People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research
Mhamed Bouguera University of Boumerdes
Faculty of Engineering
Industrial Maintenance Department

IN PARTIAL FULLFILLMENT OF THE REQUIREMENTS FOR THE DEGREE MASTER IN MECHANICS AND ENGINEERING SYSTEMS

TOPIC

Design and thermodynamic study of a STIRLING engine

Realized by:

LADJ Yassine
MOUHOUBI Riyadh

Supervised by :

Mr. DJEDID Toufik

Farmed by:

Mr.ANÇA Farouk

2016/2017

Thanks

First of all, we thank God the Almighty who provided us with the help and confidence to do this work.

We would also like to thank our promoter **Mr DJEDDID** and our framer **Mr.ANÇA Farouk** for their invaluable help, their remarks and their advice which enabled us to improve our work.

We thank the **ALGEK** company which gives us all its materials and machine to do our work in a good states.

Finally, we would like to thank all the teachers who contributed to our training during our university course.

Yassine and Riyadh

Dedicate

I am very pleased to dedicate this modest work to:

My dear father and my dear mother, who have all done for my good, and support me during all my years of studies, and who have directed my way.

My brothers and sisters and their children.

My dear friends: Riyadh, Abdenour, Mohamed, Billal, Ilyas, Hamza, Farouk.

To all my promotional teammates with whom we spent three wonderful years of study.

Yassine.L

Dedicate

I dedicate this modest work to the most tender beings in my eyes and in my heart: my mother and all my brothers and sisters and their childrens, for her support during all my years of studies.

For my friends: yassine, abdenour, mohamed, amine, atmane, nabil, fethi, morad, khaled , abdelhak ,seddik ,lotfi ,Mohamed Samir , and sidali.

To all my promotional teammates with whom we spent three wonderful years of study.

Riyadh

Summary

List of figures

Chapter I :

Chapter II :

Chapter III :

Chapter IV :

List of symbols

Symbols	Representation	Units
C_v	Specific heat at constant volume	J / (Kg*K)
e	Regenerator effectiveness	
η	Thermal efficiency	%
K	Factor defined by the equation [9]	
k	Specific heat ratio	
K_{Mh}	Report dead volume hot on the total dead volume	
K_{Mc}	Report dead volume cold on the total dead volume	
k_{MR}	Report dead volume of the regenerator on the total dead volume	
k_{MT}	Report total dead volume on total volume	
k_{CDM}	Report dead volumes on scanned volumes	
M	Total mass of the fluid in the engine	Kg
P	Absolute pressure	N/m^2
P_m	Mean effective pressure	N/m^2
Q_a	Heat added from external sources	Joules
Q_r	Heat rejected to the outside by the engine	Joules
R	Constant of the working gas	J/(Kg*K)
T_3	Temperature of the working gas in the hot space	K
$T_{3'}$	Temperature of the working gas at the outlet of the regenerator	K
T_1	Temperature of the working gas at the cold space	K
$T_{1'}$	Temperature of the working gas at the inlet of the regenerator	K

T_R	Temperature of the working gas in the regenerator	K
T_F	Temperature of the cooler	K
T_C	Temperature of the heater	K
V_T	Total volume of the working gas in he engine	m^3
V_{mh}	Hot dead volume	m^3
V_{mc}	Cold dead volume	m^3
V_{MR}	Dead volume of the regenerator	m^3
V_{MT}	Total dead volumes	m^3
V_D	Volume swept by the displacement	m^3
V_P	Volume swept by the working piston	m^3
W_{net}	Net engine power	Joules
V_e	Expansion volume	Cc
V_c	Compression volume	Cc
U	Engine frequency	Hz
P_{o-r}	Reel output power	W
P_{o-p}	Perfect output power	W

Abstract

In this memory a low temperature differential Stirling engine type gamma was studied, this engine has a volume ratio $V_{max}/V_{min} = 5$ and 0.75 regenerator effectiveness. A theorical thermodynamics study of an ideal cycle was done to calculate the efficiency and to show the effect of temperature and the presence of regenerator on thermal efficiency. A modeling of the engine and all its parts with SOLIDWORKS software, then a finite element analysis was done with ANSYS for known the effect of the moment of inertia applied on its flywheel. Finally, we end our study by manufacturing the engine and make an experience to obtain a real result and compare it to the ideal result.

Résumé

Dans ce mémoire, on a étudié un moteur sterling de faible température différentielle de type gamma, ce moteur a un rapport en volume $V_{max} / V_{min} = 5$ et 0,75 d'efficacité du régénérateur. Une étude thermodynamique théorique d'un cycle idéal a été effectuée pour calculer l'efficacité et pour montrer l'effet de la température et la présence de régénérateur sur l'efficacité thermique. Une modélisation du moteur et de toutes ses pièces avec le logiciel SOLIDWORKS, une analyse par éléments finis a été effectuée avec ANSYS pour connaître l'effet du moment d'inertie appliqué sur son volant. Enfin, nous finissons notre étude en fabriquant le moteur et en nous faisant une expérience pour obtenir un résultat réel et le comparer au résultat idéal.

ملخص:

في هذه المذكرة، قمنا بدراسة محرك ستيرلينغ ذو درجة حرارة تفاضلية منخفضة من نوع جاما ، هذا المحرك يحتوي على نسبة حجم $\frac{V_{max}}{V_{min}} = 5$ و 0.75 كفاءة استرجاع . وقد أجريت دراسة حرارية نظرية لدورة مثالية لحساب الكفاءة وإظهار تأثير درجة الحرارة و وجود إعادة التوليد على الكفاءة الحرارية. كما أجريت نمذجة للمحرك وجميع أجزائه ببرنامج سوليدوركس، ثم تحليل العناصر المحدودة لدولاب الموازنة ببرنامج أنسيس. وأخيرا، فإننا ننتهي دراستنا من خلال تصنيع المحرك و القيام بتجربة للحصول على نتيجة حقيقية ومقارنتها بالنتيجة المثالية.

Introduction

In the current energy and environmental contexts (tension in energy markets, greenhouse effect, pollution, etc.), interest in development the production of clean energy is relaunched. Thus, new spectra such as solar energy or waste recovery are offered to research on "renewable energy".

There are several types of engines capable of transforming thermal energy into mechanical energy. The most commonly used combustion engines are internal combustion engines, gas turbines and Rankine steam plants. Unfortunately, none of these systems is well adapted to the valorization of certain types of "hot source" (solar energy, biomass, high-temperature gaseous effluents, etc.).

Hot air engines with external heat input by exchanger, with or without recuperate exchanger, with separate alternating compression and expansion machines, with open or closed single-phase cycle, with or without valves, are, however, very suitable for The use of renewable energy. Indeed, the contribution of heat can be of diverse origin like the solar, the wood . . .

Moreover, in the field of low electrical power (500We, 50kWe), cogeneration does not seem to have the same development as for the large powers. This lack of success, while the residential and tertiary domain could be conducive, is due to the lack of systems adapted to this niche: For these power levels the most commonly used machines are internal combustion engines that generate a lot of noise and vibration. Few individuals are willing to install an internal combustion engine at home instead of their gas boiler, even if some manufacturers propose domestic cogeneration systems that are well insulated from the sound and vibration point. The market therefore appears to be more promising for systems based on external combustion heat engines. Among the latter, Cogeneration based on Stirling kinematic or free-piston engines have been the subject of many developments, and even some of them have already been marketed.

In this work, we are interested in the heat engine with external heat input that the STIRLING engine (1816). For this, we have divided this work into four chapters.

In the first chapter, we discus about all heat engines and specially about Stirling engine and types, its advantages and disadvantages and applications.

In the second chapter, we try to see the formulas and equations that characterize this type of engine, for which we take the example of a low temperature differential engine type gamma that we determine its thermal efficiency.

In the third chapter we make a modelling of our engine and its parts by using Solid works software, and also a finite element analysis of its flywheel with Ansys workbench.

For chapter four we have done an experimental in which we have used an LTD Stirling gamma type and calculate its power and compare it with the power of the ideal cycle studied in the second chapter.

Chapter I:

Generalities on heat engines

Heat engines

1-1- History of heat engines

An engine is a mechanical contrivance by means of which some form of energy is converted to useful work. The first mechanical utilization of an energy source to do work dates back to the 1st-c BC, when simple water wheels were used to lift water from rivers and to mill grain. Although it might be considered that these were examples of simple hydraulic ‘engines’, it is generally accepted that the term engine is associated with a much later period of technological development; that of the Industrial Revolution. Engines are normally associated with the conversion of fossil fuels (thermal energy sources) into useful work, and so we may fully refer to them as heat engines. And we can say that the begin of the heat engines was when **DENIS PAPIN** has invented the first external combustion machine [8].

Fig1-1: The first external combustion machine of DENIS PAPIN [8].

In 1673, Christian **HUYGHENS** produces a sort of vertical black powder machine, from which, due to the lack of proper mechanics (connecting rod and crank), it is not possible to recover the energy produced. In 1698, the English SAVERY perfects the PAPIN’s machine. After in 1705, another English who is named **TOHMAS NEWCOMEN** developed the first steam engine for industrial use. Three years after, POTTER succeeds in automating the Newcomen machine. In 1769, **Joseph CUGNOT** realizes the fardier, the first steam engine after the scots **James WATT** perfected the steam engine by inventing the ball regulator, the double-acting piston, the condenser, etc.

In Scotland 1816, Pastor **Robert STIRLING** invents a hot-air engine capable of using all sources of heat. After it was the turn of fabrication of electric engines such as the electric engine of **Montz JACOBI** in 1834. In 1854, the Italians **Eugenio BARSABTI** and **Felice MATTEUCI** File the patent of a gas engine using two free pistons driving a steering wheel [11].

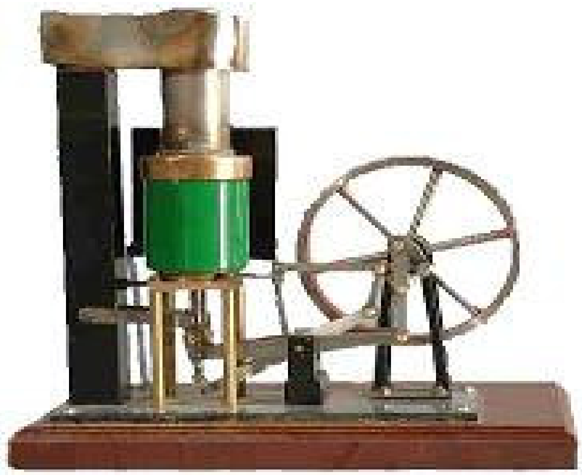


Fig 1-2 : Stirling engine [8].

In 1860, Joseph **LENOIR** invented a new type of engine which relates to an air motor system expanded by the combustion of gas ignited by electricity and **Pierre HUGON** patented another engine of the same type, it was the beginning for the motor-driven engines built by the German **OTTO**. And in the same year the Italian **PACINOTTI** manufactures the first electric engine with an alternator. Two years later , the French **Alphone BEAU DE ROCHAS** invented the cycle of the 4-times cycle (admission - compression – combustion – exhaust) , but the fuel capable of operating this engine doesn’t yet exist. Until when the German **Nichaulas OTTO** use the cycle of beau de rochas and releases the first compression internal combustion engine; it was the start of vocations cars and industrial engine. In 1897, the other German engineer **Rudolf DIESEL** invented the diesel engine that still carries its name [8].

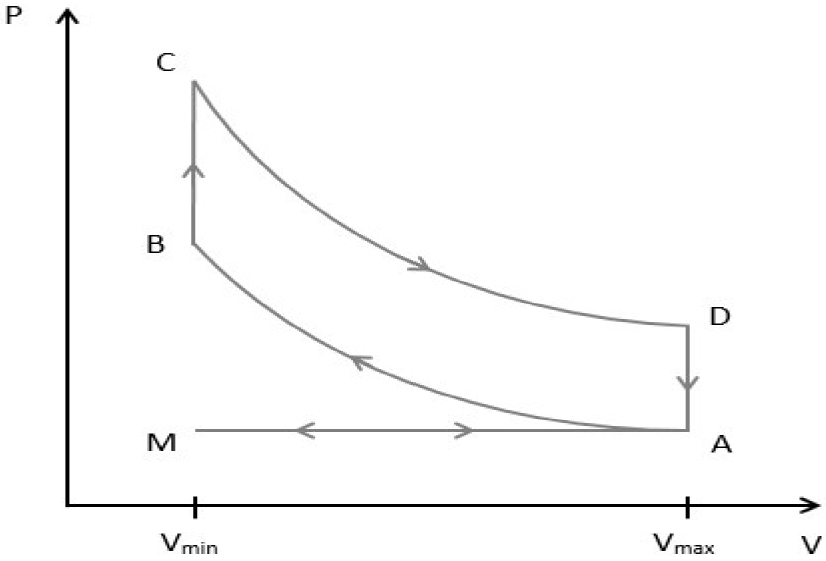


Fig 1-3 : Beau DE ROCHAS ‘s cycle [9].

1-2- Classification of heat engines

The heat engines can be classified in many different ways. Then classify heat engines, taking into account six different aspects:

- They Depending on where is combustion (internal or external).
- Depending on the fuel used and the type of lighting.
- Depending on the movement of moving parts.
- Depending on how realizes the cycle.
- Depending on the number of cylinders.
- Depending on the arrangement of the cylinders.

2- Types of thermal motors by combustion

In this classification the thermal motors divides into two types:

- **Internal combustion:** The endothermic engine combustion process is carried out in the interior of the engine. They are used in self-propelled vehicles (cars, trucks, boats ...).
- **External combustion:** Exothermic combustion engine takes place outside of the machine (steam). Such engines are not used in automotive industry.

2-1- Internal combustion engines

2-1-1- Reciprocating engines

Heat is produced by combustion in a variable volume chamber and is used to increase the pressure in a gas that fills this room (this gas is also composed initially of fuel and oxidant: air). This increase in pressure results in a force exerted on a piston, which converts the strength of the piston translation movement shaft rotational motion (crank shaft). The engines are classified into two categories according to the ignition of the fuel-air mixing technique:

- **Spark ignition engines (gasoline engine):** the suitable fuel-air mixture is obtained with a carburetor and it is admitted into the combustion chamber of the cylinder where the inflammation is produced by a spark.
- **Compression- ignition engines (diesel engine):** the fuel is diesel one injects it into under pressure into the combustion chamber containing air, previously compressed and hot, in contact with which it ignites spontaneously.

Ignitions engines, and controlled by compression, are internal combustion engines because combustion takes place inside the engine. These engines currently constitute the majority of mechanical power production units in many fields, especially the field of transport where they are especially developed because of their advantages: good performance, reliability ... compactness; this explains the expansion that took nowadays motor industry and all its related branches in all countries of the world [9].

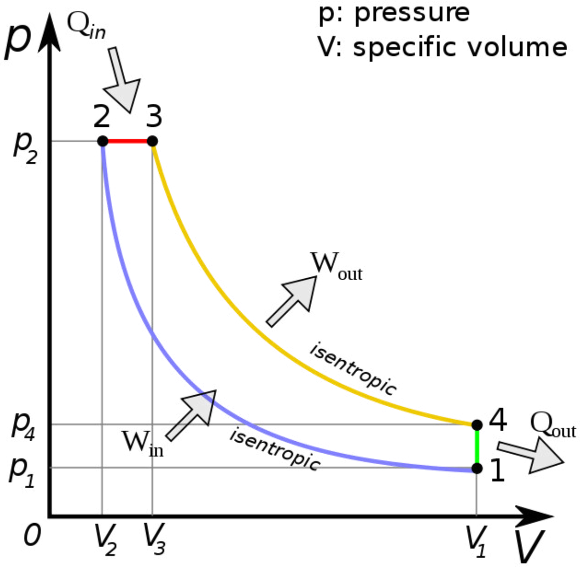


Fig1-4 ; PV diagram of a diesel cycle [9].

2-1-2- Turbomachinery

Unlike previous engines, turbomachines are continuous flow machines. In recent machines, motor fluids developments take place in successive walls and juxtaposed, unlike reciprocating engines which these transformations take place in the same space, the cylinder. Heat produced by combustion in a combustion chamber of a generally liquid fuel (kerosene for example). This augment combustion gas pressure (air + fuel). This pressurized gas flows through a constant volume expansion chamber consists of a drive shaft provided with vanes (expansion turbine). Energy is then supplied to the shaft as a torque motor that will be used firstly to consumers, on the other hand to a compressor (turbine compressor) allows the power output. In effect the air pressure increases, the air mass sucked increases, can burn more kerosene, and the available power is thus increased (relative to a turbine which does not possess input compressor stage).

2-1-3- Wankel engine (with a rotary piston)

The Wankel rotary engine is the result of extensive study conducted from 1945 to 1954 by the Wankel engineer on different rotary motor solutions. In conclusion, he considered that the best was to work in the engine, the rotary compressor made by Bernard Maillard in 1943.

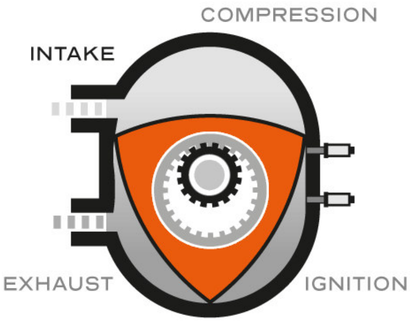


Fig 2-1 : Wankel engine [9].

- **Advantages :**
  - Small footprint to displacement equal to a conventional engine.
  - Because it does not transform linear motion into rotation, moves fewer parts, so less inertia, which allows it to reach very high regimes. (In theory max. 18000 rev / min).
  - Fewer parts allows to make mounted very quick schemes.
  - Fewer parts equal less weight.
  - The range of use begins in the early rounds and extends to breaking.

- **Disadvantages :**
  - Excessive consumption in gasoline.
  - Motor brake virtually non-existent.
  - Technically perfectible.

2-2- External combustion engines

2-2-1- Steam machine

Heat is produced in a combustion chamber (boiler) separated from the expansion chamber. This heat is used to vaporize water. The water vapor obtained by the vaporization is then sent into the expansion chamber (cylinder), where it actuates a piston. A crank rod system then retrieves the mechanical energy produced in accordance with the needs. The water supplied to the evaporator is transformed into steam by heat. The gas (water vapor pressure) is distributed to the piston which provides the work to be used by the crank-rod system. Distributors to allow each side of the piston alternately on admission or exhaust.

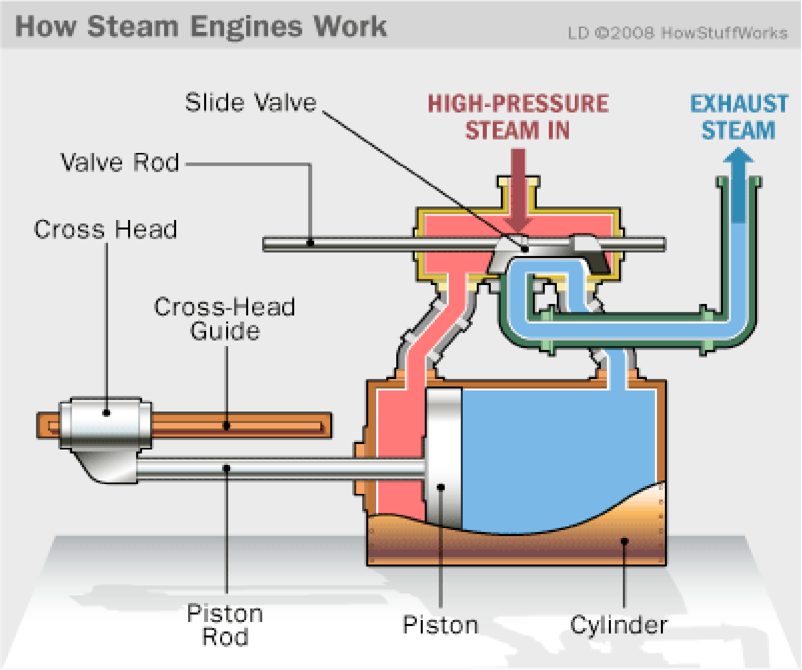


Fig2-2: Steam engine plan [9].

2-2-2- Manson engine

Manson has invented an original engine bearing his name. The latter has been published in the British journal "Newness Practical Mechanics", in March 1952. For those who study the hot air engine or external combustion like Stirling engine, they find a similarity in operating. The Manson engine has some advantages :

- The relative hardiness technology enables a motor of high reliability and low maintenance.
- Relative ease of design and production: there are few parts. Each of them is easy to produce and does not require complex and costly machining means.
- Ecological ability to best meet the environmental requirements of air pollution. It seems easier to achieve in this type of engine a complete fuel combustion.

But it has some disadvantages like:

- Poor performance: this engine has the unfortunate characteristic of rejecting warm air under pressure to the atmosphere. It is a shame not to enjoy complete relaxation to atmospheric pressure. This point is not a problem for a model, but important if we wanted to achieve power saving engine into energy.
- Noisy operation: in fact, the air discharge pressure and temperature can have a sound risk like the internal combustion engines. The fresh air intake must, too, be a source of noise.
- Gaseous medium heat exchangers are delicate and often require bulky appliances.

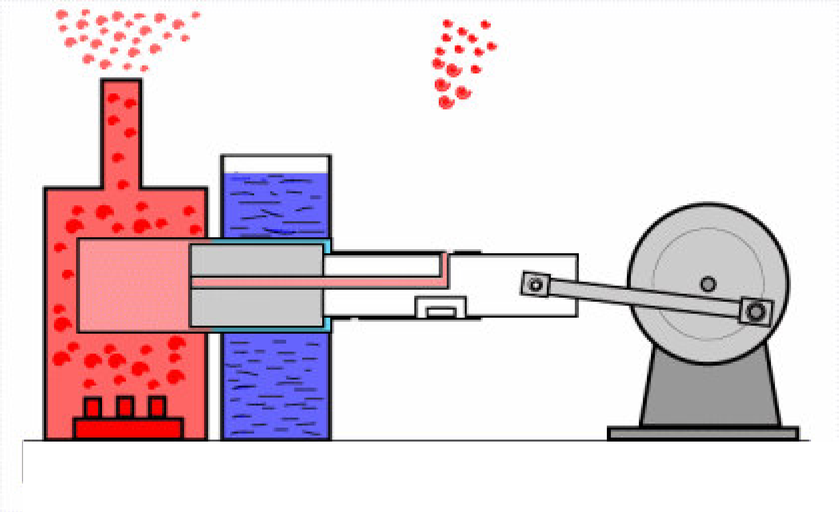


Fig 2-3: Manson engine [8].

2-2-3- Stirling engine

The Stirling engine, sometimes called external combustion engine or hot air engine was invented in 1816 which we'll talk more today. The engine pistons include two A and B and a regenerator which absorbs and releases heat during the cycle. This engine has a serval types such as the alpha engine, beta engine, and gamma [8].

3- Stirling engine history

The Stirling engine (or Stirling's air engine as it was known at the time) was invented and patented by Robert Stirling in 1816.

The life of Robert Stirling took place during the nineteenth century with many scientific and technologic discoveries. This is probably the ferment of ideas, his natural curiosity and his pragmatism that incited Robert Stirling to invent the **"hot-air engine"**. The patent was filed on September 27, 1816 and was effective on January 20, 1817.

At that time, steam boilers frequently exploded and made victims. Design problems, poor quality of materials, quasi-absence of use and maintenance rules were the reason of these disasters.

Probably, this motivated Robert Stirling to imagine an engine without boiler subjected to too strong pressures.

The principle of operation is relatively simple: combustion is external; the principal fluid is air with a modest pressure and is subjected to the following cycle: heating, expansion, cooling then compression [8].

Robert Stirling also had the idea of a heat regenerator allowing improving the overall efficiency of the installation. In good Scots, he called it an "economizer".

This invention had other applications in agriculture and industry until 1922 for pumping water, or for generating electricity. However, it probably did not have all the success that Robert Stirling and his James brother could have wished, for competitiveness reasons. The steam engine, however more dangerous at that time, took the lead. Then, one saw the first industrial applications of the electric motorization.

In 1938, the company "Philips" invested in the hot air engine, now known as "Stirling engine". Applications were developed in the automobile field. A compact engine with more than 200 horsepower, with an efficiency above 30%, was born. Unfortunately, for competitive reasons, this application did not met the expected success [8].

4- General principles

Stirling engines are mechanical devices working theoretically on the Stirling cycle, or its modifications, in which compressible fluids, such as air, hydrogen, helium, or even vapors, are used as working fluids. The Stirling engine offers possibility for having high efficiency engine with less exhaust emissions in comparison with the internal combustion engine. The earlier Stirling engines were huge and inefficient. However, over a period of time, a number of new Stirling engine models have been developed to improve the deficiencies [4].

The modern Stirling engine is more efficient than the early engines and can use any high temperature heat source. Therefore, most sources of heat can power it, including combustion of any combustible material, field waste, rice husk or the like, biomass methane and solar energy. In principle, the Stirling engine is simple in design and construction, and can be operated easily.

The Stirling engine has high performance in many applications and it is suitable where:

- Multi-fueled characteristic is required.
- A very good cooling source is available.
- Quiet operation is required.
- Relatively low speed operation is permitted.
- Constant power output operation is permitted.
- Slow changing of engine power out-put is permitted.
- A long warm-up period is permitted.

In its simplest description, a Stirling engine consists of a cylinder containing a gas and a piston recovering the mechanical energy.

First observation: the gas used is confined, it's always the same. Another feature: energy is supplied from outside of the cylinder, from where the designations “hot air engine” or “external combustion engine” which one can read sometimes.

5- Functional description

The engine is designed so that the working gas is generally compressed in the colder portion of the engine and expanded in the hotter portion resulting in a net conversion of heat into work. An internal regenerative heat exchanger increases the Stirling engine's thermal efficiency compared to simpler hot air engines lacking this feature.

A Stirling engine and its components are shown in figure below.

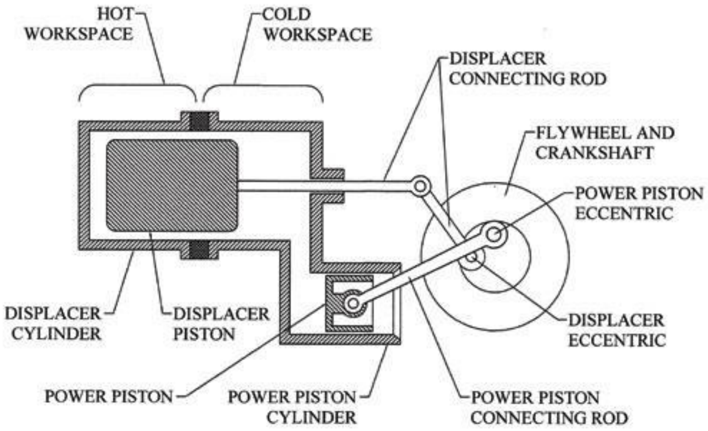


Fig 4-1: Stirling engine and its components [4].

5-1- Heat source

The heat source may be provided by the combustion of a fuel, since the combustion products do not mix with the working fluid and hence do not come into contact with the internal parts of the engine, a Stirling engine can run on fuels that would damage other engines type's internals, such as landfill gas, Other suitable heat sources include concentrated solar energy, geothermal energy, nuclear energy, waste heat and bioenergy [1].

5-2- Heat sink

The heat sink is typically the environment at ambient temperature. In the case of medium to high power engines, a radiator is required to transfer the heat from the engine to the ambient air. Alternatively, heat may be supplied at ambient temperature and the heat sink maintained at a lower temperature by such means as cryogenic fluid or iced water [1].

5-3- Regenerator

In a Stirling engine, the regenerator is an internal heat exchanger and temporary heat store placed between the hot and cold spaces such that the working fluid passes through it first in one direction then the other, taking heat from the fluid in one direction, and returning it in the other. It can be as simple as metal mesh or foam, and benefits from high surface area, high heat capacity, and low flow friction. Its function is to retain within the system that heat that would otherwise be exchanged with the environment at temperatures intermediate to the maximum and minimum cycle temperatures, thus enabling the thermal efficiency of the cycle to approach the limiting Carnot efficiency [6].

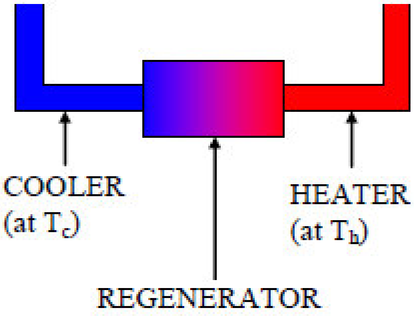


Fig 4-2: Schematic of a regenerator [1].

5-4- Displacer

The displacer is a special-purpose piston, used in Beta and Gamma type Stirling engines, to move the working gas back and forth between the hot and cold heat exchangers.

Depending on the type of engine design, the displacer may or may not be sealed to the cylinder, i.e. it may be a loose fit within the cylinder, allowing the working gas to pass around it as it moves to occupy the part of the cylinder beyond [1].

6- Configurations

There are three major configurations, namely the alpha, beta, and gamma configurations, are commonly used. Each configuration has the same thermodynamic cycle but has different mechanical design characteristics.

6-1- Alpha

In the alpha-configuration a displacer is not used. Two pistons, called the hot and cold pistons, are used on either side of the heater, regenerator, and cooler. These pistons move uniformly in the same direction to provide constant-volume heating or cooling processes of the working fluid. When all the working fluid has been transferred into one cylinder, one piston will be fixed and the other piston moves to expand or compress the working fluid. The expansion work is done by the hot piston while the compression work is done by the cold piston [4].

6-2- Beta

In the beta-configuration, a displacer and a power piston are incorporated in the same cylinder. The displacer moves working fluid between the hot space and the cold space of the cylinder through the heater, regenerator, and cooler. The power piston, located at the cold space of the cylinder, compresses the working fluid when the working fluid is in the cold space and expands the working fluid when the working fluid is moved into the hot space [4].

6-3- Gamma

The gamma-configuration uses separated cylinders for the displacer and the power pistons, with the power cylinder connected to the displacer cylinder. The displacer moves working fluid between the hot space and the cold space of the displacer cylinder through the heater, regenerator, and cooler. In this configuration, the power piston both compresses and expands the working fluid. The gamma-configuration with double-acting piston arrangement has theoretically the highest possible mechanical efficiency [4].

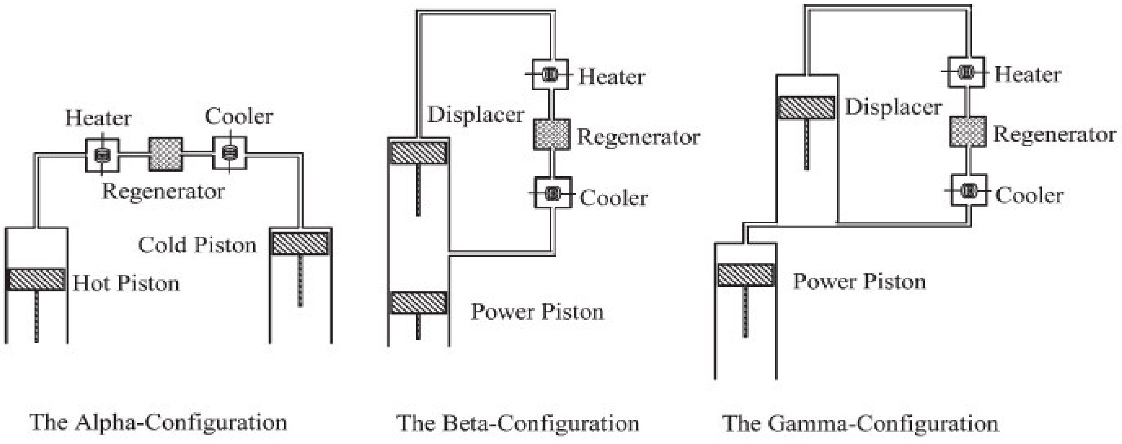


Fig. 5-1: Three basic mechanical configurations for Stirling engine [4].

6-4- Others configurations

- **The two-cylinder Stirling:** with Ross yoke is a two cylinder Stirling engine (not positioned at 90° , but at 0°) connected with a special yoke. The engine configuration/ yoke setup was invented by Andy Ross (engineer).

- **The rotary Stirling engine:** seeks to convert power from the Stirling cycle directly into torque, similar to the rotary combustion engine. No practical engine has yet been built but a number of concepts, models and patents have been produced for example the Quasiturbine engine.
- **Free piston Stirling engines:** include those with liquid pistons and those with diaphragms as pistons. In a “free piston” device, energy may be added or removed by an electrical linear alternator, pump or other coaxial device. This avoids the need for a linkage, and reduces the number of moving parts. In some designs, friction and wear are nearly eliminated by the use of non-contact gas bearings or very precise suspension through planar springs.

7- The operating principles

7-1- Stirling cycle

The ideal Stirling cycle has three theoretical advantages. First, the thermal efficiency of the cycle with ideal regeneration is equal to the Carnot cycle. During the transfer strokes, the regenerator, which is a typical temporary energy storage, rapidly absorbs and releases heat to the working fluid which is passing through. Therefore, the quantity of heat taken from the external heat source is reduced; this results in improving the thermal efficiency Figure 6-1.

The second advantage, over the Carnot cycle, is obtained by substitution of two isentropic processes with two constant-volume processes. This results in increasing the p–v diagram area. Therefore, a reasonable amount of work from the Stirling cycle is obtained without the necessity to use very high pressures and large swept volumes, as in the Carnot cycle. The Stirling cycle compared with the Carnot cycle between the same given limits of pressure, volume, and temperature, is shown in Figure 6-1. The shaded areas (2C-2-3) and (1-4C-4) indicate the additional work available by replacing two isentropic processes with two constant-volume processes. The Carnot cycle isothermal processes (1-2C and 3-4C) are, respectively, extended to process 1–2 and 3–4. The amount of available work is increased in the same proportion as the heat supplied to and rejected from the Stirling cycle [4].

The third advantage has recently been discovered. Compared with all reciprocal piston heat engines working at the same temperature limits, the same volume ratios, the same mass of ideal working fluid, the same external pressure, and mechanism of the same overall effectiveness, the ideal Stirling engine has the maximum possible mechanical efficiency. These three advantages reveal that the Stirling engine is a theoretical equivalent of all heat engines [4].

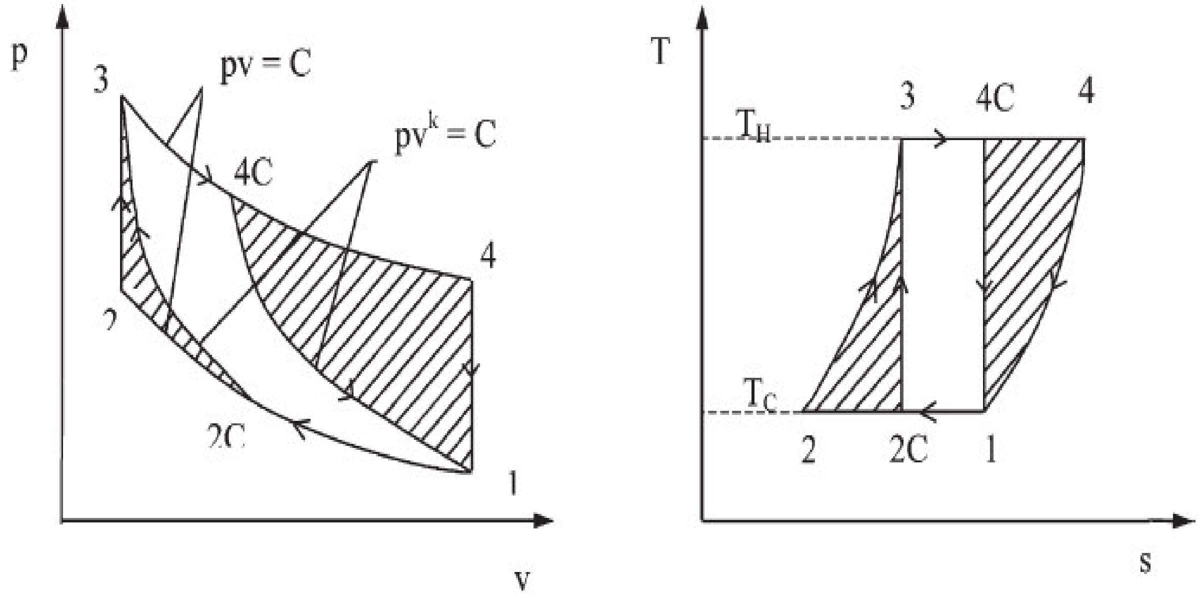


Fig. 6-1: Stirling and Carnot cycle [4].

7-2- The real cycle of Stirling

In fact, the indicator diagram noted experimentally on a Stirling engine will be significantly different from the theoretical diagram shown previously. It has a more rounded shape than the latter. This is essentially due to the continuous movements of the pistons are different to theoretical movements, Large heterogeneities of instantaneous temperature in the engine and the many irreversibility present during engine operation. Due to the unevenness of the instantaneous temperature field in a real Stirling engine, one cannot speak of a real thermodynamic cycle [7].

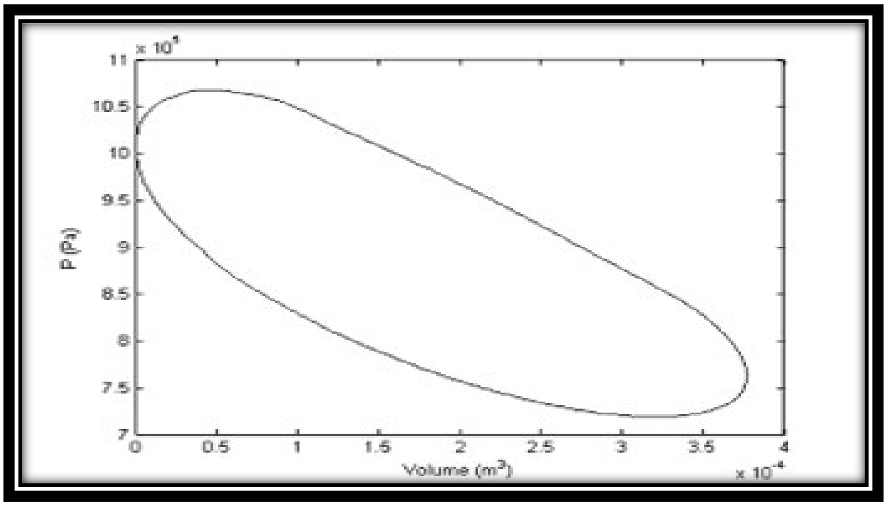


Fig 6-2: Diagram of a real Stirling cycle [7].

The mechanical work produced is the result of an infinity of different thermodynamic cycles experienced by the fluid particles in the engine. Note that the area of the diagram real indicator is smaller than the theoretical diagram indicator. This decrease the effect of reducing the power provided by the engine because the work is represented by the area of the closed diagram.

8- The basic phases

The thermodynamic cycle of the Stirling engine is very simple: it includes 4 phases during which the gas undergoes the following transformations:

8-1- An isochoric heating (with constant volume):

The burner (the hot source) provides thermal energy. We easily imagine that the pressure and the gas temperature increase during this phase.

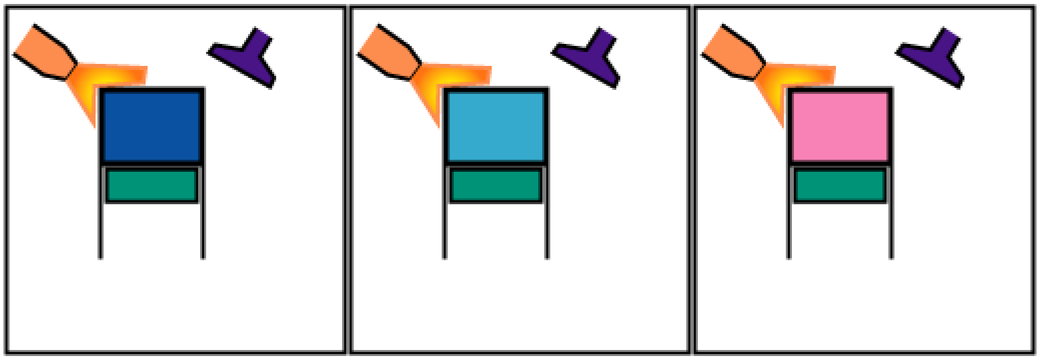


Fig 7-1: Isochoric heating process [8].

8-2- An isothermal expansion (at constant temperature):

The volume increases whereas the pressure decreases. It is during this transformation that driving energy is produced.

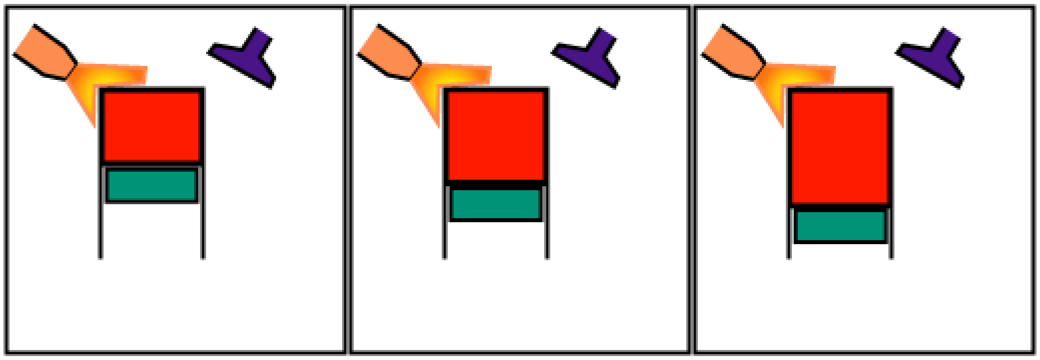


Fig 7-2: Isothermal expansion process [8].

8-3- An isochoric cooling:

The projected water (the cold source) recovers thermal energy. The temperature and the pressure decrease during this phase.

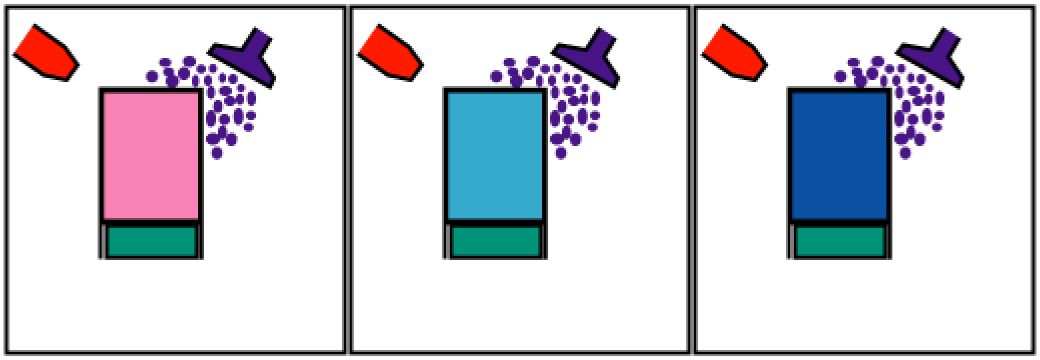


Fig 7-3: Isochoric cooling process [8].

8-4- An isothermal compression:

The pressure of gas increases whereas its volume decreases. One must provide mechanical energy to gas for this period.

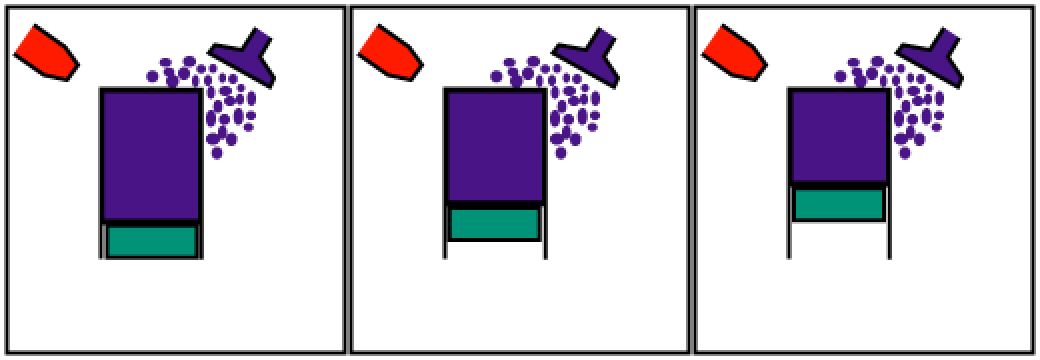


Fig 7-4: Isothermal compression process [8].

9- Adventages and Desadventges

9-1- The adventages

- **The multitude of possible « hot sources »**: because the Stirling engines can run directly on any available heat source, not just one produced by combustion, so they can run on heat from solar, geothermal, biological, nuclear sources or waste heat from industrial processes.

- **Good performance**: it can be around 40% (or 80% of the maximum Carnot cycle), against about 35% for combustion engines if the difference of 5 points seem low, it still means nearly 15% (5 / 35) additional energy.

- **Reliability and easy maintenance:** the technological simplicity makes it possible to have engines with a very great reliability and requiring little maintenance.

- **Reversible:** The Stirling cycle is invertible: a Stirling engine driven by another motor becomes a heat pump capable of cooling to -200 ° C or heating above 700 ° C, depending on the driving direction. They can be used as CHP (combined heat and power) in the winter and as coolers in summer. This without using gas with special properties that make them practical disadvantages or chemical such as Freon refrigeration machines of previous generations, destroyer of the ozone layer. However, in practice it is the effective heat pump function that allows a few machines to exist.

- **The silence of operation:** producing little vibration due to the absence of explosion, the absence of valves which Open and close, and the absence of gas escaping. This makes it silent and Reduces mechanical stress.

- **Safety:** a Stirling engine uses a single-phase working fluid that maintains an internal pressure close to the design pressure, and thus for a properly designed system the risk of explosion is low. In comparison, a steam engine uses a two-phase gas/liquid working fluid, so a faulty overpressure relief valve can cause an explosion.

- **Potentially lower pollution :** The heat coming from the outside, it is possible, non-fossil fuels, to provide it in a less polluting way than in thermal engines in which combustion is imperfect.

9-2- The desadventages

- **The price:** its cost is probably the most important problem; it is not yet competitive with other means well established. A generalization of its employees should solve this problem inherent in any novelty.

- **The ignorance of this type of engine by the general public:** Only a few fans know it exists. It is therefore necessary to promote it.

- The variety of models prevents standardization and consequently lower prices.

- The problems of sealing are difficult to solve as soon as one wishes to have high pressures of operation. The choice of “ideal” gas would be hydrogen for its lightness and its capacity to absorb the calories, but its ability to diffuse through materials is a great disadvantage.

- Heat transfers with a gas are delicate and often require bulky apparatuses.

- The lack of flexibility: the fast and effective variations of power are difficult to obtain with a Stirling engine. This one is more qualified to run with a constant nominal output. This point is a great handicap for an utilization in car industry.

10- Applications and Uses

Applications of the Stirling engine range from heating and cooling to underwater power systems. A Stirling engine can function in reverse as a heat pump for heating or cooling. Other uses include combined heat and power, solar power generation, Stirling cry coolers, heat pump, marine engines, low power aviation engines, and low temperature difference engines.

- In Spain, at the Plataforma Solar de Almería, a Stirling engine installed in the focus of a parabolic mirror.
- The Stirling engine has niche applications, in situations where the initial cost of the system is not a serious disadvantage compared to the advantages (military, research, advanced) applications.
- The main commercial application of Stirling is in the field of refrigeration.
- Industry and military. It serves as a machine for liquefying gases and as a cooler for military infrared guidance systems.
- Used as electricity generator in Iceland, Japan and in extreme environments such as the Australian and Arctic deserts by numerous scientific and military missions.
- It is used by the Swedish, Australian and soon US attack submarines as the main propulsion system, notably for the Gotland class Swedish submarines, not only because of its silence, which is a crucial Submarines, but also for the much lower production of unburned gases necessary to provide a thermal gradient (a temperature difference) to a Stirling engine; A diving submarine can evacuate gases only by compressing them at a pressure at least equal to that of the ambient environment, requiring (and thus wasting) a non-negligible part of the energy available on board.
- This engine also equips certain classes of American frigates, the cooling system of the nuclear reactor of numerous submarines and aircraft carriers as well as drones with great autonomy.

- Due to its poly-fuel capacity it has been successfully tested by the USSR on some prototype heavy tanks including "object 167" before being abandoned for reasons political as well as economic, during the transition to the market economy at the fall of the Soviet bloc.

- NASA and other space agencies use it to provide power to satellites and space probes in addition to solar panels, which it helps to guide in order to optimize its performance.

- The manufacturer of motherboards of personal computer **MSI** introduced in early 2008 a cooling system whose fan is powered by a Stirling engine using as a source of heat the energy released by the chip to be cooled.

- In 2009, most of the major boiler manufacturers proposed a micro-cogeneration plant using a Stirling engine. This type of boiler the size of a water heater not only heats water for domestic use (heating, domestic water) but also generates electricity locally.

11- Conclusion

From the previous informations we deduce that:

- The stirling engine was invented by the Scottish Robert Stirling in 1816 ,in addition to stop the problems caused by the steam boilers .
- The stirling engine work with the stirling cycle which consist of four different phases: isochoric heating , isothermal expansion , isochoric cooling and isothermal compression .
- Stirling engine has three major types: Alpha , Beta and Gamma .
- The advantages of Stirling engines make it useble in different domain such as military, industry, marine engines and solar power generation.

Chapter II:

Thermodynamics study of an ideal engine performance

1- Introduction

The Stirling engine has the main characteristic of forming a closed system; the fluid is contained in a closed enclosure and is heated by a heat source. The combustion is therefore external, which is in fact one of the advantages of this engine, the diversity of fuels usable for its operation. Its second advantage is its efficiency; the engine contains unavoidable dead volumes and is equipped with regenerators whose effectiveness varies from 0 to 99%.Usually, the literature offers theories assuming no dead volumes and impossible perfection regenerator .in this chapter we will analyze this imperfect engine by relying on the laws of classical thermodynamics and extracting important rules to calculate the efficiency of the cycle by taking account of the dead volumes and the imperfection of regenerator.

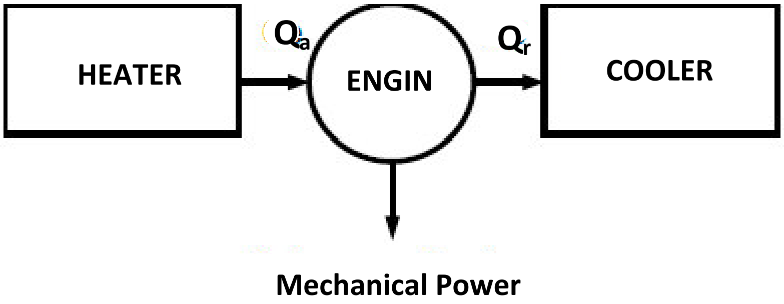


Fig 1: A simplified diagram of a Stirling engine equipped with an Imperfect regenerator [3].

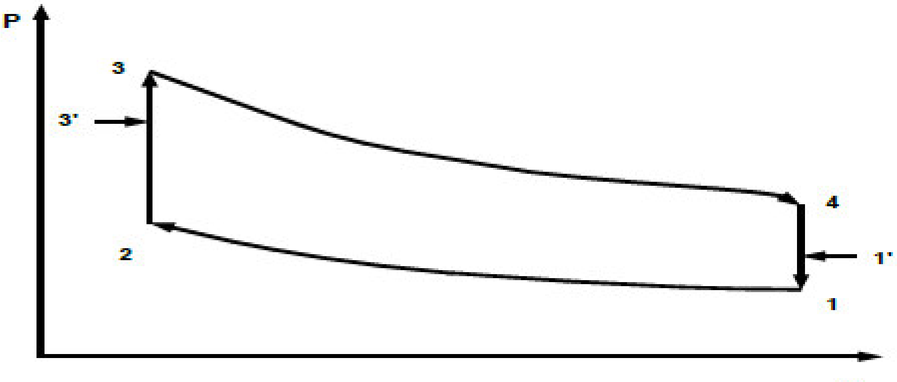


Fig 2: The P-V diagram of an ideal Stirling engine [3].

The figure (2) shows a simplified diagram of a Stirling engine with an imperfect regenerator. For an ideal regeneration, the total heat released during the processing 4-1would be absorbed by a perfect regenerator and returned to the working fluid duringprocessing 2-3. In other words, perfect regenerator has an infinite surface or it would benecessary to have a time of infinite regeneration.

For an imperfect regenerator, working fluid temperature at the inlet and at the outlet of theregenerator corresponds to T1 'and T3', respectively. External heat exchangers must then increase thetemperature of T3 'to T3 and the decrease of T1' to T1. The Stirling engine developer must be able toevaluate the effectiveness of its regenerative and analyze this imperfection to calculate the finalperformance of its realization.

2- The low temperature differential Stirling engine

Similar to a typical Stirling engine an LTD Stirling engine runs when one part of the engine is heated and another part is cooled. LTD is short for Low Temperature Differential. When a Stirling engine has a Low Temperature Differential it means that it can run when the warm side is not much warmer than the cool side. Put another way, the difference between the warm and cool side is very small. Some LTD engines can run on as little as 0.5°c.

By another definition, it is a motor whose geometry has been optimized to operate with small temperature differences between hot and cold sources with an efficiency lower than in the case of a standard Stirling engine.

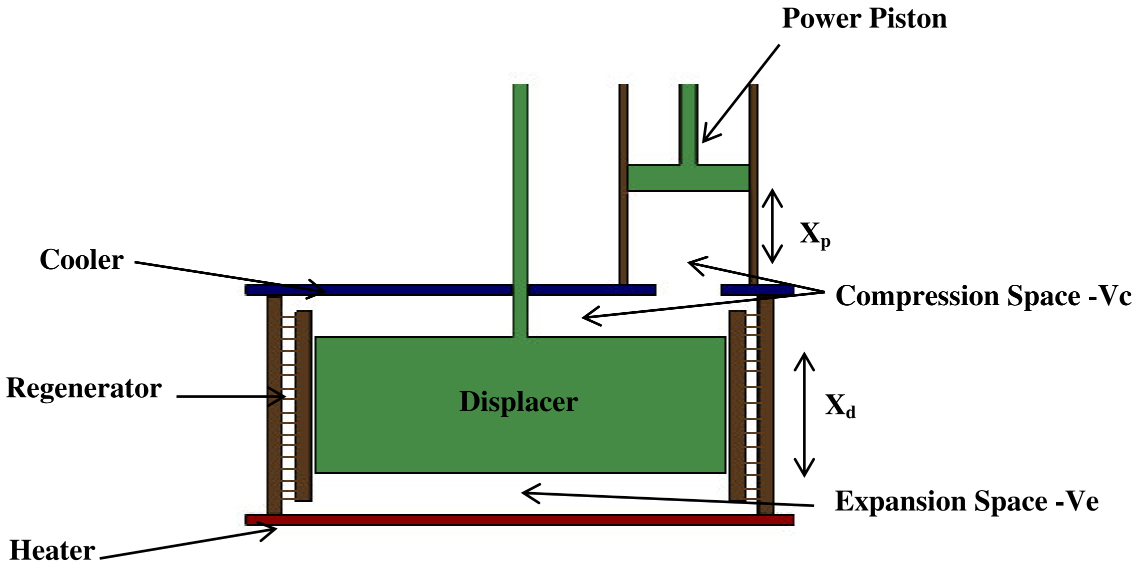


Fig 3: The LTD Stirling configuration

Where:

X_d= The stroke length of the displacer.

X_p= The stroke length of the power piston.

V_e= The volume in the expansion space.

V_c= The volume in the compression space.

2-1- Swept and unswept volumes

The dead volume is the sum of the working fluid volumes in the exchangers, regenerator and in the connections between these different elements. Obviously a Stirling engine contains irreducible dead volumes.Inpractice; there are engines with a dead volume representing 58%of total volume.

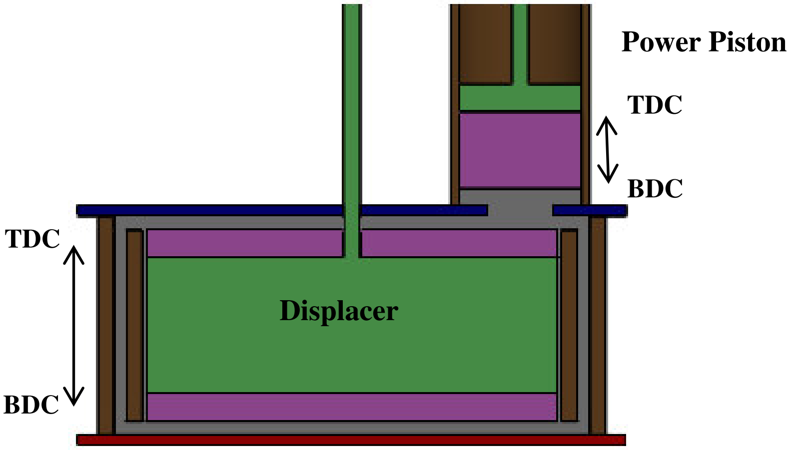


Fig 4: Illustrate the concept of "swept" and "unswept" volume.

As we can see, the swept (purple) regions in the expansion and compression space are the volumes that are "swept" by the motion of the pistons and displacers. And the dead/unswept (grey) regions in the expansion and compression space are the volumes that remain "untouched", or "dead". These regions are "untouched" by the displacer and the power piston, during their motion.

Where:

The swept volume of both displacer and power piston is equal to Vd=π*rd2*Xd/Vp=π*rp2*Xp

$$V_d=\pi * r_d^2 * X_d \text{ and } V_p = \pi * r_p^2 * X_p \tag{a}$$

Where: r_d and r_p are the displacer radius and the power piston radius respectively.

2-2- Expansion and compression volumes

For LTD Stirling engines the expression for volume in the expansion space is the same as for alpha engines. But the expression for volume in the compression space is more complex because it's related to the combined motion of the displacer and power piston. The easiest way to evaluate it is by expressing it mathematically and then graphing it alongside the expansion space volume. The graph can then be used to visually determine the swept volumes, minimum volumes, and phase angle.

In the expansion space the volume is:

$$\mathbf{V_{exp} = V_{Mh} + \frac{1}{2}V_d - \frac{1}{2}V_d \sin(\theta + \alpha)} \tag{b 1}$$

In the compression space the volume is:

$$\mathbf{V_{com} = V_{Mc} + \frac{1}{2}V_p + \frac{1}{2}V_d - \frac{1}{2}V_p \sin(\theta) + \frac{1}{2}V_d \sin(\theta + \alpha)} \tag{b 2}$$

Where :

α = the phase angle (a constant=90°) between the displacer and the power piston.

θ = the crank angle of rotation (this changes with time).from 0° to 360°.

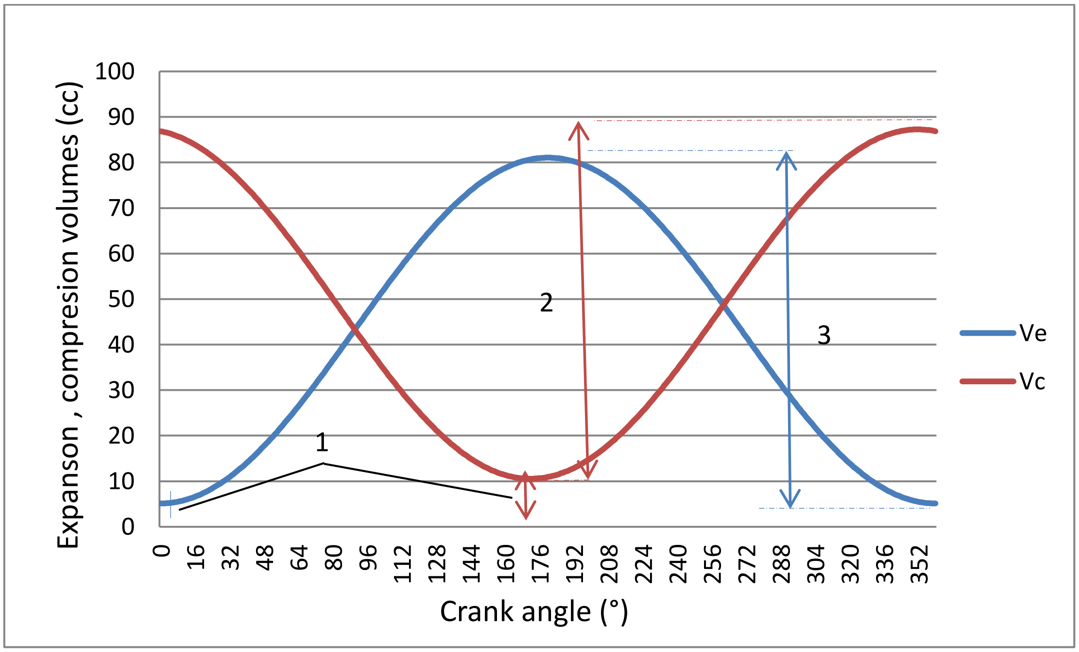


Fig 5: Expansion-compression volumes variation as a function of crank angle.

The graph below shows an example of the volume variations Vexp and Vcom as a function of crank angle θ. In this case we chose cubic-centimeter as the units for the volume where:

1: Minimum Volume in Expansion and Compression space.
2: The swept Volume in the Compression space.
3: The swept Volume in the Expansion space.

3- Stirling cycle analysis

Analyse and theoretical calculation of the Stirling cycle of the proposed configuration.The initial parameters of our Stirling engine is:

Table 1: Presentation of the initial Stirling engine parameters.

Hot temperature	$\mathbf{T_3}$ = 323.15 K (50°C)
Cold temperature	$\mathbf{T_1}$ = 283.15 K (10°C)
Volume of gas displaced by the working piston	$\mathbf{V_p}$ = 12.4 cc
Volume of gas displaced by the displacer	$\mathbf{V_D}$ = 75.93 cc
Hot Dead Volume	$\mathbf{V_{Mh}}$ = 5.1 cc
Regenerator Dead volume	$\mathbf{V_{MR}}$ = 6.7 cc
Cold Dead volume	$\mathbf{V_{Mc}}$ = 5.5 cc

3-1- The dead volumes

Let's say that the dead volumes in the heater, regenerator and cooler are expressed in cubicmeters and are respectively expressed by V_{Mh}; V_{MR} and V_{Mc} then the total dead volume is expressed by :

$$\mathbf{V_{MT} = V_{Mh} + V_{MR} + V_{Mc} = (k_{Mh} + k_{MR} + k_{Mc})\, V_{MT}} \qquad \mathbf{(1)}$$

$\mathbf{V_{MT} = 5.1+6.7+5.5 = 17.3\ cc}$

When :

$\mathbf{k_{Mh} = V_{Mh}/ V_{MT} = 0.29}$: represents the ratio of the hot dead volume on the total dead volume.

$\mathbf{k_{MR} = V_{MR}/V_{MT} = 0.39}$: represents the ratio of the regenerator dead volume on the total dead volume.

$\mathbf{k_{MC} = V_{MC} / V_{MT} = 0.32}$**:** represents the ratio of the cold dead volume on the total dead volume .

Now we can represent the ratio between the total dead volume and the total volume of the gas contained in the engine by: $\mathbf{k_{MT} = V_{MT} / V_T}$, then the total dead volume can be expressed by :

$$\mathbf{V_{MT} = k_{MT} * V_T = k_{MT} * (V_{MT} + V_D + V_P)} \qquad \mathbf{(2)}$$

Or $\mathbf{V_D}$ and $\mathbf{V_P}$represent the volume displaced by the displacement piston and the working piston in cubic meter.

The dead volume of strling engine is often expressed with reference to the total volume of the displaced gas it gives us so:

$$\mathbf{V_{MT} = k_{MDP} * (V_D + V_P)} \qquad \mathbf{(3)}$$

Where:

$\mathbf{K_{MDP}}$ **:** Is The ratio of the dead volumes to the swept volumes.

$\mathbf{K_{MDP} = 0.196}$

$\mathbf{K_{MDP} = 19.6\ \%}$

As a result the ratio of dead volume on the total volume is that of the dead volume to totalswept volumes. It is expressed by :

$$k_{MT}= k_{MDP} /(1+k_{MDP}) \text{or} \quad k_{MDP}= k_{MT}/(1-k_{MT}) \qquad (4)$$

$$k_{MT} = 0.164$$

3-2- Imperfect regenerator

We take e = 0.75

The effectiveness of the regenerator '**e**' is expressed as follows:

$$e= (T_3' - T_1) / (T_3 - T_1) \qquad (5)$$

The value of "**e**" is from 1 to 100% of efficiency, this corresponds to an ideal regeneration. A value of zero corresponds to a total absence of regenerative or worse, an efficiency of 0%. The temperature ofthe working gas at the outlet of the regenerator can be calculated from the effectiveness thereof.This gives us then:

$$T_3'= T_1 + e\ (T_3 - T_1) \qquad (5a)$$

For a regenerator having the same efficiency of heating and cooling $Q_{2\text{-}3'} = Q_{4\text{-}1'}$, the temperature of the working gas at the entrance will be:

$$T_1=T_3 + e\ (T_1 - T_3) = T_3 - e\ (T_3-T_1) \qquad (6)$$

For a Stirling engine containing substantial dead volume, it is important to have a correcttemperature of the working fluid in the regenerator. The actual temperature of the gas in thedead volume of the regenerator can be determined by a simple average:

$$T_R= \frac{T_{3\prime}+T_{1\prime}}{2} \qquad (7)$$

Substituting equations (5a) and (6) into equation (7) we get:

$$T_R=\frac{T_3+T_1}{2} \qquad (7a)$$

T_R = 303.15 K

We observe that when we use the arithmetic average of the effective, the temperature of the regenerator becomes independent of its effectiveness.

3-3- Equations of state

We start with the equation of state corresponding to the isothermal transformation 1-2 .where we have hot and cold areas, respectively V_h and V_c, and the temperatures of the working fluid respectively correspond to T_3 and T_1. Without forget the temperature of the regenerator T_R .in this equation we consider the dead volumes V_{Mh}, V_{Mc} and V_{MR}, this gives us:

$$P=\frac{mR}{\frac{V_h}{T_3}+\frac{V_{Mh}}{T_3}+\frac{V_{MR}}{T_R}+\frac{V_{Mc}}{T_1}+\frac{V_c}{T_1}}=\frac{mR}{\frac{V_h}{T_3}+K+\frac{V_c}{T_1}} \qquad \textbf{(8)}$$

Or:

$$K=\frac{V_{Mh}}{T_3}+\frac{V_{MR}}{T_R}+\frac{V_{Mc}}{T_1} \qquad \textbf{(9)}$$

m corresponds to the total mass of gas contained in the engine (in kg).

Substituting equation (1) and (7a) into equation (9) we get:

$$K=\left(\frac{k_{Mh}}{T_3}+\frac{2k_{MR}}{T_3+T_1}+\frac{k_{Mc}}{T_1}\right)V_{MT} \qquad \textbf{(9a)}$$

K = 5.75*10^- 8

It becomes clear now that for the working fluid temperature data **K** factor is a function of dead volumes.

We fill our engine with air at atmospheric pressure.The volume then correspond to the total maximum volume is:

$V_T = V_{MT} + V_D + V_P$ = 105.7 cc

The filling temperature being set at (20 °C), it is valid for the three volumes.

So, the mass of gas containing in the engine is:

$$\mathbf{m}=\frac{P}{R}*\frac{\mathrm{Vt}}{T}\mathbf{(c)}$$

$$\mathbf{m}=\frac{101300*105{,}7*10\text{^}-6}{287*293}$$

m = 1.27*10^-4 Kg

3-3-1- Isothermal compression (1-2)

We will now compute the different state transitions for a cycle Complete taking into account the efficiency of the regenerator, arbitrarily set at 75% (e = 0.75), and the dead volumes present.

During the processing corresponding to the isothermal compression, the working fluid contained inthe cold space is compressed from$\mathbf{V_{c1} = V_D + V_p}$ to $\mathbf{V_{c2} = V_D}$ .where $\mathbf{V_{c1}}$ and $\mathbf{V_{c2}}$ represent the volumes corresponding to pistons 1 and 2 in the diagram P-V of the figure 2-1.

So the heat released during isothermal transformation 1-2 is:

$$Q_{1\text{-}2} = W_{1\text{-}2} = \int_{V_{c1}}^{V_{c2}} p\,dV_c = mRT_1\int_{V_{c1}}^{V_{c2}} dV_c/(V_c + KT_1)$$

$$= mRT_1\ln\left(\frac{V_{c2}+KT_1}{V_{c1}+KT_1}\right)$$

$$= mRT_1\ln\left(\frac{V_D+KT_1}{V_D+V_P+KT_1}\right) \qquad \mathbf{(10)}$$

N.A:

$$Q_{1\text{-}2} = -\ 1.30\ \mathrm{J}$$

It is worth noting that the compression work depends only on the factor K, which is a function of the dead volumes.

3-3-2- Isochoric heating (2-3)

In principle, the heat added during the isochoric heating (2-3) is:

$$Q_{2-3} = mC_v\,(T_3 - T_{3'}) = mC_v\,(T_3 - T_1) \qquad (11)$$

N.A:

$$Q_{2-3} = 2.7\ \mathrm{J}$$

Or C_V is the specific heat at constant volume (j / kg K) assumed to be a constant. Without regenerator, this supply of heat is obtained from an external source. With an ideal regenerator, this contribution would come only from it.

The heat restored by an imperfect regenerator during this transformation is:

$$Q_{2-3'} = C_v\,(T_3 - T_{3'}) = emC_v\,(T_3 - T_1) \qquad (12)$$

N.A:

$$Q_{2-3'} = 2.03\ \mathrm{J}$$

So the heat provided by the external source for 3'-3 becomes:

$$Q_{3-3'} = mC_v\,(T_3 - T_{3'}) = (1-e)\,mC_v\,(T_3 - T_1) \qquad (13)$$

N.A:

$$Q_{3-3'} = 0.92\ \mathrm{J}$$

It is clear that during this phase of the cycle, the heat provided depends on the efficiency of the regenerator only.

3-3-3- The isothermal expansion (3-4)

During the isothermal transformation (3-4) the hot volume containing the working gas varies from $V_{h3} = V_D$ to $V_{h4} = V_D + V_p$ .The volume containing the cold gasV_c is assumed to be equal to zero .During this transformation, the heat provided is:

$$Q_{3-4} = W_{3-4} = m\int_{V_{h3}}^{V_{h4}} p\,dV_h = mRT_3 \int_{V_{h3}}^{V_{h4}} \frac{dV_h}{V_h + KT_3}$$

$$= mRT_3 \ln\left(\frac{V_{h4} + KT_3}{V_{h3} + KT_3}\right)$$

$$= RT_3 \ln\left(\frac{V_D+V_P+KT_3}{V_D+KT_3}\right) \quad (14)$$

N.A:

$$Q_{3\text{-}4} = 1.46 \text{ J}$$

It is now clear that the work done is dependent on dead volumes.

3-3-4- Isochoric cooling (4-1)

The heat released during the isochoric transformation (4-1), phase of fluid cooling is:

$$Q_{4\text{-}1} = mC_v (T_1 - T_4) = - mC_v (T_3 - T_1) \quad (15)$$

N.A:

$$Q_{4\text{-}1} = - 3.7 \text{ J}$$

Without regenerator, this heat is dissipated in the cooler. With a perfect regenerator it would be completely absorbed by it. With imperfect regeneration, the heat absorbed by the regenerator is then expressed by the following formula:

$$Q_{4\text{-}1'} = mC_v (T_{1'} - T_4) = - e\, m\, Cv (T_3 - T_1) \quad (16)$$

N.A:

$$Q_{4\text{-}1'} = - 2.8 \text{ J}$$

The heat released during processing 1-1’ is then:

$$Q_{1\text{-}1'} = mC_v (T_1 - T_{1'}) = - (1-e)\, mC_v (T_3 - T_1) \quad (17)$$

N.A:

$$Q_{1\text{-}1'} = - 0.9 \text{ J}$$

We see here that the heat transfer during the cooling phase depends on the efficiency of theregenerator.

3-4- The total heat added

For an imperfect regeneration, the total heat added from an external source in the cycle is:

$$Q_a = Q_{3\text{-}3'} + Q_{3\text{-}4} \quad (18)$$

With more details:

$$\mathbf{Q_a = m\,C_v\,((T_3 - T_{3'}) + (k-1)\,T_3 \ln(\frac{V_{h4}+KT_3}{V_{h3}+KT_3})\,)}$$

$$\mathbf{= m\,C_v\,((1-e)\,(T_3 - T_1) + (k-1)\,T_3 \ln(\frac{V_D+V_P+KT_3}{V_D+KT_3}\,)\,)} \qquad \mathbf{(18a)}$$

N.A:

$$\mathbf{Q_a = 2.4\ J}$$

It is worth recalling that k represents the specific heat ratio. It now appears obvious that the total heat input to the engine depends on the quality of the regenerator and the amount of dead volume present.

Without regeneration or with a regenerator of zero efficiency, heatTo the engine from an external source would

$$\mathbf{Q_a = Q_{2\text{-}3} + Q_{3\text{-}4}} \qquad \mathbf{(19)}$$

On the other hand, with a perfect regenerator (e = 100%), it will be:

$$\mathbf{Q_a = Q_{3\text{-}4}} \qquad \mathbf{(20)}$$

3-5- The total heat rejected to the outside

With an imperfect regenerator the heat extracted from the cycleis:

$$\mathbf{Q_r = Q_{1\text{-}1'} + Q_{1\text{-}2}} \qquad \mathbf{(21)}$$

This gives:

$$\mathbf{Q_r = m\,C_v\,((T_1 - T_{1'})\,(k-1)\,T_1 \ln\,(\frac{V_{c1}+KT_1}{V_{c2}+KT_1})\,)}$$

$$\mathbf{= m\,C_v\,((1-e)\,(T_3 - T_1) + (k-1)\,T_3 \ln(\frac{V_D+V_P+KT_1}{V_D+KT_1})\,)} \qquad \mathbf{(21a)}$$

N.A:

$$\mathbf{Q_r = -\,2.2\ J}$$

Here we have a strong resemblance between added heat and extracted heat. The dependence of these phenomena on the quality of the regenerator and the presence of dead volumes becomes indisputable.

Without any regeneration it would give:

$$Q_r = Q_{4\text{-}1} + Q_{1\text{-}2} \tag{22}$$

Whereas with the ideal regenerator it would be:

$$Q_r = Q_{1\text{-}2} \tag{23}$$

It seems obvious that symmetry of operation of the regenerator on the hot side and cold side also appears here as indispensable.

3-6- Effective work:

The excess energy of the two isothermal transformations 1-2 and 3-4 is transformed into useful mechanical work. This work, with a motor equipped with an imperfect regenerator and comprising dead volumes can be calculated like this:

$$W_{net} = \sum Q = Q_a + Q_r = Q_{3\text{-}3'} + Q_{3\text{-}4} + Q_{1\text{-}1'} + Q_{1\text{-}2}$$

$$= Q_{3\text{-}4} + Q_{1\text{-}2}$$

$$= mR\left[\ T_3 \ln\left(\frac{V_{h4}+KT_3}{V_{h3}+KT_3}\right) - T_1 \ln\left(\frac{V_{c1}+KT_1}{V_{c2}+KT_1}\right)\ \right] \tag{24}$$

Note that V_{h4} (warm volume at position 4 on the p-v diagram) is equal to V_{c1} (isochoric) that gives $V_D + V_p = V_T$. And $V_{h3} = V_{h2} = V_D = V_2$, we can deduce that :

$$W_{net} = mR\left[T_3 \ln\left(\frac{V_D+V_P+KT_3}{V_D+KT_3}\right) - T_1 \ln\left(\frac{V_D+V_P+KT_1}{V_D+KT_1}\right)\right] \tag{24a}$$

N.A:

$$W_{net} = 0.15\ J$$

It is obvious that the work done by the cycle depends on the dead volumes. If they were reduced to nothing it would give us this:

$$W_{net} = m\,R \ln\left(\frac{V_1}{V_2}\right)(T_3 - T_1) \tag{25}$$

This equation is the one found in thermodynamic works. Let us also note that in the absence of dead volumes the work provided by the cycle is totally independent of the efficiency of the regenerator.

3-7- Mean effective pressure

The mechanical work may be determined by the mean effective pressure of the cycle (Pm) and volume changes, which ultimately boil down to the volume swept by the working piston:

$$\mathbf{V_{h4} - V_{h3} = V_{c1} - V_{c2} = V_1 - V_2 = V_p}$$

Which gives:

$$\mathbf{W_{net} = P_m V_p} \qquad \mathbf{(26)}$$

With the equation (24) we get:

$$\mathbf{P_m = \left(\frac{m\,R}{V_p}\right)\left[T_3 \ln\left(\frac{V_D+V_P+KT_3}{V_D+KT_3}\right) - T_1 \ln\left(\frac{V_D+V_P+KT_1}{V_D+KT_1}\right)\right]} \qquad \mathbf{(27)}$$

N.A:

$$\mathbf{P_m = 222663\ Pa}$$

It should also be noted that the actual pressure depends on the dead volumes. If there were none, it would give us this. Using the law of perfect gases as well as remembering that $T_1 = T_2$ and that $V_1 = V_3$:

$$\mathbf{P_m = \left[\frac{(P_3-P_2)V_3}{V_1-V_2}\right] \ln\left(\frac{V_1}{V_2}\right)}$$

$$\mathbf{= \frac{(P_3-P_2)\,(V_{2-1}-1)}{V_1} \ln\left(\frac{V_1}{V_2}\right)} \qquad \mathbf{(28)}$$

Another solution is to calculate the pressure at the four "corners" of the cycle and to average them. This solution involves calculating the density of the gas relative to the specific volume. This is the principle adopted in the practice set out below.

3-8- Thermal efficiency

The thermal efficiency of a Stirling engine η can be seen as:

$$\eta = \frac{W_{net}}{Q_a} \tag{29}$$

This gives to our imperfect engine:

$$\eta = \frac{\left[T_3 \ln\frac{(V_D - V_P - KT_3)}{(V_D - KT_3)} - T_1 \ln\frac{(V_D - V_P - KT_1)}{(V_D - KT_1)}\right]}{\left[T_3 \ln\frac{(V_D - V_P - KT_3)}{(V_D - KT_3)} + (T_3 - T_1)\frac{(1-e)}{k-1}\right]} \tag{29a}$$

N.A:

$\eta_{thermal}$ = 6.34 %

It is noted that the thermal efficiency of the Stirling engine depends on both the quality of the regenerator and the proportion of dead volumes present.

Without dead volumes equation (29a) would bereduced to:

$$\eta = \frac{(T_3 - T_1)(k-1)\ln\frac{V_1}{V_2}}{T_3 + (T_3 - T_1)} \tag{30}$$

Even when a different notation is used as in Equation (30), the results are identical using conventional thermodynamics or thermodynamics in finite time. In the absence of regenerator (e = 0), which represents the worst case for the efficiency of the Stirling engine cycle. The thermal efficiency would then be:

$$\eta = (T_3 - T_1)\frac{(k-1)\ln\frac{V_1}{V_2}}{T_3 + (T_3 - T_1)} \tag{31}$$

Where: « k » is the specific heat ratio.

N.A:

η= 1.5 %

Note that:

- Without the regenerator the performance of our low temperature differential Stirling engine will be very low.

It will be noted that equations (30) and (31) are identical in the absence of Dead volumes.

If now we had the perfect regenerator e = 1 and a total absence of dead volumes, which would be ideal for a Stirling engine, the thermal efficiency would be:

$$\eta = \frac{T_3 - T_1}{T_3} = 1 - \frac{T_1}{T_3} \qquad (32)$$

N.A:

$\eta_{carnot} = 12.38\ \%$

This makes us say that the Stirling cycle has the efficiency of the Carnot cycle operating between two respective thermal sources T_3 and T_1.

It is now obvious that in theory, the Stirling engine can be a very cost-effective machine to convert heat into mechanical work. The effectiveness of a design and manufacture that would exclude dead volumes while including a quasi-perfect regenerator would be equivalent to realizing a Carnot machine. In other words, it would be the achievement of the most efficient in terms of energy efficiency.

4- The temperature effects on the engine efficiency

It is clear that the power and the efficiency of the sitirling engine depends on the changes of the temperature of the hot and the cold sources. The temperature of the hot source is a very important parameter since it determines the operating range of the Stirling engine.

To show the variation of our Stirling engine efficiency who it's the initial configuration indicated in Table N°1. For ease the mathematical calculations we enter the engine data and equations into an Exel spreadsheet. The results are shown as a graph :

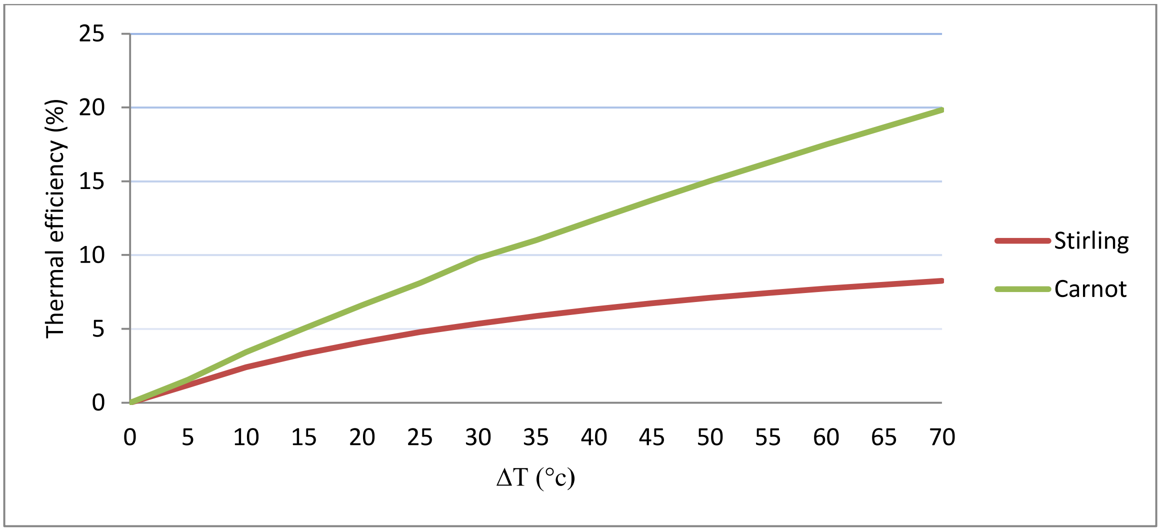


Fig6: Thermal efficiency in function of the difference of temperature.

The previous figure represents a graphical illustration of the changes of the thermal efficiency in the Stirling and Carnot cycles with the variation of Δ**T**, we can notice that the efficiency increases with the augmentation of the difference of temperature in the both cases .But we can see that the Stirling cycle cannot reach Carnot cycle (the perfect case). This contrast is due to the presence of the imperfect regenerator and the dead volumes, because it is not possible to obtain 100% efficient regenerator and hence there will be always 10 to 20% loss of heat in the regenerator, which decreases the cycle efficiency.

5- Conclusion:

From this study we can conclude that:

1. For a Stirling engine with a given dead volume, inefficient regenerator will not affect the actual work if the actual temperature of the regenerator is an arithmetic mean of the extreme temperatures of the working gas. However, a Stirling engine with a poor regenerative require more heat and require better cooling if it was equipped with an efficient regenerator.
2. The dead volumes will degrade both energy efficiency and increase the contribution of external heat. Note that the construction of a Stirling engine inevitably leads to the presence of unavoidable dead volumes.
3. A low mechanical work can still be obtained if the engine contains a rate of large dead volumes.

4. To achieve high performance, a Stirling engine must be equipped with the most efficient regenerator. If it was not, it would be better not to have a regenerator at all. Which saves us its dead volume.

Chapter III :

Engine modeling and FEA analysis of flywheel

1- Engine modeling

1-1- Introduction

To aid in the creation, modification, analysis, or optimization of a design, CAD software is used to increase the productivity of the designer, improve the quality of design, improve communications through documentation, and to create a database for manufacturing.In the case studied, it helps us a lot in representing and knowing the volumes and taking a conceptual view of the engine and its parts and allows the possibility of revision or change in the mechanical system of design.

1-2 Drawings

The scheme of this engine was selected which corresponds to the table data 1 in terms of the absolute temperature ratio and gas volume. For our configuration the Stirling engine consists of 20 parts.
The following two assembly drawings show the arrangement of the parts and their names.

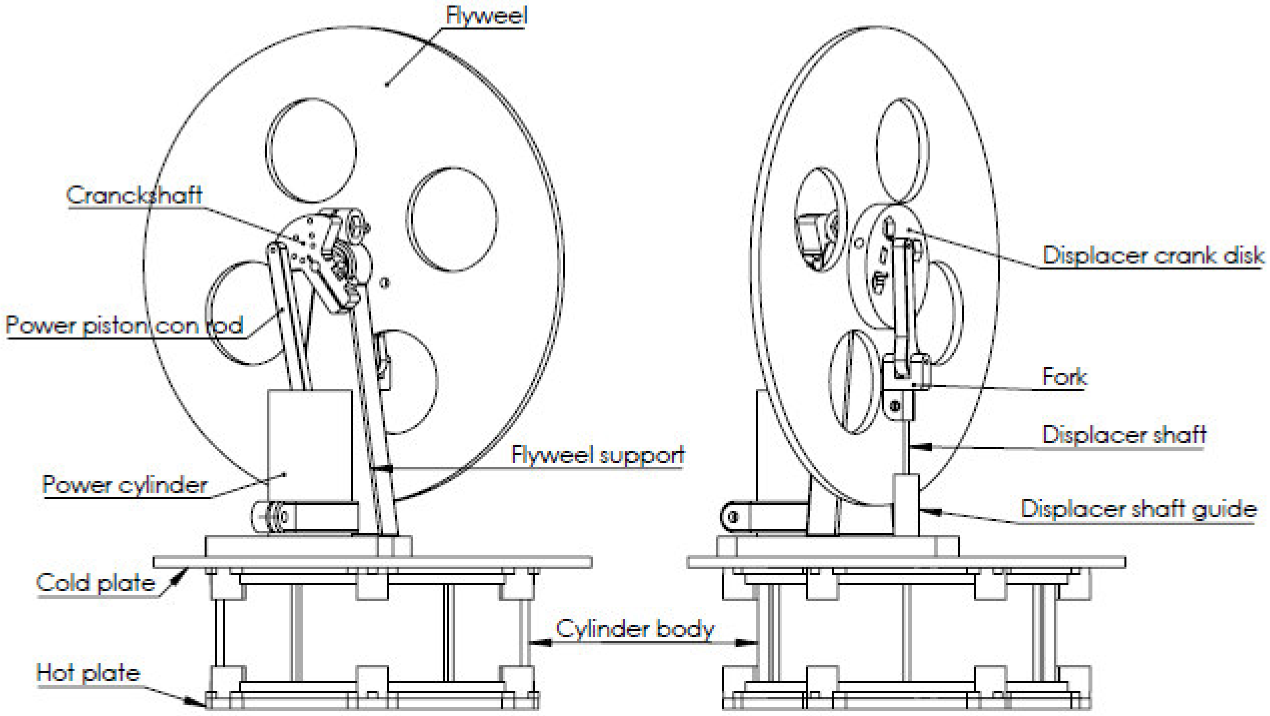


Fig 1: The arrangement of the parts.

The second assembly drawing has various parts made invisible so we can see some of the internal parts.

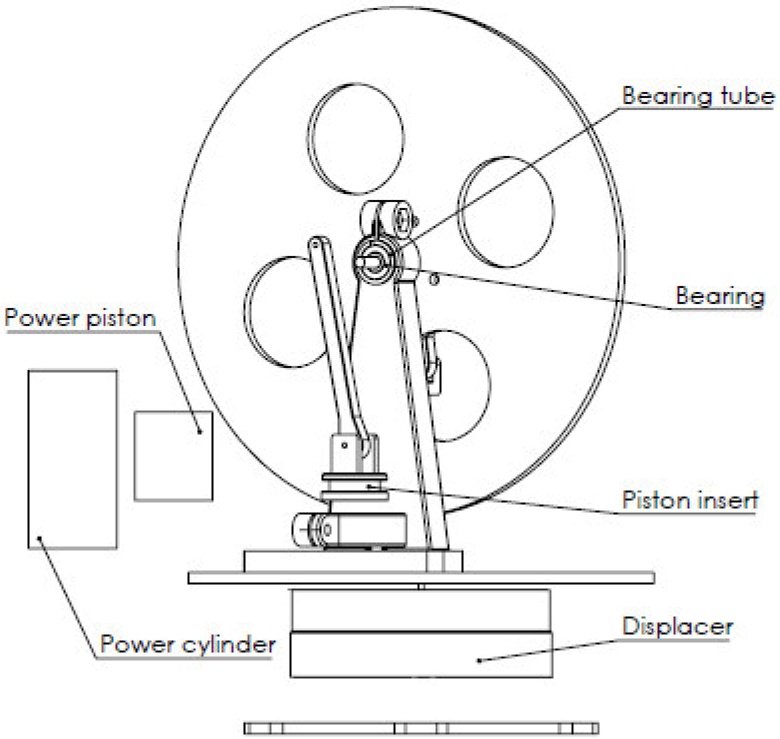


Fig 2: The internal parts of stirling engine.

1-3- Modeling

Based on the drawings we represent the 20 engine components all in one by using the CAD software Solidworks. After the representation of the parts we collect the pieces using the assembly property available in the software to show the engine parts formed a 3D assembly.

The following shows the all engine parts.

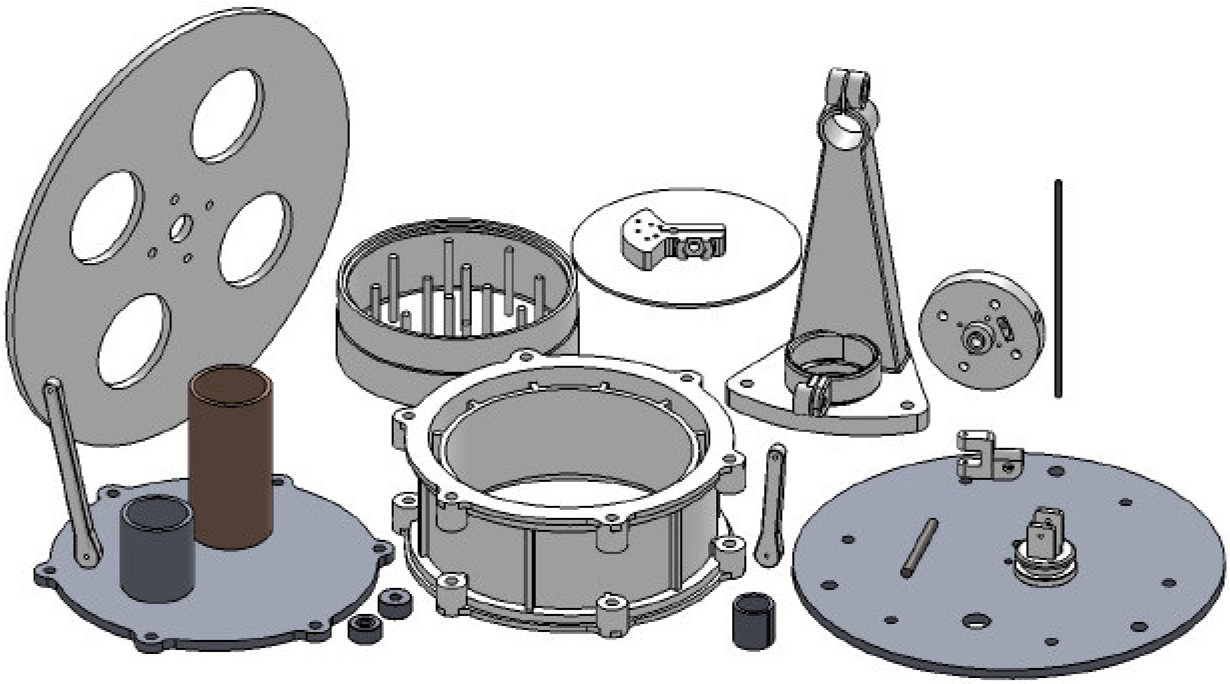


Fig 3: All modeled parts of stirling engine.

The following assembly parts show the final arrangement of the parts.

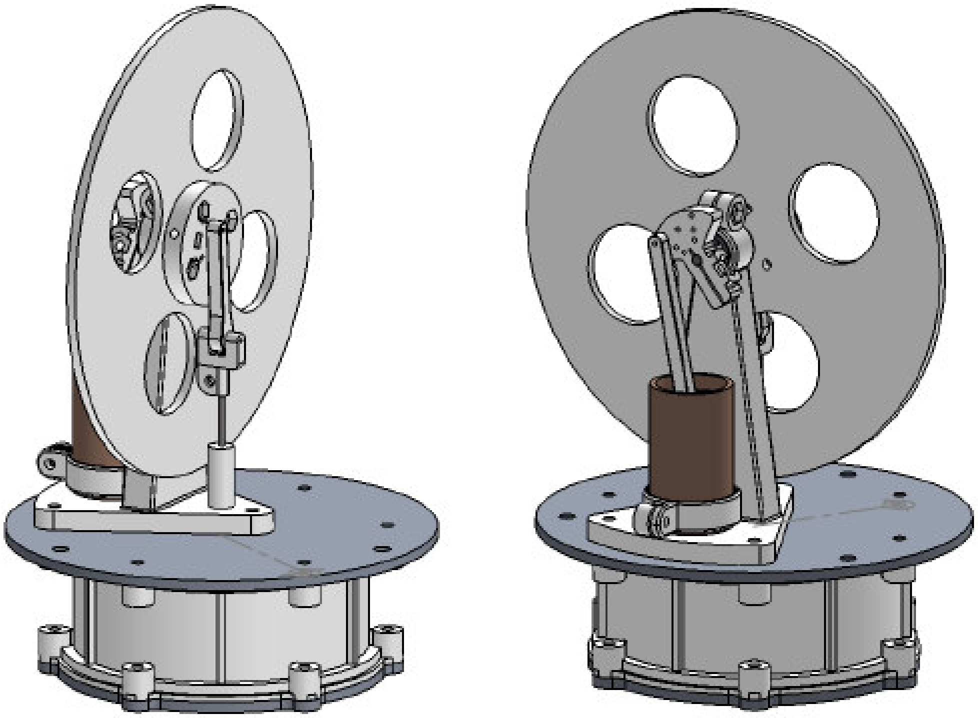


Fig 4: Final engine assembly of stirling engine.

2- Finite element analysis of flywheel with ansys

2-1- Introduction

A flywheel is an inertial energy-storage device. It absorbs mechanical energy and serves as a reservoir, storing energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than the supply A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply.

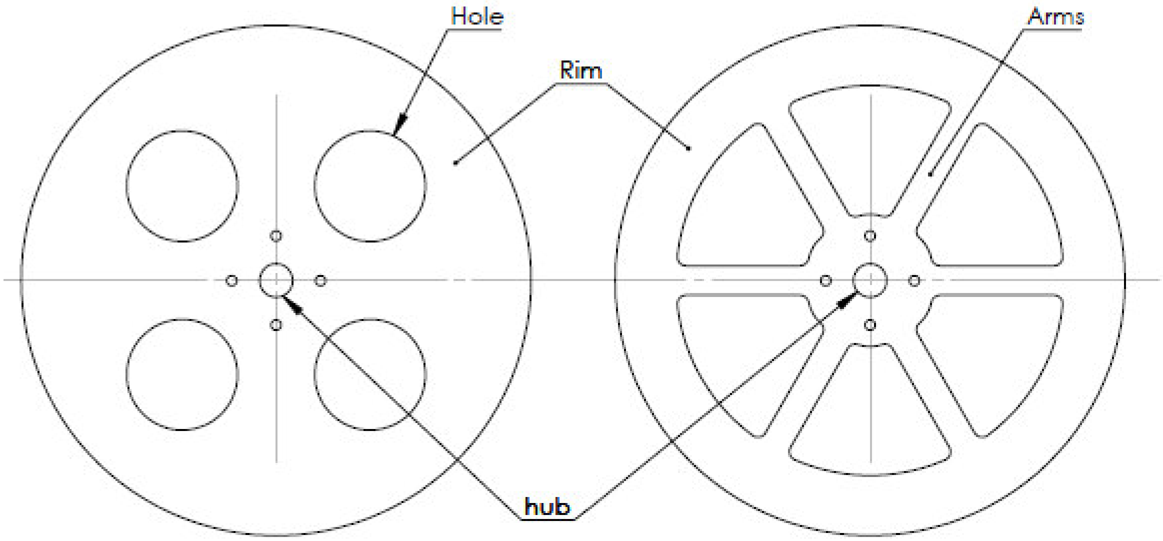


Fig 5: Types of flywheel.

2-2- Finite element analysis

The ANSYS Workbench environment is an intuitive up-front finite element analysis tool that is used in conjunction with CAD systems and/or Design Modeler. ANSYS Workbench is a software environment for performing structural, thermal, and electromagnetic analyses. The class focuses on attaching existing geometry, setting up the finite element model, solving, and reviewing results. The class will describe how to use the code as well as basic finite element simulation concepts and results interpretation. The finite element method (FEM) is a method for dividing up a very complicated problem into small elements that can be solved in relation to each other. Its practical application is often known as finite element analysis (FEA).

2-3- Generic Steps to Solving any Problem in ANSYS

Like solving any problem analytically, you need to define (1) your solution domain, (2) the physical model, (3) boundary conditions and (4) the physical properties. You then solve the problem and present the results. In numerical methods, the main difference is an extra step called mesh generation. This is the step that divides the complex model into small elements that become solvable in an otherwise too complex situation. Below describes the processes in terminology slightly more attune to the software.

1. **Build Geometry**: Construct a two or three dimensional representation of the object to be modeled and tested using the work plane coordinates system within ANSYS.
2. **Define Material Properties:** Now that the part exists, define a library of the necessary materials that compose the object (or project) being modeled. This includes thermal and mechanical properties.
3. **Generate Mesh:** At this point ANSYS understands the makeup of the part. Now define how the modeled system should be broken down into finite pieces.
4. **Apply Loads:** Once the system is fully designed, the last task is to burden the system with constraints, such as physical loadings or boundary conditions.
5. **Obtain Solution:** This is actually a step, because ANSYS needs to understand within what state (steady state, transient… etc.) the problem must be solved.

2-4- Physical system analyzed

The flywheel is attached to the drive-shaft for the crank. The physical system is illustrated in Figure 2.

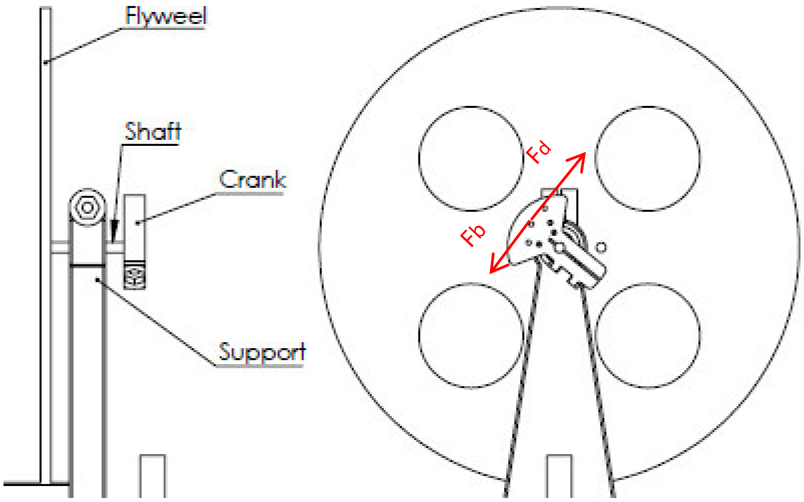


Fig 6: Flywheel in a mechanism system.

To build the physical system into a finite element model easily, some assumptions are needed:

- A rigid installation is used to connect the flywheel and its drive-shaft, no key-ways are needed to fix/drive the flywheel and there exist no slide, built-in stress and deformations on the connection surface; therefore, displacement constraints can be simply applied on the shaft hole.
- The flywheel only works in the vertical plane (X-Y plane) so that the gravity can be simply applied.
- The material used, is isotropic.
- The aero dynamical resistance can be neglected.
- There exists no vibration.
- The fillets/chamfers can be neglected unless dimensioned.

2-5- Flywheel analysis

- **Solid model**

The geometry of flywheel was modeled previously by SOLIDWORKS software, and it was imported to ansys geometry to be analyzed with a static structural.

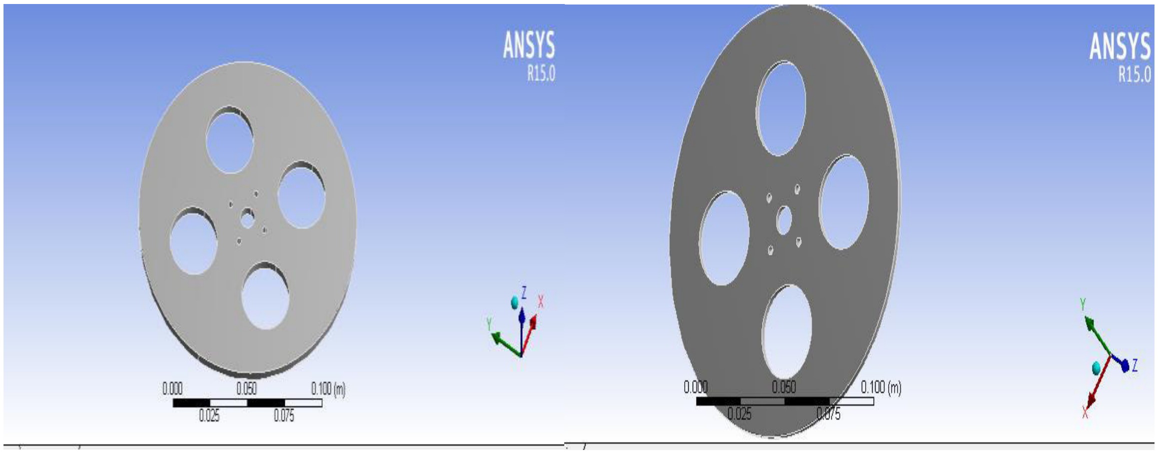


Fig 7: A solid model in Ansys.

- **Material Properties**

Due to its excellent high and low temperature performance, creep resistant, dimensional stability, very high impact strength, lightness and low cost in price, Acrylonitrile butadiene styrene (ABS) plastic is used to make the flywheel, which has the following properties ;

Table 1 : Material properties.

Material	Acrylonitrile butadiene styrene (ABS)
Ultimate strength	40 MPa
Elastic modulus	E=2.3 GPa ; G= 0.8 GPa
Density	1052kg/m^3
Poisson's ratio	0.35

- **Meshing method**

Free mesh with smart element sizing is adopted to automatically and flexibly mesh the model. Compared to mapped mesh, which is restricted to only quadrilateral (area) or only hexahedron (volume) elements, free mesh has no restrictions in terms of element shapes. Smart sizing gives the meshed a greater opportunity to create reasonably shaped element during automatic element generation. To further improve the meshing, thus to get good accuracy and economy of the analysis, a meshed model is shown in figure 8.

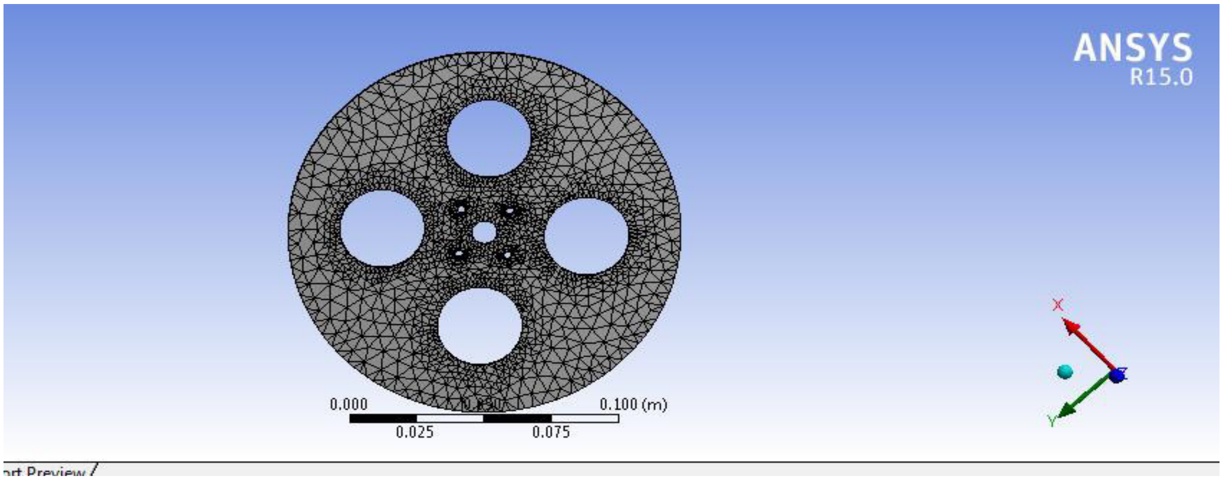


Fig 8 : A meshed model.

Table 2: Meshing properties

Type of finit element	Number of noueds	Number of elements
Tetrahedral	9904	4862

- **Boundary conditions and loads**

➢ Step

Corresponding to the physical system, symmetric constraints have been applied on symmetric middle plane of the flywheel and displacement constraints with u_x=u_y=u_z=0 on the surface of the shaft hole.

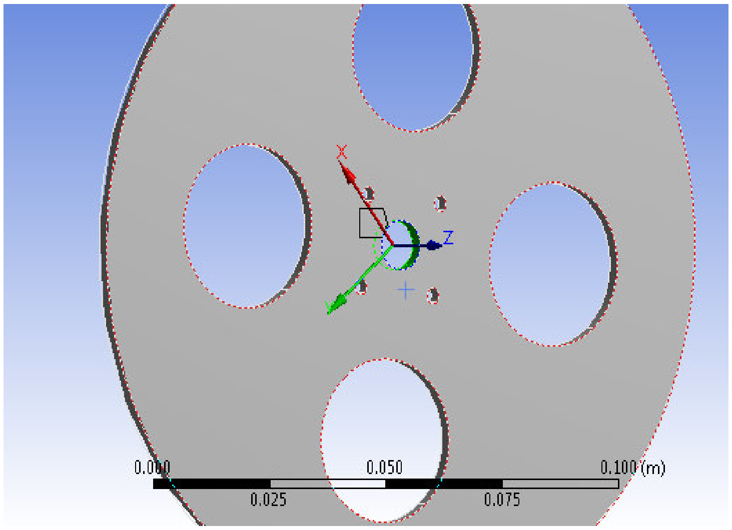
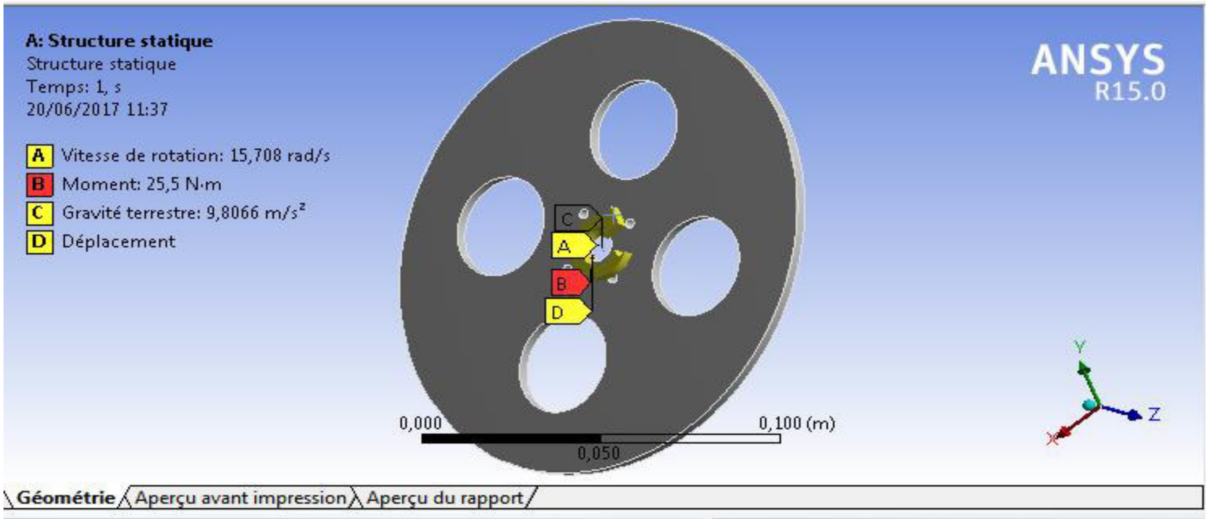


Fig 9: Displacement constraints.

The type of load includes:

- Gravity: acceleration in -Y direction = 9.8 m/s^2.
- Moment: M_Z, applied on the nodes on the shaft-hole surface.
- Rotational velocity equal to 15.708 rad/s.


A: Structure statique
Structure statique
Temps: 1, s
20/06/2017 11:37

A Vitesse de rotation: 15,708 rad/s
B Moment: 25,5 N·m
C Gravité terrestre: 9,8066 m/s^2
D Déplacement

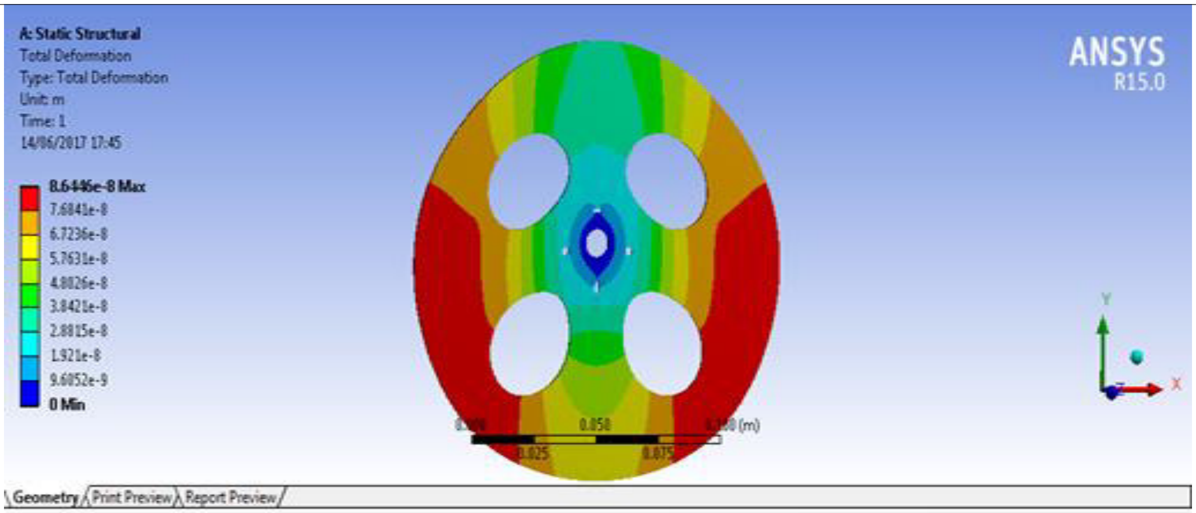
Fig 10 : Boundary conditions.

- **Solutions**

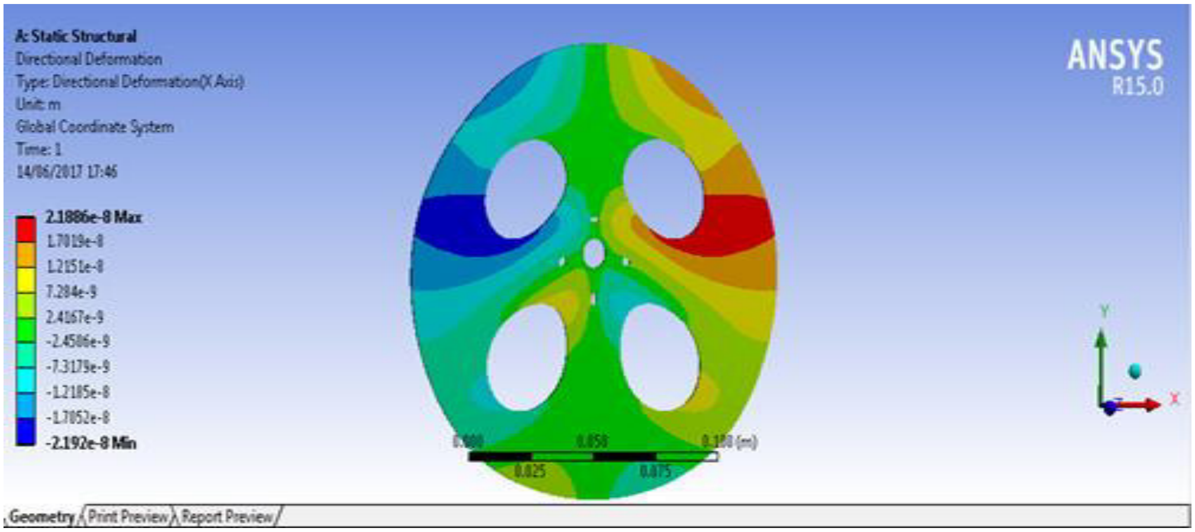
The results of analysis are given in the table 3 below and sketched in the figures attached.

Table 3 : Solution results.

	Total deformation. (m)	directional deformation (m)	Equivalent stress (Von mises) (Pa)	Maximum principal stress (Pa)	Eivalent elastic strain
Minimum	0	-2.192e^-8	12.949	-1807.8	6.0267e^-9
maximum	8.6446e ^-8	2.1886e ^-8	13197	14288	5.8504e^-6

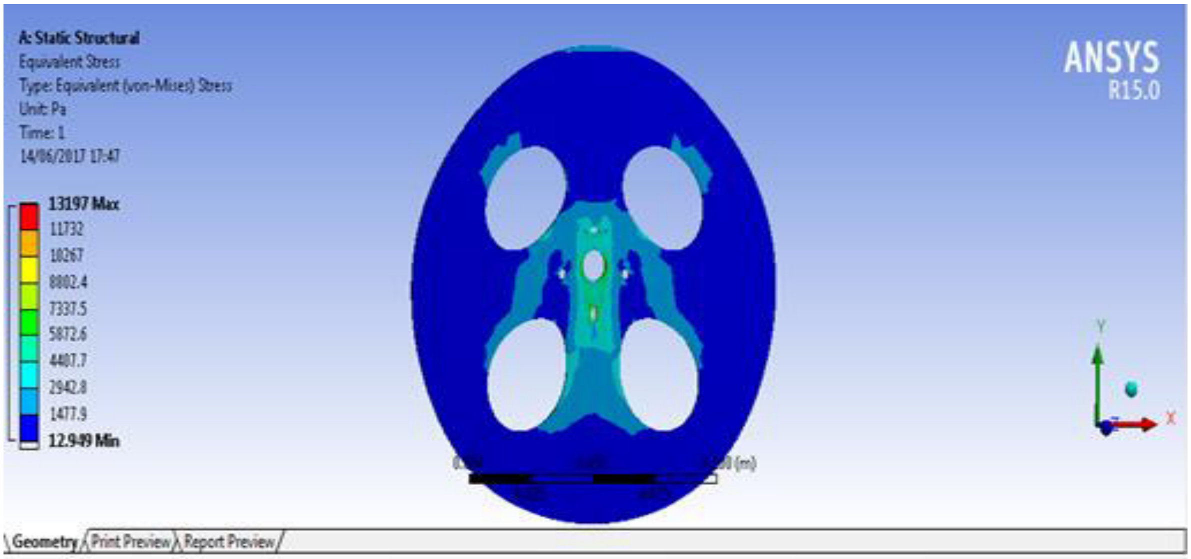


a) Total deformation.

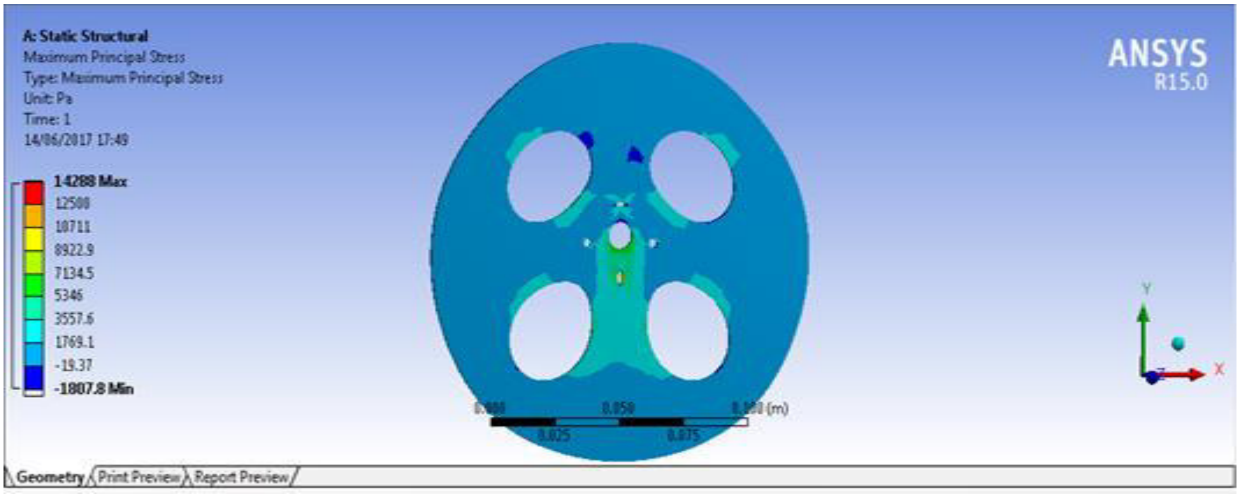


b) Directional deformation.

Fig 11 : Deformation results.



c) Equivalent stress (von mises).



d) Maximum principl stress.

Fig 12 : Stress results.

Comment

From the previous results we notice that:

- There are rotational deformations due to the torsion effect of the drive moment M_Z.
- The maximum stress occurs on the hub and near shaft-hole surface.
- The maximum deformation is along the edge of the rim.

- **Results**

In the second step we keep the same part and same material with the same boundary conditions, but by varying the rotational velocity to know its effect on stress and deformation. All the results are in the table 4 and the two graphs below.

Table 4 : Simulation results.

Rotational velocity (rad/s)	Equivalent stress (Von mises) (Pa)	Total deformation. (m)
15.788	13197	8.6446e ^-8
19.89	14306	9.1442e^-8
24.08	15700	9.8105e^-8
28.08	17279	1.0603e^-7
32.05	19060	1.1656e^-7
36	21120	1.2974e^-7

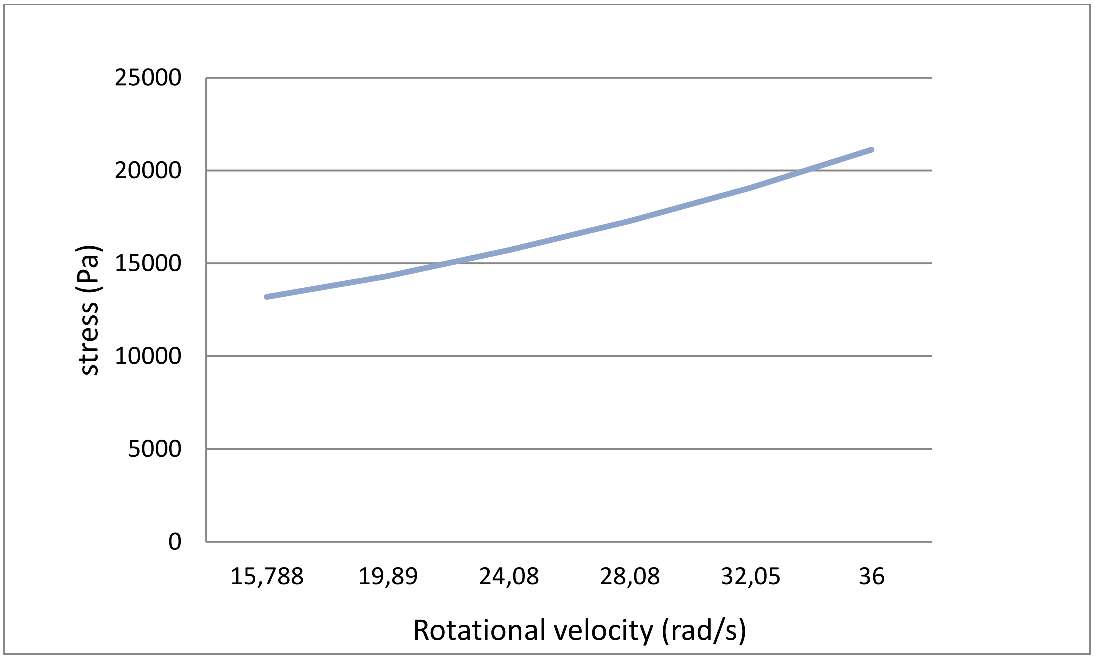


Fig 13: Stress versus velocity variation.

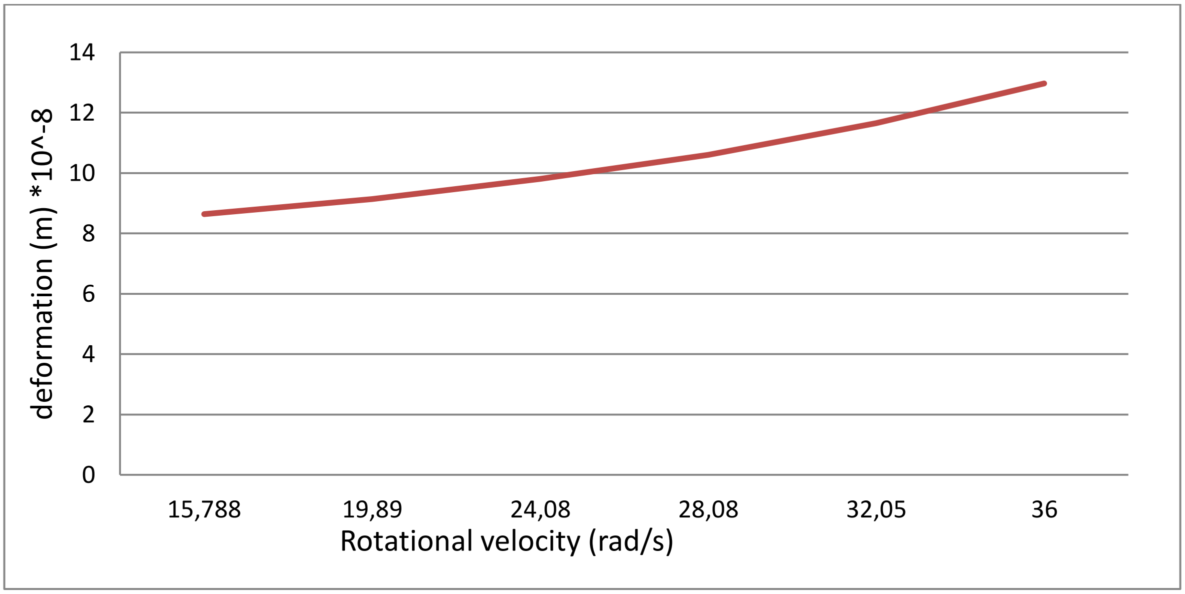


Fig 14: Deformation versus velocity variation.

We notice that the stress and deformation increase when the rotational velocity increases, so we can say that there is a relationship between the stress and the rotational velocity.

2-6- Conclusions

From the results obtained from the analysis, the following conclusions can be drawn:

- The stress and deformation usually occur at the speed-changing stages.
- The stress in the rim is not as large as estimated.
- The wheel-shaped structure contributes to decrease stress concentration.
- There exist certain relationships between rotational velocity and stress.
- The good geometry will contribute to decrease the stress concentration in the product and thus improve its lifetime.

Chapter IV:

Experimental results

Introduction

The aim of our work on this chapter is to calculate the output power experimentally by manufacturing the engine parts and make the assembly of these parts to form our engine.This experience consists to make comparaision between the perfct cycle studied on chapter two and the experiment results.

1-Parts manufacturing

Engine parts are manufactured according to the characteristics of each part and the role it plays. And we can deduce it as follows :

- **Plastic parts :**

The plastic material was chosen for its hardness, light weight and there are parts that are not affected by the heat of the engine or have not a direct contact with it, and ease of making the pieces, especially as they are modeled in the computer.
To make the sepieces, a 3D printer was used to print 10 engine parts.

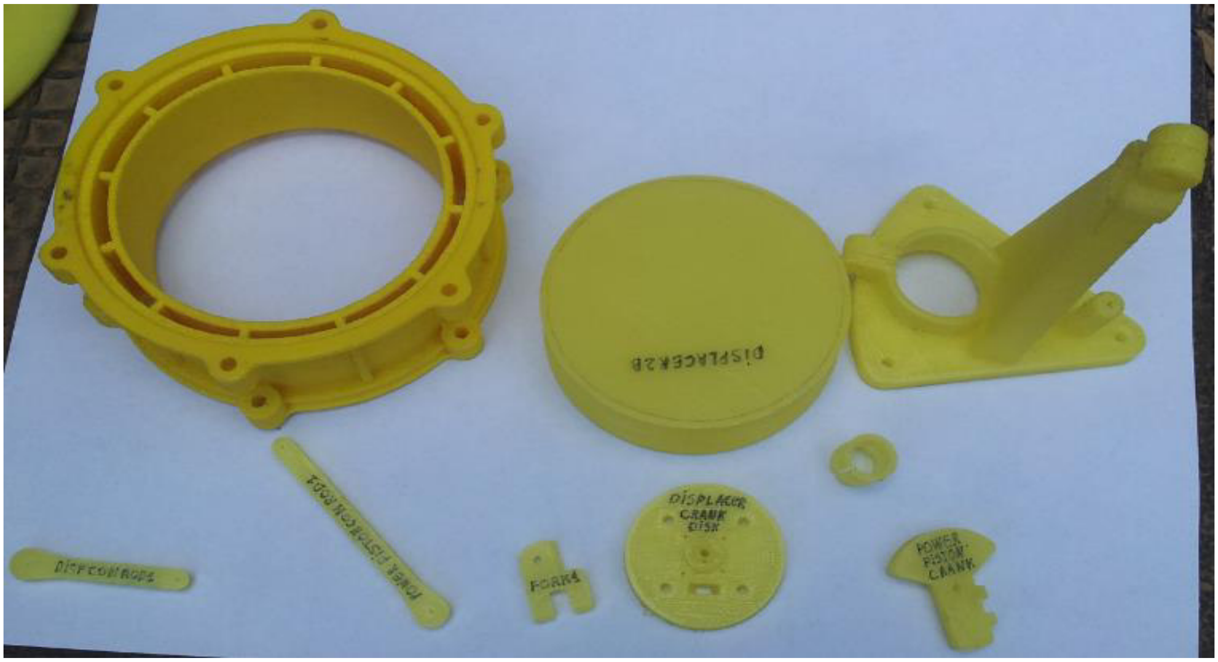


Figure 1: The plastic engine parts.

Figure 2 : The flyweel.

- **Metal parts :**

- The power piston was manufactured using the aluminum material because of its lightness and mechanical resistance.

- The power cylinder was manufactured using the bronze material because of its mechanical resistance and special self lubrication property due to friction with the power piston.

- For the hot and cold plates we used the aluminum material specially these two parts have direct contact with the source of heat and cold and its very high thermal conductivity helps us a lot.

- The power piston and power cylinder were manufactured by a lathe machine, for the hot and the cold plates we used a drill and a grinder.

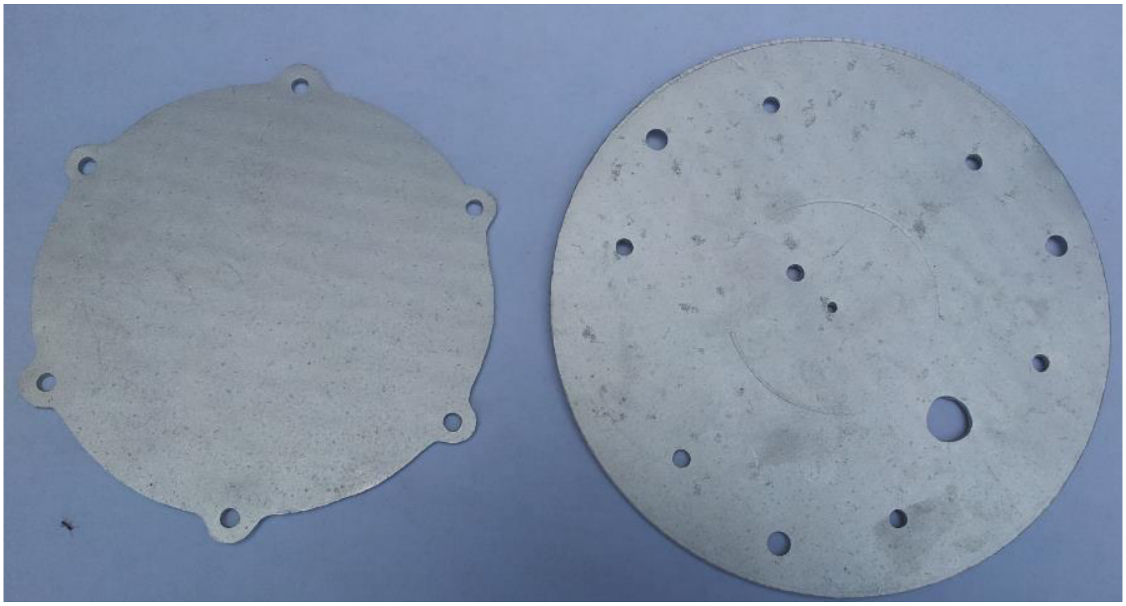


Figure 3: The hot and the cold plates.

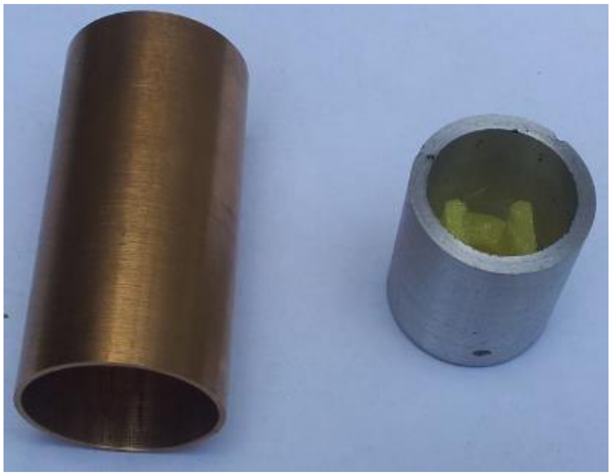


Figure 4: The power piston and cylinder.

2-Assembly

After the manufacturing stage, the first step is to provide the following supplies:

- Crankshaft: copper rod 3 mm diameter * 30 mm long.
- Displacer shaft and wrist pins: steel rod 1 mm diameter.
- Bearings.

Fgure5 : Bearing.

- Fastening and sealing.

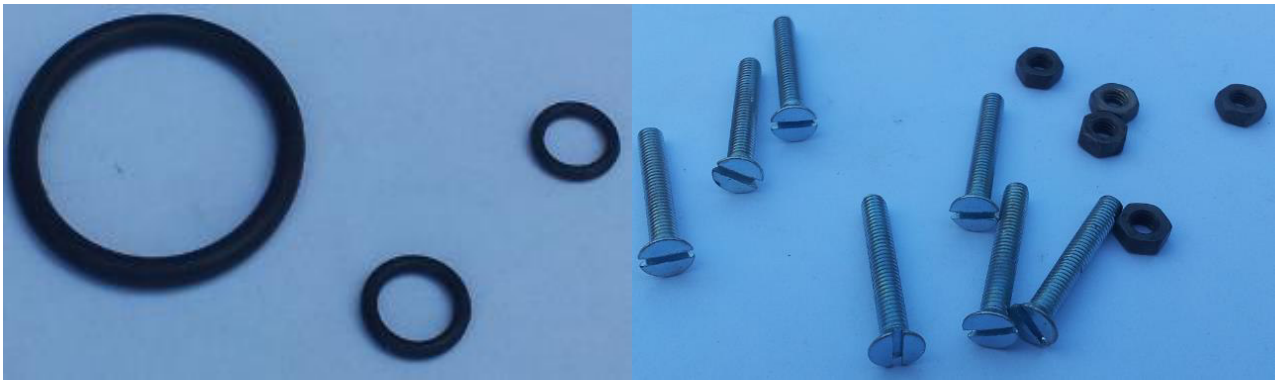


Figure 6: The o-ring and the screws.

- Regenerator: For a material that had a high thermal energy storage, absorb and release it quickly, we choose copper wire as a material for the regenerator.

Figure 7: The copper wire.

Before assembly we must lubricate the power cylinder, shafts and wrist pin with oil to reduce friction and loss of energy.

Figure 8: The Stirling engine assembly.

3- Experience steps

Our experince consists of determine the engine output power.

1. We expose the hot plate with constant heat and wetry to keepa constant difrential temperature between hot and cold plates.For the success of this step we can use hot water and iced water as a heat source and hea tsink respectively.We use a thermometer to note the temperature degree.
2. After the engine started to run we wait until the stability of engine rotation.We use a stroboscope to notice the flyweel speed 1.
3. We connect the engine with a generator dynamo which is connected to a circuit that contains a variable resistor ,voltmeter and ammeter to form the next circuit :

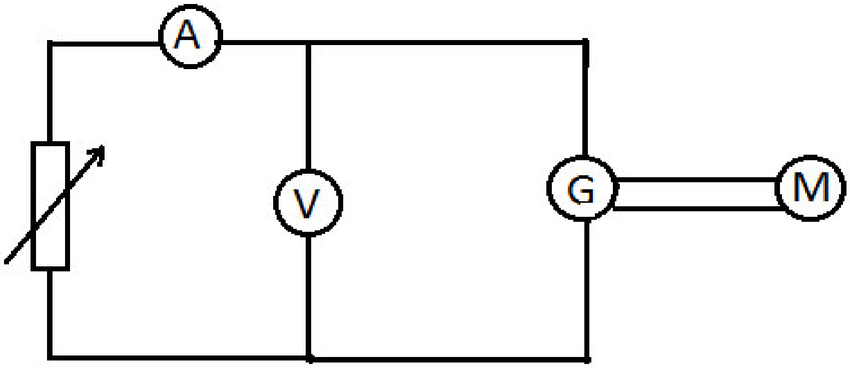


Figure 9: Schematic of the circuit.

We change the variable resistor manually that increased the intensity of the electric current who makes the dynamo consume moreby braking the stirling engine .We stay vary the resistor until the engine speed reduced to the half (speed 1=1/2).After we can take the data displayed on the voltmeter and ampeter (electric tension and intensity) to calculate the power.

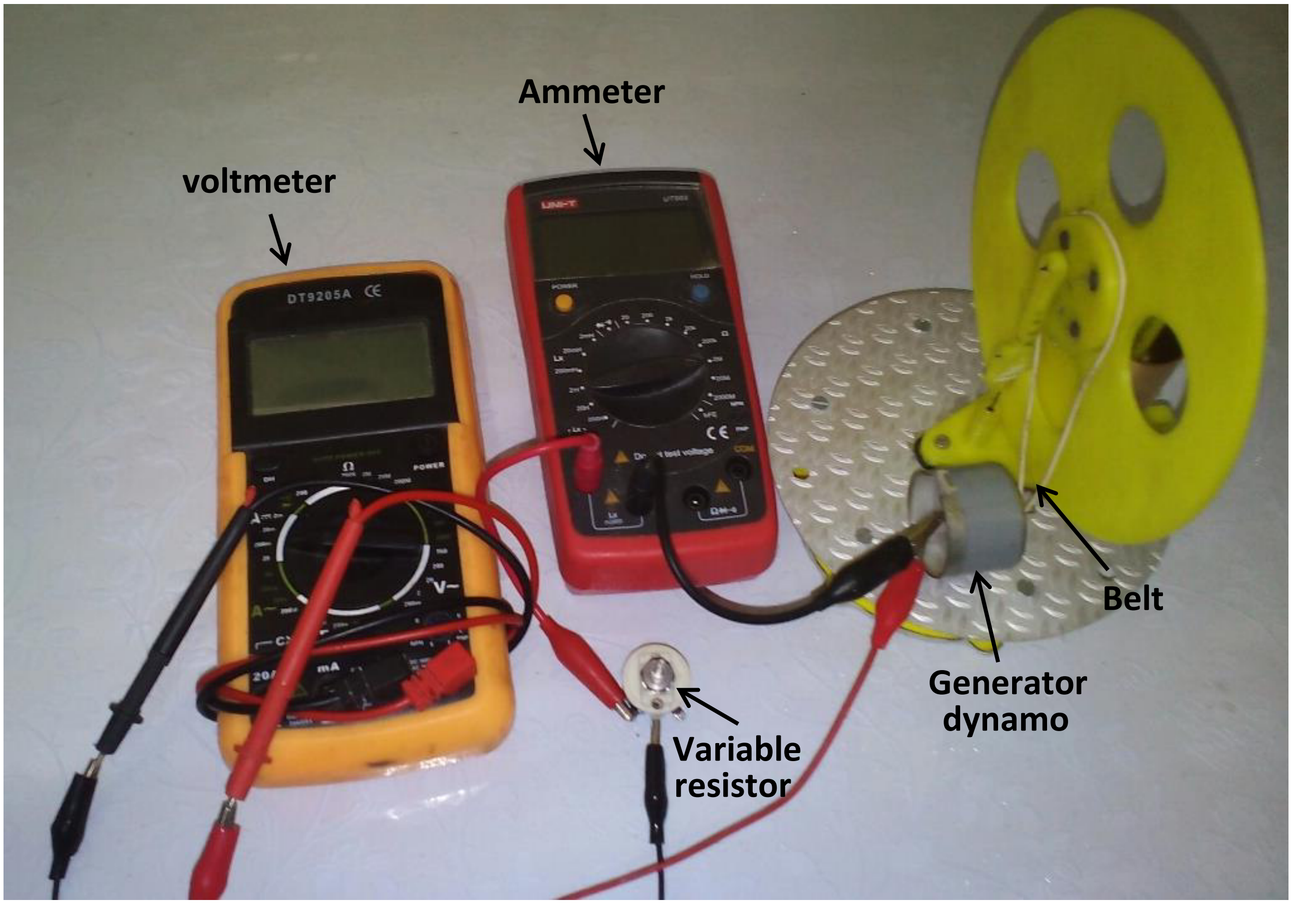


Figure 10: The experience installation.

4- Results and Comparison

4-1- Experience results

The calculation of the output power is done by the following equation:

$$\mathbf{P_{o\text{-}p} = P_{dyn}\,(1+L)*2} \qquad \mathbf{(33)}$$

Where:

P_{dyn} =U*I.

L= 40%: I t is the losses of the generator dynamo given by the manufacturer.

The values displayed by the voltmeter, the ammeter and the stroboscope are:

- The electric tension : U= 4V.
- The intensity : I= 47mA.
- The frequency : u= 5.4 Hz.

We enter the values in the equation (33) we get:

$$\mathbf{P_{o\text{-}r} = 526\ mW}$$

The comparison of the output power with the perfect cycle study done on the chapter 2 has a same rotation speed by the following equation:

$$\mathbf{P_{o\text{-}p} = W_{net} * u} \qquad \mathbf{(34)}$$

Using the effective work that was obtained in Chapter 2 equation (24) and the speed of the experiment we get:

$$\mathbf{P_{o\text{-}p} = 0.15 * 5.4 = 0.810\ W}$$

4-2- Comparison and discussion

From the results of the perfect cycle and the experimental power obtained we can calculate the power losses as follow:

$$\mathbf{\Delta P = (P_{o\text{-}p} - P_{o\text{-}r}) = (0.81 - 0.526) = 0.284\ W}$$

ΔP= 0.284 W

From the results obtained in this comparaison we can conclude that for ΔT = 40 ° C the engine power is 0.526 W with 0.284 W loss compared to the perfect cycle of Stirling, this loss of power that appear due to mechanical frictions between the engine parts , loss of pressures and heat loss.

The other values of difference temperature give us the following figure:

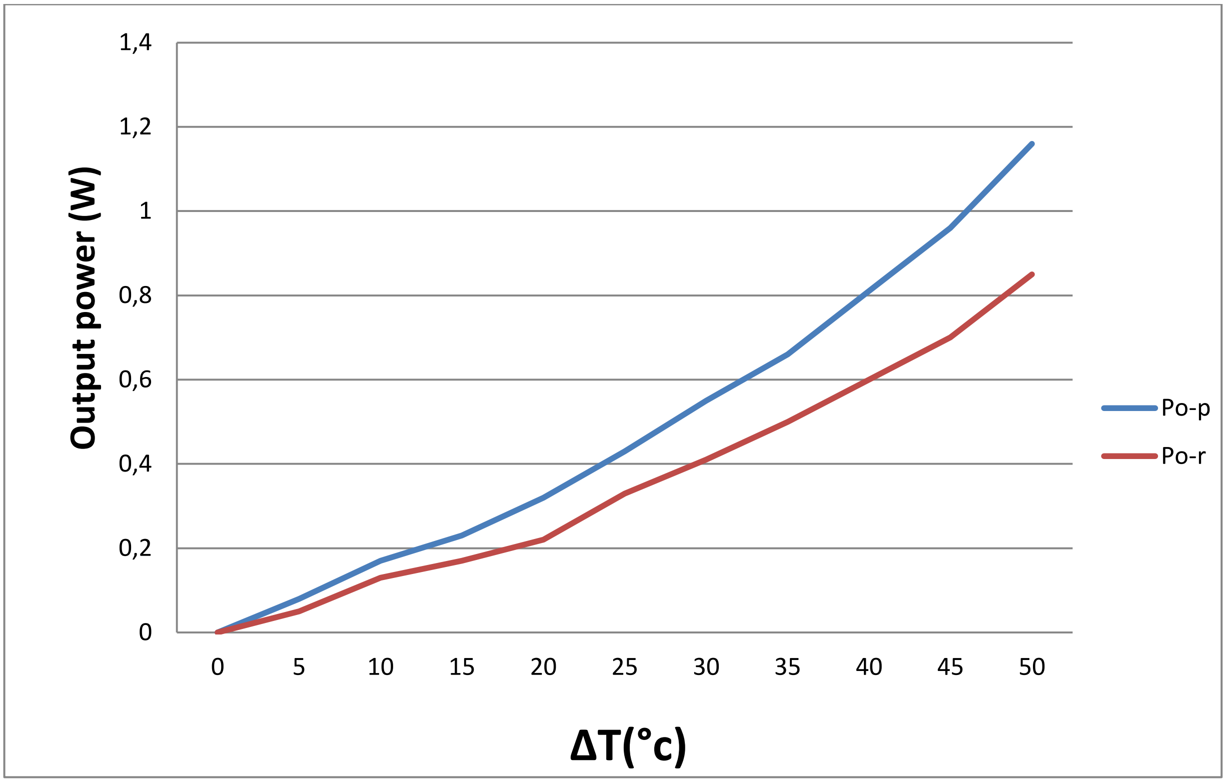


Figure 11: The perfect and the real output power in function of temperature.

We note that:

The output power varying with temperature, in which when difference of temperature increases it increases.

5- Conclusion

From the experimental results obtained we can say that there is an approaching compare it to the perfect result obtained in the ideal cycle with output difference achieves 35% due to the mechanical friction between parts and losses of heat and pressure.

General conclusion

From our study we conclude that :

- The dead volumes will degrade both energy efficiency and increase the contribution of external heat.
- To achieve high performance, a Stirling engine must be equipped with a perfect efficient regenerator.
- When we achieve a high temperature ratio the effictve work increases.

From the experience done in our work we can say that the choice of parts materials and the good operating reduce energy losses and gives a good chance to approache to the perfect power of the engine.Our works allowed us to increase our knowledge in thermodynamic study of engines, especially in stirling engine, the design of its parts under solidworks designing and finally in the mechanical analysis of the different parts of the engine applying the finite element method.

.

References:

[1]: Stirling Engine Manual, Editor: Franco Normani. Book

[2]: INTERNAL COMBUSTION ENGINE FUNDAMENTALS, John B.Heywood. Book

[3]: Analyse du cycle de Stirling convenant aux vrais moteurs, Editor Serge Klutchenko. Book

[4]: A review of solar-powered Stirling engines and low temperature differential Stirling engines article. Article in Renewable and Sustainable Energy Reviews · April 2003

[5]: “Free-Piston Stirling Engines”, G. Walker et al.,Springer 1985, reprinted by Stirling Machine World, West Richland WA

[6]: REGENERATIVE HEAT EXCHANGERS, Willmott, A.J.

[7]: Stine WB. Stirling engines. In: Kreith F, editor. The CRC handbook of mechanical engineers. Boca Raton: CRC Press; 1998

[8]: http://www.robertstirlingengine.com/

[9]: Les moteurs Stirling: Une véritable alternative aux méthodes classiques de conversion de l'énergie des Biogaz, par JP Vernet président d'EOSgen-technologies.