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Title:

Modelling, Simulation and Control of Quadruple Tank Process

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ABSTRACT

In control engineering, a single-input and single-output (SISO) system is considered as a simple system. However, numerous systems are not such uncomplicated, they have various data sources and returns. These systems are called multi-input and multi-output (MIMO) systems.

Basic MIMO frameworks have some challenges since they are enormous and complex. Moreover, they have nonlinearities and loop interactions between their inputs and outputs.

On the motivation behind considering multivariable systems, the quadruple tank process (QTP) is selected as a benchmark. This process is suitable for examining linear and nonlinear controllers and exhibits minimum and non-minimum system behaviour by essentially changing the setup of valve position.

The PID and fuzzy Logic controllers of the nonlinear process (QTP) model is simulated in MATLAB/Simulink. The obtained results from simulation of the two controllers as well as system dynamic performances are discussed.

Keywords: SISO system, MIMO system, Loop interactions, QTP, PID Control, Fuzzy Logic Control.

DEDICATION

This effort is dedicated to our parents who have taught us the most important lessons of life and gave us the thirst for new knowledge. We would like to thank all our friends and close people, those who answer the call in the middle of the day or the night, from near and from far, those who answer the call for help with no expectation of personal gain.

B.Mohammed/O.Massil

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ACRONYMS

SISO: Single Input Single Output.

MIMO: Multi Input Multi Output.

QTP: Quadruple Tank Process.

PID: Proportional-Integral-Derivative.

PLC: Programmable Logic Controller.

SCADA: Supervisory Control and Data Acquisition.

General Introduction

Multivariable process includes more than one control loop, these loops interact with each other in such a manner that single input not only affects its own output but also affects other process outputs [1, 2].

The quadruple system control is an important process for tough control techniques and focuses on the development of practical, robust and flexible system for a further experimental issue, parameters and an unlimited model change.

The quadruple tank process is a two inputs, two outputs system. The two inputs are the voltages to the water pumps, and the outputs are the two water levels of the lower tanks. This was introduced by Johansson (2000) as a teaching laboratory suitable for teaching multivariable control [3].

Due to various reasons, the quadruple tank process can be regarded as a prototype for many MIMO control applications in industry such as paper production processes, chemical processes, metallurgy and biotechnological areas, and medical industries.

This project deals with the mathematical modelling of the quadruple tank process by linearization principles and Jacobian matrix formation to represent the system in state space model. PID control and fuzzy logic control are then introduced to control the system. The simulations are conducted in MATLAB/Simulink and results are discussed. Finally, it finishes by a general conclusion.

Chapter I:

Principle Definitions & Controllers Overview

1.1. Introduction

This chapter introduces some principle definitions about control systems in general then it gives an overview about PID control. Finally, it goes through fuzzy logic control.

1.2. Principle Definitions

Dynamic process: A system that depends on both the input applied to the process and the current state of the process.

Nonlinear model: A mathematical model of a system which equations can't be represented as a polynomial equation of the form:

$$f(x) = \sum_{i=1}^{n} m_i x_i \tag{1.1}$$

State variable: the smallest number of variables that can represent a whole dynamic system at any time. The state variables have to be linearly independent, and the minimal number of them is the order of the differential equation that represents the system.

Linearization: A technique applied to approximate a non-linear model into a linear one. There are 2 techniques used mainly: linearization around and equilibrium point and manipulation of a linear model to control a process. Actually, the majority of observation and control techniques are based on linear systems.

Equilibrium points: Particular values of the process state used to linearize a model with the technique of the linearization around equilibrium point. An equilibrium point x_0 is characterized by the following expression:

$$\mathrm{df(x)/dx}|_{\mathrm{x=x0}} = 0 \tag{1.2}$$

Linearization around equilibrium point: A technique used to linearize a model, and valid with small variations in the process state variables around that equilibrium point. This technique is based on the Taylor series expansion:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x0)}{n!} (\mathrm{x} - \mathrm{x0})^{n} \tag{1.3}$$

Where x_0 is the chosen equilibrium point. As the equilibrium point equation is an infinite expression, to approximate the model to a linear one the first two terms are used.

Continuous time model: A model of a process that describes system dynamics in the continuous time, i.e., with infinitesimal time variations to consider the evolution of the system.

State space representation: A matrix representation of a linear model that defines the evolution of all the state variables function of the own state variables and the input. With a continuous model:

$$dx/dt = A_c x(t) + B_c U(t)$$

$$y(t) = C_c x(t) + D_c U(t) \tag{1.4}$$

Transfer function matrix representation: A technique to represent a process linear model based just in the relation between the inputs and the outputs. To apply it, in continuous time the Laplace transform is used, which discrete equivalent is the Z transform obtaining the following representations. In continuous time:

$$\frac{Y(s)}{U(s)} = G(s) \tag{1.5}$$

Stable system: A system which response with bounded inputs is bounded. On the contrary, an unstable system is characterized by unbounded responses to bounded inputs [4].

1.3. PID Control Overview

The PID controller is the most common form of feedback. It was an essential element of early governors and it became the standard tool when process control emerged in the 1940s. In process control today, more than 95% of the control loops are of PID type, most loops are actually PI control. PID controllers are today found in all areas where control is used [5].

The popularity of PID controller is due to its simplicity which uses only three, parameters. Proportional (Kp) term which controls the plant (system) proportional to the input error. Integral (Ti) term which provides the change in the control input proportional to the integral of the error signal and the last one is the derivative (Td) term that controls the system by providing control signal proportional to the derivative of the error signal.

Derivative action is used in some cases to speed up the response time and to stabilize the system behaviour [6]. The standard PID control configuration is as shown in **Fig 1.1**.

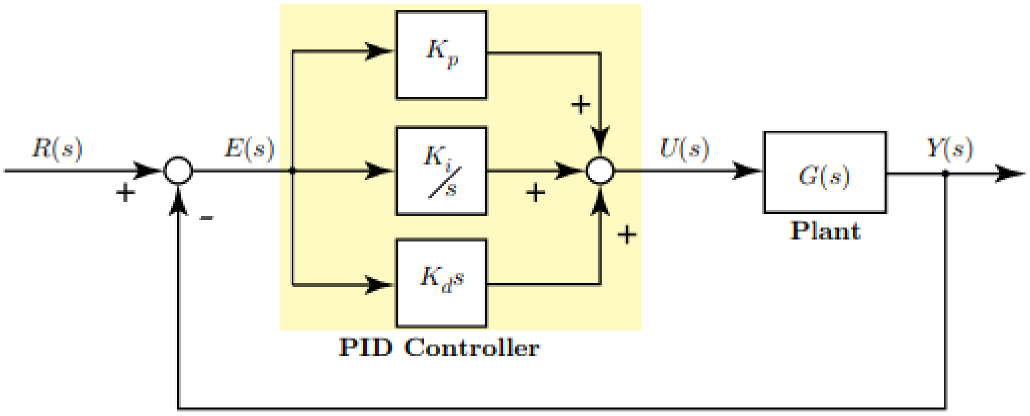


Figure 1.1: *PID Control Configuration.*

The Transfer function of the PID controller is given as follows:

$$C(s) = \frac{U(s)}{E(s)} = Kp + \frac{Ki}{s} + Kd.s \qquad (1.6)$$

Where U is the controller output (The voltage V), and E is the control error which is defined as:

$$error = u - y \qquad (1.7)$$

PID tuning comprises the selection of best values of Kp, Ti and Td of the PID controller so that the system performance can be increased.

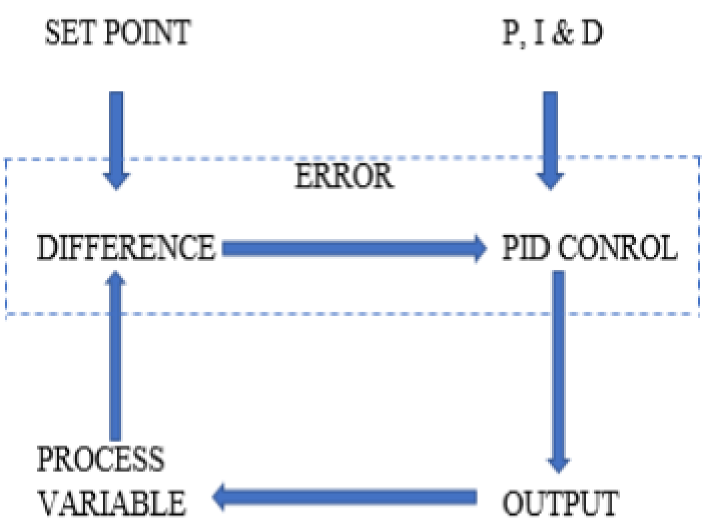


Figure 1.2: *Basic Control Loop.*

1.3.1. Role of a proportional controller

The role of a proportional depends on the present error, I on the accumulation of past error and D on prediction of future error. The weighted sum of these three actions is used to adjust Proportional control is a simple and widely used method of control for many kinds of systems. In a proportional controller, steady state error tends to depend inversely upon the proportional gain (ie: if the gain is made larger the error goes down). The proportional response can be adjusted by multiplying the error by a constant Kp, called the proportional gain. The proportional term is given by:

$$P = Kp.error(t) \tag{1.8}$$

A high proportional gain results in a large change in the output for a given change in the error. If the proportional gain is very high, the system can become unstable. In contrast, a small gain results in a small output response to a large input error. If the proportional gain is very low, the control action may be too small when responding to system disturbances. Consequently, a proportional controller (Kp) will have the effect of reducing the rise time and will reduce, but never eliminate, the steady-state error [7].

1.3.2. Role of an Integral controller

An Integral controller (IC) is proportional to both the magnitude of the error and the duration of the error. The integral in in a PID controller is the sum of the instantaneous error over time and gives the accumulated offset that should have been corrected previously. Consequently, an integral control (Ki) will have the effect of eliminating the steady-state error, but it may make the transient response worse [7]. The integral term is given by:

$$I = Ki\int_0^t error(t)dt \tag{1.9}$$

1.3.3. Role of a Derivative Controller

The derivative of the process error is calculated by determining the slope of the error over time and multiplying this rate of change by the derivative gain Kd. The derivative term slows the rate of change of the controller output. A derivative control (Kd) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response [7]. The derivative term is given by:

$$D = Kd.\frac{derror(t)}{dt} \tag{1.10}$$

Effects of each of controllers Kp, Kd, and Ki on a closed-loop system are summarized in the table shown below in T**able 1.1** [7].

Table 1.1: *Effect of PID parameters on Closed Loop Response.*

Parameter	Rise time	Overshoot	Settling time	Steady state error
Kp	Decrease	Increase	Small change	Decrease
Ki	Decrease	Increase	Increase	Decrease significantly
Kd	Minor decrease	Minor decrease	Minor decrease	No effect in theory

1.4. Fuzzy Logic Control

In everyday language, everyone uses a great deal of vagueness and imprecision, that can also be called fuzziness. People are concerned with how they can represent and manipulate inferences with this kind of information. Some examples are: a person's size is *tall*, and their age is classified as *young*. Terms such as *tall* and *young* are fuzzy because they cannot be crisply defined, although as humans they use this information to make decisions.

When someone wants to classify a person as tall or young it is impossible to decide if the person is in a set or not. By giving a degree of pertinence to the subset, no information is lost when the classification is made [8].

1.4.1. Linguistic Variables and Terms

Linguistic variables represent, in words, the input variables and output variables of the system to be controlled. Linguistic variables usually have an odd number of linguistic terms, with a middle linguistic term and symmetric linguistic terms at each extreme. Each linguistic variable has a range of expected values. The linguistic variables current temperature and desired temperature each might include the linguistic terms cold, moderate, and hot. The linguistic variable heater setting might include the linguistic terms off, low, and high [8].

1.4.2. Membership Function

Membership functions are numerical functions corresponding to linguistic terms. A membership function represents the degree of membership of linguistic variables within their linguistic terms. The degree of membership is continuous between 0 and 1, where 0 is equal to 0% membership and 1 is equal to 100% membership. There are several types of membership functions available, namely, Λ-type (triangular shape), Π-type (trapezoidal shape), singleton-type (vertical line shape), Sigmoid-type (wave shape), and Gaussian-type (bell shape) membership functions [8]. The various membership functions are shown in the **Fig 1.3**.

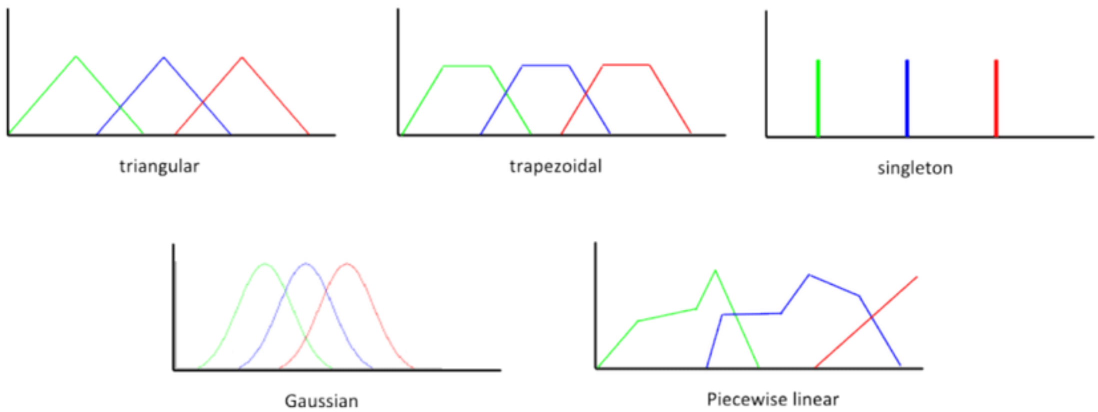


Figure 1.3: *Membership Functions for Fuzzy Controller*

1.4.3. Notation of Linguistic Rule

The principal idea of fuzzy logic systems is to express the human knowledge in the form of linguistic if-then rules. Every rule has two parts:

• Antecedent part (premise), expressed by if...

• Consequent part, expressed by: then...

The antecedent part is the description of the state of the system, and the consequent is the action that the operator which controls the system must take. There are several forms of if-then rules. The general is:

If (a set of conditions is satisfied) ***then*** (a set of consequences can be inferred).

Zadeh was the first who introduced a notion of fuzzy rule in the form:

Example: ***If*** the temperature is high, ***then*** the pressure is small.

The general form of this rule is:

Rule: ***If*** x is A, ***then*** y is B.

Temperature (x) and pressure (y) are linguistic variables. x represents the state of the system, and y is control variable and represents the action of the operator. High (A) and small (B) are linguistic values or labels characterised by appropriate membership functions of fuzzy sets. They are defined in the universe of discourse of the linguistic variables x and y [9].

1.4.4. General Structure of Fuzzy System

Every fuzzy system is composed of four principal blocks as shown in **Fig 1.4**:

1. K**nowledge base** (rules and parameters for membership functions).

2. **Decision unit** (inference operations on the rules).

3. **Fuzzification interface** (transformation of the crisp inputs into degrees of match with linguistic variables).

4. **defuzzification interface** (transformation of the fuzzy result of the inference into a crisp output) [9].

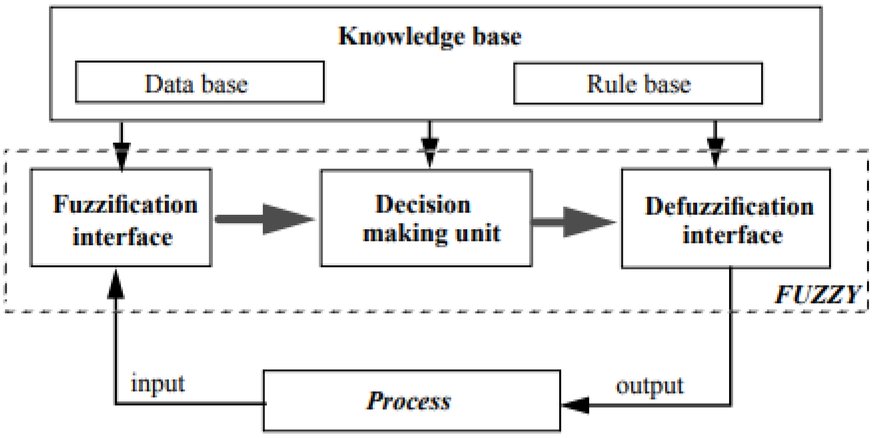


Figure 1.4: *General Structure of Fuzzy Inference System.*

1.4.5. Procedure of Fuzzy Reasoning

The steps in fuzzy reasoning are:

1. Fuzzification: to every measure of input variable is attributed the degree of membership (membership value) for all the fuzzy sets defined in the universe of discourse.

2. Application of the t - norm (usually, this operator is min or product) on the membership values of the premise part of the rules to get firing strength or the weight for each rule

3. Generation of the consequent value of each rule.

4. Defuzzification: generate the crisp output values.

The first step in the application of fuzzy reasoning is a fuzzification of inputs in the controller. It means that to every crisp value of input we attribute a set of degrees of membership to fuzzy sets defined in the universe of discourse for that input. Next step is the application of the linguistic rules. A fuzzy controller consists of a set of control rules which are combined using the sentence connectives. Suppose that fuzzy system has two inputs x, y and one output z, and that we defined n linguistic rules as follows:

If x = A1 and y = B1 then z = C1

If x = A2 and y = B2 then z = C2

...

If x = An and y = Bn then z = Cn

Where x, y and z are linguistic variables representing the process state variables and the control variable; Ai, Bi and Ci (i=1,n) are fuzzy sets defined in the universes of discourse for x, y and z respectively. In mathematical sense, activation of the rules is the application of t-norms in order to get a firing strength for every rule. Usually, it means that we apply the operator min or product on membership values.

After, the firing strengths are combined using the compositional operator which expresses the sentence connective and the consequent value (crisp or fuzzy) is generated. At last, the defuzzification is performed in order to get crisp output. The scheme of fuzzy part of the system from **Fig 1.4** is represented on **Fig 1.5** [9].

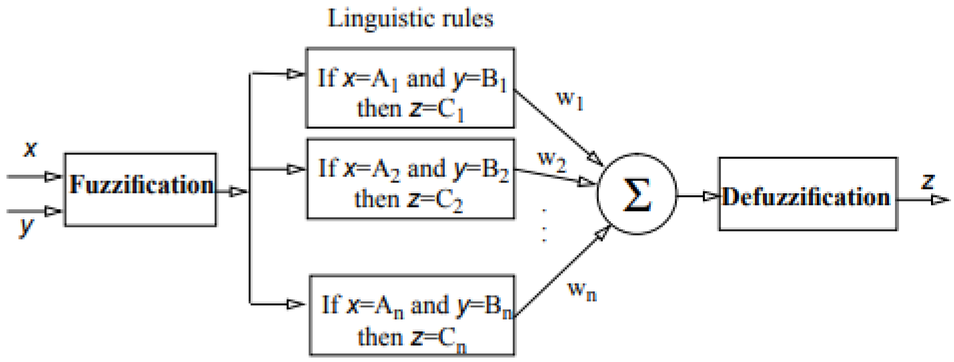


Figure 1.5: *General Structure of Fuzzy Parts of the System.*

1.4.6. Implementation of Fuzzy Logic

Fuzzy system consists of three main parts: linguistic variables, membership functions and rules. The basic steps in designing fuzzy logic control is as follows:

- Identifying the input and output variables.
- Partitioning the interval of each input and output into number of fuzzy subsets, assigning each a linguistic label.
- Determining a membership function for each fuzzy subset.
- Assigning the fuzzy relationship between the "input fuzzy subsets" on one hand and the "output fuzzy subsets" on the other hand, thus forming the Rule Base.
- Interpreting the rules using fuzzy "AND" and "OR», operators. In fuzzy systems, more than one rule may fire at the same time, but with varied strengths.
- Translating the processed fuzzy data into the crisp data suitable for real time applications [8].

1.5. Conclusion:

In this chapter, principle definitions concerning control systems are given. Besides, an overview of PID control is introduced. At the end of this chapter, the fuzzy logic control is explained.

In the next chapter, a quadruple tank system description will be given and its mathematical model will be developed.

Chapter II:

System's Description & Modelling

2.1. Introduction

This chapter gives the quadruple tank process description. Besides, it shows the development of the non-linear mathematical model of the system then it gives the linearized model of QTP. Finally, it shows the zeros' location of the system.

2.2. Quadruple Tank Process Description

The project is focused on the quadruple tanks apparatus. This was introduced by Johansson as a teaching laboratory suitable for teaching multivariable control. Four tanks are arranged as in **Fig 2.1**.

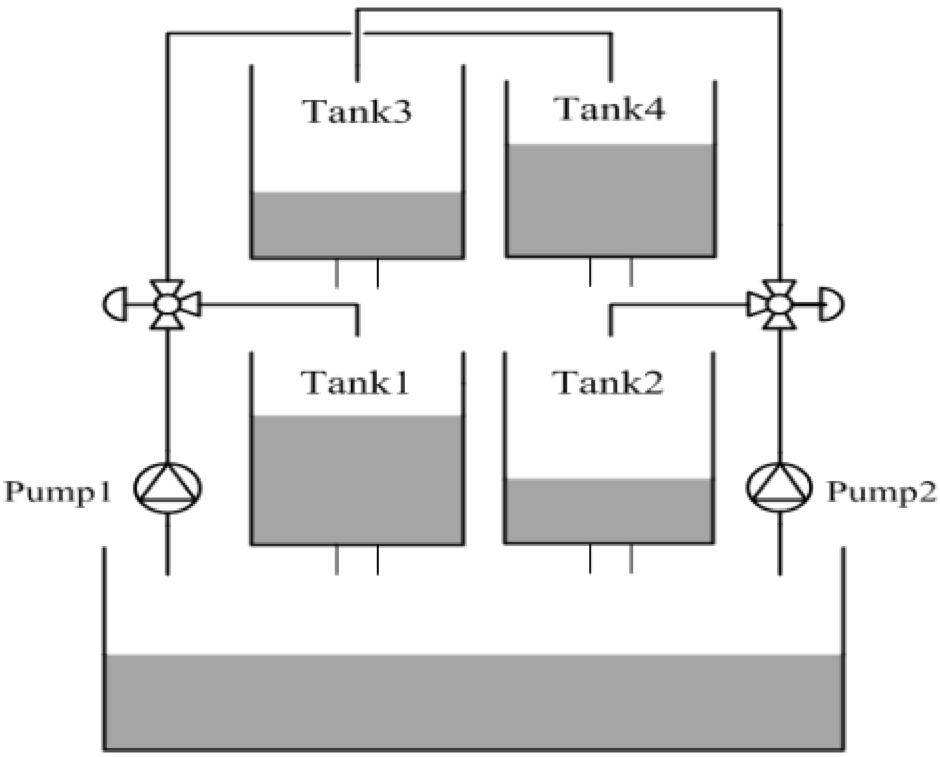


Figure 2.1: *Quadruple Tank Process.*

The quadruple tank process is a two input, two output system. The two inputs are the voltages to the water pumps, and the outputs are the two water levels of the lower tanks. In addition, there are valves between the pumps and the tanks. By adjusting these valves, the proportions of flow going to the upper and lower tanks are changed.

Each tank output goes to two tanks, one is the lower tank and the other is the upper diagonal tank. Pump 1 is shared by tank 1 and tank 4 while pump2 is shared by tank 2 and tank 3. Each tank has a discharge outlet at the bottom. The discharge from tank 3 goes to tank 1 while the discharge from tank 4 goes to tank 2. This interaction creates a strong coupling between the tanks which makes it a multivariable control system. The discharges from tank 1 and tank 2 go to the reservoir at the bottom.

2.3. Mathematical Model

As it is mentioned in section 1, the process inputs are the input voltages and the outputs are the lower tanks water levels. For each tank, the model is obtained by using Bernoulli's law and mass balance law. Tank numbers are represented by "i", which may be 1,2,3,4.

2.3.1. The Non-linear Model

The equivalent mathematical model of the process is given by Bernoulli's law and mass balance law as follow: [10]

Rate of accumulation = (Rate of in-flow) – (Rate of out-flow)

$$\frac{dVi}{dt} = Q_{in} - Q_{out}$$

$$A_i \frac{dhi}{dt} = Q_{in} - Q_{out} \tag{2.1}$$

Where:

Vi: Volume of the tank.

Ai: Cross sectional area of the tank.

hi: Water level of the tank.

Q_{in}: In-flow of the tank.

Q_{out}: Out-flow of the tank.

2.3.1.1. Model of the Output Pipe

Based on Bernoulli's law, the total energy at the beginning of the pipe equals to the total energy at the output of the pipe. We suppose that water is stationary at the input of the pipe (No kinetic energy), the only energy that remains is pressure energy. At the output of the pipe there is kinetic energy due to the flow of water through the pipe. The potential energy is negligible due to the small height of the pipe (we suppose that the water moves through the pipe at the same height).

$$Q_{out\_i} = a_i v_o(t) \tag{2.2}$$

Applying Bernoulli's law:

$$P_{energy} = K_{energy}$$

$$P_{energy} = PV_i \tag{2.3}$$

$$K_{energy} = \frac{1}{2} m_i v^2{}_o(t) \tag{2.4}$$

Hence:

$$PV_i = \frac{1}{2} m_i * v^2{}_o(t)$$

$$P = \rho g h(t) \tag{2.5}$$

$$m_i = \rho V_i \tag{2.6}$$

Thus:

$$\rho g h(t) * V_i = \frac{1}{2} m_i v^2{}_o(t)$$

$$g h(t) = \frac{1}{2} v^2{}_o(t)$$

$$v_o(t) = \sqrt{2ghi(t)} \tag{2.7}$$

Hence:

$$Q_{out\_i} = a_i\sqrt{2ghi(t)} \tag{2.8}$$

Where:

a_i: Cross sectional area of the pipe.

$v_o(t)$: Velocity of water at the output of the pipe.

P_{energy}, K_{energy}: Pressure energy, kinetic energy.

$h_i(t)$: Water level of tank.

P: Pressure.

m_i: Mass of water.

ρ: Density of water.

g: Gravity constant.

2.3.1.2. Model of the Pumps and the Valves

The in-flow of a tank (q_{in_i}) depends on the input voltages of the pumps and the valves' positions.

$$q_{in\_1} = \gamma_1 k_1 V_1 \tag{2.9}$$

$$q_{in\_2} = \gamma_2 k_2 V_2 \tag{2.10}$$

$$q_{in\_3} = k_2 V_2 (1-\gamma_2) \tag{2.11}$$

$$q_{in\_4} = k_1 V_1 (1-\gamma_1) \tag{2.12}$$

Where:

k_1: Pump 1 constant.

k_2: Pump 2 constant.

γ_1: Ratio of valve 1 position.

γ_2: Ratio of valve 2 position.

2.3.1.3. Full Model of the Tanks

Tank 1:

Using the law of conservation of mass:

$$A_1 \frac{dh1}{dt} = q_{in\_1} + q_{out\_3} - q_{out\_1}$$

$$A_1 \frac{dh1}{dt} = \gamma_1 k_1 V_1 + a_3\sqrt{2gh3(t)} - a_1\sqrt{2gh1(t)} \tag{2.12}$$

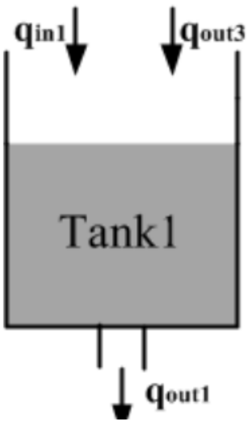


Figure 2.2: *Tank 1 Model.*

Tank 2:

$$A_2\frac{dh2}{dt} = q_{in\_2} + q_{out\_4} - q_{out\_2}$$

$$A_2\frac{dh2}{dt} = \gamma_2 k_2 V_2 + a_4\sqrt{2gh4(t)} - a_2\sqrt{2gh2(t)} \tag{2.13}$$

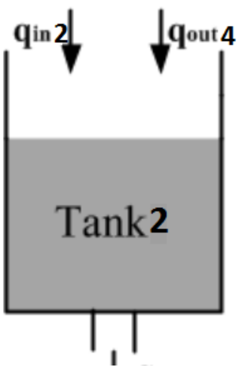


Figure 2.3: *Tank 2 Model.*

Tank 3:

$$A_3\frac{dh3}{dt} = q_{in\_3} - q_{out\_3}$$

$$A_3\frac{dh3}{dt} = k_2V_2(1\text{-}\gamma_2) - a_3\sqrt{2gh3(t)} \quad (2.14)$$

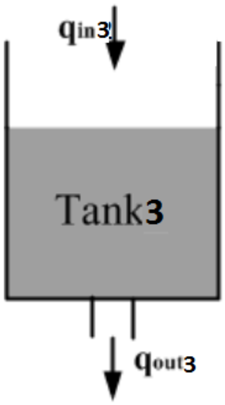


Figure 2.4: *Tank 3 Model.*

Tank 4:

$$A_4\frac{dh4}{dt} = q_{in\_4} - q_{out\_4}$$

$$A_4\frac{dh4}{dt} = k_1V_1(1\text{-}\gamma_1) - a_4\sqrt{2gh4(t)} \quad (2.15)$$

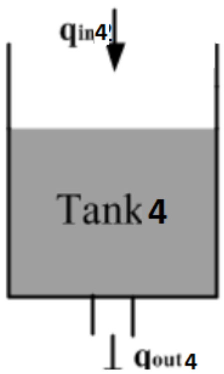


Figure 2.5: *Tank 4 Model.*

The non-linear equations of the quadruple tanks system are given as follows:

$$A_1\frac{\mathrm{dh1}}{\mathrm{dt}} = -a_1\sqrt{2gh1(\mathrm{t})} + a_3\sqrt{2gh3(\mathrm{t})} + \gamma_1 k_1 V_1$$

$$A_2\frac{\mathrm{dh2}}{\mathrm{dt}} = -a_2\sqrt{2gh2(\mathrm{t})} + a_4\sqrt{2gh4(\mathrm{t})} + \gamma_2 k_2 V_2$$

$$A_3\frac{\mathrm{dh3}}{\mathrm{dt}} = -a_3\sqrt{2gh3(\mathrm{t})} + k_2 V_2(1-\gamma_2)$$

$$A_4\frac{\mathrm{dh4}}{\mathrm{dt}} = -a_4\sqrt{2gh4(\mathrm{t})} + k_1 V_1(1-\gamma_1) \tag{2.16}$$

By considering V1 = u1(t) and V2 = u2(t) and using the state space representation, the following set of matrices is obtained:

$$\begin{bmatrix} \frac{dh1(t)}{dt} \\ \frac{dh2(t)}{dt} \\ \frac{dh3(t)}{dt} \\ \frac{dh4(t)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-a1}{A1}\sqrt{2g} & 0 & \frac{a3}{A1}\sqrt{2g} & 0 \\ 0 & \frac{-a1}{A2}\sqrt{2g} & 0 & \frac{a4}{A2}\sqrt{2g} \\ 0 & 0 & \frac{-a3}{A2}\sqrt{2g} & 0 \\ 0 & 0 & 0 & \frac{-a4}{A4}\sqrt{2g} \end{bmatrix} * \begin{bmatrix} \sqrt{h1(t)} \\ \sqrt{h2(t)} \\ \sqrt{h3(t)} \\ \sqrt{h4(t)} \end{bmatrix} + \begin{bmatrix} \frac{\gamma1\mathrm{k1}}{A1} & 0 \\ 0 & \frac{\gamma2\mathrm{k2}}{A2} \\ 0 & \frac{\mathrm{k2}(1-\gamma2)}{A3} \\ \frac{\mathrm{k1}(1-\gamma1)}{A4} & 0 \end{bmatrix} * \begin{bmatrix} u1 \\ u2 \end{bmatrix} \tag{2.17}$$

The previous set of matrices represent respectively the matrices A B C and D.

2.3.2. The linearized Model

The previous system represents a highly nonlinear system due to Bernoulli law so the system requires a linearization around the equilibrium point using either Taylor series method or Jacobian method.

The following steps represent the procedure of getting the equilibrium points, at steady state, the level of tanks remains constant, so the partial derivative of the function is found at steady state and equated to zero as follows:

$$\frac{dhi(t)}{dt} = 0 \quad \text{for i = 1,2,3,4.}$$

The following set of equations is obtained: [11]

$$\frac{-a1}{A1}\sqrt{2gh10} + \frac{a3}{A1}\sqrt{2gh30} + \frac{\gamma 1k1}{A1}u10 = 0$$

$$\frac{-a2}{A2}\sqrt{2gh20} + \frac{a4}{A2}\sqrt{2gh40} + \frac{\gamma 2k2}{A2}u20 = 0$$

$$\frac{-a3}{A3}\sqrt{2gh30} + \frac{(1-\gamma 2)k2}{A3}u20 = 0$$

$$\frac{-a4}{A4}\sqrt{2gh30} + \frac{(1-\gamma 1)k1}{A4}u10 = 0 \qquad (2.18)$$

The following set of equilibrium points is obtained after developing the set of equations **(2.18):**

$$H10 = \frac{1}{2g}[\frac{(1-\gamma 2)k2}{a1}u20 + \gamma 1k1u10]^2$$

$$H20 = \frac{1}{2g}[\frac{(1-\gamma 1)k1}{a2}u10 + \gamma 2k2u20]^2$$

$$H30 = \frac{1}{2g}[\frac{(1-\gamma 2)k2}{a3}u20]^2$$

$$H40 = \frac{1}{2g}[\frac{(1-\gamma 1)k1}{a4}u10]^2 \qquad (2.19)$$

The non-linear relationship in the equation **(2.16)** is due to the square root term present in those equations which makes the controller design difficult. To overcome the difficulty the linearization is required. The equation is solved using Taylor series followed by Jacobian matrix transformation to obtain a state space form of the QTP. After obtaining the State space model of QTP the state space to transfer function conversion is done by using a simple conversion technique [12].

The initial step is to obtain a linear approximation of the differential equations which is done by Taylor series. If the mathematical model of QTP is being integrated to obtain h1, h2, h3 and h4 it produces an infinite series of values [13]. It is common practice to approximate a function by using a finite number of terms of its Taylor series. The general form of differential equation can be represented by:

$$\frac{dx1}{dt} = f_1\,(h_1, h_2, \ldots, h_n, u_1, u_2, \ldots, u_n)$$

$$\vdots$$

$$\frac{dxn}{dt} = f_n\,(h_1, h_2, \ldots, h_n, u_1, u_2, \ldots, u_n) \tag{2.20}$$

The general vector form:

$$\dot{x} = f(x\,,u) \tag{2.21}$$

Let:

$$H_e = h_e + \Delta h \tag{2.22}$$

$$U_e = u_e + \Delta u \tag{2.23}$$

Using Taylor series yields the linear approximation:

$$\dot{x} = f(H_e\,,U_e) = f(h_e + \Delta h,\, u_e + \Delta u\,) \tag{2.24}$$

$$f(x,\,u) = f(h_e,\,u_e) + \frac{df}{dh}\,(h_e,\,u_e)\,\Delta h + \frac{df}{du}\,(h_e,\,u_e)\,\Delta u + \text{higher order terms}$$

For simplification, higher order terms are neglected. Then the Jacobian matrices are constructed as follows:

$$A = \frac{\partial f}{\partial h}\,(h_e,\,u_e) = \begin{bmatrix} \frac{\partial f1}{\partial h1} & \cdots & \frac{\partial f1}{\partial hn} \\ \vdots & \ddots & \vdots \\ \frac{\partial fn}{\partial h1} & \cdots & \frac{\partial fn}{\partial hn} \end{bmatrix} \tag{2.25}$$

$$B = \frac{\partial f}{\partial u}\,(h_e,\,u_e) = \begin{bmatrix} \frac{\partial f1}{\partial u1} & \cdots & \frac{\partial f1}{\partial un} \\ \vdots & \ddots & \vdots \\ \frac{\partial fn}{\partial u1} & \cdots & \frac{\partial fn}{\partial un} \end{bmatrix} \tag{2.26}$$

Computing the Jacobian matrices with respect to h and u yields the following results:

$$A = \begin{bmatrix} \frac{-a1\sqrt{2g}}{2A1\sqrt{h10}} & 0 & \frac{a3\sqrt{2g}}{2A1\sqrt{h30}} & 0 \\ 0 & \frac{-a2\sqrt{2g}}{2A2\sqrt{h20}} & 0 & \frac{a4\sqrt{2g}}{2A2\sqrt{h40}} \\ 0 & 0 & \frac{-a3\sqrt{2g}}{2A3\sqrt{h30}} & 0 \\ 0 & 0 & 0 & \frac{-a4\sqrt{2g}}{2A4\sqrt{h40}} \end{bmatrix} \tag{2.27}$$

$$B = \begin{bmatrix} \frac{\gamma 1k1}{A1} & 0 \\ 0 & \frac{\gamma 2k2}{A2} \\ 0 & \frac{k2(1-\gamma 2)}{A3} \\ \frac{k1(1-\gamma 1)}{A4} & 0 \end{bmatrix} \tag{2.28}$$

Hence, the system can be represented in state space form as follows:

$$\dot{x}_1 = \frac{-a1}{A1}\sqrt{\frac{g}{2h10}}x_1 + \frac{a3}{A1}\sqrt{\frac{g}{2h30}}x_3 + \frac{\gamma 1k1}{A1}u_1$$

$$\dot{x}_2 = \frac{-a2}{A2}\sqrt{\frac{g}{2h20}}x_2 + \frac{a4}{A2}\sqrt{\frac{g}{2h40}}x_4 + \frac{\gamma 2k2}{A2}u_2$$

$$\dot{x}_3 = \frac{-a3}{A3}\sqrt{\frac{g}{2h30}}x_3 + \frac{(1-\gamma 2)k2}{A3}u_2$$

$$\dot{x}_4 = \frac{-a4}{A4}\sqrt{\frac{g}{2h40}}x4 + \frac{(1-\gamma 1)k1}{A4}u_1 \tag{2.29}$$

For a simpler matrix representation, a time constants notation is given as follows: [14]

$$\frac{dx}{dt} = \begin{bmatrix} \frac{1}{T1} & 0 & \frac{A3}{A1T3} & 0 \\ 0 & \frac{1}{T2} & 0 & \frac{A4}{A2T4} \\ 0 & 0 & \frac{1}{T3} & 0 \\ 0 & 0 & 0 & \frac{1}{T4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma 1k1}{A1} & 0 \\ 0 & \frac{\gamma 2k2}{A2} \\ 0 & \frac{k2(1-\gamma 2)}{A3} \\ \frac{k1(1-\gamma 1)}{A4} & 0 \end{bmatrix} u$$

$$Y = \begin{bmatrix} Kc & 0 & 0 & 0 \\ 0 & Kc & 0 & 0 \end{bmatrix} x \tag{2.30}$$

Where:

$$Ti = \frac{Ai}{a1}\sqrt{\frac{2hi0}{g}} \qquad \text{for } i = 1, \ldots, 4$$

2.3.3. Transfer Function Matrix

Using the matrices given in the state space representation we get the following transfer function:

$$G(s) = C(sI - A)^{-1}B \tag{2.31}$$

By developing the equation (2.31), the following result is obtained:

$$G(s) = \begin{bmatrix} \frac{\gamma 1 c1}{1+sT1} & \frac{(1-\gamma 2)c1}{(1+sT3)(1+sT1)} \\ \frac{(1-\gamma 1)c2}{(1+sT4)(1+sT2)} & \frac{\gamma 2 c2}{1+sT2} \end{bmatrix} \tag{2.32}$$

Where:

$$c1 = \frac{T1K1Kc}{A1} \tag{2.33}$$

$$c2 = \frac{T2K2Kc}{A2} \tag{2.34}$$

Here the ration k1/k2 is approximately equal to 1. The parameters $\gamma 1, \gamma 2 \in (0,1)$ are determined from how the valves are set prior to an experiment [3].

The transfer function for each tank is given as follows:

$$G1(s) = \frac{\gamma 1 c1}{1+sT1} \tag{2.35}$$

$$G2(s) = \frac{\gamma 2 c2}{1+sT2} \tag{2.36}$$

$$G3(s) = \frac{(1-\gamma 1)c2}{(1+sT4)(1+sT2)} \tag{2.37}$$

$$G4(s) = \frac{(1-\gamma 2)c1}{(1+sT3)(1+sT1)} \tag{2.38}$$

2.4. Zero Location

The zeros of G(s) are the zeros of the numerator polynomial of the rational function:

$$\det G(s) = \frac{c1c2}{\gamma 1\gamma 2 \prod_{i=1}^{4}(1+sTi)} * \left[(1+sT3)(1+sT4) - \frac{(1-\gamma 1)(1-\gamma 2)}{\gamma 1\gamma 2}\right] \tag{2.39}$$

It follows that the system is non-minimum phase for:

$$0 < \gamma 1 + \gamma 2 < 1$$

And minimum phase for:

$$1 < \gamma 1 + \gamma 2 < 2$$

The multivariable zero being in the left or in the right half plane has a straightforward physical interpretation. Let qi denotes the flow through pump 1 and assume that q1 = q2. Then the sum of the flows to the upper tanks is [2- $(\gamma 1 + \gamma 2)$]q1 and the sum of the flows to the lower tanks is $(\gamma 1 + \gamma 2)$q1. Hence, the flow to the lower tanks is greater than the flow to the upper tanks if the system is minimum phase. The flow to the lower tanks is smaller than the flow to the upper tanks if the system is non-minimum phase. It is intuitively easier to control y1 with v1 and y2 with v2, if most of the flows goes directly to the lower tanks. There is thus an immediate connection between zero location and physical intuition. The control problem is particularly hard if the total flow to the upper tanks (Tanks 3 and 4) is approximately equal to the total flow going to the lower tanks (Tank 1 and 2). This corresponds to $\gamma 1 + \gamma 2 = 1$, i.e, a multivariable zero close to the origin [15].

Table2.1: *Valve Settings*

Valve Values	Process	Zero Location
$1 < \gamma 1 + \gamma 2 < 2$	Minimum phase	Zero in left hand plane
$0 < \gamma 1 + \gamma 2 < 1$	Non-minimum Phase	Zero in right hand plane
$\gamma 1 + \gamma 2 = 1$		Zero is located at the origin

2.5. Conclusion

In this chapter, the non-linear mathematical model of the quadruple tank process is developed then it is linearized using Taylor series. Besides, a transfer function matrix of the system is given. At the end of this chapter, the zero location of the system is discussed according to valves settings.

In the next chapter, the mathematical model is simulated in MATLAB/Simulink and the results are discussed.

Chapter III:

Simulations & Results

3.1. Introduction

In this chapter, the simulation of the quadruple tank process is conducted via MATLAB/Simulink. Firstly, the PID control of the non-linear model simulation is discussed. Then, the same model is simulated using fuzzy logic control. Finally, the results are discussed and compared in terms of rise time, settling time and overshoot.

3.2. PID control of QTP simulation

3.2.1. Simulink Block

A general Simulink block is shown in **Fig 3.1** with different parameters required for the system control and analysis as set point, valve constant scope, PID controller and the four tank dynamics.

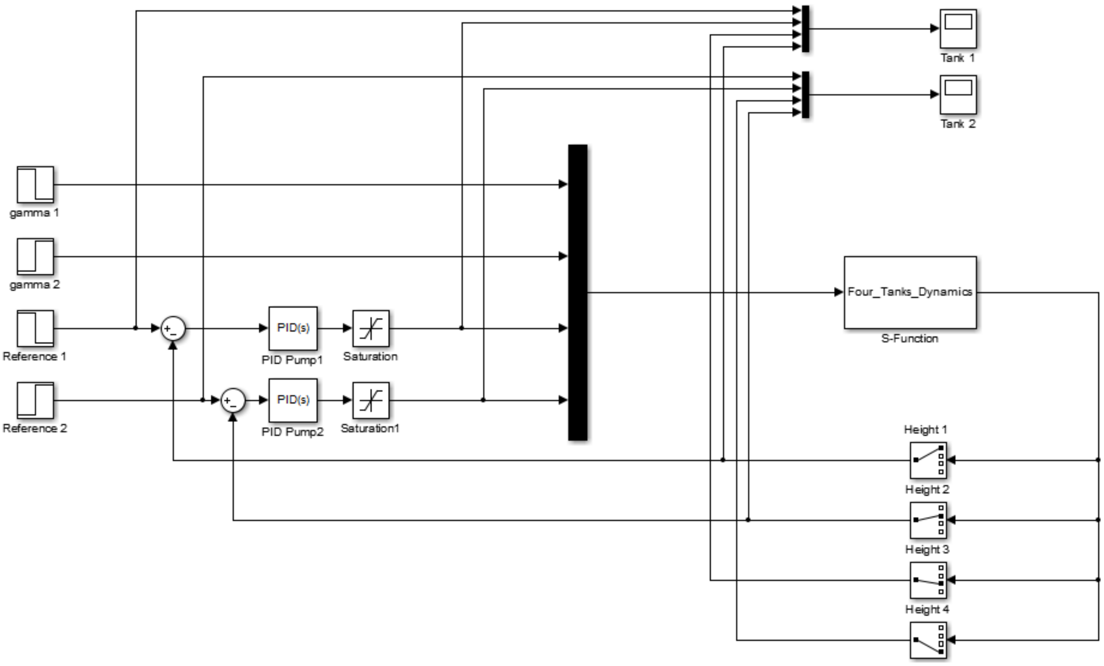
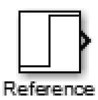
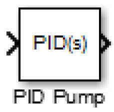


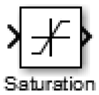
Figure 3.1: *Simulink Block Diagram for PID Control of QTP.*



It is used to specify the valves ratio and the set points of the system.



The PID block generates the signal which controls the speed of the pumps.



It is used to limit the signal generated by the PID block.

It contains the state space model of the system. It has four inputs: the two valves ratios and the PID signals, and four outputs: the four tanks height (h1, h2, h3, h4).

Using the system dynamics in Simulink requires a script file, in this case, two have been used. The 1st one is to initialize the tanks parameters used in the program. The 2nd is the system dynamics represented by equation **(2.16)**.

(See the m.files in APPENDIX A).

3.2.2. Simulation Results

The previous program is run, and simulated with different set points and valves ratios which ensure that our system is minimum phase. There are two cases:

Case 01: The simulation is started with a set point 1 of 7 cm and set point 2 of 3 cm, then it is changed to 6 cm and 4 cm respectively at time equals to 250 s and see how system reacts to these changes. Valve 1 and valve 2 ratios are set to 0.9 and 0.5 respectively..

Case 02: The simulation is started with a set point 1 of 7 cm, set point 2 of 3 cm, valve 1 ratio equals to 0.9 and valve 2 ratio equals to 0.5, then it is changed to 6 cm, 4 cm, 0.8 and 0.6 respectively, and see how the system reacts to these changes.

The specification are summarized in **Table 3.1**.

Table 3.1: *Simulation Specifications*

Cases	Case 01 0 – 250s	Case 01 250s – 500s	Case 02 0 – 250s	Case 02 250s – 500s
Set point 1	7 cm	6 cm	7 cm	6 cm
Set point 2	3 cm	4 cm	3 cm	4 cm
Valve 1 ratio	0.9	0.9	0.9	0.8
Valve 2 ratio	0.5	0.5	0.5	0.6

Simulation results are shown in **Fig 3.2**, **Fig 3.3, Fig 3.4** and **Fig 3.5**.

Case 01:

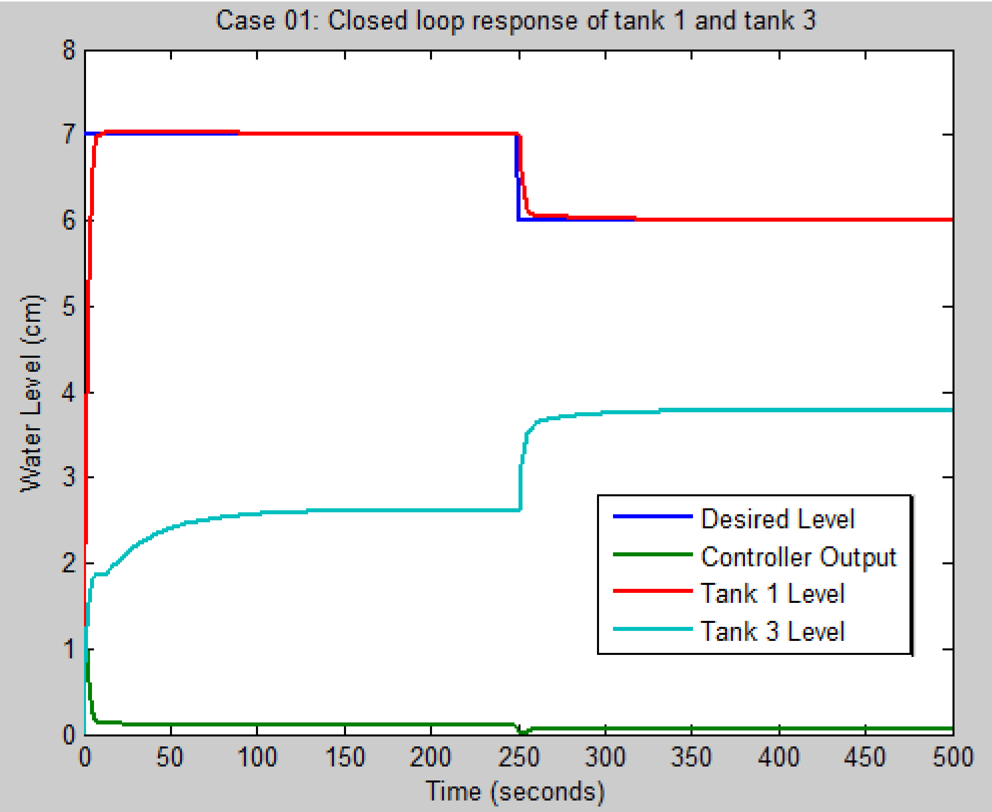


Figure 3.2: *Case 01: Closed loop response of tank 1 and tank 3*

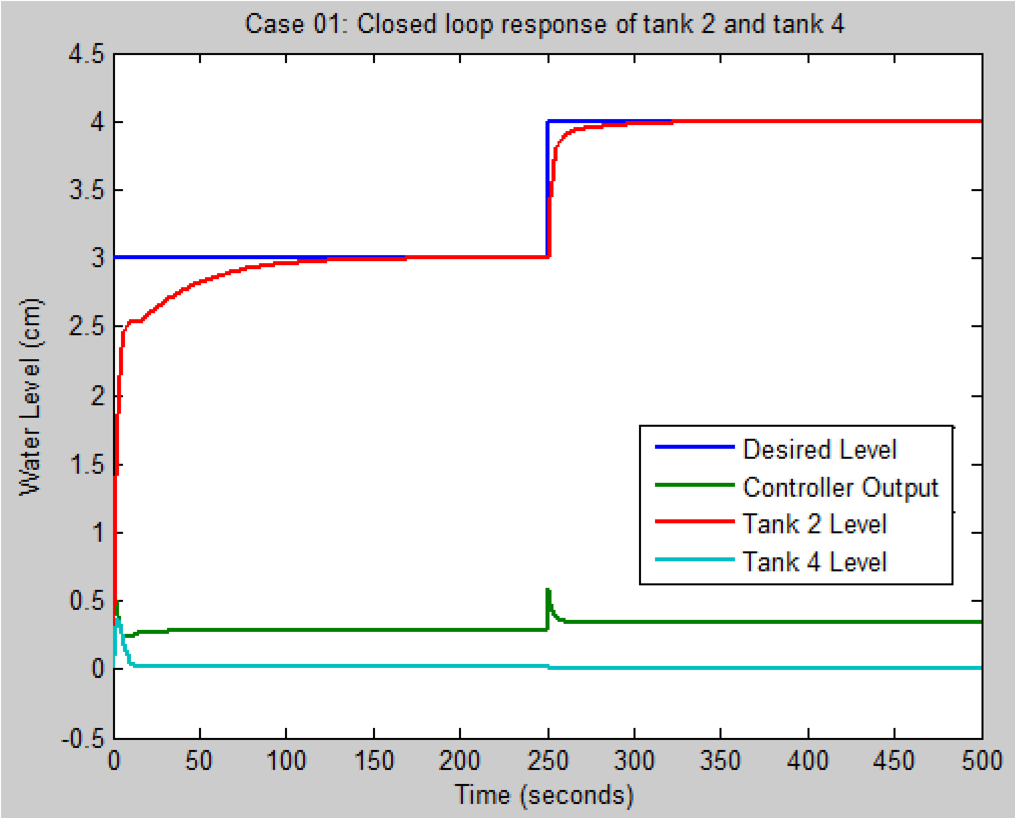


Figure 3.3: *Case 01: Closed loop response of tank 2 and tank 4*

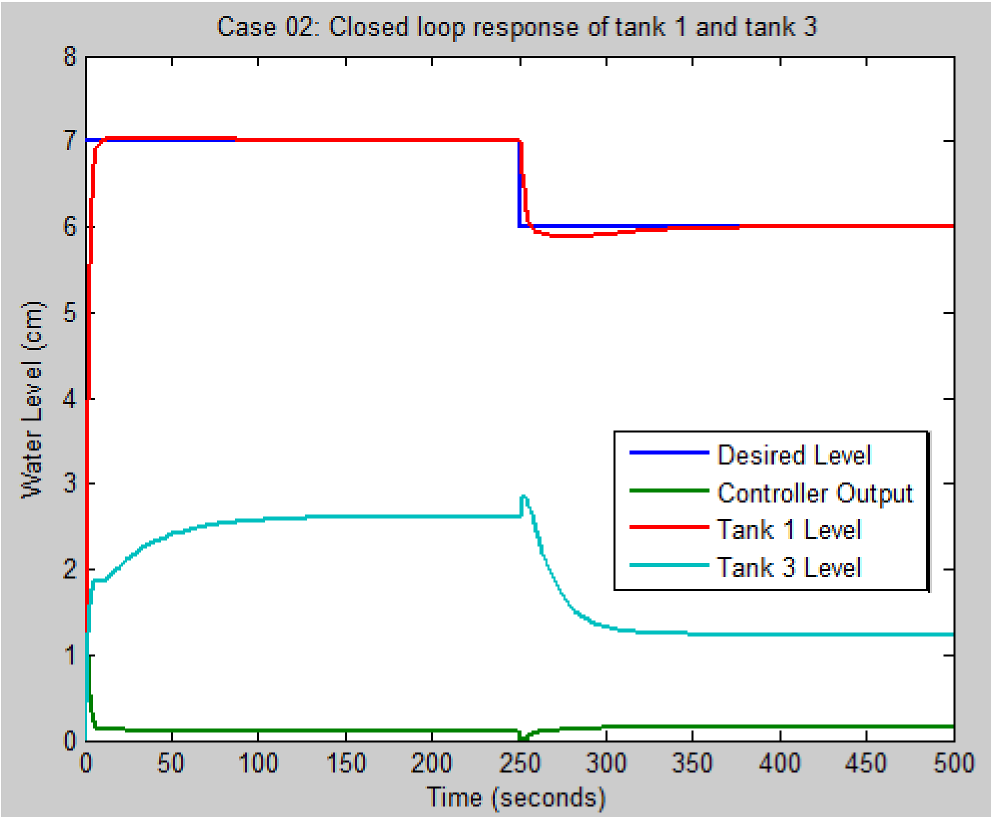


Figure 3.4: *Case 02: Closed loop response of tank 1 and tank 3*

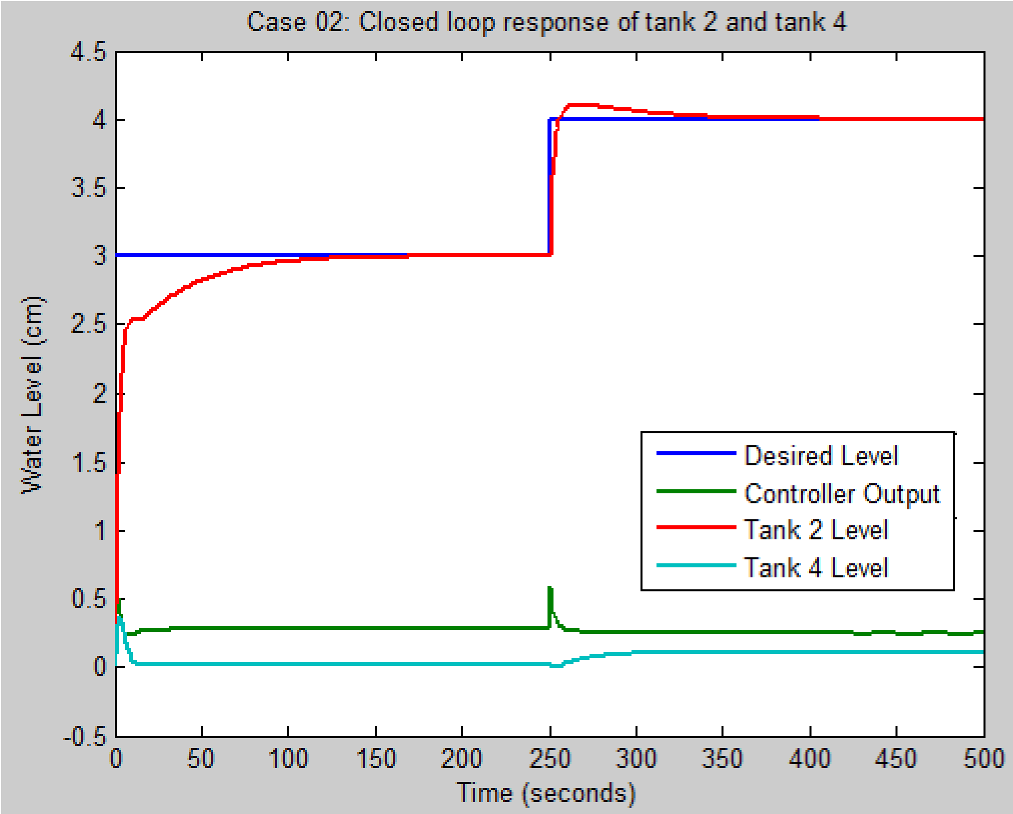


Figure 3.5: *Case 02: Closed loop response of tank 2 and tank 4*

3.2.3. Results Discussion

The results obtained from the closed loop control simulation of the quadruple tank process and performance of the system is studied for the PI controller. The Parameters of the PI controller are obtained using the help of auto tuner of MATLAB (Kp = 0.3 and Ki = 0.01).

Fig 3.2: It represents the closed loop response of tank 1 and tank 3 with valve 1 ratio = 0.9 and valve 2 ratio = 0.5. From t =0 to t = 250 s, tank 1 desired level is set to 7 cm, the rise time of the system is approximately 10 s. The output achieves the set point value at time equals to 15 s. Clearly, the output signal of the PI controller starts at a value of 1 then it decreases until it stabilizes at 0.1. From t = 250s to t = 500 s, the set point is changed to 6 cm. Clearly the output signal of the controller decreases in order to decrease the speed of the pump and that result in decreasing the water level to the desired point then it settles.

Fig 3.3: It represents the closed loop response of tank 2 and tank 4 with valve 1 ratio = 0.9 and valve 2 ratio = 0.5. From t =0 to t = 250 s, tank 1 desired level is set to 3 cm, the rise time of the system is approximately 50 s. The output achieves the set point value at time equals to 120 s. Clearly, the output signal of the PI controller starts at a value of 1 then it decreases until it stabilizes at 0.25. From t = 250s to t = 500 s, the set point is changed to 4 cm. Clearly the output signal of the controller increases in order to increase the speed of the pump and that result in increasing the water level to the desired point then it settles.

Fig 3.4: It represents the closed loop response of tank 1 and tank 3. From t =0 to t = 250 s, valve 1 ratio = 0.9 and valve 2 ratio = 0.5, tank 1 desired level is set to 7 cm, the rise time of the system is approximately 10 s. The output achieves the set point value at time equals to 15 s. Clearly, the output signal of the PI controller starts at a value of 1 then it decreases until it stabilizes at 0.1. From t = 250s to t = 500 s, the set point is changed to 6 cm and valve 1 ratio = 0.8 and valve 2 ratio = 0.6. Clearly the output signal of the controller decreases in order to decrease the speed of the pump and that result in decreasing the water level which is got bellow the desired point then it rises until it achieves the desired point then it settles.

Fig 3.5: It represents the closed loop response of tank 2 and tank 4. From t =0 to t = 250 s, valve 1 ratio = 0.9 and valve 2 ratio = 0.5, tank 1 desired level is set to 3 cm, the rise time of the system is approximately 50 s. The output achieves the set point value at time equals to 120 s. Clearly, the output signal of the PI controller starts at a value of 1 then it decreases until it stabilizes at 0.25. From t = 250s to t = 500 s, the set point is changed to 4 cm and valve 1 ratio = 0.8 and valve 2 ratio = 0.6. Clearly the output signal of the controller increases in order to increase the speed of the pump and that result in increasing the water level which is got above the desired point then it decreases until it achieves the desired point then it settles.

3.3. Fuzzy Logic Control of QTP Simulation

3.3.1. Simulink Block

A general Simulink block is shown in **Fig 3.6** with different parameters required for the system control and analysis as set point, valve constant scope, fuzzy logic controller and the four tank dynamics.

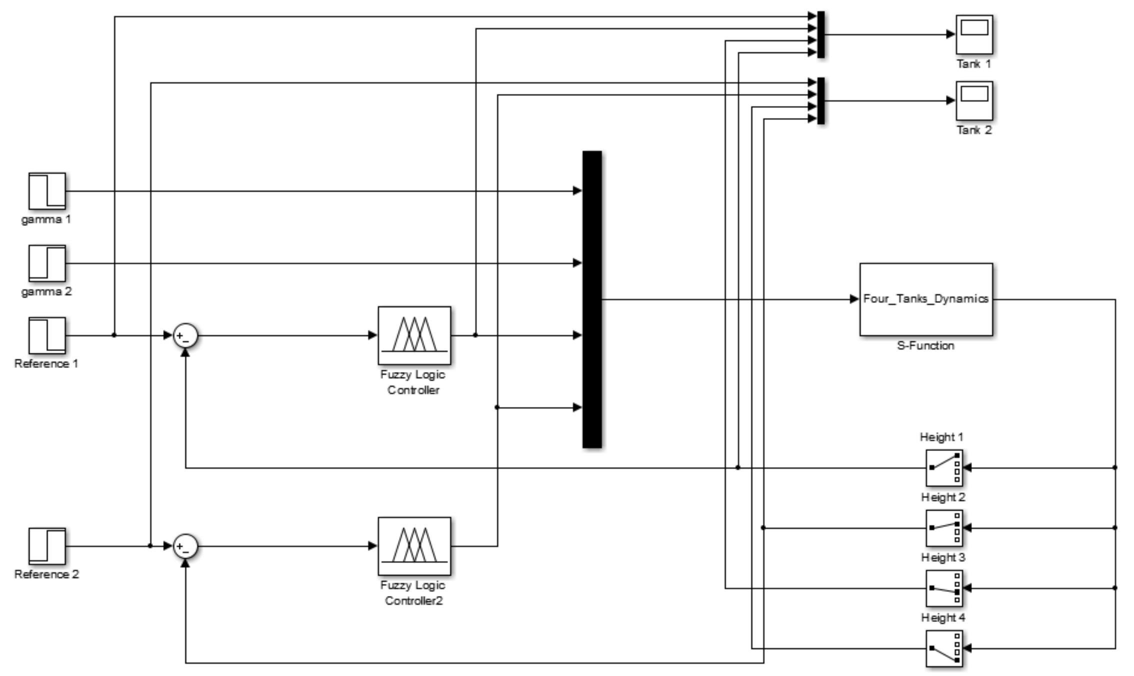


Figure 3.6: *Simulink Block Diagram of Fuzzy Logic Control of QTP*

3.2.3. Membership functions

The input to the fuzzy logic controller is the error between the desired level and the actual level of the lower tank and the output is the signal that controls the speed of the pump. The input membership function range is specified from -10 to 10 whereas the output membership function range from 0 to 1. Both controllers have the same specifications. These specifications are shown in **Fig 3.7** and **Fig 3.8**.

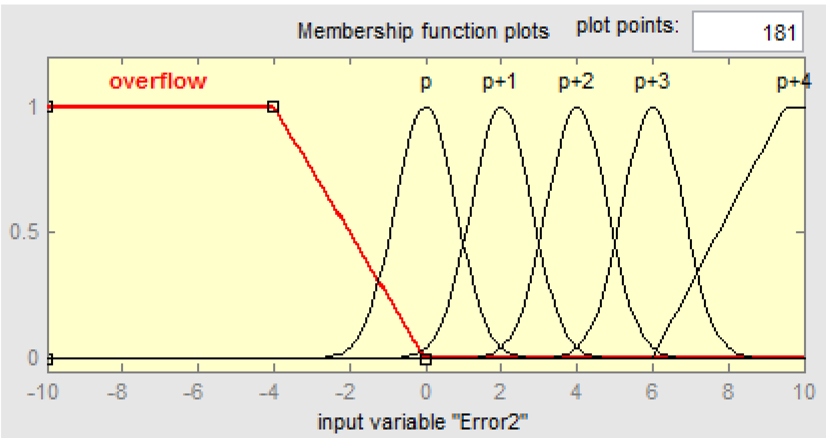


Figure 3.7: *Membership Function plot of the input variable.*

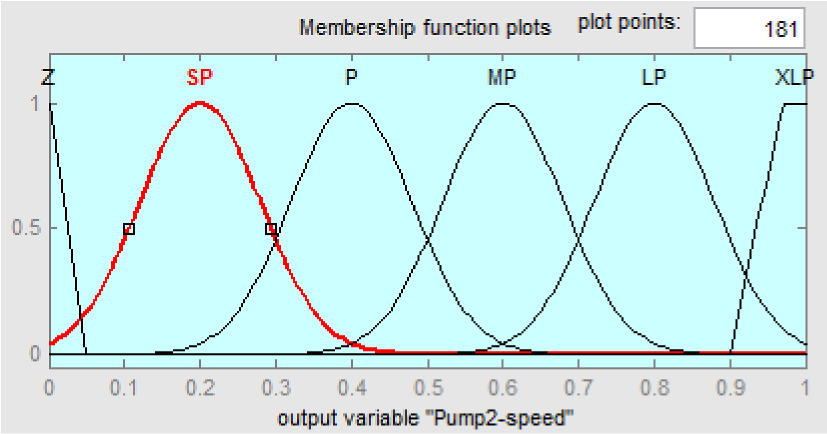


Figure 3.8: *Membership function Plot of output Variable.*

If the error is positive with a high value (P+4), then the output is XPL in order to speed up the flow of the pumps. If the error is less than 0, then the pumps must be stopped and the output is Z. Otherwise different rules are fired as shown in the **Fig 3.9**.

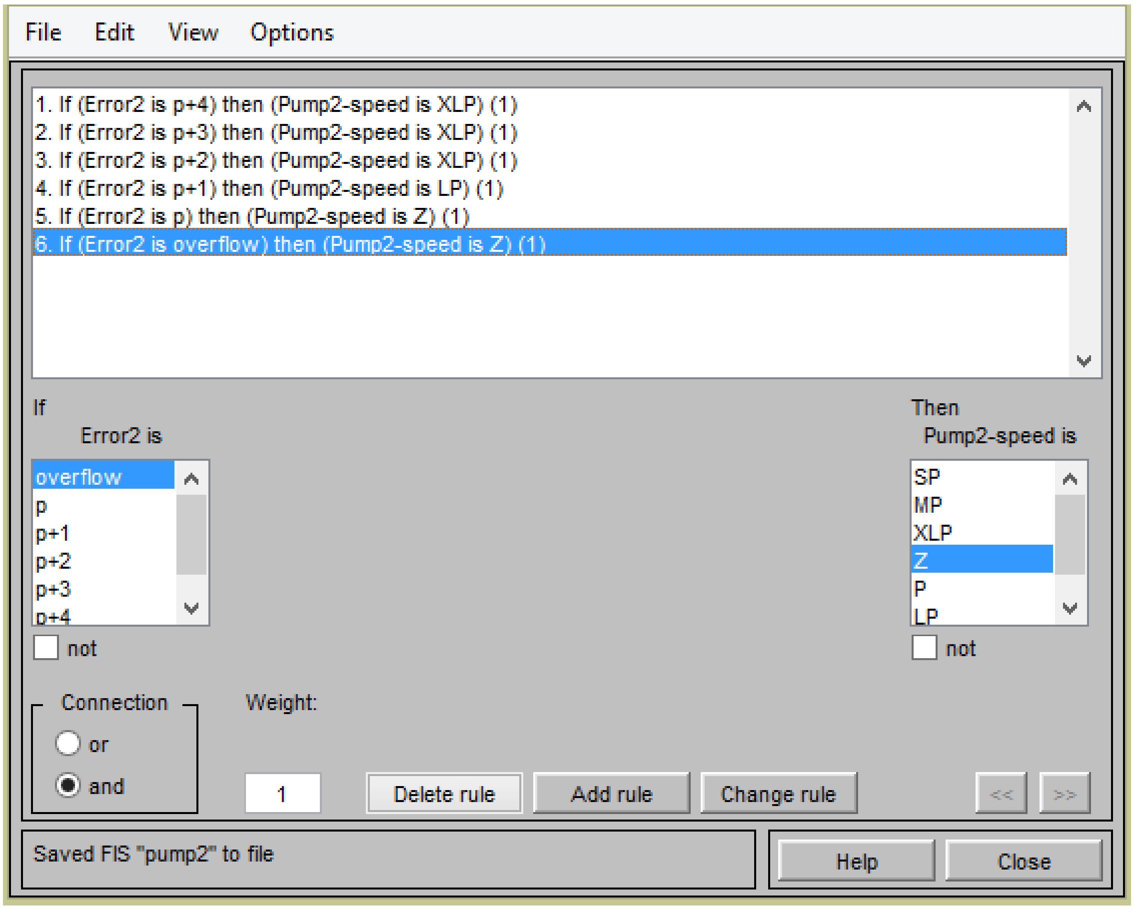


Figure 3.9: *Rules of Fuzzy Logic Controller*

The Output variable versus the input variable plot is then generated. The surface is shown in **Fig 3.9.**

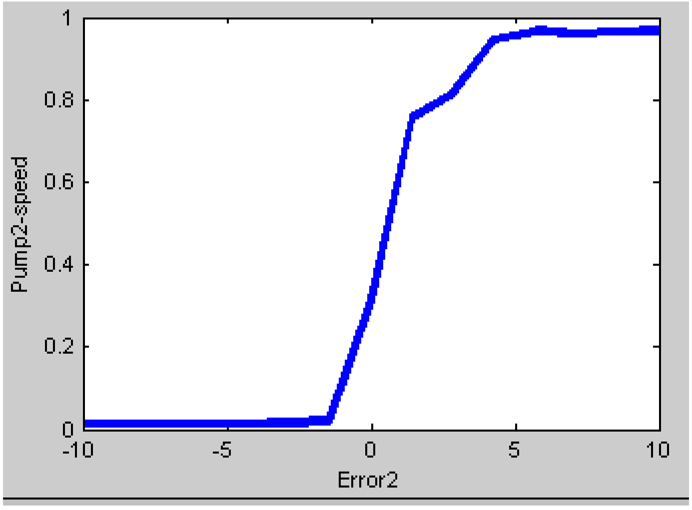


Figure 3.10: *Pumps Speed Versus Error.*

3.2.4. Simulation Results

The previous program is run, and simulated with different set points and valves ratios which ensure that our system is minimum phase. There are two cases as shown in **Table 3.1**.

Simulation results are shown in **Fig 3.11**, **Fig 3.12, Fig 3.13** and **Fig 3.14**.

Where:

Red: Lower tank level.

Dark Bleu: Desired level.

Bleu: Upper tank Level.

Purple: Controller output.

Case 01:

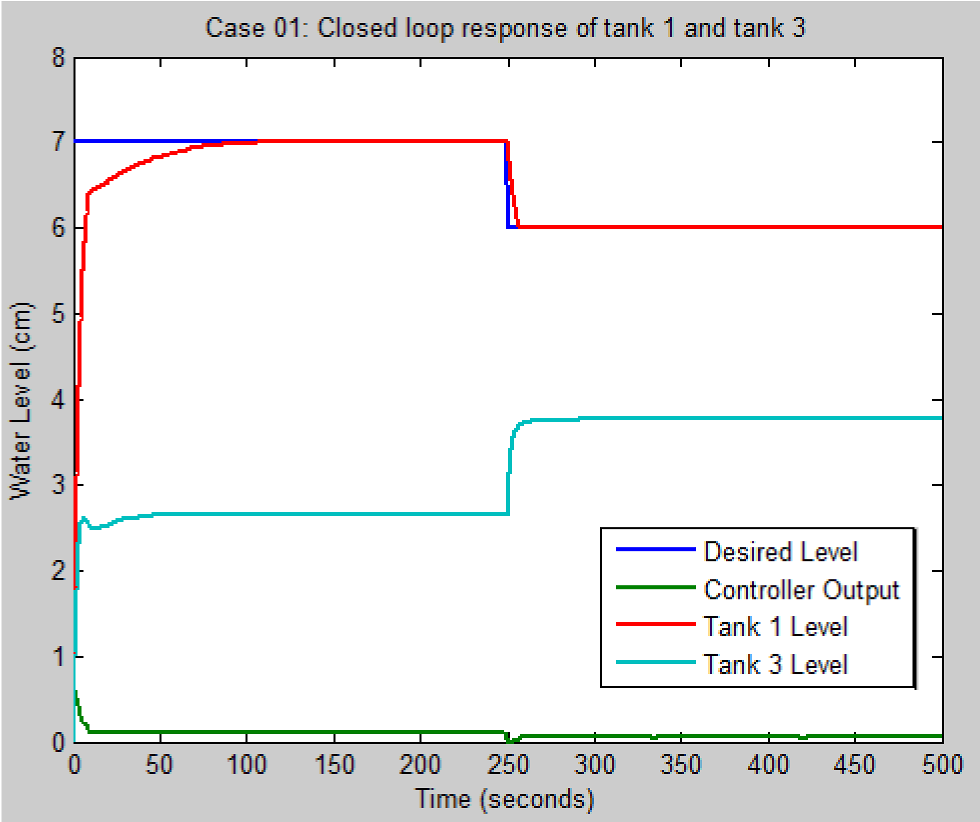


Figure 3.11: *Case 01: Closed loop response of tank 1 and tank 3*

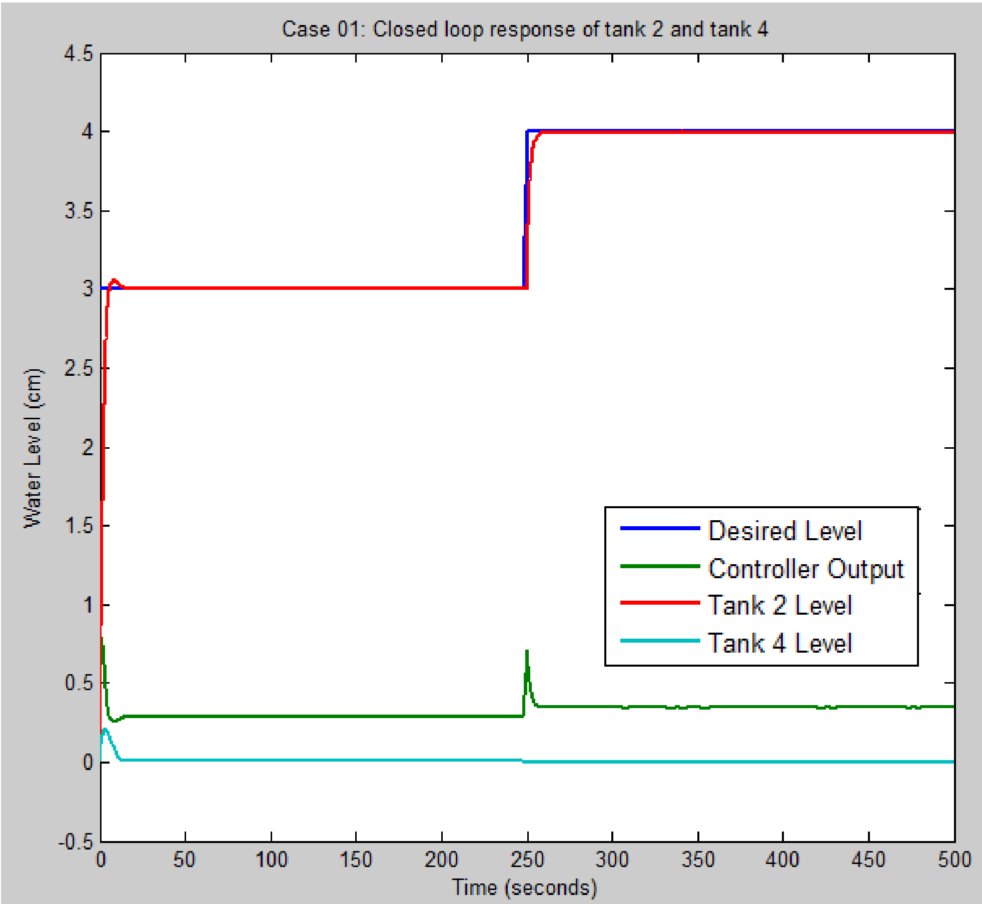


Figure 3.12: *Case 01: Closed loop response of tank 2 and 4*

Case 02:

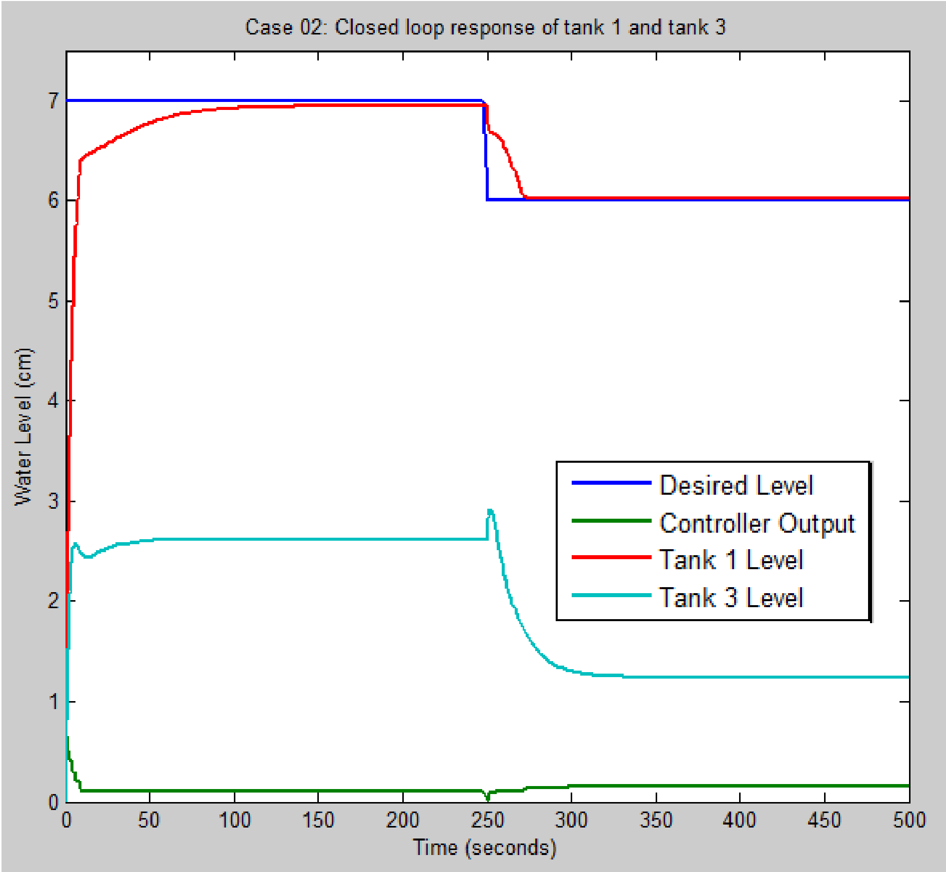


Figure 3.13: *Case 02: Closed loop response of tank 1 and tank 3*

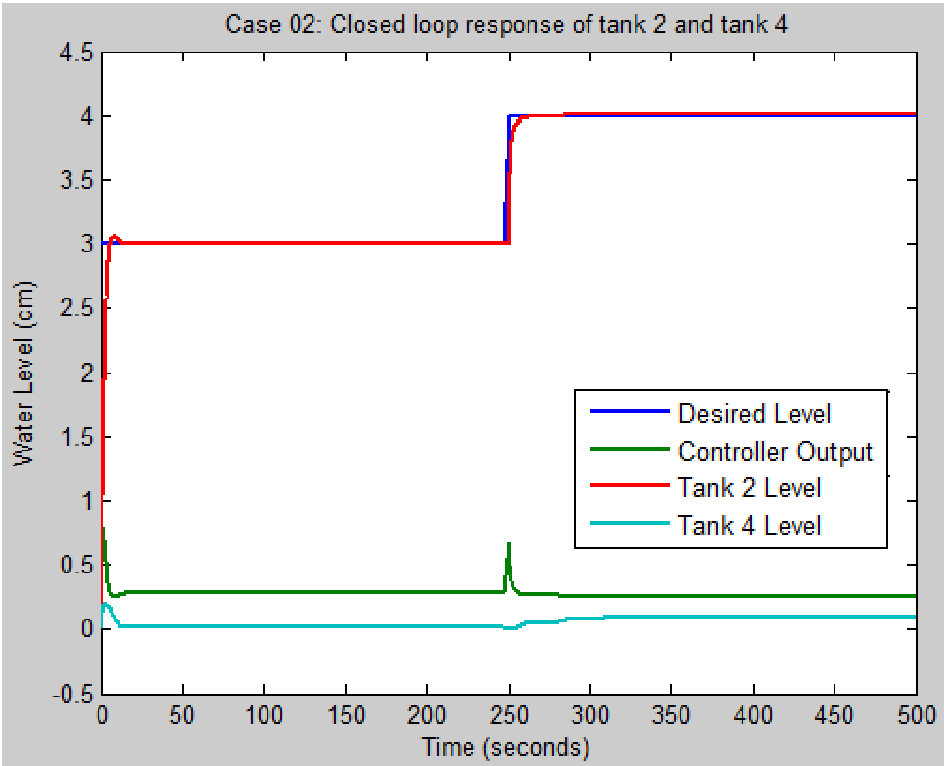


Figure 3.14: *Case 02: Closed loop response of tank 2 and 4*

3.2.5. Results Discussion

The results obtained from the closed loop control simulation of the quadruple tank process and performance of the system using fuzzy logic controller are discussed below:

Fig 3.11: It represents the closed loop response of tank 1 and tank 3 with valve 1 ratio = 0.9 and valve 2 ratio = 0.5. From t =0 to t = 250 s, tank 1 desired level is set to 7 cm, the rise time of the system is approximately 10 s. The output achieves the set point value at time equals to 60 s. Clearly, the output signal of the fuzzy logic controller starts at a value of 0.8 then it decreases until it stabilizes at 0.1. From t = 250s to t = 500 s, the set point is changed to 6 cm. Clearly the output signal of the controller decreases in order to decrease the speed of the pump and that result in decreasing the water level to the desired point then it settles.

Fig 3.12: It represents the closed loop response of tank 2 and tank 4 with valve 1 ratio = 0.9 and valve 2 ratio = 0.5. From t =0 to t = 250 s, tank 1 desired level is set to 3 cm, the rise time of the system is approximately 8 s. The output have slightly overshoot before it stabilizes at time equal to 18 s. Clearly, the output signal of the fuzzy logic controller starts at a value of 0.8 then it decreases until it stabilizes at 0.25. From t = 250s to t = 500 s, the set point is changed to 4 cm. Clearly the output signal of the controller increases in order to increase the speed of the pump and that result in increasing the water level to the desired point then it settles.

Fig 3.13: It represents the closed loop response of tank 1 and tank 3. From t =0 to t = 250 s, valve 1 ratio = 0.9 and valve 2 ratio = 0.5, tank 1 desired level is set to 7 cm, the rise time of the system is approximately 10 s. The output achieves the set point value at time equals to 60 s within an error of 0.7%. Clearly, the output signal of the fuzzy logic controller starts at a value of 0.8 then it decreases until it stabilizes at 0.1. From t = 250s to t = 500 s, the set point is changed to 6 cm and valve 1 ratio = 0.8 and valve 2 ratio = 0.6. Clearly the output signal of the controller decreases in order to decrease the speed of the pump and that result in decreasing the water level to reach the desired point then it settles.

Fig 3.14: It represents the closed loop response of tank 2 and tank 4 with valve 1 ratio = 0.9 and valve 2 ratio = 0.5. From t =0 to t = 250 s, tank 1 desired level is set to 3 cm, the rise time of the system is approximately 8 s. The output have slightly overshoot before it stabilizes at time equal to 18 s. Clearly, the output signal of the fuzzy logic controller starts at a value of 0.8 then it decreases until it stabilizes at 0.25. From t = 250s to t = 500 s, the set point is changed to 4 cm. Clearly the output signal of the controller increases in order to increase the speed of the pump and that result in increasing the water level to the desired point then it settles within an error of 0.3%.

3.3. Comparison between PID control and Fuzzy Logic Control

The table below shows the rise time and the settling time of the system when using a PI Controller and fuzzy logic controller.

Table 3.2: *System Performance Results*

	PI Controller	**Fuzzy Logic Controller**
Case 01:	Rise time of tank 1: 10 s Settling time of tank 1: 15 s Rise time of tank 2: 50 s Settling time of tank 2: 120 s	Rise time of tank 1: 10 s Settling time of tank 1: 60 s Rise time of tank 2: 08 s Settling time of tank 2: 18 s
Case 02:	Rise time of tank 1: 10 s Settling time of tank 1: 15 s Rise time of tank 2: 50 s Settling time of tank 2: 120 s	Rise time of tank 1: 10 s Settling time of tank 1: 60 s Rise time of tank 2: 08 s Settling time of tank 2: 18 s

- PID controller has only three parameters to adjust. Controlled system shows good results in terms of response time and precision when these parameters are well adjusted.
- Fuzzy controller has a lot of parameters. The most important is to make a good choice of rule base and parameters of membership functions. Once a fuzzy controller is given, the whole system can actually be considered as a deterministic system. When the parameters are well chosen, the response of the system has very good time domain characteristics.
- Fuzzy controlled system doesn't have much better characteristics in time domain like PID controlled system.

- One of the most important problems with fuzzy controller is that the computing time is much more long that for PID, because of the complex operations as fuzzification and particularly defuzzification.

3.4. Conclusion

This chapter dealt with the PID control and fuzzy logical control of the quadruple tank process simulation. The obtained results are discussed and compared. At the end of this chapter a comparison between PID control and fuzzy logical control is given.

General Conclusion

In this project, the nonlinear mathematical model of the quadruple tank system was developed by applying Bernouli's law and mass balance law. The obtained set of equations is then linearized using Taylor series expansion and Jacobian matrix. Moreover, the transfer function matrix of the system was derived and the multivariable zero physical interpretation was discussed.

Our principle objective was met. We were able to simulate and control the quadruple tank system in MATLAB/Simulink using two different techniques: PID control and fuzzy logic control. A comparison between the two techniques was made based on closed loop system performance.

PID controllers are commonly used to regulate the time-domain behaviour of many different types of dynamic plants. These controllers are extremely popular because they can usually provide good closed loop response characteristics when the PID parameters are well adjusted, and can be tuned using relatively simple rules.

Fuzzy controller has a lot of parameters. The most important is to make a good choice of rule base and parameters of membership functions. Once a fuzzy controller is given, the whole system can actually be considered as a deterministic system. When the parameters are well chosen, the response of the system has very good time domain characteristics.

PID controller cannot be applied with the systems which have a fast change of parameters, because it would require the change of PID constants in the time. It is necessary to further study the possible combination of PID and fuzzy controller. It means that the system can be well controlled by PID which is supervised by a fuzzy system.

As a further work, the quadruple tank process will be implemented using PLC and will be supervised using a SCADA system.

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APPENDIX A: m.file of MATLAB Simulation

The following program is the MATLAB M.file for the Simulink block.

```
% Diameters

d_P = 0.5;       % inner pipe diameter (cm)
d_T = 7;        % inner tank diameter (cm)

% height

h_T = 10;        % inner tank height (cm)

%--------------------------------------------------------

rho = 1;        % g/cm^3
g = 9.8*100;           % acceleration of gravity (cm/s^2)
A = (pi*(d_T/2)^2); % cross sectional area of the tank
a = (pi*(d_P/2)^2); % cross sectional area of the pipe

% coefficient of flow out of the bottom of the tank

C_F = a*sqrt(rho)*sqrt(2*g);

% constants to make calculations easier

C1 = a/A;
C2 = C_F/A;

% Max speed of the pump

v1_max = 500;         % cm/s
v2_max = v1_max;      % cm/s
```