58.6889 105.2784 $2.5765*10^5$ $2.5765*10^5$	1.3738 $*10^{-7}$	7.8314 $*10^{-5}$
Case 9.2.2	-0.2 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	627.86 99	520.53 65	669.131 5	7.2708 $*10^5$	131.7880 105.5874 59.0729 59.0729 105.2785 $2.5765*10^5$ $2.5760*10^5$	2.9418 $*10^{-7}$	7.8314 $*10^{-5}$

Case 10.2.2	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	635.54 83	522.52 11	669.131 5	7.6111 $*10^5$	535.4455 566.4192 555.9742 555.9742 105.2819 $2.5765*10^5$ $2.5760*10^5$	6.4803 $*10^{-8}$	5.9037 $*10^{-5}$
Case 11.2.2	-0.1 -1 -1-i -1+i -3.6152 -20.2 -20.2	635.86 38	522.61	669.131 5	7.6086 $*10^5$	884.977 592.1156 567.9393 567.9393 105.282 $2.5765*10^5$ $2.5760*10^5$	1.3043 $*10^{-7}$	7.8314 $*10^{-5}$
Case 12.2.2	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	636.49 48	522.78 85	669.131 5	7.6025 $*10^5$	997.5172 653.0404 592.374 592.374 105.2822 $2.5765*10^5$ $2.5765*10^5$	2.618 $*10^{-7}$	7.8314 $*10^{-5}$

**Table 4.9:** Performance analysis corresponding to 2 blocks random distribution

Cases	Eigenvalues	Norm1	Norm2	Norm $\infty$	S(V)	S( $\lambda_i$ )	M <sub>2</sub>	M <sub>3</sub>
Case 1.2.3	-0.05 -1 -0.4224-i3.0633 0.4224+i3.0633 -3.6152 -20.2 -20.2	3.0342 $*10^3$	3.0428 $*10^3$	3.4106 $*10^3$	4.2197 $*10^6$	12.5761 48.7209 99.6684 99.6684 18.09 $1.385*10^6$ $1.385*10^6$	1.1854 $*10^{-8}$	1.4585 $*10^{-5}$
Case 2.2.3	-0.1 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	3.0342 $*10^3$	3.0428 $*10^3$	3.4106 $*10^3$	4.2193 $*10^6$	13.5439 48.1334 99.6684 99.6684 18.0056 $1.385*10^6$ $1.385*10^6$	2.3702 $*10^{-8}$	1.4585 $*10^{-5}$
Case 3.2.3	-0.2 -1 -0.4224-i3.0633 -0.4224+i3.0633 -3.6152 -20.2 -20.2	3.0342 $*10^3$	3.0428 $*10^3$	3.4106 $*10^3$	4.2168 $*10^6$	15.9199 46.8233 99.6684 99.6684 17.8307 $1.385*10^6$ $1.385*10^6$	4.7429 $*10^{-8}$	1.4585 $*10^{-5}$

Case 4.2.3	-0.0167 -1 -0.8-i3.0633 -0.8+i3.0633 -3.6152 -20.2 -20.2	3.1354 *10 <sup>3</sup>	3.1446 *10 <sup>3</sup>	3.5187 *10 <sup>3</sup>	4.9341 *10 <sup>6</sup>	11.9980 49.0888 126.3709 126.3709 18.1451 1.1451 1.5599*10 <sup>6</sup> 1.5599*10 <sup>6</sup>	3.3882 *10 <sup>-9</sup>	1.295 *10 <sup>-5</sup>
Case 5.2.3	-0.0167 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.9302 *10 <sup>3</sup>	1.9438 *10 <sup>3</sup>	2.2665 *10 <sup>3</sup>	3.4952 *10 <sup>6</sup>	11.998 49.0889 110.2396 110.2396 18.1452 1.0152*10 <sup>6</sup> 1.0152*10 <sup>6</sup>	4.7831 *10 <sup>-9</sup>	1.9897 *10 <sup>-5</sup>
Case 6.2.3	-0.0167 -1 -1-i -1+i -3.6152 -20.2 -20.2	276.43 49	390.54 27	593.56	8.56 *10 <sup>5</sup>	11.9983 49.0877 122.1799 122.1799 18.1454 2.432*10 <sup>5</sup> 2.432*10 <sup>5</sup>	1.953 *10 <sup>-8</sup>	8.3058 *10 <sup>-5</sup>
Case 7.2.3	-0.05 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.9302 *10 <sup>3</sup>	1.9438 *10 <sup>3</sup>	2.2665 *10 <sup>3</sup>	3.4949 *10 <sup>6</sup>	12.5762 48.7210 110.2396 110.2396 18.0901 1.0152*10 <sup>6</sup> 1.0152*10 <sup>6</sup>	1.4313 *10 <sup>-8</sup>	1.9897 *10 <sup>-5</sup>
Case 8.2.3	-0.1 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.9302 *10 <sup>3</sup>	1.9438 *10 <sup>3</sup>	2.2665 *10 <sup>3</sup>	9.4942 *10 <sup>6</sup>	13.5439 48.1335 110.2396 110.2396 18.0056 1.0152*10 <sup>6</sup> 1.0152*10 <sup>6</sup>	2.862 *10 <sup>-8</sup>	1.9897 *10 <sup>-5</sup>
Case 9.2.3	-0.2 -1 -0.4224-i2.5 -0.4224+i2.5 -3.6152 -20.2 -20.2	1.9301 *10 <sup>3</sup>	1.9438 *10 <sup>3</sup>	2.2665 *10 <sup>3</sup>	3.4917 *10 <sup>6</sup>	15.9199 46.8234 110.2396 110.2396 17.8308 1.0152*10 <sup>6</sup> 1.0152*10 <sup>6</sup>	5.7279 *10 <sup>-8</sup>	1.9897 *10 <sup>-5</sup>

Case 10.2.3	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	276.43 1	390.54 19	593.555 2	8.5603 $*10^5$	12.5764 48.7197 122.1799 122.1799 18.0903 $2.432*10^5$ $2.432*10^5$	5.8435 $*10^{-8}$	8.3058 $*10^{-5}$
Case 11.2.3	-0.1 -1 -1-i -1+i -3.6152 -20.2 -20.2	276.42 52	390.54 08	593.548	8.5597 $*10^5$	13.5442 48.1321 122.1799 122.1799 18.0058 $2.432*10^5$ $2.432*10^5$	1.1683 $*10^{-7}$	8.3058 $*10^{-5}$
Case 12.2.3	-0.05 -1 -1-i -1+i -3.6152 -20.2 -20.2	276.41 35	390.53 85	593.533 7	8.5562 $*10^5$	15.9204 46.8219 122.1799 122.1799 17.8310 $2.432*10^5$ $2.432*10^5$	2.3375 $*10^{-7}$	8.3058 $*10^{-5}$

## 5- Discussion

In each of the following sections we are going to analyze the effect of number of blocks either one or two blocks, the effect of permutation: the first or the second permutation and the effect of distribution either by taking a dominant block and a non-dominant block or by distributing the poles (eigenvalues) randomly on the gain magnitude, individual and overall sensitivities and the robust stability.

### 5.1-The Effect of Number of Blocks

#### I-First Permutation

##### I.1- Feedback Gain Magnitude

It is noticed that the magnitude is smaller for two blocks, where the best case is case 8.1.2 with  $n_1=1.1620*10^4$ ,  $n_2=9.4822*10^3$ ,  $n_\infty=1.0529*10^4$ .

### I.2- Overall & Individual Sensitivity

- The overall sensitivity is smaller for two blocks, where the best is case 4.1.2 with:  $S(V) = 1.67644 * 10^7$ .
- The poles have smaller individual sensitivity for two blocks, where the best case is case 4.1.2 with:  $\{S_1 = 6.8461, S_2 = 702.2489, S_{3,4} = 453.6747, S_4 = 2.0224 * 10^3, S_6 = 5.9614 * 10^6, S_7 = 5.9060 * 10^6\}$ .

### I.3- Robust Stability

- The robust measure  $M_2$  is bigger for two blocks, where the best case is case 3.1.2 with:  $M_2 = 0.1182 * 10^{-7}$ .
- The robust measure  $M_3$  is bigger for two blocks, where its value is identical for all cases and it is equal to  $0.3385 * 10^{-5}$ .

## II-Second Permutation

### II.1- Feedback Gain Magnitude

It is noticed that the magnitude is smaller for two blocks, where the best case is case 5.2.2 with:  $n_1 = 627.37613, n_2 = 520.4054, n_\infty = 669.1315$ .

PS:  $n_\infty$  has the same value for all cases.

### II.2- Overall & Individual Sensitivity

- The overall sensitivity is smaller for two blocks, except for case 4 and the best is case 4.2.2 with:  $S(V) = 2.3105 * 10^5$ .
- The poles have smaller individual sensitivity for two blocks, except for cases from 5 to 12 where the most dominant pole has smaller individual sensitivity for one block, and the best case is case 1.2.2 with:  $\{S_1 = 24.0858, S_2 = 676.2117, S_{3,4} = 29.8121, S_4 = 105.2778, S_6 = 2.5765 * 10^5, S_7 = 2.576 * 10^5\}$ .

### II.3- Robust Stability

- The robust measure  $M_2$  is bigger for two blocks, except for cases 4 and 5, and the best case is case 5.2.1 with:  $M_2 = 1.8165 * 10^{-6}$ .
- The robust measure  $M_3$  is bigger for two blocks, except for case 4 where the best case is case 4.2.1 with  $M_3 = 2.6416 * 10^{-4}$ .

PS:  $M_3$  has the same value which is equal to  $7.8314 \cdot 10^{-5}$ , except for cases 6 and 10.

## 5.2-The Effect of Permutation: [b1, b2], [b2, b1]

### I- One Block

#### I.1- Feedback Gain Magnitude

It is noticed that the magnitude is smaller for the second permutation, where the best case is case 6.2.1 with:  $n_1 = 1.0691 \cdot 10^4$ ,  $n_2 = 1.1057 \cdot 10^4$ ,  $n_\infty = 1.4954 \cdot 10^4$ .

#### I.2- Overall & Individual Sensitivity

- The overall sensitivity is smaller for the second permutation, where the best is case 4.2.1 with:  $S(V) = 2.3105 \cdot 10^5$ .
- The poles have smaller individual sensitivity for the second permutation, where the best case is case 1.2.1 with:  $\{S_1 = 44.4043, S_2 = 89.8437, S_{3,4} = 526.0910, S_5 = 4.6649 \cdot 10^3, S_6 = 3.2436 \cdot 10^6, S_7 = 3.2436 \cdot 10^6\}$ .

#### I.3- Robust Stability

- The robust measure  $M_2$  is bigger for the second permutation, where the best case is case 5.2.1 with:  $M_2 = 1.8165 \cdot 10^{-6}$ .
- The robust measure  $M_3$  is bigger for the second permutation, where best case is case 4.2.1 with:  $M_3 = 2.6416 \cdot 10^{-4}$ .

### II- Two Blocks

#### II.1- Feedback Gain Magnitude

It is noticed that the magnitude is smaller for the second permutation, where the best case is case 5.2.2 with:  $n_1 = 627.37613$ ,  $n_2 = 520.4054$ ,  $n_\infty = 669.1315$ .

#### II.2- Overall & Individual Sensitivity

- The overall sensitivity is smaller for the second permutation, where the best is case 3.2.2 with:  $S(V) = 6.0645 \cdot 10^5$ .
- The poles have smaller individual sensitivity for the second permutation, where the best case is case 1.2.2 with:  $\{S_1 = 24.0858, S_2 = 67.2117, S_{3,4} = 29.8121, S_5 = 105.2778, S_6 = 2.5765 \cdot 10^5, S_7 = 2.5765 \cdot 10^5\}$ .

### II.3- Robust Stability

- The robust measure  $M_2$  is bigger for the second permutation, where the best case is case 3.2.2 with:  $M_2 = 3.295 * 10^{-7}$ .
- The robust measure  $M_3$  is bigger for the second permutation, where the best case is cases 1/2/3/4/5/7/8/9/11/12 .2.2 with:  $M_3 = 7.8314 * 10^{-5}$ .

### 5.3- The Effect of Distribution: "Dominant/Non-dominant" & "Random"

#### I- First Permutation

##### I.1- Feedback Gain Magnitude

It is noticed that from case 1 to 3: the magnitude is smaller for dominant/non-dominant distribution. Where from case 4 to 12: the magnitude is smaller for random distribution.

The best case is case 12.1.3 with:  $n_1 = 1.1215 * 10^3$ ,  $n_2 = 1.4541 * 10^3$ ,  $n_\infty = 2.1946 * 10^3$ .

##### I.2- Overall & Individual Sensitivity

- The overall sensitivity is smaller for random distribution, where the best is case 6.1.3 with:  $S(V) = 2.6805 * 10^6$ .
- The dominant poles have a smaller individual sensitivity for dominant/non-dominant distribution, except for cases: 6,10,11,12 and the non-dominant poles have a bigger individual sensitivity for dominant/non-dominant distribution. The best case is case 4.1.2 with:  $\{S_1 = 6.84612, S_2 = 702.4089, S_{3,4} = 453.6747, S_5 = 2.0224 * 10^3, S_6 = 5.9614 * 10^6, S_7 = 5.9062 * 10^6\}$ .

##### I.3- Robust Stability

- The robust measure  $M_2$  is bigger for random distribution, except for case 4, and the best case is case 12.1.3 with:  $M_2 = 76.2856 * 10^{-8}$ .
- The robust measure  $M_3$  is bigger for random distribution, where best case is case 6.1.3 with:  $M_3 = 2.3526 * 10^{-5}$ .



## II- Second Permutation

### II.1- Feedback Gain Magnitude

It is noticed that the magnitude is smaller for the dominant/non-dominant distribution, except for the cases 10, 11, 12 where the best case is 12.1.3 with:  $n_1 = 276.4135$ ,  $n_2 = 390.5385$ ,  $n_\infty = 593.5337$ .

### II.2- Overall & Individual Sensitivity

- The overall sensitivity is smaller for dominant/non-dominant distribution, where the best is case 3.2.2 with:  $S(V) = 4.2168 \cdot 10^6$ .

The poles (-20.2 and the complex pair) have smaller individual sensitivity for dominant/non-dominant distribution, except for the cases: 6, 10, 11, 12. The other poles have smaller individual sensitivity for random distribution, where the best case is case 1.2.3 with:  $\{S_1 = 12.5761, S_2 = 48.7209, S_{3,4} = 99.6684, S_5 = 18.09, S_6 = 1.385 \cdot 10^6, S_7 = 1.385 \cdot 10^6\}$ .

### II.3- Robust Stability

- The robust measure  $M_2$  is bigger for dominant/non-dominant distribution, where the best case is case 3.2.2 with:  $M_2 = 3.295 \cdot 10^{-7}$ .
- The robust measure  $M_3$  is bigger for dominant/non-dominant distribution, except for cases: 6/10/11/12 .2.3 with:  $M_3 = 8.3058 \cdot 10^{-5}$ .

## 6-The Effect of the Similarity Transformation $T_c$

### 6.1- Norm of $T_c$

As we can notice from the previous section, the similarity transformation of the second permutation gives the smallest gain magnitudes compared to the results gotten by the similarity transformation of the first permutation. We can explain this by the relationship that exists between  $T_c$  and  $K$ , which is expressed by the equation:  $K = K_c \cdot T_c$ . This implies that:  $\|K\| = \|K_c\| \|T_c\|$ .

To confirm that, the norm of  $T_c$  has been calculated:

- The first permutation gives a  $T_c$  that has a norm-2 equal to:  $\|T_c\|_2 = 27.3032$ .
- The second permutation gives a  $T_c$  that has a norm-2 equal to:  $\|T_c\|_2 = 7.6419$ .

**6.2- Condition number of  $T_c$** 

To see the effect more and more we introduce the condition number of the similarity transformation  $T_c$ , which has an effect on the sensitivity. We have noticed that the similarity transformation of the second permutation gives the smallest individual and overall sensitivities compared to the results gotten by the similarity transformation of the first permutation.

**PS:** With the similarity transformation of the second permutation, we notice that the individual sensitivity of the pole “-1” is enhanced.

- The first permutation gives a  $T_c$  that has a condition number equal to:  
 $\chi(T_c)=1.1196 *10^4$ .
- The second permutation gives a  $T_c$  that has a condition number equal to:  
 $\chi(T_c)=2.6679 *10^3$ .

As said previously in chapter 2, the smaller the condition number the better or the well-conditioned. Hence ones again the second permutation gives better system performance.

# Conclusion

The main objective of this work was to see the effect of the similarity transformation on the system performance in order to select the best control design that satisfies the requirements, in the case of airplane “F-16 fighter”.

Different cases has been simulated and evaluated in terms of performance stability, robustness measures and gain magnitudes. This was done in three different ways considering the number of blocks, permutation and the distribution of the closed-loop poles.

During the work, the general controller canonical form was used since it is simple and easy for use.

Throughout the simulation, we noticed that the second permutation with two blocks gives the best results. However, there was a tradeoff between the feedback gain magnitude & overall sensitivity and individual sensitivity. Since they are inherently different sets of requirements, improvement of one of them must inevitably result in a degradation of the other. For the two blocks “dominant/non-dominant distribution”, the smallest gain magnitudes and overall sensitivity were obtained. Whereas, for the two blocks “random distribution”, the smallest individual sensitivities were introduced; in addition to that, the individual sensitivity of the poles “-1” was decreased so the performance of the system was enhanced.

As a further work, we may suggest to do the control design using the second permutation with two blocks. And also, we may suggest calculating the time domain criteria for more concrete results.

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# Appendix A

- **Simulation code:**

In order to facilitate our work, the following MATLAB program was used to proceed with the simulation.

```
program.m x +
1 - a = [ -0.3220  0.0640  0.0364  -0.9917  0.0003  0.0008  0
2         0  0  1 -0.0037  0  0  0
3        -30.6492  0 -3.6784  0.6646 -0.7333  0.1315  0
4         8.5396  0 -0.0254 -0.4764 -0.0319 -0.0620  0
5         0  0  0  0  0 -20.2  0
6         0  0  0  0  0  0 -20.2  0
7         0  0  0  0  57.2958  0  0 -1 ];
8
9 - b = [ 0  0
10        0  0
11        0  0
12        0  0
13       20.2  0
14        0  20.2
15        0  0 ];
16
17 - b1= b(:,1);
18 - b2= b(:,2);
19 - R= [b1 a*b1 a^2*b1 a^3*b1 b2 a*b2 a^2*b2];
20 - r= inv (R);
21 - p1= r(4,:);
22 - p2= r(7,:);
23 - Tc=[p1; p1*a; p1*a^2; p1*a^3; p2; p2*a; p2*a^2];
24 - tc= inv (Tc);
25 - Ac= Tc*a*tc;
26 - e= eig (Ac);
27 - Bc= Tc*b;
28
29 - Drhinv= [1 -3.462; 0 1];
30 - Drh= inv(Drhinv);
31
32 - Kcbar = [ ];
33 - Kc= Drh*Kcbar;
34 - K=Kc*Tc;
35 - g = a- b*K;
36 - e1= eig (g);
37
38 - n1=norm(K,1);
39 - n2=norm(K,2);
40 - ninf=norm(K,inf);
41
```

```

41
42 - [V,D,W]=eig(g);
43 - [d,ind] = sort(diag(D));
44 - Ds = D(ind,ind);
45 - Vs = V(:,ind);
46 - Ws = W(:,ind);
47 - vinv=inv(Vs);
48 - s=(norm(Vs,2))*(norm(vinv,2));
49 - D1=d(1,:);
50 - W1=Ws(:,1);
51 - V1=Vs(:,1);
52 - X1=(W1')*V1;
53 - s1=((norm(W1,2))*(norm(V1,2)))/abs(X1);
54 - m31=(abs(real(D1)))/s1;
55 - D2=d(2,:);
56 - W2= Ws(:,2);
57 - V2= Vs(:,2);
58 - X2=(W2')*V2;
59 - s2=((norm(W2,2))*(norm(V2,2)))/abs(X2);
60 - m32=(abs(real(D2)))/s2;
61 - D3=d(3,:);
62 - W3= Ws(:,3);
63 - V3= Vs(:,3);
64 - X3=(W3')*V3;
65 - s3=((norm(W3,2))*(norm(V3,2)))/abs(X3);
66 - m33=(abs(real(D3)))/s3;
67 - D4=d(4,:);
68 - W4= Ws(:,4);
69 - V4= Vs(:,4);
70 - X4=(W4')*V4;
71 - s4=((norm(W4,2))*(norm(V4,2)))/abs(X4);
72 - m34=(abs(real(D4)))/s4;
73 - D5=d(5,:);
74 - W5= Ws(:,5);
75 - V5= Vs(:,5);
76 - X5=(W5')*V5;
77 - s5=((norm(W5,2))*(norm(V5,2)))/abs(X5);
78 - m35=(abs(real(D5)))/s5;
79 - D6=d(6,:);
80 - W6= Ws(:,6);
81 - V6= Vs(:,6);
82 - X6=(W6')*V6;
83 - s6=((norm(W6,2))*(norm(V6,2)))/abs(X6);
84 - m36=(abs(real(D6)))/s6;
85 - D7=d(7,:);
86 - W7= Ws(:,7);
87 - V7= Vs(:,7);
88 - X7=(W7')*V7;
89 - s7=((norm(W7,2))*(norm(V7,2)))/abs(X7);
90 - m37=(abs(real(D7)))/s7;
91
92 - m2=(abs(real(D1)))/s;
93
94 - M= [m31; m32; m33; m34; m35; m36; m37];
95 - m3= min (M);
96

```

- This code was used while working with the first permutation  $[b_1, b_2]$ .
- In order to use the program with the second permutation  $[b_2, b_1]$ , we have done some modification presented as follows:

```
17 - b1= b(:,1);
18 - b2= b(:,2);
19 - R= [b1 a*b1 a^2*b1 b2 a*b2 a^2*b2 a^3*b2];
20 - r= inv (R);
21 - p1= r(3,:);
22 - p2= r(7,:);
23 - Tc=[p1; p1*a; p1*a^2; p2; p2*a; p2*a^2; p2*a^3];
24 - tc= inv (Tc);
25 - Ac= Tc*a*tc;
26 - e= eig (Ac);
27 - Bc= Tc*b;
28
29 - Drhinv= [1 -3.462; 0 1];
30 - Drh= inv(Drhinv);
```