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3D antenna array optimization using Firefly Algorithm

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Abstract

The design of antenna arrays in a cubic geometry is presented in this project. The decision variables considered for this synthesis problem is the amplitude excitations. The synthesis process is carried out by the technique of a new type of nature-inspired global optimization methodology in the design of an optimized cubic antenna array which ensures minimum side lobes and high directivity, this new optimization method is based on the reaction of a firefly to the light of other fireflies and it is known as **Firefly Algorithm** (FA) a population based iterative heuristic global optimization algorithm technique, developed by Xin-She Yang, for multi-dimensional and multi-modal problems, with the potential to implement constraints on the search domain. Simulation results, detailed by using an antenna array with isotropic elements and with cubic antenna optimized by Firefly algorithm, show that side lobe level is reduced significantly in non-uniform case. Besides, the directivity is not worse than that of the uniform one.

Dedication

I dedicate this work to my brother and father who has pushed me through it all, to my beloved mother who has always encouraged me and has been praying for me every time, and lastly to my all family and friends and teachers who have been there by my side during my whole academic life. May GOD bless you all.

Abdelali Laouar

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General Introduction

In the 1890s, there were only a few antennas in the world. These rudimentary devices were primary a part of experiments that demonstrated the transmission of electromagnetic waves. By World War II, antennas had become so ubiquitous that their use had transformed the lives of the average person via radio and television reception. The number of antennas in the United States was on the order of one per household, representing growth rivaling the auto industry during the same period. By the early 21st century, thanks in large part to mobile phones, the average person now carries one or more antennas on them wherever they go (cell phones can have multiple antennas, if GPS is used, for instance). This significant rate of growth is not likely to slow, as wireless communication systems become a larger part of everyday life. In addition, the strong growth in RFID devices suggests that the number of antennas in use may increase to one antenna per object in the world (product, container, pet, banana, toy, cd, etc.). This number would dwarf the number of antennas in use today. Hence, learning a little (or a large amount) about antennas couldn't hurt, and will contribute to one's overall understanding of the modern world. [1]

Antennas are basic components of any electric system and are connecting links between the transmitter and free space or free space and the receiver. it acts as a transducer that converts the current or voltage generated by the feeding-based circuit, such as a transmission line, a waveguide or coaxial cable, into energy field propagating through space and vice versa. Thus, antennas play very important role in finding the characteristics of the system in which antennas are employed.

Antennas are employed in different systems in different forms. For wireless communication systems, the antenna is one of the most critical components. A good design of the antenna can relax system requirements and improve overall system performance. So, to meet the particular need at hand, usually antenna required to optimize the radiation energy in some directions and suppress it in others, it must then take various forms, it may be a piece of conducting wire, an aperture, a patch, an assembly of elements (array), a reflector, a lens, and so forth. In long distance communication, there is great need for very directive antennas with very high gain.

Due to the radiation pattern limitations of a single antenna, several single antenna elements can be combined to form an array. The technology based in antenna arrays is a key solution for current and future wireless communication systems. Therefore, several antenna arrays have been developed to meet the special needs of the applications, such as: beam steering, directivity (DIR), gain, angular coverage, side lobe level (SLL), among others. However, there exist applications where the traditional antenna arrays cannot achieve the objectives such as tracking, direction finding and satellites communications. By these situations, 3D geometric antenna arrays have raised as an attractive solution [2] when Research reports concerning antenna array designs for 3D geometries are increasingly receiving attention for their capabilities for forming different radiation shapes and for scanning over all angles. Nevertheless, a synthesis of the pattern needs to be performed to achieve the maximum performance of the antenna system, Theoretically, the array should be designed with a maximum directivity and minimum side lobe level so as to achieve maximum signal to noise plus interference ratio at the output of the array antenna.

Due to the amazing development of computers, the application of numerical optimization techniques to antenna design has become possible. Among these techniques, bio inspired algorithms like the firefly algorithm (FA) have been found to be effective in optimizing difficult multidimensional problems in a variety of fields.

In the last two decades, more than a dozen of new metaheuristic algorithms such as particle swarm optimization, differential evolution, bat algorithm, firefly algorithm and cuckoo search have appeared and they have shown great potential in solving tough engineering optimization problems. Among these new algorithms, it has been shown that firefly algorithm is very efficient in dealing with multimodal, global optimization problems. Firefly algorithm (FA) is a new population-based metaheuristic approach developed by Xin-She Yang, which is nature-inspired by behavior of the flashing characteristics of fireflies. In this context, the flashing light can be formulated in such a way that it is associated with the objective function to be optimized, which makes it possible to formulate the firefly algorithm.

In our project we concentrated on designing directivity and side-lobe levels of uniformly distributed antenna elements along a cubic by varying the amplitude, then by turning some of the antenna elements on while the others are off in what is called “Thinning”, using firefly algorithm optimization.

The project report is organized in three chapters as follows:

Chapter 1: Generalities about Antennas then about antenna arrays, focusing on the cubic antenna array which is going to be optimized in the project.

Chapter 2: Generalities about optimization and Metaheuristic algorithms, then the definition of the firefly Algorithm, its assumptions and how it works.

Chapter 3: Result, Discussion and comments, about the design of the SLL and the Directivity of a uniformly distributed cubic antenna array by varying the amplitude only, then by thinning, using the firefly algorithm.

And finally, the Conclusion and suggestions for future work are also presented.

Chapter One: Generalities about antenna

I. Introduction

The field of antennas is vigorous and dynamic, and over the last 60 years antenna technology has been an indispensable partner of the communications revolution. Many major advances that occurred during this period are in common use today; however, many more issues and challenges are facing us today, especially since the demands for system performances are even greater. For wireless communication systems, it is very important to design antenna in such a way that improve overall system performance. for example, designing antenna in term of maximizing directivity and minimizing side lobes can achieve high-performance antenna. A typical example is TV for which the overall broadcast reception can be improved by utilizing a high-performance antenna. So, the antenna serves to a communication system the same purpose that eyes and eyeglasses serve to a human.

II. Antenna definition

The antenna is defined by the IEEE as "that part of a transmitting or receiving system that is designed to radiate or to receive electromagnetic waves". [3] In other word, it's the device that's used to couple energy from a guiding structure (transmission line, waveguide, etc.) into a propagation medium, such as free space, and vice versa. It provides directivity and gain for the transmission and reception of electromagnetic waves. [4] It's usually an arrangement of wires, metal rods, and so on for radiating and receiving radio signals. A transmitting antenna converts electrical current to electromagnetic radio waves projected into free space or a waveguide. A receiving antenna converts electromagnetic radio waves into electric current. An antenna commonly performs both transmit and receive functions. [5]

III. Types of Antennas

Antennas play an important role in modern technology, so it is not surprising that there are so many types, some of which are simple, others are more complicated.

Among the many types we mention, to name a few:

1. Wire Antennas
   1.1. Dipole Antennas
      1.1.1. Straight Wire Dipoles Antennas
      1.1.2. Biconical Dipoles Antennas

1.1.3. Bow-Tie Dipoles Antennas
1.1.4. Slot Dipoles Antennas
1.1.5. Folded Dipoles Antennas
1.1.6. Sleeve Dipoles Antennas
1.1.7. Shunt-Fed Dipoles Antennas
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1.2. Monopoles Antennas
1.2.1. Folded Monopole Antennas
1.2.2. Parasitic Monopole Antennas
1.3. Loop Antennas
1.3.1. Small Loop Antennas
1.3.2. Ferrite Loaded Loop Antennas
1.3.3. Larger Diameter Loop Antennas
1.4. Helix Antennas
1.5. Yagi-Uda Antennas
2. Aperture Antennas
2.1. Horn Antennas
2.1.1. Pyramidal Horn Antennas
2.1.2. Conical Horn Antennas
2.1.3. Dual-Mode Horn Antennas
2.1.4. Conical Corrugated Horn Antennas
2.1.5. Low-Gain Axially Corrugated Horn Antennas
2.2. Waveguide Antennas
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3. Microstrip Antennas
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3.2. Circular Microstrip Patch Antennas
4. Reflector Antennas
4.1. Parabolic Reflector Antennas with Front Feed
4.2. Parabolic Reflector Antennas with Cassegrain Feed
4.3. Corner Reflector Antennas
4.4. Offset-Fed Parabolic Reflector Antennas
4.5. Axisymmetric Dual-Reflector Antennas
4.6. Offset-Fed Dual-Reflector Antennas
4.7. Spherical Reflectors
4.8. Torus Antenna
5. Lens Antennas
5.1. Lens Antennas with Index of Refraction N>1
5.1.1. Convex-Plane
5.1.2. Convex-Convex
5.1.3. Convex-Concave
5.2. Lens Antennas with Index of Refraction N<1
5.2.1. Concave-Plane
5.2.2. Concave-Concave
5.2.3. Concave-Convex
6. Frequency-Independent Antennas
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IV. Fundamental Parameters of Antennas

With so many types, antennas characteristics must be determined. A set of parameters are used to provide a description of the performance of the antennas so that they can be understood, developed and improved. These parameters can be used as criteria for comparing different antenna designs. In the following, we will define some of the most important antennas parameters that we will be using in our optimization later on.

IV.1. Radiation pattern

According to IEEE, radiation pattern is the spatial distribution of a quantity that characterizes the electromagnetic field generated by an antenna. The distribution can be expressed as a mathematical function or as a graphical representation. The quantities that are most often used to characterize the radiation from an antenna are proportional to or equal to power flux density, radiation intensity, directivity, phase, polarization, and field strength. The spatial distribution over any surface or path is also an antenna pattern. When the amplitude or relative amplitude of a specified component of the electric-field vector is plotted graphically, it is called an amplitude pattern, field pattern, or voltage pattern. When the square of the amplitude or relative amplitude is plotted, it is called a power pattern. When the quantity is not specified, an amplitude or power pattern is implied. [3]

In other word, Antenna pattern or radiation pattern is a graph or chart representing the absolute or normalized antenna gain as a function of angle (typically azimuth or elevation) and used to describe the directional properties of an antenna. In the near field, the antenna pattern is a function of the distance from the antenna whereas in the far field, the pattern is independent of distance from the antenna. [4].

From the antenna pattern we can define two kinds of lobes: main lobe and side lobe. The main lobe (major lobe) is the radiation lobe containing the direction of maximum radiation. While the side lobe (minor lobe) is the radiation lobe in any direction other than that of the major lobe, which means any radiation lobe except the major lobe. [3]

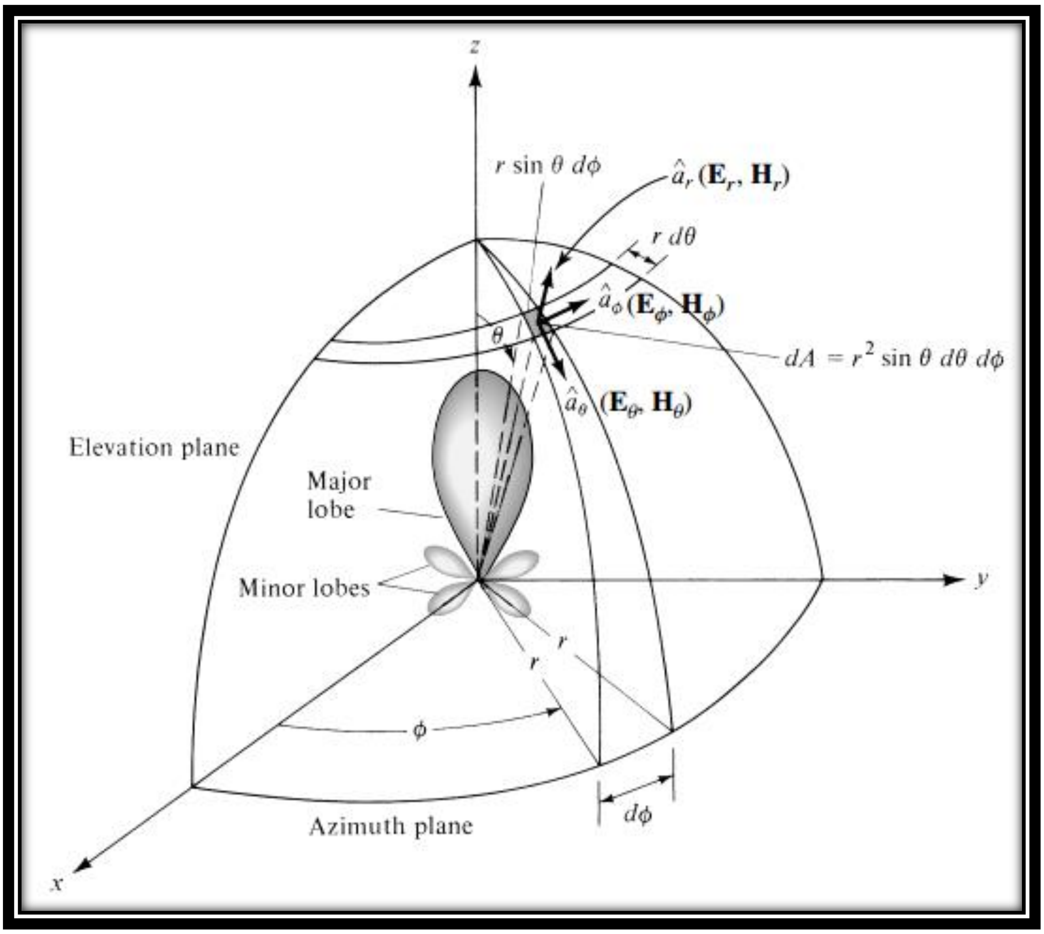

Figure 1. 1 Radiation pattern

IV.1.1. Radiation lobes:

Various parts of a radiation pattern referred to as lobes are classified into:

IV.1.1.1. Main/major Lobe

The major lobe of an antenna contains the direction of maximum radiation. In some antennas, such as split-beam antennas, there may exist more than one major lobe. A perfect beam antenna with only the main lobe is only hypothetical. Our main goal in designing is to maximize the main lobe while minimizing the side lobes and back lobes. [6]

IV.1.1.2. Side/Minor Lobe

All the lobes other than the main lobe are called the minor lobe. These lobes represent the radiation in undesired directions and they should be minimized. The level of minor lobes is usually expressed as a ratio of the power density in the lobe in question to that of the major lobe. This ratio is called the side lobe ratio or side lobe level (SLL). [6]

IV.2. Directivity and Gain

Directivity is the maximum ratio of an antenna's ability to focus or receive power in a given direction relative to a standard; the standard is usually an isotropic radiator or a dipole. Only depends on the radiation pattern shape and does not include the efficiency of the antenna. [4] The directivity of an antenna in a given direction is given by the ratio of the radiation intensity in the given direction from the antenna to the radiation intensity averaged over all directions. The average radiation intensity is equal to the total power radiated by the antenna divided by 4π. If the direction is not specified, the direction of maximum radiation intensity is implied. [3]

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} \tag{1.1}$$

where:

D=directivity (dimensionless).

U=radiation intensity (W/unit solid angle).

U_0 =radiation intensity of isotropic source (W/unit solid angle).

P_{rad} =total radiated power (W).

the antenna gain takes into account loss so that the gain of an antenna will always be less than the directivity. [6]

$$G(\theta,\varphi) = \frac{U(\theta,\varphi)}{P_{in}/4\pi} = \frac{4\pi\, U(\theta,\varphi)}{P_{in}} \tag{1.2}$$

P_{in}: Total input power, i.e. the power accepted by the antenna.

IV.3. Radiation intensity

In antenna theory, it is the far-field quantity that is a function of angle that gives the level of radiation in a specific direction. Radiation intensity is the radial component of the time average Poynting vector with all the terms associated with the distance from the antenna normalized out. [4] Simply, in a given direction, it is the power radiated from an antenna per unit solid angle. [3] Thus, the units are watts per square radian.

The radiation intensity is a far-field parameter, and it can also be obtained by simply multiplying the radiation density by the square of the distance. [6] In mathematical form it is expressed as:

$$U = r^2 W_{rad} \tag{1.3}$$

U=radiation intensity (W/unit solid angle)

W_{rad} =radiation density (W/m^2)

IV.4. Power density

Power density generally refers to the average power density, which is a measure of the power per unit area of a propagating EM wave. Mathematically, it is defined as the time average of the Poynting vector.it can be written as [4]

$$P_{rad} = P_{av} = \oiint_S W_{rad}.dS = \oiint_S W_{av}.\hat{n}\,da \tag{1.4}$$

IV.5. Efficiency

In general, the overall efficiency can be written as [6]

$$e_0 = e_r e_c e_d \tag{1.5}$$

Where

e_0 = Total efficiency (dimensionless)

e_r = Reflection (mismatch) efficiency (dimensionless)

e_c = Conduction efficiency (dimensionless)

e_d = Dielectric efficiency (dimensionless)

V. Antenna Arrays

V.1. Introduction

In the previous section, the radiation characteristics of single-element antennas were discussed. Usually the radiation pattern of a single element is relatively wide, and each element provides low values of directivity (gain). In many applications it is necessary to design antennas with very directive characteristics (very high gains) to meet the demands of long-distance communication. This can only be accomplished by increasing the electrical size of the antenna. Enlarging the dimensions of single elements often leads to more directive characteristics.

Another way to enlarge the dimensions of the antenna, without necessarily increasing the size of the individual elements, is to form an assembly of radiating elements in an electrical and geometrical configuration. This new antenna, formed by multi elements, is referred to as an array.

V.2. Antenna array definition

The array antenna is an antenna comprised of a number of radiating elements the inputs (or outputs) of which are combined. The possible arrangements often include arrangements in which the elements can be made congruent by simple translation or rotation. In an array antenna, the array element is a single radiating element or a convenient grouping of radiating elements that have fixed relative excitations. [3] In most cases, the elements of an array are identical. This is not necessary, but it is often convenient, simpler, and more practical. The individual elements of an array may be of any form (wires, apertures, etc.). [6]

V.3. Antenna array pattern

The total field of the array is determined by the vector addition of the fields radiated by the individual elements. This assumes that the current in each element is the same as that of the isolated element (neglecting coupling). This is usually not the case, and it depends on the separation between the elements. To provide very directive patterns, it is necessary that the fields from the elements of the array interfere constructively (add) in the desired directions and interfere destructively (cancel each other) in the remaining space. Ideally, this can be accomplished, but, practically, it is only approached.

In an array of identical elements, there are at least five controls that can be used to shape the overall pattern of the antenna. These are:

1. the geometrical configuration of the overall array (linear, circular, rectangular, spherical, etc.)

2. the relative displacement between the elements

3. the excitation amplitude of the individual elements

4. the excitation phase of the individual elements

5. the relative pattern of the individual elements

For an arbitrary N elements 3D antenna array, all elements have the same directional function. Since different elements have different propagation path r, and the phase lag $e^{jk\hat{r}\cdot r}$, thus the exciting current amplitude I_n is different and so is the exciting phase $e^{j\alpha_n}$.

Hereby, the radiation pattern for this array is given by: [7]

$$P(\theta, \varphi) = f(\theta, \varphi)\, AF(\theta, \varphi) \tag{1.6}$$

Where $f(\theta, \varphi)$ denotes the element pattern. $AF(\theta, \varphi)$ denotes the array factor and it is given by: [8]

$$AF(\theta, \varphi) = \sum_{n=1}^{N} I_n e^{j\alpha_n + jk\hat{r}\cdot r} \tag{1.7}$$

after mathematical computation, it is given by:

$$AF(\theta, \varphi) = \sum_{n=1}^{N} I_n e^{j[k(x_n \sin\theta \cos\varphi + y_n \sin\theta \sin\varphi + z_n \cos\theta) + \alpha_n]} \tag{1.8}$$

Equation 1.5 represents the multiplication principle:

“the array pattern can be considered to be the product of the element pattern and the array factor. The array factor depends only on the position of each element and amplitude and phase of the exciting current.”

In the following section, we assume that the radiated pattern of each array antenna element is isotropic, thus $f(\theta, \varphi) = 1$, and the array pattern is determined by the array factor $AF(\theta, \varphi)$.

An array of antenna elements is a spatially extended collection of a number of elements that have the same polar radiation patterns, orientated in the same direction in the 3D space. It is not necessary for the elements to be placed on a regular grid, neither to have the same terminal voltages, but it is assumed that they are all fed with the same frequency and that one can define a fixed amplitude and phase angle for the drive voltage of each element.

phased array antenna is an antenna composed of an aperture of individual radiating elements. Beam scanning is implemented by changing the phases of the signals at the antenna elements with the weights remaining fixed as the beam is steered. [4]

Phased arrays have the advantage that by controlling the phase angle of each array element, it is feasible to control the radiation pattern of the array and to steer the lobe in a desirable direction.

V.4. Type of antenna array

If we consider a uniformly distributed antenna array, there are three main configurations by which the antenna array can be designed, and they are: [9]

V.4.1. 1D-Linear antenna array

The field of an isotropic radiator located at the origin may be written as (assuming θ- polarization):

$$E_\theta = I_0 \frac{e^{-jkr}}{4\pi r} \tag{1.9}$$

Where:

I_0 = The complex excitation of the isotropic radiator.

k= the free space wave number.

r= distance of the observation point from the origin.

In our approach, we assume that the N elements of the array are uniformly – spaced with a separation distance d, see Figure 1.2.

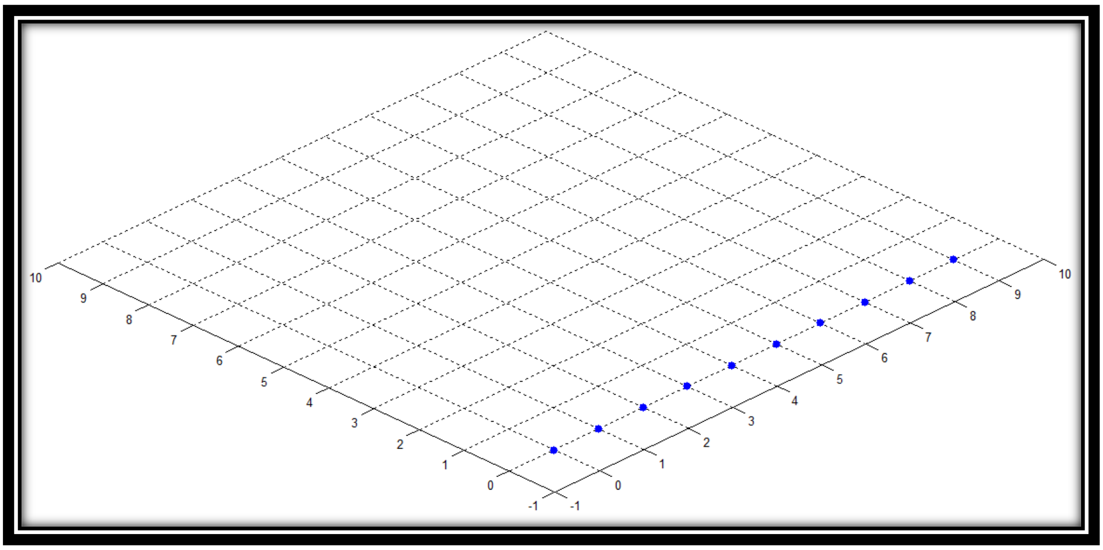

Figure 1. 2 1D-Linear antenna array

The current magnitudes of the array elements are supposed to be equal and the current on the array element located at the origin is used as the phase reference (zero phase).

$$I_1 = I_0 \quad I_2 = I_0 e^{j\emptyset_2} \quad \cdots \quad I_N = I_0 e^{j\emptyset_N} \tag{1.10}$$

The far electromagnetic fields of the individual array elements are:

$$E_{\theta_1} \approx I_0 \frac{e^{-jkr}}{4\pi r} = E_0 \tag{1.11}$$

$$E_{\theta_2} \approx I_0 e^{j\emptyset_2} \frac{e^{-jk(r-d\cos\theta)}}{4\pi r} = E_0 e^{j(\emptyset_2 + kd\cos\theta)} \tag{1.12}$$

$$\vdots$$

$$E_{\theta_N} \approx I_0 e^{j\emptyset_N} \frac{e^{-jk(r-(N-1)d\cos\theta)}}{4\pi r} = E_0 e^{j(\emptyset_N + (N-1)kd\cos\theta)} \tag{1.13}$$

The overall array far field is found using superposition and could be express as:

$$E_\theta = E_{\theta_1} + E_{\theta_2} + E_{\theta_3} + \cdots + E_{\theta_N} = E_0 \times AF \tag{1.14}$$

The array factor for a uniformly spaced N element linear array is:

$$AF = \left[1 + e^{j(\emptyset_2 + kd\cos\theta)} + \cdots + e^{j(\emptyset_N + (N-1)kd\cos\theta)}\right] \tag{1.15}$$

A uniform array is defined by uniformly – spaced identical elements of equal magnitude with a linearly progressive phase from element to element:

$$\emptyset_1 = 0 \quad \emptyset_2 = \alpha \quad \emptyset_3 = 2\alpha \quad \cdots \quad \emptyset_N = (N-1)\alpha \tag{1.16}$$

Inserting this linear phase progression into the formula for the general N– element array (equation 1.14), we obtain:

$$AF = \left[1 + e^{j(\alpha + kd\cos\theta)} + \cdots + e^{j(N-1)[\alpha + kd\cos\theta]}\right] = \sum_{n=1}^{N} e^{j(n-1)\psi} \tag{1.17}$$

The function $\psi = \alpha + kd\cos\theta$ is defined as the array phase function and is a function of the element spacing, phase shift, frequency and elevation angle. If the array factor from (1.16) is multiplied by $e^{j\psi}$, the result is:

$$AF \times e^{j\psi} = \left[e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \cdots + e^{jN\psi}\right] \tag{1.18}$$

Subtracting the array factor from the equation above, we obtain:

$$AF = e^{j(N-1)\frac{\psi}{2}} \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \tag{1.19}$$

The complex exponential term in (1.18) represents the phase shift of the array phase center relative to the origin. If the position of the array is shifted so that the center of the array is located at the origin, this phase term goes away. The array factor after phase shift and normalization becomes:

$$AF = \frac{1}{N} \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \tag{1.20}$$

V.4.2. 2D-Planar antenna array

Unlike linear arrays that can only scan the main beam in one polar plane (θ - the elevation plane or ϕ - the azimuth plane), planar arrays scan the main beam along both θ and ϕ. Planar arrays offer more gain and lower side lobes than linear arrays, at the expense of using more elements.

The design principles for planar arrays are similar to those presented earlier for the linear arrays. Since the elements are placed in two dimensions, see Figure 1.3, The array factor of N by M planar array can be expressed as the multiplication of the array factors of two linear arrays: one along the x-axis and one along the y-axis.

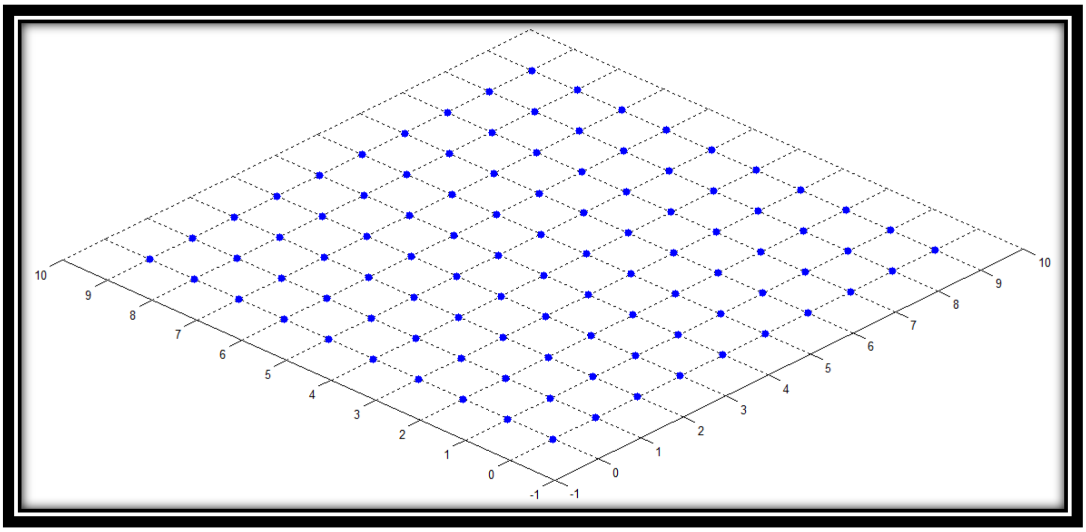

Figure 1. 3 2D-Planar antenna array

$$AF = AF_x \times AF_y = \frac{\sin\left(\frac{N\psi_x}{2}\right)\sin\left(\frac{M\psi_y}{2}\right)}{N\sin\left(\frac{\psi_x}{2}\right)M\sin\left(\frac{\psi_y}{2}\right)} \quad (1.21)$$

Where:

$$\psi_x = \alpha_x + kd_x \sin\theta\cos\varphi \quad (1.22)$$

$$\psi_y = \alpha_y + kd_y \sin\theta\sin\varphi \quad (1.23)$$

V.4.3. 3D-Cubic antenna array

Starting from the planar array, where it is considered that the system has a rectangular configuration of elements, the 3D array antenna, see Figure 1.4, is achieved by introducing a number of planar arrays on the z axis. In this case, the array factor of N by M by H uniform array is:

$$AF = AF_x \times AF_y \times AF_z = \frac{\sin\left(\frac{N\psi_x}{2}\right)\sin\left(\frac{M\psi_y}{2}\right)\sin\left(\frac{H\psi_z}{2}\right)}{N\sin\left(\frac{\psi_x}{2}\right)M\sin\left(\frac{\psi_y}{2}\right)H\sin\left(\frac{\psi_z}{2}\right)} \quad (1.24)$$

Where:

$$\psi_x = \alpha_x + kd_x \sin\theta \cos\varphi \quad (1.25)$$

$$\psi_y = \alpha_y + kd_y \sin\theta \sin\varphi \quad (1.26)$$

$$\psi_z = \alpha_z + kd_z \cos\theta \quad (1.27)$$

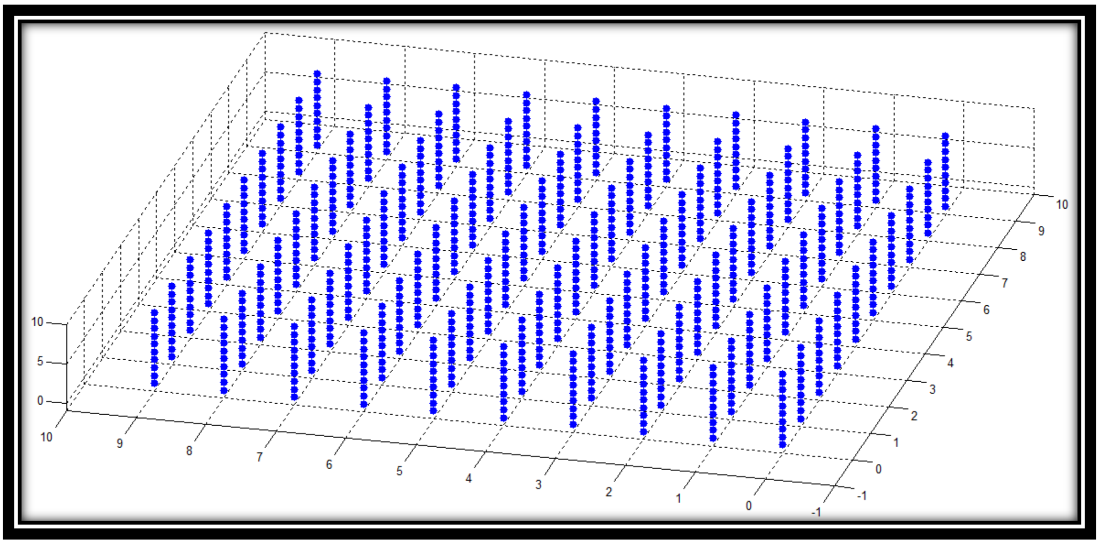

Figure 1. 4 3D-Cubic antenna array

In case of three – dimensional array antennas there are restrictions regarding the distance between elements on z axis which could not be less than elements dimensions.

VI. Conclusion

This chapter is divided into two parts, in the first one, we have showed generalities about antennas and its importance in the domain of communication, we have presented also different explanations of various types of antennas and their fundamental parameters as directivity, gain, side lobes, and radiation patterns …etc. In the second part, we have showed the advantages of array antennas in wireless long communication distance being largely directive and high gain, moreover we have dealt with different geometries as linear, planar and 3-D cubic arrays.

Chapter Two: Firefly Optimization

I. Introduction

Optimization is one of the basic mathematical techniques that lie at the heart of modern competitive management, design and development. It is the act of achieving the best possible result under given circumstances. In design, construction, maintenance...etc., engineers have to take many technological and managerial decisions at several stages.

The main objective of the optimization methods is to determine the maximum or the minimum of mathematical functions, called objective functions, which may be, or may not be, subject to constraints on its variables. Due to the wide variety of practical applications, optimization algorithms have been increasingly studied in the area of engineering and applied mathematics. The optimization methods can be divided into two major groups: deterministic and stochastic methods, which may use or not the derivatives of the objective and constraint functions.

The most of deterministic methods are local search methods. These methods are characterized for producing always the same set of solutions (optimal points) if the algorithm start under the same initial conditions. In turn, the stochastic methods are characterized by having one or more components of randomness, called stochastic components. This implies that for the same problem, and subject to the same initial conditions, these algorithms may not generate the same optimal solutions.

There is a range of possibilities of how to form this stochastic component. For example, one way is to make a simple randomization by randomly sampling the search space or by making random walks. The majority of this type of methods is considered as metaheuristic.

Firefly Algorithm (FA) is an algorithm that belongs to the second group, that is, a stochastic and metaheuristic algorithm, and it was developed by Yang. It is a recent nature inspired optimization algorithm, inspired by the social behavior of fireflies, and is based on their flashing and attraction characteristics. [10]

II. Metaheuristic optimization algorithms

II.1. Introduction

Metaheuristic algorithms are becoming an important part of modern optimization. A wide range of metaheuristic algorithms have emerged over the last two decades, [11] because of computational intelligence and soft computing. These algorithms are usually nature-inspired with multiple interacting agents. A subset of metaheuristics is often referred to as swarm intelligence (SI) based algorithms, and these SI-based algorithms have been developed by mimicking the so-called swarm intelligence characteristics of biological agents such as birds, fish, humans and others. For example, particle swarm optimization was based on the swarming behavior of birds and fish, while the firefly algorithm was based on the flashing pattern of tropical fireflies, and cuckoo search algorithm was inspired by the brood parasitism of some cuckoo species.

In the last two decades, more than a dozen new algorithms such as particle swarm optimization, differential evolution, bat algorithm, firefly algorithm and cuckoo search have appeared and they have shown great potential in solving tough engineering optimization problems. Among these new algorithms, we can find firefly algorithm which is very efficient in dealing with multimodal, global optimization problems. [12]

II.2. Classification of Metaheuristic algorithms

In recent years, several optimization methods, especially metaheuristic optimization methods, have been developed by scientists. People have utilized power of nature to solve problems. [13] Real-world optimization problems are often very challenging to solve. To solve such problems, optimization tools have to be used, though there is no guarantee that the optimal solution can be obtained. New algorithms have been developed to see if they can cope with these challenging optimization problems. Among these new algorithms, many algorithms such as particle swarm optimization, cuckoo search and firefly algorithm, have gained popularity due to their high efficiency.

Nature has inspired many researchers in many ways and thus is a rich source of inspiration. Nowadays, most new algorithms are nature-inspired, because they have been developed by drawing inspiration from nature. Obviously, we can divide all existing algorithms into four major categories: swarm intelligence (SI) based, bio-inspired (but not SI-based), physics/chemistry-based, and others. [14]

II.2.1. Physics and chemistry Based Metaheuristic Optimization

In solving of complex, multimodal, high dimensional and nonlinear problems; the metaheuristic optimization methods are used. Generally, these problems can be seen in engineering, industry, business and many other areas. Scientists utilize several physical, chemical, biological laws which are helped them to improve new optimization methods.

Although there are many metaheuristic optimization methods that are based on physics, many of them are not known by scientist and there are very limited works about these methods. [13] At this chapter, we will try to introduce some physics-based algorithms proposed by different scientists, and we record briefly some of them, see the Table (last page of appendices)

II.2.2. biology Based Metaheuristic Optimization

From the set theory point of view, SI-based algorithms are a subset of bio-inspired algorithms, while bio-inspired algorithms are a subset of nature-inspired algorithms. That is Conversely, not all nature-inspired algorithms are bio-inspired.

Many bio-inspired algorithms do not use directly the swarming behavior. Therefore, it is better to call them bio-inspired, but not SI-based [14], We will record some of bio-inspired algorithm based later on, see the Table (last page of appendices)

II.2.3. Swarm intelligence-based metaheuristic optimization

Swarm intelligence (SI) concerns the collective, emerging behavior of multiple, interacting agents who follow some simple rules. While each agent may be considered as unintelligent, the whole system of multiple agents may show some self-organization behavior and thus can behave like some sort of collective intelligence. Many algorithms have been developed by drawing inspiration from swarm-intelligence systems in nature.

All SI-based algorithms use multi-agents, inspired by the collective behavior of social insects, like ants, termites, bees, and wasps, as well as from other animal societies like flocks of birds or fish [14]. A list of swarm intelligence algorithms is recorded at the Table (last page of appendices)

There are some other types of algorithms that don’t belong to our three main classification, we will record some of them, see the Table (last page of appendices)

II.3. Some examples of metaheuristic algorithms

II.3.1. Genetic Algorithms

In artificial intelligence, genetic algorithms (GA) use mechanisms inspired by biological evolution, such as reproduction, mutation and selection (Wright, 1932; Goldberg, 1989). The candidate solutions of the optimization problem are represented as individuals in a population. The fitness function is the function to optimize and determines the quality of an individual. To apply a GA to an optimization problem, the parameters that define a candidate solution are considered as the genotype of the individual. The genotype is the footprint that uniquely identifies every possible solution. Evolution of the population takes place after multiple iterations where biological operators are applied:

Reproduction and mutation search the space of solutions by creating new candidate solutions, while the selection operator ensures that better candidate solutions survive to the next generation more often than worse ones [15].

II.3.2. Differential evolution algorithm:

Differential Evolution Algorithm is inspired by natural evolution. In the year of 1997 Storn and Price introduced this Evolutionary Algorithm namely Differential Evolution Algorithm. [16] The DE algorithm is a population-based algorithm like genetic algorithms using the similar operators; crossover, mutation and selection. The main difference in constructing better solutions is that genetic algorithms rely on crossover while DE relies on mutation operation. This main operation is based on the differences of randomly sampled pairs of solutions in the population.

The algorithm uses mutation operation as a search mechanism and selection operation to direct the search toward the prospective regions in the search space. The DE algorithm also uses a non-uniform crossover that can take child vector parameters from one parent more often than it does from others. By using the components of the existing population members to construct trial vectors, the recombination (crossover) operator efficiently shuffles information about successful combinations, enabling the search for a better solution space.

An optimization task consisting of D parameters can be represented by a D-dimensional vector. In DE, a population of NP solution vectors is randomly created at the start. This population is successfully improved by applying mutation, crossover and selection operators [17]. The main steps of the DE algorithm are given in Figure 2.1.

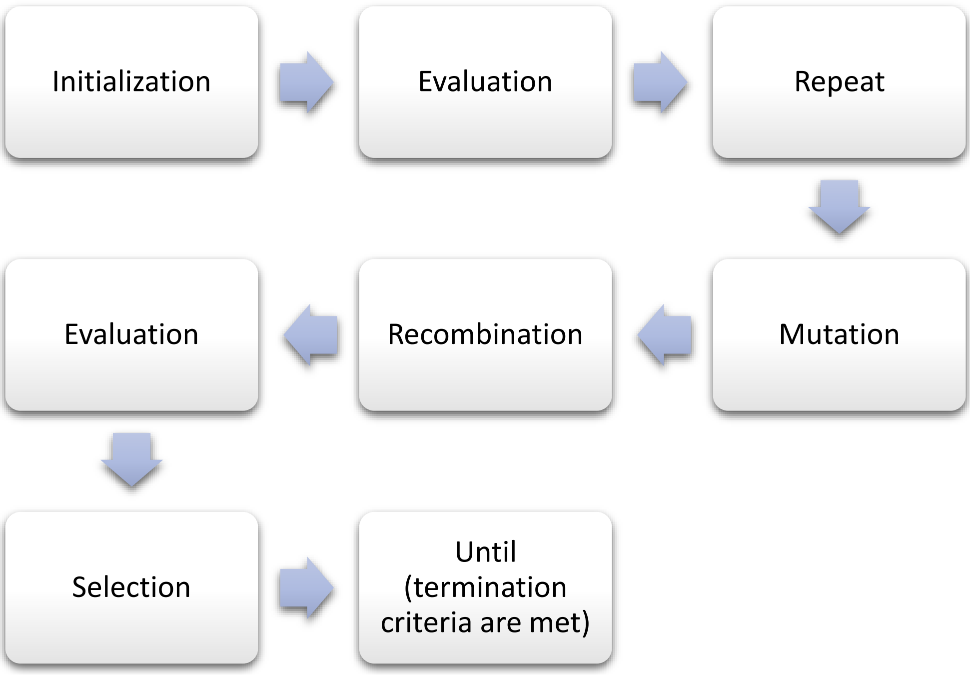

Figure 2. 1 Differential evolution algorithm

II.3.3. Particle swarm optimization algorithm

Particle Swarm Optimization (PSO) is a biologically inspired optimization technique. It was first proposed by Eberhart and Kennedy in 1995. PSO is inspired by the social behavior of bird flocking or fish schooling. [18]

The PSO algorithm searches the space of the objective functions by adjusting the trajectories of individual agents, called particles, as the piecewise paths formed by positional vectors in a quasi-stochastic manner. There are now as many as about 20 different variants of PSO. Here, we only describe the simplest and yet popular standard PSO.

The particle movement has two major components: a stochastic component and a deterministic component. A particle is attracted toward the position of the current global best and its own best location while at the same time it has a tendency to move randomly.

When a particle finds a location that is better than any previously found locations, it updates it as the new current best for particle I. There is a current global best for all n particles.

The aim is to find the global best among all the current best solutions until the objective no longer improves or after a certain number of iterations [19].

II.3.4. Gravitational Search Algorithm

RASHEDI et al. proposed Gravitational Search Algorithm (GSA) in 2009. GSA is based on law of gravity and law of motion. According to the gravity law, every particle in universe attracts each other with a force and this force is proportional to their masses and inversely proportional to the square of the distance between them. In this algorithm, every agent is considered as objects. These objects' performances are measured by their masses. In this algorithm, it is expecting that at the end of the GSA run, position of the object with the heaviest mass will show the global solution. [13] Main steps of the algorithm are given in Figure 2.2.

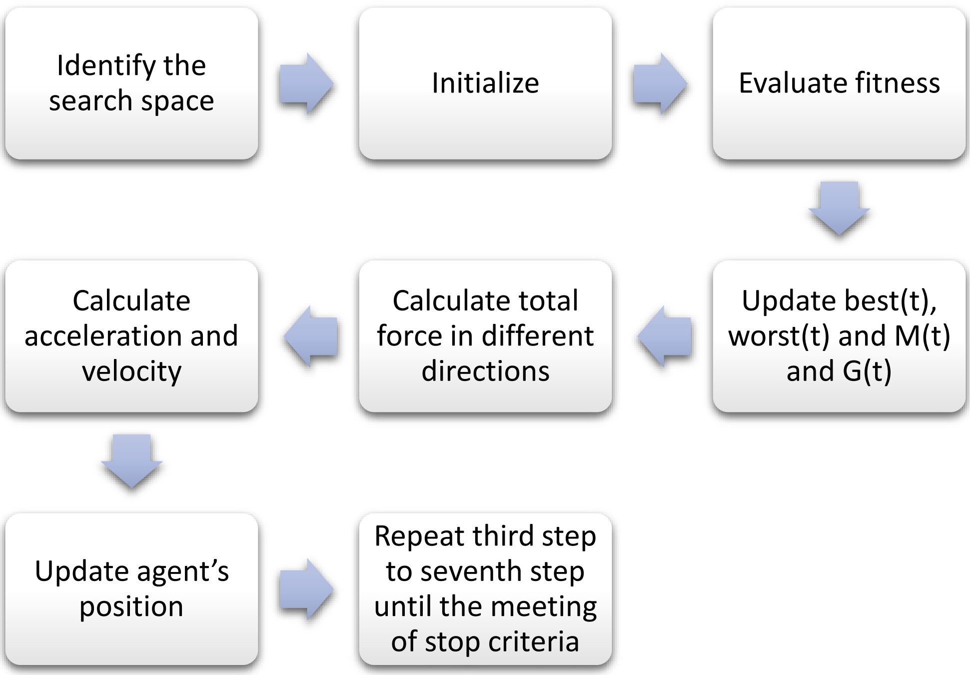

Figure 2. 2 Gravitational Search Algorithm

II.3.5. Central Force Optimization Algorithm

Central Force Optimization (CFO) Algorithm was proposed by FORMATO in 2007. CFO is a metaheuristic algorithm based on metaphor of gravitational kinematics. Unlike the many other stochastic algorithms, CFO is a deterministic method.

CFO doesn't require randomness in any of its calculation. CFO searches a multi-dimensional decision space.

In this algorithm, probes fly along the decision space under the impression of gravity and they change their positions according to the equations of motion. In process of time, it is expecting that all probes will be trapped in close orbits of big masses with largest gravitational field. That means they slowly move towards the probe that has achieved the highest mass or fitness. [13] Main steps of CFO are:

Step 01: compute the initial probe positions, the corresponding fitness, and assign the initial accelerations.

Step 02: compute each probe's new position based on previously computed accelerations.

Step 03: verify that each probe is located inside the decision space, making correction as required.

Step 04: update the fitness at each new probe position.

Step 05: compute the accelerations for the next time step based on new positions.

Step 06: loop over all time steps.

II.4. Conclusion

The sources of inspiration for algorithm development are very diverse, and consequently the algorithms are equally diverse.

III. Firefly Algorithm

III.1. Definition

Firefly Algorithm (FA) is one of the recent swarm intelligence methods developed by Xin-She Yang in 2008. FA is a stochastic, nature-inspired, metaheuristic algorithm that can be applied for solving the hardest optimization problems. [10]

III.2. Overview of Firefly Algorithm

FA is an optimization algorithm inspired by behavior and motion of fireflies. It is a population-based optimization algorithm which uses swarm intelligence to converge. It is similar to other optimization algorithms employing swarm intelligence such as PSO, GA and DEA. But FA is found to have superior performance in many cases. [20]

FA initially produces a swarm of fireflies located randomly in the search space. The initial distribution is usually produced from a uniform random distribution. The position of each firefly in the search space represents a potential solution of the optimization problem. The dimension of the search space is equal to the number of optimizing parameters in the given problem.

The fitness function takes the position of a firefly as input and produces a single numerical output value denoting how good the potential solution is. A fitness value is assigned to each firefly.

The FA uses a phenomenon known as bioluminescent communication to model the movement of the fireflies through the search space. [20] The velocity or the pull of a firefly towards another firefly depends on the attractiveness. The attractiveness depends on the relative distance between the fireflies. It is a function of the brightness of the fireflies as well. A brighter firefly that is far away may not be as attractive as a less bright firefly that is closer. In each iterative step, FA computes the brightness and the relative attractiveness of each firefly. Depending on these values, the positions of the fireflies are updated. After a sufficient amount of iterations, all fireflies converge to the best possible position on the search space. [21]

The firefly algorithm is based on three main principles [10]:

1- All fireflies are unisex, implying that all the elements of a population can attract each other.

2- The attractiveness between fireflies is proportional to their brightness. The firefly with less bright will move towards the brighter one. If no one is brighter than a particular firefly, it moves randomly. Attractiveness is proportional to the brightness which decreases with increasing distance between fireflies.

3- The brightness or light intensity of a firefly is related with the type of function to be optimized. In practice, the brightness of each firefly can be directly proportional to the value of the objective function.

Based on these three rules, the basic steps of the firefly algorithm (FA) can be summarized as the pseudo code shown in Figure 2.3.

Firefly Algorithm

Objective function $f(\mathbf{x})$, $\mathbf{x} = (x_1, ..., x_d)^T$
Initialize a population of fireflies $\mathbf{x}_i$ $(i = 1, 2, ..., n)$
Define light absorption coefficient γ
while (t <MaxGeneration)
for $i = 1 : n$ all n fireflies
    for $j = 1 : i$ all n fireflies
        Light intensity I_i at $\mathbf{x}_i$ is determined by $f(\mathbf{x}_i)$
        if $(I_j > I_i)$
        Move firefly i towards j in all d dimensions
        end if
        Attractiveness varies with distance r via $\exp[-\gamma r]$
        Evaluate new solutions and update light intensity
    end for j
end for i
Rank the fireflies and find the current best
end while
Postprocess results and visualization

Figure 2. 3 Pseudo code of the firefly algorithm (FA). [22]

III.3. Attractiveness [19]

In the FA, there are two important issues: the variation of light intensity and formulation of the attractiveness. For simplicity, we can always assume that the attractiveness of a firefly is determined by its brightness or light intensity which in turn is associated with the encoded objective function.

In the simplest case for maximum optimization problems, the brightness I of a firefly at a particular location x can be chosen as I (x) ∝ f (x). However, the attractiveness β is relative, it should be seen in the eyes of the beholder or judged by the other fireflies. Thus, it should vary with the distance rij between firefly i and firefly j. As light intensity decreases with the distance from its source, and light is also absorbed in the media, we should allow the attractiveness to vary with the degree of absorption.

In the simplest form, the light intensity I(r) varies according to the inverse square law $\mathrm{I(r)} = \frac{I_s}{r^2}$, where I_s is the intensity at the source.

For a given medium with a fixed light absorption coefficient γ, the light intensity(I) varies with the distance r. that is:

$$I = I_0 e^{-\gamma r} \tag{2.1}$$

Where:

I_0 = is the original light intensity.

In order to avoid the singularity at r = 0 in the expression $\mathrm{I(r)} = \frac{I_s}{r^2}$, the combined effect of both the inverse square law and absorption can be approximated using the following Gaussian form:

$$I(r) = I_0 e^{-\gamma r^2} \tag{2.2}$$

Sometimes, we may need a function which decreases monotonically at a slower rate. In this case, we can use the following approximation:

$$I(r) = \frac{I_0}{1 + \gamma r^2} \tag{2.3}$$

At a shorter distance, the above two forms are essentially the same. This is because the series expansions about r = 0 are equivalent to each other up to the order of $O(r^3)$.

$$e^{-\gamma r^2} \approx 1 - \gamma r^2 + \frac{1}{2}\gamma^2 r^4 + \cdots \tag{2.4}$$

$$\frac{1}{1 + \gamma r^2} \approx 1 - \gamma r^2 + \gamma^2 r^4 + \cdots \tag{2.5}$$

As a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, we can now define the attractiveness β of a firefly by:

$$\beta(r) = \beta_0 e^{-\gamma r^2} \tag{2.6}$$

Where β_0 is the attractiveness at r = 0.

As it is often faster to calculate $\frac{1}{(1+r^2)}$ than an exponential function, the above function, if necessary, can conveniently be replaced by:

$$\beta(r) = \frac{\beta_0}{1 + \gamma r^2} \tag{2.7}$$

The above Equation defines a characteristic distance:

$$\Gamma = \frac{1}{\sqrt{\gamma}} \tag{2.8}$$

Over which the attractiveness changes significantly from β_0 to $\beta_0 e^{-1}$.

In the implementation, the actual form of attractiveness function $\beta(r)$ can be any monotonically decreasing functions such as the following generalized form:

$$\beta(r) = \beta_0 e^{-\gamma r^m}, \qquad (m \geq 1) \tag{2.9}$$

For a fixed γ, the characteristic length becomes:

$$\Gamma = \gamma^{-1/m} \qquad \to 1 \text{ as } m \to \infty \tag{2.10}$$

Conversely, for a given length scale Γ in an optimization problem, the parameter γ can be used as a typical initial value. That is:

$$\gamma = \frac{1}{\Gamma^m} \tag{2.11}$$

III.4. Distance and Movement: [19]

The distance between any two fireflies i and j at x_i and x_j, respectively, is the Cartesian distance:

$$r_{ij} = \|\boldsymbol{x_i} - \boldsymbol{x_j}\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \tag{2.12}$$

Where $x_{i,k}$: is the k^{th} component of the spatial coordinate x_i of i^{th} firefly.

In 2-D case, we have:

$$r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{2.13}$$

The movement of a firefly i is attracted to another more attractive (brighter) firefly j is determined by:

$$\boldsymbol{x_i} = \boldsymbol{x_i} + \beta_0 e^{-\gamma {r_{ij}}^2}(\boldsymbol{x_j} - \boldsymbol{x_i}) + \alpha\left(rand - \frac{1}{2}\right) \tag{2.14}$$

where the second term is due to the attraction while the third term is randomization with α being the randomization parameter, rand is a random number generator uniformly distributed in [0, 1]. For most cases we can take β_0 = 1 and α ∈ [0, 1].

Furthermore, the randomization term can easily be extended to a normal distribution N (0, 1) or other distributions.

IV. FA Summarized:

The important steps are summarized below. [23]

IV.1. Step 1: Initialization:

The number of fireflies in the population space is K. The position of the n^{th} firefly is denoted by a vector W_n, where each measurement indicates the weight of an array element.

$$\boldsymbol{W}_n = (w_n^1, w_n^2, w_n^3, \cdots w_n^t, w_n^N) \tag{2.15}$$

Where n=1, 2, 3…K and t=1, 2, 3…N. To initialize the location of K fireflies in N-dimensional search space, which are randomly selected with in the search boundary by the next equation:

$$w_n^t = w_L^t + (w_H^t - w_L^t) \times rand(\,) \tag{2.16}$$

Where w_L^t and w_H^t represents the lower and upper values of the t^{th} variable in the population respectively, and $rand(\,)$ is a uniform random variable with values ranging from 0 to 1.

IV.2. Step 2: Fitness Function

Calculate the fitness for each firefly position in the population and sort the population from brightest to lightest. The brightness of each firefly is calculated at current generation by the fitness function at their current location. The brightest or light intensity is inversely proportional to cost function of individual firefly for minimization problem.

IV.3. Step 3: Update the Location of Fireflies

Depending on the attractiveness, each firefly in the population will move toward the adjacent firefly with more light intensity and its position is updated for the next generation. The firefly i (less intensity) will move toward the other fireflies j that are brighter. There are two important issues in the FA, the deviation of brightness or light intensity and formation of the attractiveness. The attractiveness of a firefly is calculated by its brightness or light intensity which is directly associated with the cost function. The brightness of the n^{th} firefly B_n is given by the equation:

$$B_n = f_{fitness}(w_n) \tag{2.17}$$

The attractiveness between the i-th and j-th firefly is given by:

$$\beta_{\mathrm{ij}} = \beta_0 e^{-\gamma {r_{ij}}^2} \tag{2.18}$$

Where:

β: is a constant whose value is 1.

γ: is dynamic range of search space.

r_{ij}: is a distance between wi and wj given by the equation:

$$r_{ij} = \|\boldsymbol{w_j} - \boldsymbol{w_i}\| = \sqrt{\sum_{n=1}^{N} (w_j^t - w_i^t)^2} \tag{2.19}$$

The position of firefly is updated in each iterative step. If the intensity/brightness of j-th firefly is greater than the brightness of the i-th firefly, then the i-th firefly moves towards the j-th firefly. The motion of the fireflies is denoted by the following equation:

$$w_n = w_n + \beta_{ij}(w_j - w_i) + \alpha\varepsilon_n \quad (2.20)$$

Where: α is a constant parameter of scale whose value depends on the dynamic range of the solution space. The third term is randomization with the vector of random variables ε_n being drawn from a Gaussian distribution. [22]

IV.4. Step 4: Ranking and Computation of Global Best

On the basis of their brightness, the fireflies are ranked in the current generation, and the brightness of each firefly is compared with all other fireflies, and the location of the brightest firefly in the population is taken as current global best, and, in this way, for the brightest firefly we received a best fitness value in the recent generation.

IV.5. Step 5: Termination of Program

When fitness function achieves a certain prescribed value, or when maximum number of cycles (NOC) is reached, the program terminates and stores the best value, otherwise it goes back to step 2 to 4. The location of the best firefly gives the optimum solution and the corresponding brightness of the firefly that gives the optimum fitness value of the fitness function. The flowchart of FA is shown in Figure 2.4.

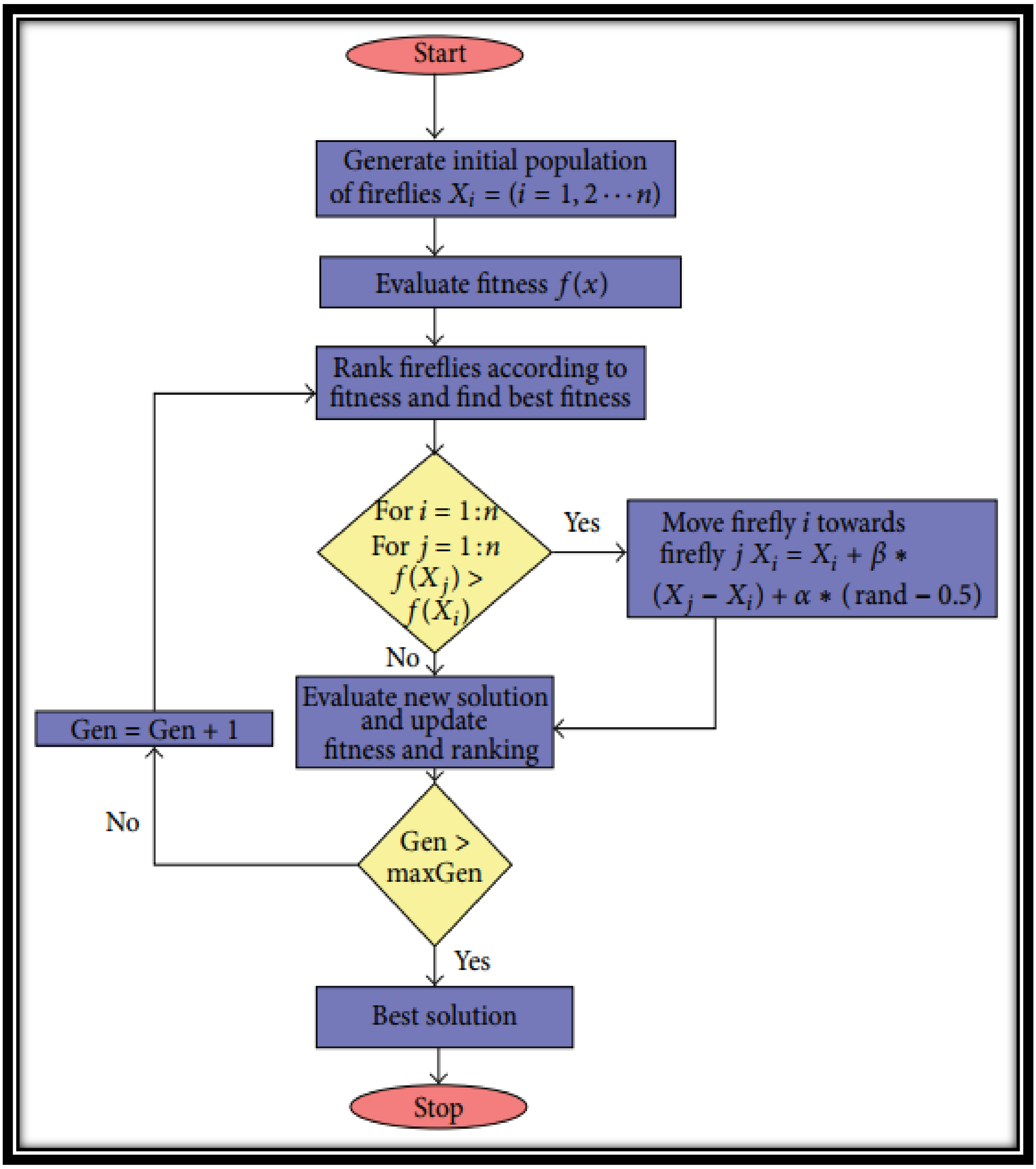

Figure 2. 4 The flowchart of Firefly Algorithm

V. Conclusion

This chapter is divided into two parts, the first part shows the general optimization and its definition, it shows also the metaheuristic optimization algorithms and its classification based on chemistry and physics, biology, and swarm intelligence. It shows also some examples of algorithms such as genetic algorithm, particle swarm optimization algorithm (PSO) and differential evolution algorithm (DEA) …etc. In the second part, we define the firefly algorithm (FA), a new type of nature-inspired global optimization algorithm inspired by the social behavior of fireflies, and it is based on their flashing and attraction characteristics.

Chapter Three: Results and Discussion

I. Problem statement:

In this chapter, we will be considering a 3D cubic antenna array while optimizing its performance using the firefly algorithm.

In the uniform case, the amplitude I=1 for all elements. For the non-uniform case, the optimization will be in two parts, the amplitude (I) varies in the interval $[0 \quad 1]$ in the first part, while in the second part, it's either '0' or '1' in what's called "thinning". In each part, the objective function, or the cost function, will be the sidelobe level only, then the sidelobe level and the directivity at the same time, which means:

$$f_1 = SLL_{max}(I) = 20log_{10}\left|\frac{AF(\theta)}{AF(\theta)_{max}}\right| \; in\ the\ side\ lobe\ region \tag{3.1}$$

$$f_2 = -DIR(I) = -10log_{10}\left(\frac{4\pi \max(\max(|AF|^2))}{\sum\sum|AF|^2 \sin(\theta)\, d\theta d\varphi}\right) \tag{3.2}$$

And

$$f_3 = f_2 + 10\, f_1 \tag{3.3}$$

Where:

AF: is the array factor of the cubic antenna array

SLL_{max}: is the maximum side lobe level in dB

DIR: is the directivity of the cubic antenna array in dB

$d\theta = \frac{\pi}{A}$ and $d\varphi = \frac{2\pi}{B}$ while A and B are positive real numbers.

The antenna array is a uniformly spaced $(d_x = d_y = d_z)$ cubic antenna array of 10 by 10 by 10 elements, see Figure 3.1.

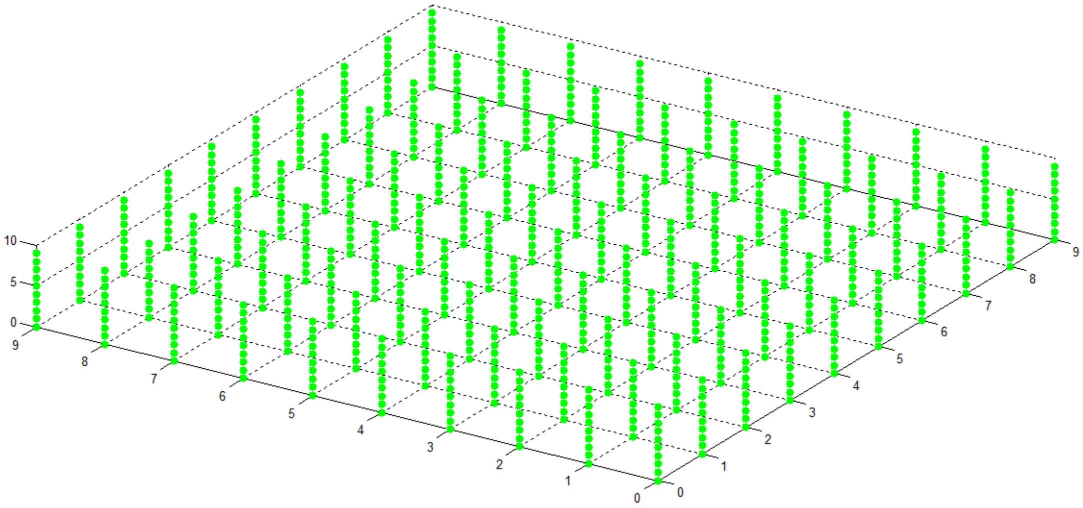

Figure 3. 1 3D Cubic antenna array

The array factor of array antennas is presented in the first chapter by the equation (1.7). For 10 by 10 by 10 cubic antenna array, the array factor becomes:

$$AF(\theta,\varphi) = \sum_{h=1}^{H}\sum_{n=1}^{N}\sum_{m=1}^{M} I_{m,n,h} e^{j[(h-1)\psi_z+(n-1)\psi_y+(m-1)\psi_x]} \tag{3.4}$$

Where:

$I_{m,n,h}$: is the excitation current of the elements.

$$\psi_x = kd_x \sin\theta \cos\varphi + \alpha_x \tag{3.5}$$

$$\psi_y = kd_y \sin\theta \sin\varphi + \alpha_y \tag{3.6}$$

$$\psi_z = kd_z \cos\theta + \alpha_z \tag{3.7}$$

$$\alpha_x = -kd_x \sin\theta_s \cos\varphi_s \tag{3.8}$$

$$\alpha_y = -kd_y \sin\theta_s \sin\varphi_s \tag{3.9}$$

$$\alpha_z = -kd_z \cos\theta_s \tag{3.10}$$

$$k = \frac{2\pi}{\lambda} \tag{3.11}$$

We have:

$$M = N = H = 10$$

$$d_x = d_y = d_z = \frac{\lambda}{2}$$

$$\theta_s = \varphi_s = 45^o$$

The firefly algorithm has been discussed in the second chapter and its flowchart is presented in Figure 2.4. We will be using 50 fireflies with 1000 iterations while starting with $\alpha = 0.5, \beta = 0.2, and\ \gamma = 1$.

II. The uniform case:

The relative array factor of the uniform case is shown in Figure 3.2 and Figure 3.3 for φ=0 and φ=90 respectively. We have got $\mathbf{25.3567}$dB for the directivity while the side lobe level is $\mathbf{-8.0139}$dB for the uniform case when φ=0, and the side lobe level is $\mathbf{-6.8876}$dB when φ=90.

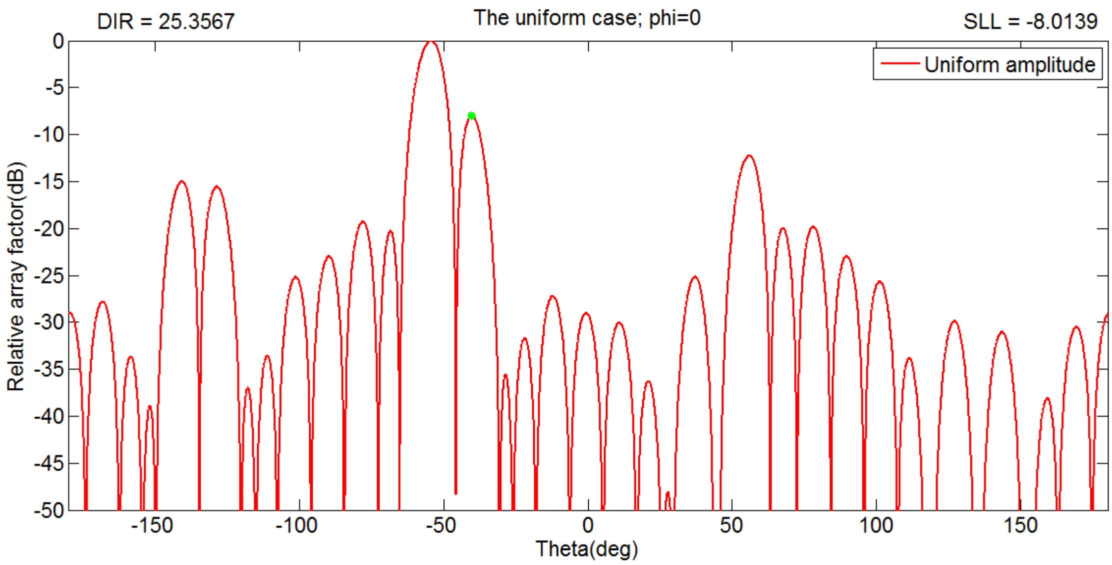

Figure 3. 2 The Uniform case; phi=0

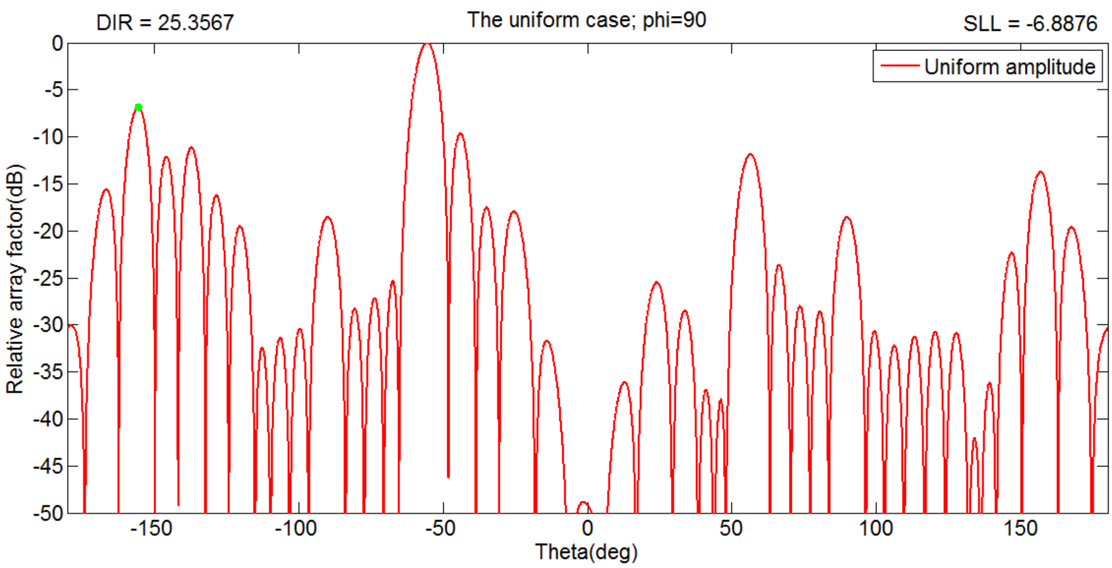

Figure 3. 3 The Uniform case; phi=90

III. The non-uniform case: Amplitude Variation

III.1. Optimization of the Side Lobe Level only

III.1.1. When φ=0

By setting the objective function to f_1, which is presented in the equation 3.1, and varying the amplitudes of the excitation current of the elements in the interval [0 1], we got the relative array factor, after the optimization for φ=0 using the parameters that have been presented in the beginning of this chapter, and it is shown in Figure 3.4. We have reduced the side lobe level from **-8.0139**dB to **-37.371**dB which is a better value, but at the same time, the directivity has been reduced from **25.3567**dB to **25.14**dB.

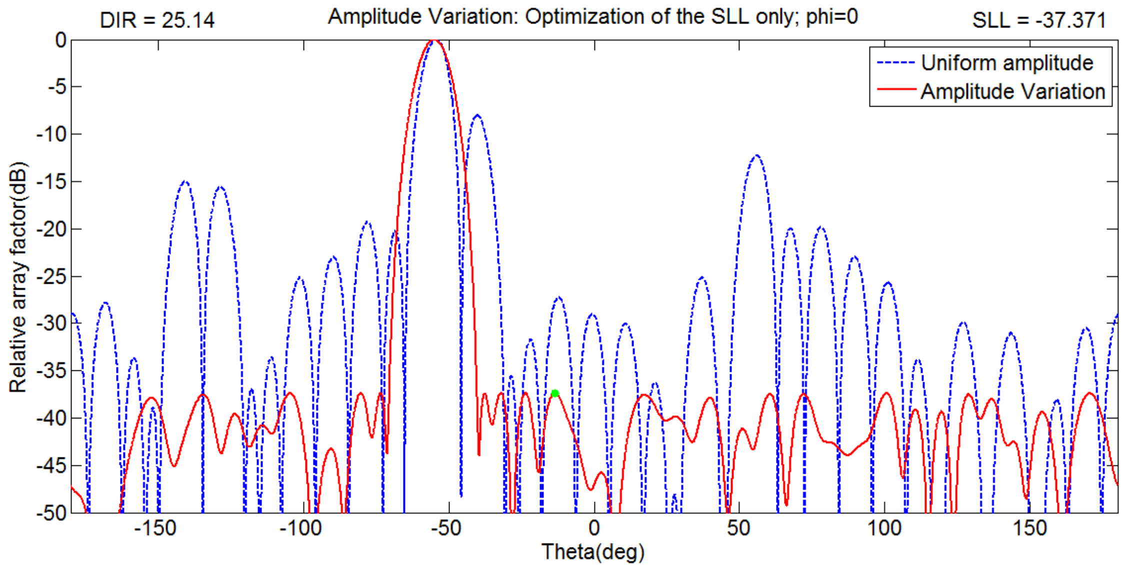

Figure 3. 4 Amplitude Variation: SLL Only; phi=0

III.1.2. When φ=90

By setting the objective function to f_1 again, and varying the amplitudes of the excitation current of the elements in the interval [0 1], we got the relative array factor, after the optimization for φ=90 using the parameters that have been presented in the beginning of this chapter, and it is shown in Figure 3.5. We have reduced the side lobe level from **-6.8876dB** to **-31.8464dB** which is also a better value, but at the same time, the directivity has been reduced from **25.3567dB** to **25.0852dB**.

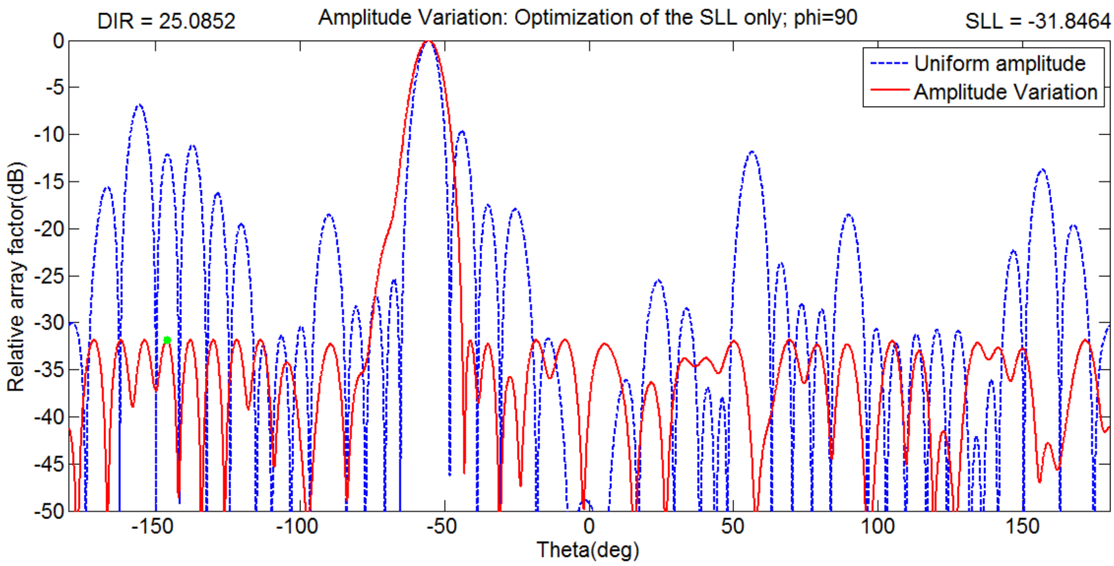

Figure 3. 5 Amplitude Variation: SLL Only; phi=90

III.2. Optimization of the Side Lobe Level and the Directivity

III.2.1. When φ=0

By setting the objective function to f_3, which is presented in the equation 3.3, and varying the amplitudes of the excitation current of the elements in the interval $[0 \quad 1]$, we got the relative array factor, after the optimization for φ=0 using the parameters that have been presented in the beginning of this chapter, and it is shown in Figure 3.6. We have reduced the side lobe level from **-8.0139dB** to **-34.8082dB** which is a better value, but at the same time, the directivity has been reduced from **25.3567dB** to **25.2013dB**.

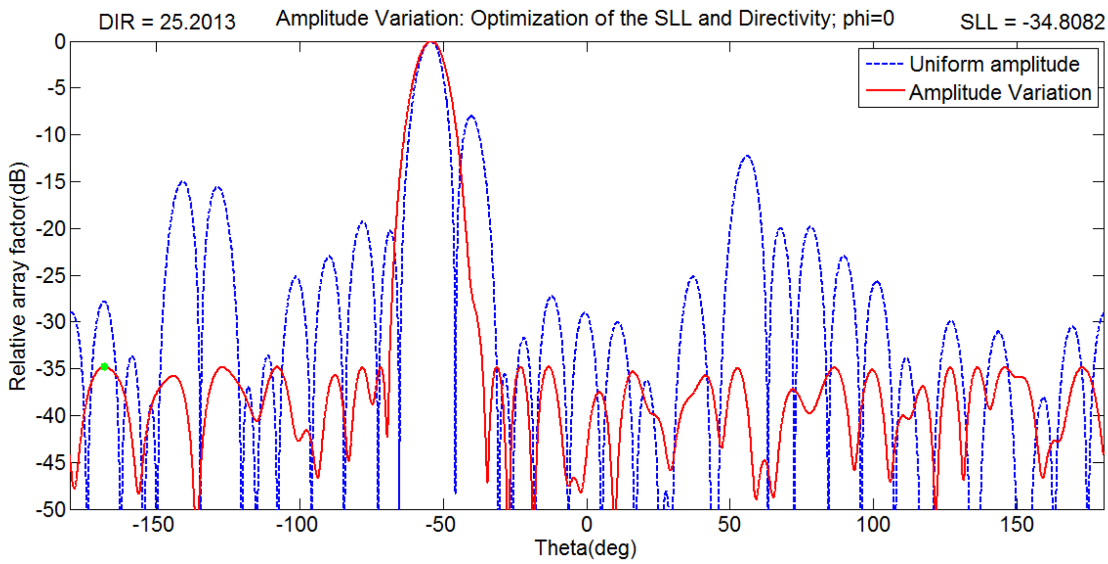

Figure 3. 6 Amplitude Variation: SLL and Directivity; phi=0

III.2.2. When φ=90

By setting the objective function to f_3 again, and varying the amplitudes of the excitation current of the elements in the interval [0 1], we got the relative array factor, after the optimization for φ=90 using the parameters that have been presented in the beginning of this chapter, and it is shown in Figure 3.7. We have reduced the side lobe level from **-6.8876dB** to **-31.7007dB** which is also a better value, but at the same time, the directivity has been reduced from **25.3567dB** to **25.072dB**.

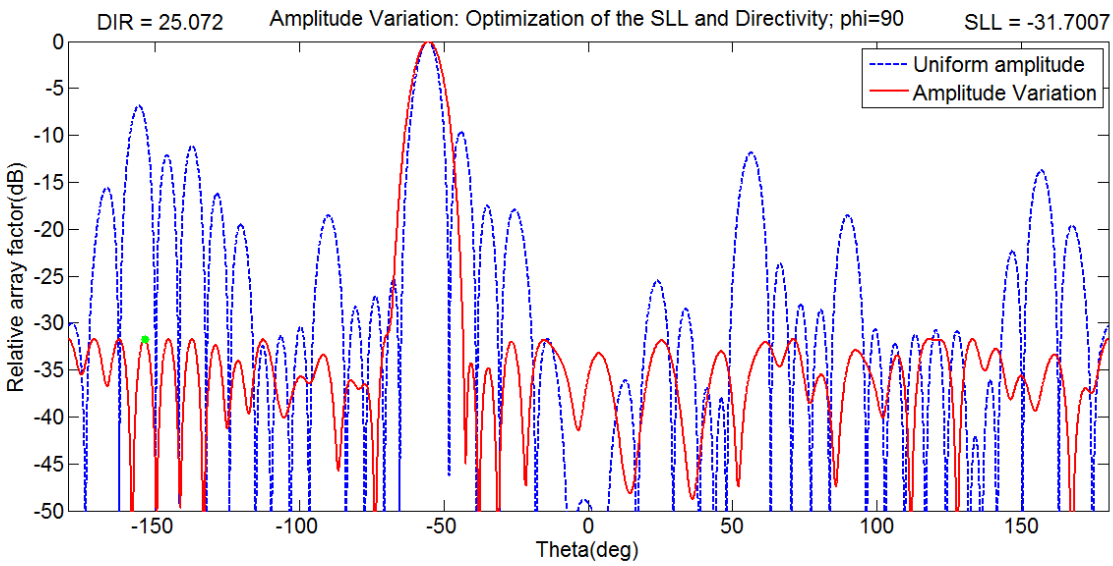

Figure 3. 7 Amplitude Variation: SLL and Directivity; phi=90

IV. The non-uniform case: Thinning

IV.1. Optimization of the Side Lobe Level only

IV.1.1. When φ=0

By setting the objective function to f_1, which is presented in the equation 3.1, and choosing the amplitudes of the excitation current of the elements to be either ‘0’ or ‘1’, we got the relative array factor, after the optimization for φ=0 using the parameters that have been presented in the beginning of this chapter, and it is shown in Figure 3.8. We have reduced the side lobe level from **-8.0139dB** to **-27.1982dB** which is a better value, but at the same time, the directivity has been reduced from **25.3567dB** to **24.1635dB**.

The remaining elements, that the firefly algorithm have kept on, are **512** out of **1000** elements while the others are turned off. The percentage of the elements that are kept on is **51.2%** and they are distributed as shown in Figure 3.9 by the green color.

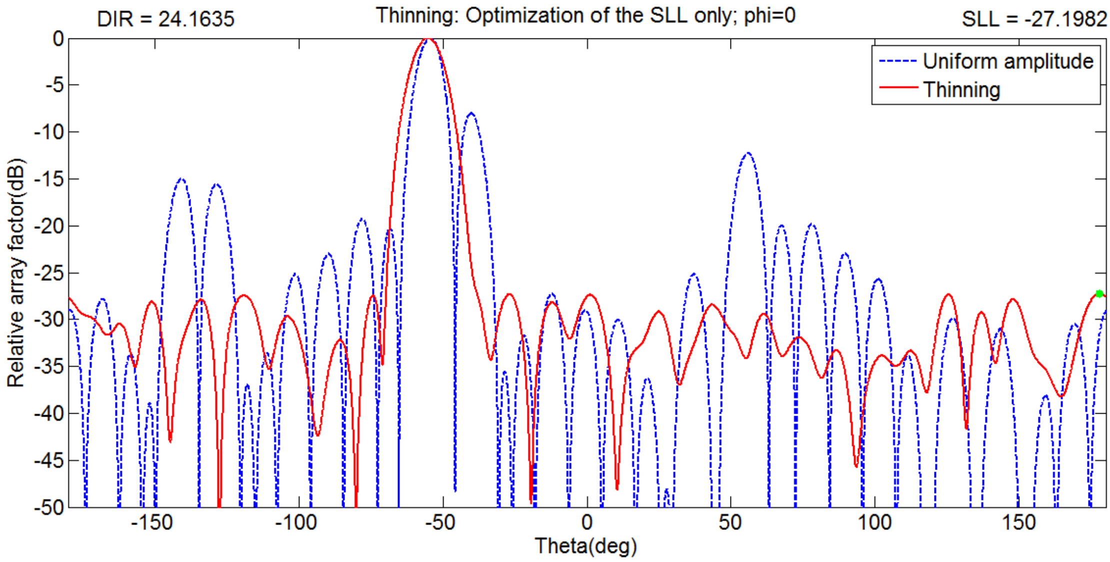

Figure 3. 8 Thinning: SLL Only; phi=0

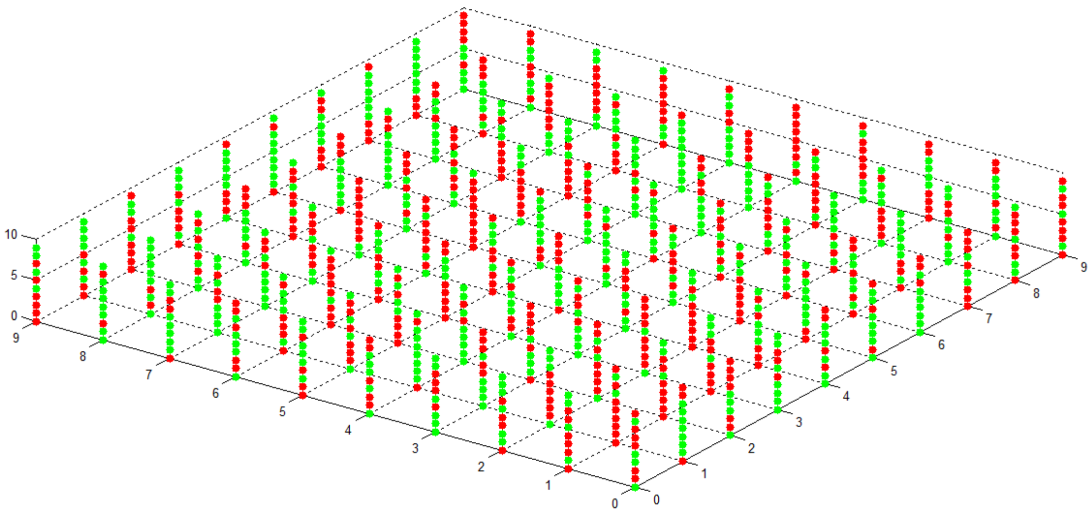

Figure 3. 9 Thinning array: SLL Only; phi=0

IV.1.2. When φ=90

By setting the objective function to f_1 again, and choosing the amplitudes of the excitation current of the elements to be either ‘0’ or ‘1’, we got the relative array factor, after the optimization for φ=90 using the parameters that have been presented in the beginning of this chapter, and it is shown in Figure 3.10. We have reduced the side lobe level from **-6.8876dB** to **-25.3654dB** which is also a better value, but at the same time, the directivity has been reduced from **25.3567dB** to **24.1075dB**.

The remaining elements, that the firefly algorithm have kept on, are **525** out of **1000** elements while the others are turned off. The percentage of the elements that are kept on is **52.5%** and they are distributed as shown in Figure 3.11 by the green color.

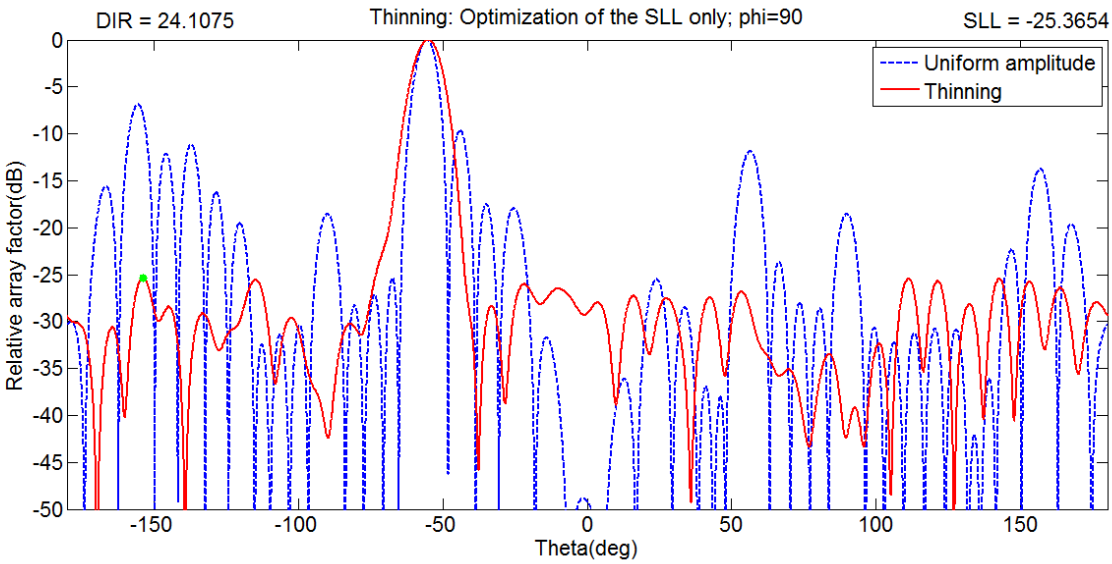

Figure 3. 10 Thinning: SLL Only; phi=90

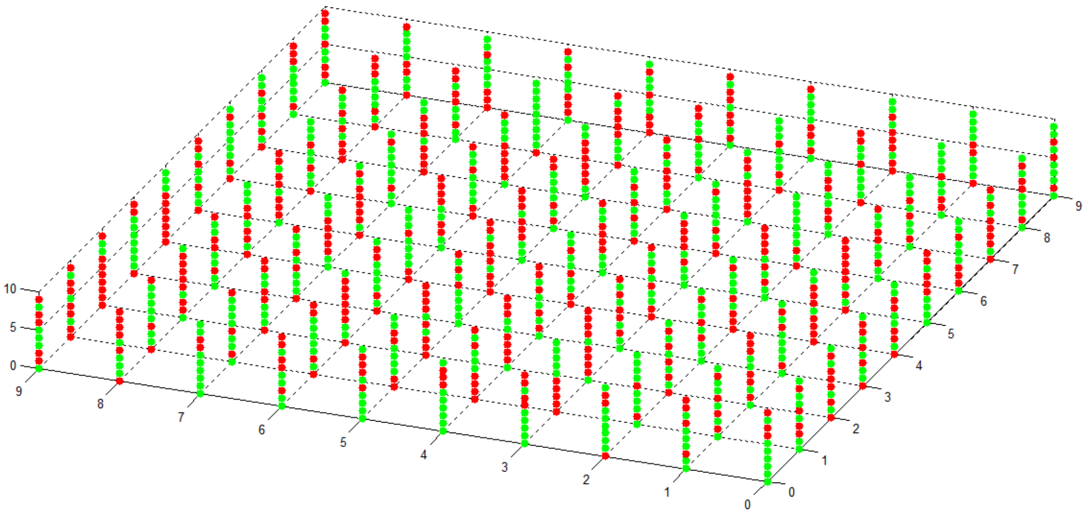

Figure 3. 11 Thinning array: SLL Only; phi=90

IV.2. Optimization of the Side Lobe Level and the Directivity

IV.2.1. When φ=0

By setting the objective function to f_3, which is presented in the equation 3.3, and choosing the amplitudes of the excitation current of the elements to be either ‘0’ or ‘1’, we got the relative array factor, after the optimization for φ=0 using the parameters that have been presented in the beginning of this chapter, and it is shown in Figure 3.12. We have reduced the side lobe level from **-8.0139dB** to **-25.5057dB** which is a better value, but at the same time, the directivity has been reduced from **25.3567dB** to **24.1814dB**.

The remaining elements, that the firefly algorithm have kept on, are **523** out of **1000** elements while the others are turned off. The percentage of the elements that are kept on is **52.3%** and they are distributed as shown in Figure 3.13 by the green color.

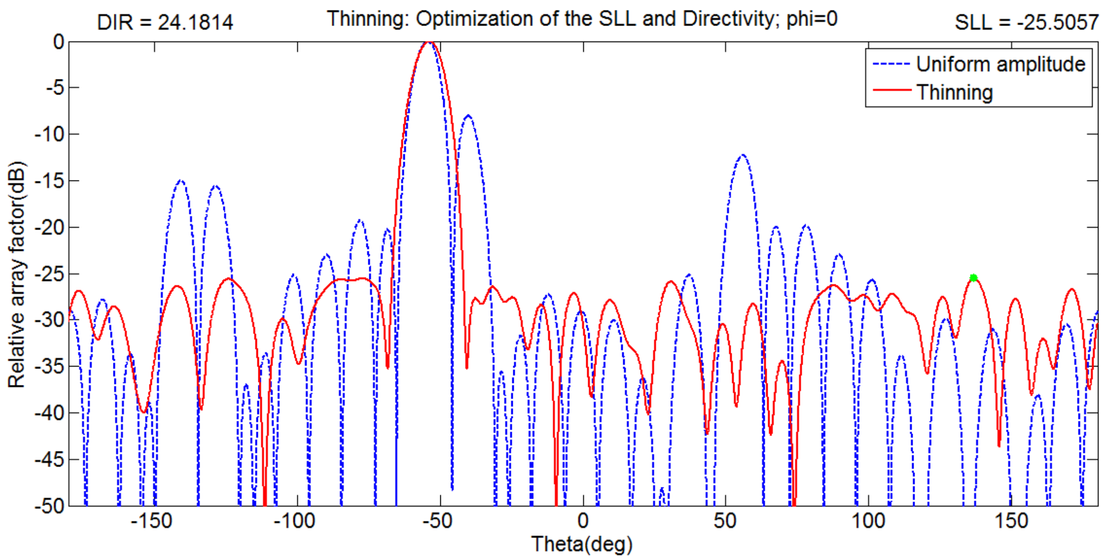

Figure 3. 12 Thinning: SLL and Directivity; phi=0

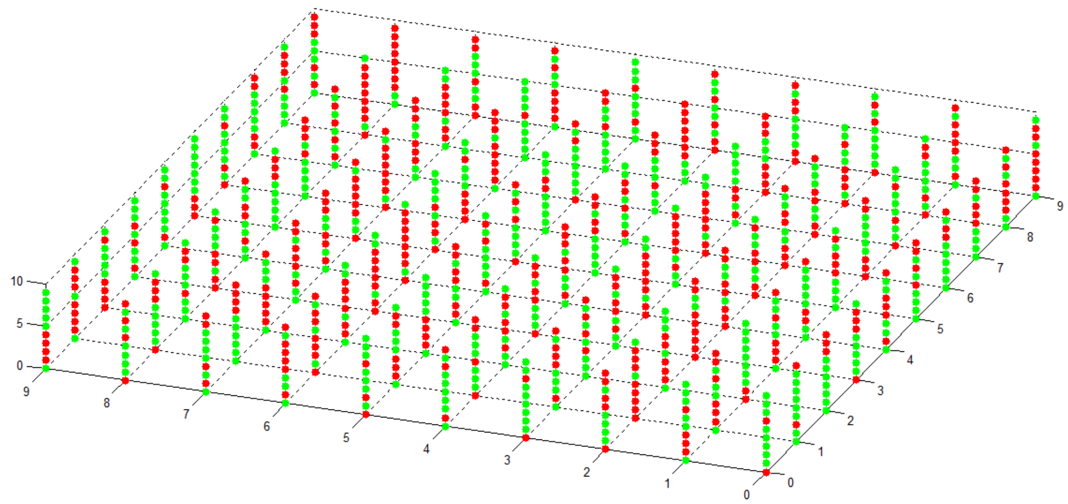

Figure 3. 13 Thinning array: SLL and Directivity; phi=0

IV.2.2. When φ=90

By setting the objective function to f_3 again, and choosing the amplitudes of the excitation current of the elements to be either ‘0’ or ‘1’, we got the relative array factor, after the optimization for φ=90 using the parameters that have been presented in the beginning of this chapter, and it is shown in Figure 3.14. We have reduced the side lobe level from **-6.8876dB** to **-26.0877dB** which is also a better value, but at the same time, the directivity has been reduced from **25.3567dB** to **24.1283dB**.

The remaining elements, that the firefly algorithm have kept on, are **546** out of **1000** elements while the others are turned off. The percentage of the elements that are kept on is **54.6%** and they are distributed as shown in Figure 3.15 by the green color.

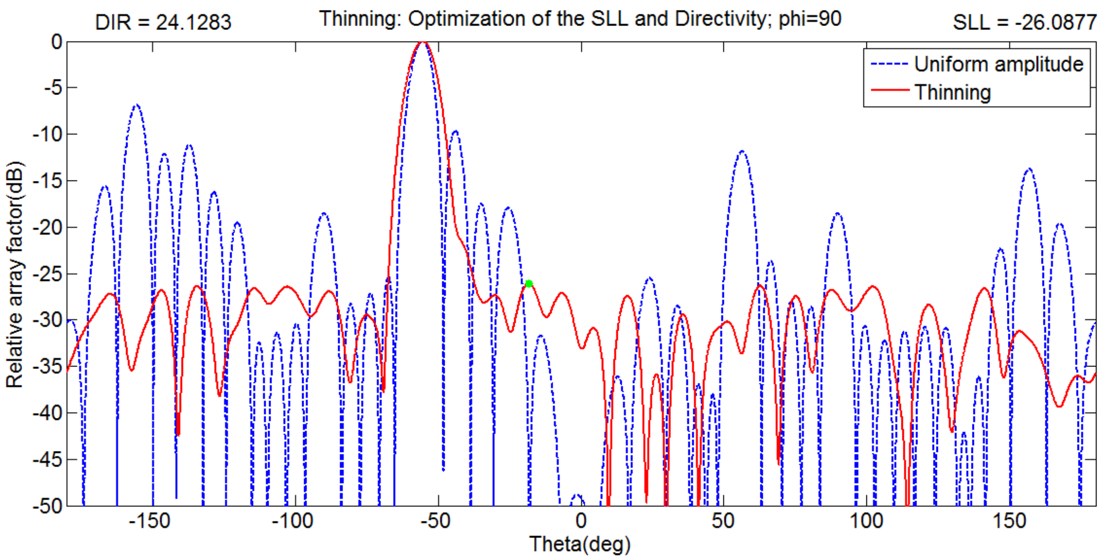

Figure 3. 14 Thinning: SLL and Directivity; phi=90

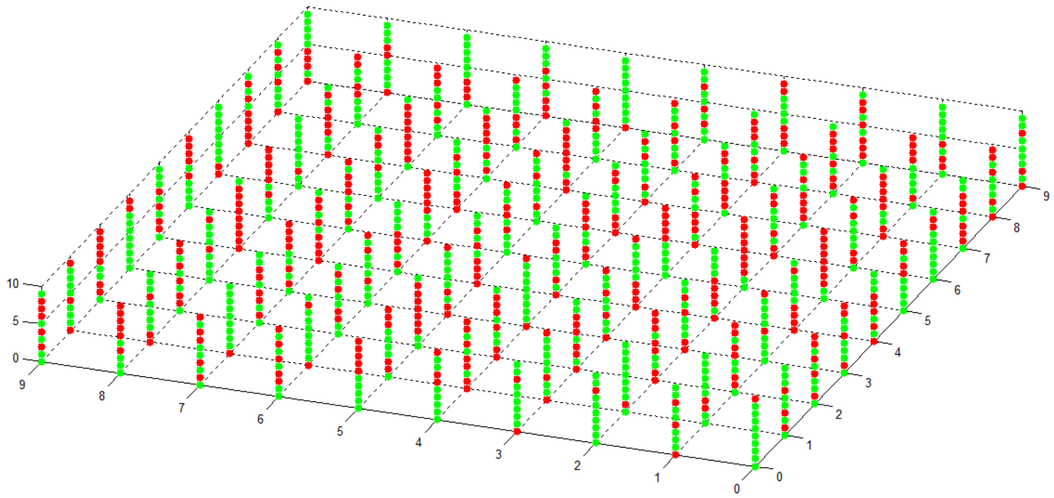

Figure 3. 15 Thinning array: SLL and Directivity; phi=90

V. Comparison

In this chapter, we have optimized the SLL and the directivity of a 10 by 10 by 10 Cubic antenna array using firefly algorithm by varying the amplitude of the excitation current of the elements, and the results are shown in Table 3.1. We can see that the Firefly algorithm have reached satisfying values of sidelobe levels in the amplitude variation part, with an acceptable slight deduction in the directivities. While in the "thinning" part, the results were acceptable especially since about half of the elements have been turned off.

Table 3. 1 The results of the optimization

<table>
<tr><th>The case</th><th>The amplitude</th><th>Parameters to be optimized</th><th>φ</th><th>SLL</th><th>DIR</th><th>Percentage of used elements</th></tr>
<tr><td colspan="3" rowspan="2">Uniform case</td><td>$\varphi = 0$</td><td>-8.0139dB</td><td>25.3567dB</td><td rowspan="6">100%</td></tr>
<tr><td>$\varphi = 90$</td><td>-6.8876dB</td><td>25.3567dB</td></tr>
<tr><td rowspan="8">Non-Uniform case</td><td rowspan="4">Amplitude Variation</td><td rowspan="2">Side Lobe Level only</td><td>$\varphi = 0$</td><td>-37.371dB</td><td>25.1400dB</td></tr>
<tr><td>$\varphi = 90$</td><td>-31.8464dB</td><td>25.0852dB</td></tr>
<tr><td rowspan="2">Side Lobe Level and Directivity</td><td>$\varphi = 0$</td><td>-34.8082dB</td><td>25.2013dB</td></tr>
<tr><td>$\varphi = 90$</td><td>-31.7007dB</td><td>25.0720dB</td></tr>
<tr><td rowspan="4">Thinning</td><td rowspan="2">Side Lobe Level only</td><td>$\varphi = 0$</td><td>-27.1982dB</td><td>24.1635dB</td><td>51.2%</td></tr>
<tr><td>$\varphi = 90$</td><td>-25.3654dB</td><td>24.1075dB</td><td>52.5%</td></tr>
<tr><td rowspan="2">Side Lobe Level and Directivity</td><td>$\varphi = 0$</td><td>-25.5057dB</td><td>24.1814dB</td><td>52.3%</td></tr>
<tr><td>$\varphi = 90$</td><td>-26.0877dB</td><td>24.1283dB</td><td>54.6%</td></tr>
</table>

VI. Conclusion

From what we have seen in this chapter, we can say that the Firefly Algorithm has successfully optimized the sidelobe level in each case while the directivity is approximately the same as the uniform case. The Firefly Algorithm have been proven to be effective in this kind of optimization problems.

General Conclusion

In this project, we used the recently developed biological inspired meta-heuristic algorithm called firefly algorithm (FA) to demonstrate through simulation experiments in the context of the cubic antenna array design problem. We formulated the design problem as an optimization task on the basis of side lobe levels reduction by varying parameters like the amplitude of the excitation current. This is applying to cubic antenna arrays. All these factors together have been considered for optimal results in our design problem and they account for the significance of this work.

The FA algorithm generates the non-uniform excitation amplitude for the cubic arrays in question with a set of dimension (1000), minimum and maximum boundaries ([0 1]), the performance of the antenna arrays was observed and studied in term of side lobe level and directivity, also their subsequence array patterns were generated for observation, from these data it was clearly shown that the optimal design is done by finding optimal excitation currents of the elements of the array. The simulated results reveal that the optimal design offers a considerable SLL reduction along with reduction of Directivity compared to the corresponding uniform array.

The FA algorithm has successfully obtained the minimum SLL and an approximation of the uniform directivity. Furthermore, for a future research, FA can be focused upon exploration of other parameters like gain, beam width, and first null by varying more parameter like spacing, phase shift, and other parameters. It can also be tried with a various number of elements. Furthermore, results generated by the firefly algorithm could be compared with other evolutionary algorithms like PSO and GA, this is because, the firefly algorithm has proved to be a very useful tool finding the best solutions in the analysis and design of antenna systems.

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Appendices

Amplitude Variation: SLL Only; φ=0				
I11=[0.504551460974769	0.448534690156753	0.557584658731151	0.437876367656879	0.454275856999491
0.592137922866744	0.911905287096658	0.375227796743996	0.503699711749731	0.413420506654741
0.595597826325765	0.146643019819884	0.409214406803387	0.273917374811951	0.352908010811130
0.201825557046713	0.164573087632553	0.420843202656202	0.736273626836777	0.521376539847622
0.521221584927468	0.646255296979943	0.431157381879625	0.575698625992865	0.665356735445828
0.682856442930748	0.551014565344244	0.468444331084932	0.433755399067510	0.0901097951543419
0.810920777571503	0.542841138735592	0.347887353743963	0.579510444376833	0.796363565918550
0.423776382329116	0.217975635034302	0.129146593199041	0.464404278560220	0.628100447883776
0.694110623600606	0.440437702927278	0.395454776769399	0.856600575619009	0.187296944820315
0.382239306482435	0.532615148084016	0.290949224178297	0.771860265147973	0.518489010993243
0.340339735199842	0.688616175129123	0.220574457092993	0.360292032266916	0.267896035349079
0.524761809974191	0.373594155009870	0.733093570089236	0.398201275981692	0.450149997937230
0.778581389796497	0.470971182661692	0.237805363401371	0.475620794743139	0.755984497488306
0.222572820015170	0.753024814754807	0.202510365523298	0.181865185876221	0.459725369665109
0.407828079881962	0.741436599702533	0.413743806641435	0.875791480893837	0.616006424343897
0.257075385591706	0.505905239490115	0.381801694655218	0.893202041157803	0.574287022953129
0.613765896556312	0.518916843689365	0.764934217900942	0.643033660634986	0.577379406035825
0.169449950212076	0.565133650014225	0.455078709029536	0.511179986747816	0.354992304114364
0.184429119828261	0.325176872440448	0.423125657540873	0.340496610658302	0.654518089583816
0.178986528468077	0.113420382225965	0.285055397851256	0.353970819674296	0.391926790325723
0.584568986442046	0.494240679489634	0.278946702672158	0.644782350939538	0.839847510293186
0.246938896321890	0.806223700399789	0.678400409864508	0.642887527670655	0.816540084990380
0.706664280929807	0.473495600163686	0.690417566021615	0.513248475569250	0.491379662527907
0.394531811657721	0.618865176164257	0.636210620779045	0.528720402712352	0.170474164428357
0.482717942190160	0.529185946551162	0.573330788328976	0.428882923060149	0.402350982880359
0.160462903112803	0.354066172363238	0.656847070495344	0.468311372748531	0.490863899682501
0.774633364765514	0.621390666921905	0.839562745642866	0.498885830553407	0.924570097974739
0.601544627584435	0.558687205662099	0.463694140317818	0.694376202727006	0.784560575313290
0.188867642425738	0.648111081369454	0.334344262211786	0.327419578793073	0.705433654685153
0.725353650995497	0.729633247178832	0.711882432071077	0.523539645468870	0.536392009532752
0.171881535167438	0.463246713116966	0.460389285491617	0.332117544357010	0.603479356838913
0.445795848715577	0.804829117613380	0.740127578208755	0.672157482130283	0.638879299990688
0.465710592956526	0.557363989454928	0.203145192709398	0.411190447585739	0.346651948510953
0.269932060011907	0.642780390421453	0.248146892355011	0.715003354922016	0.797098364387954
0.606278009485173	0.888089741427491	0.631269600453473	0.333740657774957	0.883875212032783
0.664070568884398	0.789017191119153	0.630240211930700	0.761918743729730	0.632495915095084
0.361817890344057	0.437977201776753	0.818200363968052	0.797385562504687	0.496168096469692
0.580091200378249	0.592382377896049	0.435414984099983	0.709833984579864	0.389446859904827
0.346012326408983	0.300352373308764	0.363078531665428	0.203368429016370	0.174387217891351
0.569415864818416	0.532427393650241	0.523933110024014	0.160969819642160	0.609080648951525
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0.769392718697976	0.265764454495676	0.137499045914675	0.444310681958862	0.252863740247041
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0.572500728371425	0.330333004222004	0.674406109363602	0.675461460898362	0.735926062457133
0.324691085439132	0.353492541915291	0.389310484396714	0.330120391062321	0.589960609570735
0.198231488621555	0.422044013093482	0.491254529713791	0.251217622572475	0.437747167747457
0.557234574288763	0.703104697955007	0.389117096695436	0.638732562396026	0.263483082819602
0.450301989882383	0.444052081090118	0.721713830201476	0.526161783877891	0.758952609078747
0.365850845538059	0.589818375432833	0.616053688297885	0.407368319375733	0.696631062256341
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0.576657102421742	0.509671196204254	0.527443190985504	0.216247294570921	0.544711515381170
0.473806983923865	0.709214075938576	0.897366401178793	0.516858692848640	0.761174165885197
0.474932565140928	0.156633397108867	0.156394567708712	0.776560201202884	0.592991012553897
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0.245402249604623	0.165093584869715	0.545809419714502	0.124741463047795	0.615616003636418
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0.869841035225290	0.686115225654699	0.364073472516941	0.446998018419633	0.287242147055343
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0.628785124458690	0.455501757447739	0.474972260195891	0.518086853982542	0.305881559994726
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0.559472765647606	0.339977501271445	0.191732967586062	0.558068799056689	0.216721249263996
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0.441576495369239	0.566193284133948	0.750755187549039	0.472481430308896	0.308123865417235
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0.788485109362457	0.739116443233114	0.293015272299625	0.804181696422950	0.503542826687493
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0.615834212278334	0.808672619567816	0.405037263963093	0.423721143745362	0.501439617953591
0.894308757789798	0.709479088897493	0.487153209509766	0.430380039141564	0.259692667876154
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0.476335374356144	0.369691974539183	0.447871075427340	0.696776433455406	0.691089404974068
0.776259081445529	0.630322429262995	0.586087663345489	0.422109072685514	0.768025263929509
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0.372839767713522	0.438441697837248	0.682740741106883	0.472011564296151	0.187663884124095
0.164174618531531	0.352517666417013	0.498968572522890	0.627652242967802	0.575753845206701
0.582062333319103	0.679562062654006	0.311393985431093	0.770821861355141	0.411015481703717
0.319537230125829	0.484810750163105	0.437157545138627	0.0829952036572884	0.194491220399170
0.433596971446392	0.335009776506309	0.558993969174157	0.628102908957956	0.836935184319119
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0.470452361808516	0.387512032298915	0.175207244936486	0.742273771289067	0.413554167171888
0.535465185572662	0.414962701767175	0.352332145166703	0.430486160203859	0.458972486063231
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0.155028010776283	0.345791285593570	0.279597641412468	0.533243194625900	0.388121356712793
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0.676602618365301	0.232056358904166	0.377118420194534	0.718762697728315	0.238343146211302
0.580291126831475	0.425979014363892	0.533919544230527	0.743336620003658	0.490775029471072
0.578325493301527	0.285362935827978	0.349532655745480	0.867357758664880	0.317907728614878
0.670878638467100	0.365273562676845	0.717018186671230	0.650605811737593	0.787795055189480
0.785950246657004	0.637440418622816	0.406818725661654	0.624769982489544	0.227912303879449
0.454544640695902	0.523416358245553	0.389868706003093	0.585787810632771	0.873421360562469
0.551586494867909	0.771204989881529	0.429961746621276	0.541286917788918	0.435088431148373
0.544789085274156	0.372990095200697	0.333294780643226	0.168753765181487	0.377598589972874
0.333704297785368	0.397927281548487	0.182190095146613	0.402752762637405	0.403754571663166
0.447916263961324	0.658347169198362	0.533995830902934	0.499470643695964	0.232414288855801
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0.662964972869092	0.466506313779281	0.827240977180851	0.438837062196390	0.350764295137894
0.509841574509899	0.462046662955369	0.443419591420716	0.599543979458032	0.176927415813137
0.436021727073248	0.520635912979249	0.486812680668934	0.632504870738493	0.598249733658801
0.904575559832277	0.773976498740296	0.696308820466813	0.636683847734302	0.502521524244353
0.333401986714425	0.383462705998230	0.487099519628555	0.257850121479245	0.319904336543353
0.527839973130106	0.531409123051784	0.354800895121780	0.554924226740783	0.467935338969539
0.436791203737731	0.455354087899204	0.443104272896132	0.651288447373532	0.759018756184792
0.377900614131006	0.742652862145897	0.296451722137492	0.144412725254832	0.444880507589985
0.572251527143017	0.824360367831990	0.438596571367780	0.640664657779426	0.428371288115217
0.452544706748028	0.604778969546535	0.410524385469092	0.594263271518166	0.378354504774277]

Amplitude Variation: SLL Only; φ=90

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0.720993513234507	0.531596143509954	0.648514302992601	0.497199159003426	0.501687838426937
0.350117593273546	0.514272935960341	0.421017568037547	0.773985358195942	0.502780788428597
0.419857769968652	0.533597343902492	0.419196173566247	0.376015970437293	0.234687361511053
0.253435183573723	0.209990818502784	0.714651776571636	0.684348832798684	0.245519896006785
0.575146889890266	0.313205762829594	0.367613891338450	0.383174117272336	0.364662507600128
0.610156611736966	0.722921448745208	0.546257844826457	0.384425985491784	0.384621265956967
0.703040424527310	0.536815921505429	0.820774593984070	0.551735564943563	0.704458587125231
0.757162320268748	0.494796655185760	0.444274819928124	0.481654209379312	0.340597019472089
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0.171374713777348	0.293275712113724	0.346572804699381	0.176448441558165	0.399845561819756
0.408189636156406	0.510154940921010	0.683106395494420	0.782405684173376	0.565194059903070
0.401407135142296	0.905617198381923	0.0137388371407522	0.489327564183585	0.427486497750438
0.829044210259617	0.517998848042171	0.396588146426301	0.352034091918786	0.524378587186784
0.659140901916619	0.748250594815212	0.243868682508394	0.504680573187130	0.792553974117109
0.285692674389950	0.745100841467416	0.312273523252886	0.693972047177358	0.558429811964579
0.393204653360043	0.754925039075637	0.379183356054767	0.469820225892503	0.552017703513328
0.503383948129739	0.552593165170887	0.578750694983553	0.342686715693049	0.725286710027972
0.377020332252761	0.602724448092912	0.421112742246369	0.610303056153770	0.778552686083506
0.354414553063489	0.185615733534901	0.497692927240481	0.768330599086889	0.457037912983736
0.362584302385934	0.871757781221735	0.690442530091096	0.408135631536861	0.593320534965635
0.353050971284049	0.583095850784815	0.262754386183123	0.606075483202423	0.801311430829764
0.271504005626279	0.568233676125005	0.512507832570672	0.401325803579777	0.739808644992630
0.190527161655254	0.158059233486531	0.415256658541097	0.401688495072914	0.644492441963083
0.322381782822244	0.881501674218111	0.512605136581691	0.532627020091448	0.406440602475454
0.281446971333307	0.603457016827269	0.480745234013231	0.393803466372708	0.901638303233170
0.471370654446716	0.616179525217442	0.524765468346595	0.585335006065544	0.657958569483833
0.898712153181478	0.454377491931298	0.238799052405794	0.449507017103357	0.827235203709436
0.367952156337814	0.462642098064045	0.221009145845315	0.173031356497348	0.426400546170545
0.111670860854321	0.766956659775651	0.311140580328730	0.474073844201996	0.675663908540190]

Amplitude Variation: SLL and Directivity; φ=0

I14=[0.271024060336877	0.469633409167703	0.871616053010124	0.479271568814402	0.457231568951428
0.121572837922170	0.418420599978616	0.590133414834796	0.758821186253386	0.284341468825033
0.273051943074302	0.383744210877728	0.608182968200307	0.257512968134350	0.502589483933576
0.303716495573957	0.516029008267152	0.758648178259171	0.580744099908066	0.443788627883519
0.478121165322520	0.537944185697951	0.275068563089092	0.473807315186358	0.297346018949083
0.518548057236967	0.199734653063495	0.792192138787961	0.400338581430292	0.318663351739570
0.633536051827068	0.524912329159576	0.373492683791269	0.512936666470687	0.793302371054705
0.350238297829139	0.312211302039222	0.183807576793419	0.645827462992910	0.623386437757580
0.159749158417227	0.715044070097267	0.460681907620657	0.334885123634481	0.855855336531256
0.572563933579616	0.314712615708787	0.532284900583608	0.523860588666554	0.340355079994274
0.219257860662770	0.269166654201233	0.498919837992428	0.375497874669048	0.503232990306270
0.410865429492871	0.440863356812646	0.523020799715563	0.555347102883523	0.677634482173603

0.504292460467042	0.876307123488123	0.138897294581549	0.173487208510473	0.349529891072217
0.469740984272546	0.600724704922729	0.193014990427096	0.300523709818926	0.456706699296972
0.670115098468787	0.558281612398269	0.693213998698645	0.643106705145846	0.651933890535307
0.717270764123090	0.345823463679840	0.320608627379114	0.359642278816981	0.217752032797049
0.739142448208211	0.389515977640863	0.396531609761394	0.700096203503931	0.586321429015871
0.723080555568350	0.357217135767078	0.455067165228820	0.791774099898021	0.563781641751314
0.650527831579696	0.271953238633988	0.564215250987278	0.298887484714796	0.662062655719936
0.587050298744066	0.528488576433859	0.665283787851478	0.639880980336957	0.272370531092493
0.313769354384325	0.654011306936814	0.543354028176167	0.301490610235708	0.857221733529911
0.306790586685360	0.625845775889191	0.615193921203657	0.767008876923360	0.355373485244422
0.285417215261115	0.158717514063641	0.471169163112798	0.202648500709838	0.714644164397909
0.620447135029797	0.708932071107470	0.776417155606228	0.428562504277958	0.461364317770650
0.633958117302027	0.491389810428539	0.121852499236239	0.549168620594596	0.319910697149600
0.309385506934179	0.333951423714052	0.374300987918564	0.630738979709093	0.351716662926537
0.534514052499024	0.780656904792271	0.747869872156406	0.386790372191939	0.395705202585515
0.367583835100475	0.195729377660857	0.570842092283292	0.866653450217613	0.335537826734136
0.131288281736731	0.281472448608629	0.603712987230004	0.679499996522511	0.434483034295232
0.669591250633880	0.450181134770909	0.618153215088637	0.626129945917920	0.678441774957009
0.651108747098359	0.851025900360058	0.285119083031139	0.770308147879358	0.376195189608385
0.299844885153444	0.420335125727257	0.476973869742378	0.497557705478108	0.663642025676706
0.634658977408617	0.180716547507491	0.743650724853984	0.292328201281450	0.329737336766891
0.319318389733398	0.261606173191289	0.101683113992168	0.493705021527154	0.729861898816080
0.622269611546300	0.426763664654639	0.809092369473434	0.782898469742862	0.423025609259470
0.304814883422722	0.667680201608995	0.259715720268857	0.532229055118971	0.782695565805727
0.757054942388715	0.296513301439743	0.847770498168270	0.511852000163551	0.264256573111381
0.914398011892237	0.605040108515463	0.751189823822035	0.495393806349238	0.569965715613643
0.378742052307026	0.326407584892736	0.267015065780097	0.651897030432864	0.334744944142745
0.623192954788341	0.503757106461009	0.317851848551239	0.651784822712298	0.884484981341750
0.652386059809340	0.638595839638247	0.823335624866007	0.703678211527651	0.918785514524367
0.719198142430296	0.436545544684081	0.714734501218091	0.239348509409770	0.364467784726090
0.298779967354923	0.436940039036991	0.341082819374419	0.257372358837088	0.554618057302215
0.336919873326244	0.533492122312469	0.376447431584958	0.299033355881401	0.494794294911438
0.629498731697385	0.467493915201557	0.654771913866285	0.293868601093307	0.205559881701790
0.383996286533611	0.550228300692021	0.422750940000436	0.499715692936046	0.416555034209244
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0.657952618517803	0.507560396466059	0.487224962542586	0.360639649607277	0.449571076804578
0.361827313176543	0.789595271516957	0.737396483298976	0.463122430356197	0.488511572310990
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0.121163484991869	0.543986531840754	0.476623949441659	0.383431426940057	0.529333358650442
0.485513255264854	0.409694187210644	0.628968870645267	0.587935722508592	0.281857322289961
0.545633877472468	0.577728603791448	0.391252334636812	0.0975140699383286	0.481857465369516
0.648343600467816	0.613538697271001	0.299370722708496	0.225692000108242	0.566605254680213
0.509552889171269	0.879319507144881	0.237023358304535	0.497907442491971	0.614193333909847
0.459442051415062	0.384149306841687	0.451912012677397	0.290326150581864	0.549985074256698
0.665492812118967	0.634370012217240	0.681866535794637	0.533431394242992	0.837042679107054
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0.300538705213941	0.301048980406705	0.530794896538303	0.418262924455018	0.534060018874301
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0.793044184533035	0.305034711658037	0.445550021824335	0.469509906923913	0.266368925519149
0.408006855053631	0.315383210707333	0.340592655101468	0.900772329617279	0.799617261974786
0.599336684889957	0.272317099232892	0.164335157140466	0.665876319926951	0.758506136540328
0.415519160744641	0.353569574045983	0.416914482143708	0.225724488091502	0.335497680411266
0.379061750339736	0.386591166983719	0.155420389803746	0.494491896017886	0.692241649245458
0.866854296564158	0.614271878945975	0.415650201338569	0.590200634125949	0.641197195376981
0.540361140488077	0.359520987135260	0.278474042732728	0.464441521026429	0.257721646888612
0.596811516307893	0.828811357369587	0.817559204510358	0.152340732946165	0.586122619690783
0.563835145559101	0.359515079080045	0.450269578450910	0.569415898833604	0.702621162642504
0.355139004470127	0.153825662026332	0.544049077949000	0.738897962864509	0.356290755625511
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0.501737577669020	0.598076939962711	0.224423950966598	0.803360953945537	0.349417565936671
0.706190333030110	0.870470283371420	0.797949329264441	0.321785027574712	0.833359473486644
0.692730145861862	0.829698192875979	0.714823597796553	0.649811186076453	0.675502496271067
0.302703930028350	0.621061009744610	0.457498668189466	0.502778743502019	0.801288386869060
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0.682725070942224	0.559426877923391	0.152218169566606	0.139347187368034	0.244316788373080
0.818206751961987	0.421780621844903	0.663044712683417	0.351719762813707	0.789843189683165
0.573273160242970	0.731530787726607	0.604377600674166	0.278573270261893	0.230004072763416
0.603197560838422	0.164601679066850	0.493762724765908	0.918587502629814	0.436051932100546
0.474481119872094	0.617279665529168	0.618319125097687	0.692032911656215	0.447157820933604
0.312130815099668	0.701915616148146	0.270644614790473	0.694581004995142	0.673397196300184
0.622323425518551	0.674430893034197	0.407571507049956	0.450271604950456	0.710713624699955
0.735538489090182	0.779053474875136	0.634029360213906	0.673104761531076	0.572957827719180
0.659789215989078	0.473023356447378	0.233399871219713	0.175132002192401	0.687292152562485
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0.499997702877740	0.790310334956725	0.560985456278075	0.689424043662082	0.662240772548343
0.653946680975761	0.222562592648950	0.679872587824610	0.427748123114988	0.309425754053046
0.315185260621513	0.463901422150439	0.271644259714671	0.154687661555378	0.587655382329722
0.244871816718491	0.625843467145258	0.824563824889937	0.931717472629164	0.115842936413294
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0.565293481430671	0.379417651399344	0.354805284713508	0.437809383464628	0.424514484660904
0.671915735105695	0.632285542620677	0.639408710403004	0.531417953696786	0.778510650427097
0.447216062522377	0.421306491007154	0.809614322896445	0.541111515547690	0.479171458973013
0.659838708497295	0.851680001321714	0.473521286633474	0.0958975245049239	0.242157210709400
0.467288252953862	0.114575875452525	0.744774133910612	0.850442370875993	0.621479998721245
0.631085342812439	0.432940066553204	0.373567367020891	0.419644515624991	0.273651205733875
0.0764708385572431	0.399145650186101	0.580055708230713	0.495910601182450	0.155002881278617
0.227378846531472	0.282654350762539	0.703240533461507	0.737252897660274	0.590466901755821
0.583298428552238	0.362949528255477	0.541782531606668	0.418877609021018	0.312245380411694
0.744729648891475	0.437890548830376	0.641646524042889	0.718293782066036	0.526462444216949
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0.237299961465238	0.284152080710767	0.109818362643649	0.686811160730937	0.480324882570287
0.363599035153331	0.560413059422551	0.170813723044018	0.604125954201087	0.534006890369200
0.720804302780979	0.554217470384638	0.431338275846462	0.603250573066906	0.637457225463486
0.224060609575560	0.189290924780390	0.619241036344248	0.575441268450947	0.282006186385550
0.364585706222729	0.499917072035116	0.502723521289561	0.647642498413008	0.752747007728607
0.739326464322568	0.411779968827684	0.307813286966834	0.116145266244800	0.446867215861937
0.224123808989673	0.478291882140136	0.493254875945656	0.675087754373854	0.774998679915016
0.606879994302004	0.464423973256707	0.973053549575026	0.825077540762807	0.828594739834614
0.296756489722010	0.632219227412286	0.552772719562271	0.734746385039377	0.641129910803816
0.507077622945855	0.494545912877972	0.432147235263802	0.184223969470683	0.392940689564241
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0.288248092357065	0.479715795782525	0.447975926192078	0.721371718783449	0.742703644267977
0.263227496474133	0.289632252629597	0.483929442164360	0.535553162378369	0.410537074747645
0.113007322222004	0.577603611063930	0.511764857802080	0.499627169270577	0.599696757504229
0.573410572247814	0.316672144610860	0.504055169124180	0.323633528515517	0.244033125759626
0.613307876487196	0.266541316841718	0.244394759855361	0.226523962606926	0.240555344428132
0.437248974031212	0.854005167921011	0.118320874525334	0.530450806479396	0.420656217684240
0.226402779739248	0.491124785842307	0.721624858132105	0.298176722714831	0.372211625163351
0.571524494255794	0.236053463166805	0.570678406603610	0.608473284980314	0.238546218420596
0.684449528010427	0.836174296410161	0.296003779038896	0.599244218537652	0.266316981033787

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0.507226675822596	0.471955149437820	0.643729000468574	0.380665586966566	0.453407739520770
0.556174701561958	0.488259384002993	0.611105786840803	0.369247248281542	0.741441621022965
0.510633532612961	0.0335959412226771	0.110793010588915	0.274526692997171	0.257136420631042
0.719030347550423	0.399975246075949	0.419765127436948	0.642101210932731	0.635114135065979
0.630023383589199	0.219590438376500	0.169880256000493	0.223695656811984	0.419751650599688
0.599775453808763	0.330536641050301	0.188718991287308	0.388371207761257	0.629239552720888
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0.793035412099898	0.682954228462567	0.651719559529984	0.191206781363439	0.524851898533975
0.399991426284847	0.692110936584728	0.718003964749667	0.301904841176974	0.761725632711509
0.643352371767224	0.771228938586696	0.604306228450321	0.531466870507663	0.536289836090127
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0.664029886376769	0.865319917796241	0.662649656060320	0.426801591531971	0.269894835721245
0.478500906794144	0.418066075718168	0.201961447635701	0.287478025555056	0.424181027918177
0.637826420627739	0.281566170338210	0.590414154017614	0.346345154558963	0.467240936756906
0.396124156991041	0.259764012133222	0.426989416837387	0.353542321245249	0.946676275577078
0.234250967880444	0.589372180367280	0.654289889569024	0.683434452782676	0.282995620763588
0.555503366853255	0.423506745973092	0.629038640556797	0.751298691743089	0.785925194974091
0.287832700400584	0.497135549124098	0.599204997961822	0.791187788563891	0.546265070194627
0.643899155408995	0.336416052982547	0.528465703758965	0.528084152158659	0.139166810033014
0.373713469446859	0.746645574034881	0.327520992988247	0.580021826582181	0.815687728851091
0.526704849073135	0.383679161824513	0.866161650406152	0.0882229863500374	0.553904318959168
0.444877179596281	0.368272902699386	0.398121101729950	0.392144690580212	0.599443585748329
0.350222279998923	0.627266725823269	0.204740171946638	0.449253460483084	0.439819442763996
0.263892303061648	0.270486367547704	0.503413323950802	0.618893208176878	0.608618149053294
0.741385545332984	0.593740470830585	0.730011332620033	0.649146286195946	0.664109159351090
0.797223799029723	0.614206503524064	0.430721661357675	0.155489413152457	0.734315466317694
0.621599733400687	0.377508415677981	0.633514170038372	0.379371936544577	0.245874954061160
0.301068183328648	0.0847983971951192	0.255195687762763	0.377441905386349	0.354829550505854
0.595610560191600	0.700893971310266	0.567247796446439	0.683218157998150	0.813418062639469
0.785547727655828	0.398953016499769	0.525507008352110	0.686092774430781	0.582746854501144
0.872758656061512	0.428606370989008	0.582231946759324	0.640365622707585	0.664708901070086
0.439821863302409	0.755021254071841	0.731536137258463	0.496131056650348	0.343013101558626
0.466469910683375	0.501447411240801	0.298893093804166	0.603513195038753	0.508121481476963
0.661770602568215	0.460550457134298	0.656357541772914	0.225376383396924	0.475292957424329
0.674487775803256	0.226910766102009	0.550463763743946	0.267116479641590	0.427514569781696
0.406399412060760	0.516777468588665	0.186016229632726	0.382738080976950	0.594876354238254
0.851407546616670	0.585965536705982	0.441262255635430	0.227251691937208	0.555127394933673
0.269836138661291	0.445925639244935	0.598409258196072	0.469993153639264	0.754233649958984
0.762000251136227	0.605240085907887	0.574569460004724	0.518529697885820	0.420700205621477
0.573579122551180	0.728866911319369	0.385221744548512	0.295090018252949	0.688088371867377
0.283843948852017	0.579796160847575	0.336707604885641	0.376153539362020	0.127049741961619
0.397993173139561	0.382192160568547	0.421539347429858	0.237928723716636	0.537513209265721
0.472712238530464	0.357123693137488	0.466501370794571	0.359869011938076	0.726915559909686
0.456198918296794	0.558225889111705	0.296360355849149	0.254740556531791	0.204159679845002
0.254191776434433	0.586730144735863	0.618007246525912	0.299184791241443	0.755335236861362
0.861778391019748	0.458399243347626	0.739038073937119	0.305451495254666	0.350396033200010
0.353483778072547	0.384249578919706	0.668695422169312	0.875961544746688	0.741426714703898
0.570929397615347	0.262562863281290	0.459255393640870	0.148992761331009	0.357500883888220
0.635882787728937	0.637337367579255	0.509107624333459	0.448815815466901	0.216589039004812
0.514402238398948	0.576831288904275	0.255725851321206	0.764137960361639	0.591027268355133
0.509451552863591	0.497581089118707	0.376438257119086	0.222593932857022	0.447171169343346
0.950724684326491	0.706747569310215	0.403264683768178	0.572050113043898	0.385496826884329
0.424107070927994	0.545187972619578	0.506215567852018	0.626080873504672	0.417959099118380
0.582224941234636	0.530348570546413	0.379595586411972	0.552733900186463	0.829110580563266
0.486899468438452	0.557991400091655	0.296925097256429	0.280138428355188	0.620014981296647
0.294359335534196	0.328663041415075	0.402394733745802	0.430043287294594	0.250400772444606
0.553493660557870	0.728069468933165	0.449448528586469	0.743590685324854	0.213209818712102
0.549161546431703	0.572897931345870	0.580543756521122	0.433239241545063	0.588102394350692

0.507758148759300	0.414569190764795	0.216168995821230	0.689105436119191	0.755480818270854
0.533888809838012	0.351758048580151	0.357622585034914	0.431652096556615	0.866119107371199
0.232659972185859	0.582927183454006	0.319286710339306	0.272722207825139	0.660769236859663
0.366302296418543	0.753482143701173	0.338438816644538	0.771402725969563	0.556801643145874
0.506805478629359	0.550791759959914	0.437185458774873	0.457891257587833	0.467504326906109
0.161373510444296	0.153139679362940	0.688757555399109	0.444743662822590	0.721787911036836
0.688391082253343	0.491709770854881	0.762756451249503	0.561962402476456	0.548048479573162
0.378585858986485	0.666485265710338	0.379294022195862	0.367907259644449	0.183427683311798]

Amplitude Variation: SLL and Directivity; φ=90

I15=[0.686815533847806	0.346877165333556	0.398313094502469	0.304211268919975	0.250901765799589
0.342572519605973	0.306743082480725	0.291109559642151	0.396318291555161	0.0620700649358482
0.610713265526414	0.422212121046245	0.273066327586583	0.303567696375086	0.222636437369962
0.581188353314711	0.301385163561537	0.248850079567082	0.803687344043495	0.490223915964346
0.486877880000050	0.391861803994654	0.723892104462166	0.467456299349183	0.367360981500702
0.304869195247407	0.793085747068491	0.472127690248269	0.676207884808621	0.612265848237740
0.679273436996877	0.417084686298627	0.411592610992793	0.725288248704011	0.507336302419184
0.470887543885885	0.293237190303058	0.0186633241842520	0.608626984055455	0.337287624920532
0.770696826075940	0.220378600191238	0.191819829293203	0.598956350013439	0.390860499451761
0.475456257588545	0.580189737530847	0.172150393787621	0.772617294646519	0.464744027823662
0.501058747744238	0.425261553193294	0.173548347102657	0.351923308421593	0.549074129378532
0.747386084182978	0.710640905952368	0.582943078253838	0.396376240571350	0.291626101235067
0.467681883359960	0.185495090633108	0.299253058769561	0.585635824341720	0.612879498132417
0.485258974485242	0.755818383023897	0.448499887286394	0.325603321304657	0.254187752257675
0.262316593099427	0.618482552924395	0.426382805631998	0.866276567966463	0.511399018206707
0.528830250162374	0.534332611270417	0.359731738539089	0.714184925713548	0.395817744948657
0.754063215025997	0.413101353520840	0.466514978467490	0.784012502815730	0.364794119112342
0.375338918185843	0.439953857694468	0.176377183004705	0.340240496172151	0.505773471416064
0.257584833168015	0.439641657178831	0.234665468761882	0.580617480950686	0.458691452288762
0.445461984786231	0.177102121638871	0.232625689255904	0.523346807987559	0.608745602006274
0.623263811344947	0.481251265892777	0.402970054543165	0.318798480136995	0.200441382139760
0.297427843861482	0.334916995912146	0.340512888027632	0.580712960332292	0.330544370695770
0.837047823588303	0.376183275250709	0.692256196244238	0.663933030875249	0.460055350292178
0.394749133307152	0.709170677248155	0.791048237945855	0.568591593987420	0.0877226761099125
0.673346276168477	0.499623891875773	0.500303920883255	0.457103863225099	0.512305553572116
0.212223158563824	0.711585904502979	0.832432983768405	0.385031690789527	0.260919073590491
0.803447645144056	0.729514675313645	0.684673102215941	0.301695720213991	0.660635442812378
0.673914574888010	0.458273046137758	0.423393836277753	0.746709961029633	0.337050655234534
0.719017341728343	0.346653495480424	0.488934332204615	0.329778571449659	0.360473203174017
0.559343949750447	0.777387952770616	0.537214996520313	0.646208534406505	0.507276426583181
0.567315393944734	0.281352151548426	0.360660076506298	0.309216757520940	0.611357636024645
0.328808932178903	0.778707132784180	0.680630059596392	0.786500688634807	0.327963530717304
0.299664884545689	0.484530546888577	0.330405383464451	0.668152163655627	0.500964918770634
0.684802632635757	0.617182351740554	0.475466285565020	0.403447156539794	0.880755553944322
0.476929538464601	0.617030293389321	0.510798725460136	0.446737770270435	0.595788652588583
0.719537280878002	0.630055879466775	0.272080764218277	0.726253604472109	0.378254740698115
0.594359844936935	0.333869542716636	0.575809404345129	0.764446690225386	0.298081548413151
0.358868720843186	0.523365733681343	0.436315099230915	0.784696512041379	0.360181563815792
0.723090755951587	0.219844801350318	0.451240390505715	0.508303517954074	0.155324359473963
0.767077641892831	0.342325575626426	0.477540848255608	0.381576928029621	0.483046735220650
0.529459705711804	0.288873815839700	0.398509459645089	0.868289334029926	0.391408968765953
0.717295146778392	0.358104870362308	0.238497012991772	0.499370388715563	0.221971212618201
0.327268832772650	0.723900075894567	0.229159335058587	0.349426797132659	0.261661448687266
0.624544732131717	0.441292721066335	0.547355820206400	0.840616382434844	0.703258018843152
0.546067177950795	0.292649861601442	0.833038788462245	0.553272614482997	0.418717498073050
0.573482636052841	0.611777098310727	0.528011157873263	0.364840132495912	0.737600895944073
0.456947658185603	0.503912823251141	0.269475630444869	0.548207213455433	0.402669795974868
0.450589517571601	0.607917245796046	0.417753489951370	0.652748395156848	0.752549744781125
0.610055020992284	0.457583536289997	0.287133395095371	0.728070869819421	0.275714972854952
0.334007053420894	0.315996483686069	0.190873662419116	0.497697541771149	0.464904890055555

0.937818619530014	0.262356306488167	0.444217052394504	0.518820108199939	0.157063817645174
0.564877572116724	0.669593941659658	0.472213876173826	0.831854800041190	0.840782446151774
0.502141285386554	0.195396584978463	0.351022369888288	0.800462804373223	0.758397808210725
0.363070758130216	0.564986559052178	0.483314724065591	0.852529128167768	0.415328614569846
0.714910875574774	0.878867909097478	0.432026229839239	0.778157800549147	0.354292942534838
0.259321195824682	0.851101609926598	0.396285399842763	0.792796696026455	0.431625678054689
0.489416490729624	0.575974495553100	0.205102804511427	0.574706556165031	0.327454855741415
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0.637348074796738	0.605082571188323	0.366020233887246	0.647030856671494	0.326637013407159
0.758330680549896	0.294255862189988	0.483444411787507	0.258938581908955	0.553186647047314
0.854864081359475	0.608039320398061	0.158600013856886	0.668107344108879	0.461890551521739
0.613019513730291	0.667527820325016	0.186288552782022	0.591686343352716	0.418963195564528
0.602547946921400	0.445428611278749	0.364559639204401	0.471333446364966	0.535616013215394
0.566959413749773	0.965160494727345	0.232229291468767	0.300551706714286	0.349498958578149
0.348243374986459	0.412551566852806	0.422792069846817	0.391913948702305	0.530650523252074
0.589630540510645	0.604599201980405	0.596334343408233	0.841572934254723	0.570184974942668
0.500842881577301	0.686480632450023	0.612505972000894	0.531048797291040	0.402477863246198
0.623350126872701	0.914526542738038	0.259611050660397	0.505166281522198	0.585617092646227
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0.695748665361584	0.799080824823359	0.467976101865654	0.804282448101234	0.236376137688823
0.740913829563771	0.797646175173460	0.423356908281158	0.622649081571045	0.514536611502726
0.904733141140793	0.655269892433241	0.0355851935851620	0.548804789069514	0.458045896459716
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0.466271357735814	0.674242060840615	0.557829771433610	0.299791555952873	0.286810515322139
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0.841364210887815	0.397906038452831	0.521558943309531	0.741804044018695	0.773052867272036
0.359501944394332	0.217351358972581	0.183375040191461	0.790333439120209	0.572123257203025
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0.811067306281673	0.854284002087425	0.563812745683871	0.495711231631037	0.633503802767193
0.775822671234265	0.506049441965643	0.703637673579943	0.238020339077572	0.402964221685851
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0.383802917014123	0.708168455740246	0.619569314997271	0.472383492470430	0.251337388281286
0.505478837931038	0.568141834843854	0.440371620695750	0.352139533424756	0.448007394062010
0.685975959586034	0.861335501401714	0.490742507607818	0.337742277580466	0.609801905982616
0.643399302241800	0.350477594906336	0.587160931394909	0.592094436773796	0.594728188209120
0.478040418306472	0.562186268603046	0.219340998567941	0.333799249066664	0.420484270664328
0.376022447338723	0.854421335957087	0.238555003735856	0.888383035478330	0.473684175071678
0.576873347467467	0.701759199840041	0.433347647507468	0.850102983826905	0.183700619728877
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0.553630031105215	0.682108535043957	0.292877834142924	0.749268583124381	0.590315045239928
0.663062578495010	0.658012848595757	0.483383213532293	0.662543296213492	0.159205152947243
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0.305343864131869	0.704429532440792	0.361625147808371	0.756421039199295	0.661058804539010
0.648095525783223	0.757223748153883	0.173801038655324	0.707414128755934	0.190238939973613
0.446116447006676	0.594667045736030	0.334287603781515	0.727924117265098	0.470231589798234
0.284291521556001	0.690833443906890	0.460084879900954	0.685408904051465	0.759116243700664
0.672825068058575	0.774924954183850	0.237988397388018	0.689223124750470	0.480422235758486
0.810725643323478	0.280440852177039	0.595388717037151	0.156637389653464	0.461990391311830
0.446744290007465	0.532150545875711	0.165771637310346	0.264171463874292	0.429048650918716
0.740220788337153	0.700337895607249	0.284020350042775	0.626002437433076	0.352623971315340
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0.734429533638615	0.680151482959933	0.253990678830980	0.526839008179653	0.616012130597451
0.780942857750963	0.868421734235135	0.140653367695118	0.525951540111607	0.800936878164272
0.438264798013313	0.819901331521618	0.538054276716180	0.762251171225226	0.344229689606441

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0.687284260037856	0.924678595575151	0.199069757706314	0.877640550589927	0.285991869316347
0.383465508290926	0.779537767344860	0.829588035695451	0.808317157740878	0.295507160522036
0.725018350993341	0.423675306467226	0.145978970904303	0.852252304935790	0.731933256848970
0.0752596365399235	0.168089127819425	0.742945322500259	0.391323035537277	0.374673039602011
0.623601877209377	0.612071582285036	0.725673429150947	0.918603273417690	0.552063236970805
0.249153726885426	0.820256832293304	0.171683305660358	0.647787286891364	0.452406607772242
0.518449029548593	0.498831168009759	0.295869541574987	0.400246899854740	0.440834596295590
0.287910339056362	0.673887243312163	0.281371837925433	0.799255542064586	0.707628407673494
0.347987880533460	0.781199857540469	0.261536896002982	0.706383329048600	0.716735454708336
0.511649780695828	0.563741278428607	0.446910826280298	0.275338052314668	0.603992539220210
0.429607655781519	0.646324315237269	0.177569368601488	0.244417957948782	0.223104611785413
0.300939493198455	0.683706800620537	0.584589264110201	0.639474135048359	0.642242616839654
0.423576806396131	0.806962926444178	0.442608783477619	0.367649075067917	0.806557637767136
0.296025713003049	0.208979463477120	0.797558997282915	0.457009367046724	0.522045242645094
0.359662276913900	0.782378867089012	0.278361178823098	0.201852089493709	0.473452670605257
0.716966296340033	0.750488368990901	0.147529765986039	0.488874866811020	0.331515156349567
0.609817114359764	0.669474963742318	0.0932320757408774	0.347877651107838	0.356323297784851
0.625488177912979	0.204841384409492	0.119979896886029	0.318463207376494	0.672085928030524
0.270774419840945	0.789062991088232	0.458209158581254	0.751853906012250	0.536587311640023
0.427801304260556	0.556636266932924	0.416892982650845	0.419587860151301	0.513168824653331
0.600918132106227	0.686438066970778	0.240942247739779	0.990636600561807	0.371114614557004
0.404456461933256	0.287595753441111	0.390117356075159	0.685969798584956	0.739970786769452
0.159625763457966	0.834893652729118	0.697139059664761	0.622906970050055	0.457090002281002
0.548460899804237	0.939310570229040	0.368066740233029	0.258387227859244	0.452438068201937
0.912027399465519	0.689057317788619	0.410606517186245	0.654403750654180	0.563790776604735
0.697230650584393	0.546781293500659	0.298699994117128	0.676694216389691	0.650750215199624
0.497618476170094	0.897600835293266	0.400447186546775	0.206129696999606	0.399073871427125
0.694485370416055	0.680214789640502	0.373993081612976	0.705537139600902	0.523551568628076
0.771008447797502	0.809186323065030	0.142821097688666	0.460005031367658	0.592772680795340
0.378586727646083	0.722228638428386	0.409744413652714	0.580188678938463	0.772023492604150
0.265042970195429	0.294481410200360	0.609234971440788	0.662013296623400	0.219512884016456
0.563172098849111	0.535732950197847	0.495849607148580	0.413645362665329	0.707970795377260
0.658796846756415	0.868569998403694	0.532505909148961	0.294631959722057	0.440855536594200
0.700623287695659	0.899009717995420	0.638980373269978	0.510116238151017	0.754800839074850
0.362941800322623	0.613971937583605	0.455328710816367	0.398860415254988	0.299677103678710
0.349926203571186	0.665515893541384	0.807709098145948	0.386445596598356	0.482553917635899
0.302940342970779	0.667435641997044	0.719494797342719	0.778144980793119	0.584989388637774
0.219206135015309	0.841348887018018	0.761363239984066	0.136449725036139	0.751928160508835
0.108794423366238	0.433170372357996	0.680098865929786	0.600418638909473	0.686598428186602
0.435795333480192	0.475121412128990	0.367881996974318	0.334946011502860	0.577514283975059
0.728416957489956	0.544345200123326	0.494378123991384	0.614482302899510	0.863067027069885
0.307205343780210	0.596014713754635	0.648407513091387	0.405235044061558	0.215270098486903
0.287882321370967	0.576455000090070	0.448414298646447	0.448371146270009	0.818903436161960
0.255704950770114	0.585989751948092	0.441687420603241	0.244024759203816	0.488528523456006
0.622592972705511	0.774730146350601	0.814189466261770	0.190836102978925	0.389986189217063
0.307837973459955	0.408552797349138	0.838013318256463	0.768288750227461	0.969223644715998
0.315056433180043	0.847774210290264	0.341498524130895	0.425042627902415	0.721897062941001
0.211767169937793	0.298752429313045	0.699578075833530	0.364942995363196	0.271089234810043
0.259613411754676	0.761775028484067	0.185393320114363	0.753337763244753	0.582150884141660
0.542301538078522	0.749562122824702	0.451125710345977	0.352464387604219	0.920515066268654
0.676539878386306	0.794473563822225	0.196096725214840	0.506364155528153	0.478936601202193
0.337814644475918	0.372848549408212	0.447267948960201	0.150172173517498	0.573205030441164
0.385738376474918	0.458700152084445	0.174126382024828	0.474811621374691	0.452908809127453
0.572081072782075	0.417536442817298	0.413545826625202	0.388933179657693	0.627944231730584
0.231044537684556	0.364636715703824	0.0268812057078234	0.745697524679806	0.684097992738741
0.331011754947710	0.487758775884675	0.759216493094288	0.445898238100099	0.746147688084278
0.534110254374239	0.420580582415791	0.469636583814096	0.580855847835772	0.366033912289166
0.312492793968155	0.629024070965044	0.565072830743920	0.249694573700764	0.600807565525958

0.817003890172032	0.756742692314144	0.174025645458175	0.130633065693849	0.895429234970413
0.153437823793083	0.807686031112376	0.498134782178182	0.653400964316981	0.799091650589559
0.283367180836285	0.503743639589334	0.358358185765283	0.287707200297821	0.501727164516793
0.480721774928491	0.657933333550338	0.737067421337606	0.488744983297638	0.506726378539519
0.576054059648735	0.573689326545834	0.714651748060753	0.188020183658773	0.712608769043285
0.368291797157343	0.426284234140893	0.626167049701967	0.272060803293544	0.629217557728778
0.107269045776743	0.617378310536785	0.409358674977989	0.0633003178210386	0.645857685839775
0.298217425196861	0.499530180400677	0.934904125798918	0.304783719107001	0.281708605874601
0.449541755156582	0.700450785061204	0.299283844735665	0.237327071975794	0.701030307219412
0.534358169424997	0.607542640913982	0.345178192840644	0.386621177030867	0.680472710787127
0.350237289619634	0.543728682817218	0.199906362615910	0.633045426658371	0.602831387059674
0.399273648829263	0.0958753298127665	0.552150383020145	0.455755899981585	0.411767063445332
0.701666720442644	0.389287153906000	0.750997900988925	0.629531800612580	0.580494373966082
0.526634249043186	0.505250549094345	0.348242652707453	0.253000586810333	0.530870469106264
0.359109595717280	0.588495964545677	0.411407471271521	0.621924714419062	0.825159035050448
0.538209831467412	0.470967332751771	0.485897645987574	0.391814258804568	0.458894212261965
0.475041723790998	0.762886261099502	0.186582051457202	0.100497848344401	0.536342430388391
0.283745412026818	0.347634871084844	0.445065504810534	0.266783950936893	0.667158165039518
0.139347520508908	0.624204418579334	0.868618258030945	0.293232889310171	0.631623525566505
0.600150198208300	0.401556323493489	0.323495655808138	0.350466774729311	0.574121092919749
0.388708225823787	0.521767703145931	0.548618856173093	0.447577797825036	0.685844698726600
0.204071892420525	0.216170083374696	0.538778596250192	0.154373846621816	0.562446506810952
0.154148975267563	0.734787189746190	0.548389661723863	0.418587585009582	0.583450529510561
0.767396360683644	0.784765770575044	0.651690709146615	0.432287782297277	0.705436600317903
0.288561655710040	0.570560374173181	0.531774259475590	0.599570470590221	0.348483272511464
0.352562828925405	0.483345479616155	0.454724476553290	0.369056435305708	0.800794569740884
0.467350391840943	0.665406282683095	0.485369788218337	0.660305960865782	0.724997460028400
0.576283026439533	0.874452829239692	0.765363482272822	0.192075623621967	0.588006346886301
0.440287780311595	0.437368896553446	0.256921416883337	0.271124674733128	0.386910347682482
0.242235519406358	0.892044474708797	0.493857305395846	0.418105664728987	0.715255841986454]

Thinning: SLL Only; φ=0

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0 0 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1 0 1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 1 1 1 0 1
1 0 1 1 1 1 0 0 0 0 1 0 1 0 0 0 1 0 1 0 1 0 1 1 0 1 0 0 0 0 0 0 1 0 0 1 1 1 0 0 0 0 0 1 0 0 0 1 1 1 1 0 0 0 1 0 1 0 1 1 0 1
0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 0 0 0 1 1 0 0 0 0 1 1 1 1 0 0 0 0 0 1 0 0 1 0 0 0 1 1 1 0 1 0 1 0 0 0 1 0 0 0 1 1 1 1 1 1 1
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0 1 1 1 0 0 1 0 0 0 0 1 0 1 1 1 1 1 1 1 1 0 0 0 0 0 1 1 1 1 0 1 1 1 0 0 1 1 1 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 0 0 1 1
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1 0 1 0 0 0 0 1 1 1 1 1 1 0 1 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 0 0 1 1 0 1 1 1 0 1 1 0 0 1 0 1 0 0 0 0 1 0 0 1 1 0 0 1 1 1 0
0 1 1 0 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 0 0 1 0 0 1 1 1 0 0 0 0 0 0 1 1 1 1 1 0 1 1 1 1 0 0 1 1 1 1 1 0 0 0 0 0 1 0 1 1 0 1
1 0 1 0 0 1 1 0 0 0 0 1 1 1 1 0 1 1 1 1 1 0 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 1 0 1 0 1 1 1 1 1 0 1 1 0 1 0 0
1 0 1 0 0 0 1 1 1 1 1 0 0 0 1 1 0 1 1 0 0 1 0 0 1 0 0 1 0 0 1 1 1 1 1 1 1 0 0 1 1 0 1 0 0 0 0 1 0 0 0 1 1 1 1 1 1 0 0 0 1 0
0 1 1 0 0 1 1 0 0 0 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 1 1 0 0 0 0 1 0 0 0 0 1 0 1 0 1 1 1 1 0 1 1 0 1 1 0 0 1 1 1
1 1 0 1 0 0 0 1 0 0 0 0 0 0 1 1 0 0 1 1 0 0 0 1 1 1 0 0 0 0 1 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 1 1 0 1 0 1 0 0
0 1 1 0 1 0 1 1 0 1 0**]**

Thinning: SLL Only; φ=90

I22=[1 1 0 0 0 1 1 0 1 1 1 1 1 1 0 0 1 1 1 0 0 1 0 1 0 1 0 1 0 0 1 0 1 1 0 1 0 1 0 1 1 0 1 1 0 1 0 0 1 1 1 0 0 1 0 1 1 1 0
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0 1 1 0 1 1 1 1 0 1 0 0 1 0 1 0 0 1 0 1 0 0 0 0 1 1 1 0 0 1 0 0 1 0 1 0 1 1 0 1 1 1 1 0 1 0 0 1 0 1 0 1 1 1 1 1 0 1 0 1 1 0
1 1 0 1 0 1 1 0 0 0 1 0 1 1 1 0 0 1 0 0 1 0 1 0 0 1 1 0 1 1 1 0 0 0 1 0 1 1 1 0 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 1 1 0 0 1 1
1 1 1 1 1 1 1 0 1 0 1 1 0 1 0 0 0 1 0 1 0 1 0 1 0 1 1 1 1 0 0 0 0 0 0 1 1 0 0 0 0 1 0 0 1 0 1 0 1 0 1 0 0 1 0 1 0 1 0 0 1 0
0 1 1 1 0 0 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 0 1 0 1 0 0 1 1 0 0 0 1 0 1 0 0 0 1 0 0 0 1 0 1 1 1 1 1 0 0 0 1 1 1 1 0 1 1 0 1 0 1
0 1 1 1 1 0 0 1 0 1 1 1 0 1 1 0 0 0 1 1 0 1 0 0 1 0 0 1 1 1 1 1 1 0 1 1 0 1 0 1 1 1 1 0 1 1 0 1 0 1 0 1 1 0 1 0 0 1 1 1 0 1
0 0 0 0 1 1 0 1 0 1 1 0 1 0 0 1 0 1 0 0 1 0 1 0 0 1 0 0 1 1 1 0 1 1 1 1 0 1 1 1 0 0 0 0 0 1 1 1 1 0 1 0 1 1 1 1 0 1 0 1 0 0
1 0 1 1 0 1 1 1 0 0 0 1 0 1 0 0 0 1 0 1 1 0 0 0 1 0 0 1 0 1 0 1 1 1 1 0 0 0 1 1 0 1 0 1 0 1 1 1 1 1 1 1 0 0 0 0 1 1 1 1 0 0
0 1 0 0 0 0 1 0 1 1 1 1 0 1 1 1 1 0 1 0 0 1 0 0 1 1 1 0 0 0 0 1 1 1 1 1 1 0 1 0 1 1 1 1 0 0 1 0 1 0 0 1 1 0 1 0 1 1 0 0 0 0

0 1 1 1 1 0 1 1 0 1 0 1 1 0 1 0 1 0 0 1 1 0 1 0 1 1 0 1 0 1 1 1 0 0 0 0 1 0 1 1 0 0 1 0 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0 1 0 0
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0 0 0 0 0 1 0 1 1 0 0**]**

Thinning: SLL and Directivity; φ=0

I24=[0 1 1 0 1 1 1 1 1 1 1 1 0 1 1 1 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 1 1 1 0 1 0 1 0 0 1 0 1 0 1 1 1 1 1 0 0 1 1 1 1 0 1 0 1
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0 1 1 1 0 1 1 0 0 0 0 0 0 1 0 1 0 1 0 1 1 1 1 1 1 0 0 0 0 1 0 0 1 0 1 0 1 1 0 0 0 0 1 0 1 0 1 1 1 1 0 1 0 1 1 0 1 1 1 0 1 1
1 1 1 1 1 0 0 0 0 1 1 1 1 1 0 1 0 0 1 0 0 0 0 1 0 1 1 0 0 1 1 0 0 0 1 1 1 1 1 0 1 1 1 0 0 0 0 0 1 1 0 0 0 1 1 1 1 1 1 1 1 1
1 0 0 0 0 1 0 0 1 1 0 1 1 1 0 0 0 1 1 1 1 0 1 1 1 0 1 0 1 0 0 0 1 1 1 1 1 1 1 1 0 0 0 1 0 0 1 1 1 1 1 1 1 1 0 1 0 0 0 0 0 1
0 0 1 0 1 0 0 0 0 1 1 1 1 0 0 1 1 1 0 0 0 0 0 0 1 1 1 1 1 1 0 1 1 1 1 0 0 1 0 0 0 1 1 1 1 1 0 1 1 1 0 1 0 1 0 0 0 1 0 0 1 1
1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 1 0 1 0 1 1 1 0 1 1 1 1 1 1 1 0 0 0 0 0 1 1 0 1 0 1 1 0 0 0 0 0 0 0 0 1 1 1 0 1 1 1 1 0 0 0 0 1
0 1 1 1 1 0 1 1 1 1 1 0 1 1 0 0 0 1 1 1 1 1 0 1 1 1 0 0 1 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 1 1 0 1 1 1 1 0 1
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0 1 1 1 1 1 1 1 0 1 0**]**

Thinning: SLL and Directivity; φ=90

I25=[1 1 1 1 0 1 1 1 0 0 0 1 1 1 0 1 1 0 1 1 1 1 0 0 0 1 0 1 1 1 0 0 0 0 0 1 0 1 0 1 1 0 0 0 0 1 0 0 1 1 1 1 1 1 0 0 1 1 0 1
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1 0 1 1 1 1 1 1 1 0 1 0 1 0 0 1 1 1 1 1 1 0 1 1 0 0 0 1 0 0 1 0 0 0 1 1 0 1 1 1 1 0 0 1 0 0 0 1 1 0 1 0 1 0 0 1 0 1 1 1 1 0
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0 1 1 1 0 1 1 0 0 1 1 1 1 1 1 1 1 0 0 0 0 1 0 1 0 0 0 0 1 0 0 0 0 1 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 0 1 1 0 1 0 0 0 1 1 0 0
1 0 1 0 0 0 0 1 0 1 0 0 1 1 1 0 0 1 0 1 1 1 1 1 1 1 1 1 0 1 0 1 1 0 1 0 1 1 1 1 0 1 1 0 1 0 1 0 1 0 1 1 0 1 0 0 1 0 0 1 1
1 1 1 1 1 0 0 1 1 1 0 0 1 1 1 1 1 0 0 1 1 0 1 1 1 1 0 0 1 1 1 1 1 0 0 1 1 0 0 1 1 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 0 0 1 0 0 1
1 0 0 1 0 0 0 1 1 1 0 0 0 1 0 1 1 1 1 0 1 1 0 1 0 1 1 1 1 0 0 0 0 1 1 1 0 1 1 0 1 1 0 1 1 1 1 0 1 1 1 1 0 1 1 1 1 0 1 0 1 0
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1 0 0 0 1 0 0 1 0 1 0 0 1 0 1 0 0 1 0 0 1 1 1 1 1 1 1 0 0 1 0 0 1 1 1 1 0 0 0 0 1 0 1 0 0 1 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 1
1 0 1 0 0 0 1 1 0 0 0 0 1 0 1 1 0 0 1 0 1 1 0 1 0 0 0 1 1 0 1 0 0 0 0 0 0 0 1 0 1 1 1 0 0 1 1 1 1 0 1 0 1 1 0 1 1 1 1 1 1 0
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Table of A list of algorithms [14]

Swarm intelligence-based algorithms	
Algorithm	Author
Accelerated PSO	Yang et al.
Ant colony optimization	Dorigo
Artificial bee colony	Karaboga and Basturk
Bacterial foraging	Passino
Bacterial-GA Foraging	Chen et al.
Bat algorithm	Yang
Bee colony optimization	Teodorovi´c and
Bee system	Dell'Orco
BeeHive	Lucic and Teodorovic
Wolf search	Wedde et al.
Bees algorithms	Tang et al.
Bees swarm optimization	Pham et al.
Bumblebees	Drias et al.
Cat swarm	Comellas and Martinez
Consultant-guided search	Chu et al.
Cuckoo search	Iordache
Eagle strategy	Yang and Deb
Fast bacterial swarming	Yang and Deb
algorithm	Chu et al.
Firefly algorithm	Yang
Fish swarm/school	Li et al.
Good lattice swarm	Su et al.
optimization	Krishnanand and Ghose
Glowworm swarm	Chen et al.
optimization	Gandomi and Alavi
Hierarchical swarm model	Mucherino and Seref
Krill Herd	Kennedy and Eberhart
Monkey search	Yang
Particle swarm algorithm	Yang
Virtual ant algorithm	Ting et al.
Virtual bees	
Weightless Swarm Algorithm	

Other algorithms	
Anarchic society optimization	Shayeghi and
Artificial cooperative search	Dadashpour
Backtracking optimization	Civicioglu
search	Civicioglu
Differential search algorithm	Civicioglu
Grammatical evolution	Ryan et al.
Imperialist competitive	Atashpaz-Gargari and
algorithm	Lucas
League championship	Kashan
algorithm	Xu et al.
Social emotional optimization	

Bio-inspired (not SI-based) algorithms	
Algorithm	Author
Atmosphere clouds model	Yan and Hao
Biogeography-based	Simon
optimization	Shi
Brain Storm Optimization	Storn and Price
Differential evolution	Kaveh and
Dolphin echolocation	Farhoudi
Japanese tree frogs calling	Hern´andez and
Eco-inspired evolutionary	Blum
algorithm	Parpinelli and
Egyptian Vulture	Lopes
Fish-school Search	Sur et al.
Flower pollination algorithm	Lima et al.
Gene expression	Yang
Great salmon run	Ferreira
Group search optimizer	Mozaffari
Human-Inspired Algorithm	He et al.
Invasive weed optimization	Zhang et al.
Marriage in honey bees	Mehrabian and
OptBees	Lucas
Paddy Field Algorithm	Abbass
Roach infestation algorithm	Maia et al.
Queen-bee evolution	Premaratne et al.
Shuffled frog leaping algorithm	Havens
Termite colony optimization	Jung
	Eusuff and Lansey
	Hedayatzadeh et al.

Physics and Chemistry based algorithms	
Big bang-big Crunch	Zandi et al.
Black hole	Hatamlou
Central force optimization	Formato
Charged system search	Kaveh and
Electro-magnetism optimization	Talatahari
Galaxy-based search algorithm	Cuevas et al.
Gravitational search	Shah-Hosseini
Harmony search	Rashedi et al.
Intelligent water drops	Geem et al.
River formation dynamics	Shah-Hosseini
Self-propelled particles	Rabanal et al.
Simulated annealing	Vicsek
Stochastic difusion search	Kirkpatrick et al.
Spiral optimization	Bishop
Water cycle algorithm	Tamura and
	Yasuda
	Eskandar et al.